Calculation of the field and temperature dependence of the *c*-axis plasmon in $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$

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We calculate the *c*-axis Josephson plasmon frequency $\omega_{pl}(B,T)$ versus temperature T and magnetic induction B in $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$, treating the order parameter fluctuations in the lowest Landau level (LLL) approximation. The calculation agrees qualitatively with recent field-cooled measurements of Matsuda *et al.* assuming a *c*-axis dielectric constant $\epsilon_0 \sim 20-50$. For any given B, ω_{pl} falls monotonically with increasing T, remains finite in the flux liquid state, and varies continuously across the solid-liquid transition when disorder is included. We also obtain an expression for ω_{pl} which is valid everywhere in the flux liquid state and is not restricted to the LLL approximation. This expression leads to $\omega_{pl} \sim 1/\sqrt{B}$ in the flux liquid state, in agreement with experiment. [S0163-1829(99)03705-4]

I. INTRODUCTION

The most anisotropic high-T_c superconductors behave rather like a collection of underdamped Josephson junctions stacked parallel to the *c* axis. Recent microwave experiments in these materials see clear evidence of such junctions, in the form of Josephson plasmon oscillations. For example, in *c*-axis polarized reflectivity measurements a strong absorption is measured at a frequency which is interpreted as the Josephson plasmon frequency, $\omega_{pl}(B,T)$, where B is the magnetic induction parallel to c and T the temperature. In single crystals of $La_{2-x}Sr_xCuO_4$, these experiments yield $\omega_{pl} \sim 20-50$ cm^{-1}, i.e., 600–1500 GHz,[1] but in the more anisotropic $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$ they give $\omega_{pl} \sim 30-80$ GHz,[2,3] much smaller than the superconducting gap and weakly damped. On the theory side, Tachiki *et al.*[4] have calculated the c-axis plasmon-induced reflectivity and transmissivity at finite B in $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$. Bulaevskii and co-workers[5] have derived a relation between ω_{pl} and the interlayer Josephson coupling energy, while Koshelev has obtained the leading terms in a high-T expansion for $\omega_{pl}(B,T)$.[6]

In this paper, we calculate $\omega_{pl}(B,T)$ within the Lawrence-Doniach (LD) model, incorporating thermal fluctuations within the lowest Landau level (LLL) approximation. This model, which is suitable at high B and T, accounts well for first-order flux lattice melting in clean samples of both $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$ and $YBa_2Cu_3O_{7-\delta}$.[7–9] We calculate $\omega_{pl}(B,T)$ by combining this model with a simple theory for the electrodynamics of high-T_c superconductors, based on Maxwell's equations. In some respects, but not all, our results agree qualitatively with measurements of ω_{pl} over a range of T and B. In particular, we find that ω_{pl} remains nonzero even in the *flux liquid*. In contrast to experiment, however, the calculated ω_{pl} falls discontinuously at melting into the flux liquid state, and the contours of constant ω_{pl} in the B-T plane do not cross the flux lattice melting curve. When we extend the model to include columnar disorder, we find that, in this case, $\omega_{pl}(B,T)$ is continuous in T across the irreversibility line, in agreement with experiment, though the constant ω_{pl} contours still remain parallel to the melting transition line (which in this case is generally considered to be a glass transition). Finally, we obtain a formally *exact* expression, valid beyond the LLL model and in disordered as well as ordered samples, for ω_{pl} in the flux liquid state.

II. MODEL

In the LD model, the complex superconducting order parameter $\psi_n(\mathbf{r})$ at a point $\mathbf{r} \equiv (x,y)$ within the nth CuO_2 layer is described by the Ginzburg-Landau free energy functional $\mathcal{F}[\psi] = d \int d\mathbf{r} \Sigma_n F_n[\psi]$, where

$$F_n[\psi] = a(T)|\psi_n(\mathbf{r})|^2 + \frac{b}{2}|\psi_n(\mathbf{r})|^4 + \frac{1}{2m_{ab}}\left|\left(-i\hbar\nabla_\perp - \frac{e^*}{c}\mathbf{A}_\perp\right)\psi_n(\mathbf{r})\right|^2 + t\left|\psi_{n+1}(\mathbf{r}) - \psi_n(\mathbf{r})\exp\left(\frac{2\pi i}{\Phi_0}\int_{ns}^{(n+1)s} A_z dz\right)\right|^2. \tag{1}$$

Here d is the superconducting layer thickness, s is the repeat distance between layers, $e^* = 2e$, $\Phi_0 = hc/e^*$, $\mathbf{A}_\perp$ and A_z are components of the vector potential, and $a(T)$, b, m_{ab}, and m_c are material-dependent parameters. The interlayer Josephson coherence $t = \hbar^2/(2m_c s^2) \equiv \hbar^2/[2m_{ab}s^2\gamma^2]$, where $\gamma \equiv \sqrt{m_c/m_{ab}}$. It is readily shown that the free energy (1) is invariant under the gauge transformation $\mathbf{A} \rightarrow \mathbf{A} + \nabla\Lambda$, $\psi_n(\mathbf{x}) \rightarrow \psi_n(\mathbf{x})\exp[ie^*\Lambda/(\hbar c)]$, where $\Lambda(\mathbf{x})$ is an arbitrary function of $\mathbf{x} \equiv (\mathbf{r},z)$. Thus, the free energy is applicable, in principle, to magnetic fields with components perpendicular as well as parallel to the *c* axis.

In the extreme-type-II high-T_c materials, the magnetic induction $\mathbf{B}$ and field $\mathbf{H}$ are approximately equal. Maxwell's equations for the z component of the electric field $\mathbf{E}$ then give[10]

$$\left(\nabla^2 - \frac{\epsilon_0}{c^2}\frac{\partial^2}{\partial t^2}\right)E_z = 4\pi\left(\frac{1}{\epsilon_0}\nabla_z\rho + \frac{1}{c^2}\frac{\partial}{\partial t}J_z\right), \tag{2}$$

where ρ is the charge density, J_z is the z component of current density, and ϵ_0 is an appropriate dielectric constant. We seek a wavelike solution to this differential equation, such that $\nabla_z \rho = 0$. We also include in J_z only the supercurrent density, and neglect the normal current in parallel, which damps the plasmon. The supercurrent density in the region $ns \leq z \leq (n+1)s$ can then be written

$$J_z(\mathbf{x},t) = J_{n,n+1}(\mathbf{r}) \sin\, \theta_{n,n+1}(\mathbf{r},t), \tag{3}$$

where $\theta_{n,n+1}(\mathbf{r},t)$ is the gauge-invariant phase difference between layers n and $n+1$ at position $\mathbf{r}$ and time t, and $J_{n,n+1}(\mathbf{r})$ is the critical current density between those layers. We also use the Josephson relation

$$\frac{\partial \theta_{n,n+1}(\mathbf{r},t)}{\partial t} = \frac{e^* s}{\hbar} \langle E_z(\mathbf{r},t) \rangle_{n,n+1}, \tag{4}$$

where $\langle \cdots \rangle_{n,n+1}$ denotes the average of the quantity in brackets along the z axis in the region $ns \leq z \leq (n+1)s$. Following the treatment of Ref. 5, we now assume that E_z is slowly varying along the z direction, and replace $\langle E_z(\mathbf{r},t) \rangle_{n,n+1}$ by $E_z(\mathbf{r},z,t)$; we also express the $\theta_{n,n+1}(\mathbf{r},t)$ as the sum of a time-independent part $\theta^0_{n,n+1}(\mathbf{r})$ and a small time-dependent part $\delta\theta_{n,n+1}(\mathbf{r},t)$. Then differentiating Eq. (3) and linearizing with respect to $\delta\theta_{n,n+1}$ gives in the region $ns \leq z \leq (n+1)s$

$$\frac{\partial J_z(\mathbf{x},t)}{\partial t} = J_{n,n+1}(\mathbf{r}) \cos\, \theta^0_{n,n+1}(\mathbf{r})\, \frac{\partial \delta\theta_{n,n+1}(\mathbf{r},t)}{\partial t}. \tag{5}$$

In addition, we now assume that E_z is slowly varying along the z direction, so that $\langle E_z(\mathbf{r},t) \rangle_{n,n+1}$ can be replaced by $E_z(\mathbf{r},z,t)$. Finally, we combine Eqs. (2), (4), and (5), and seek a solution of the form $E_z(\mathbf{r},z,t) = \mathrm{Re}\ [E(\mathbf{r},z,\omega) \times \exp(-i\omega t)]$. In the region $ns \leq z \leq (n+1)s$, $E(\mathbf{r},z,\omega)$ is then found to satisfy the differential equation

$$\left(\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + \frac{\epsilon_0 \omega^2}{c^2} \right) E_z(\mathbf{r},z,\omega) = \frac{e^* s J_{n,n+1}(\mathbf{r})}{\hbar c^2} \cos\, \theta_{n,n+1}(\mathbf{r}) E_z(\mathbf{r},z,\omega). \tag{6}$$

If we neglect the spatial variation of E_z in the z direction, this differential equation may be written [for $ns \leq z \leq (n+1)s$]

$$\left(\nabla_\perp^2 + \epsilon(\mathbf{r},n,\omega) \frac{\omega^2}{c^2} \right) E_z(\mathbf{r},\omega) = 0, \tag{7}$$

where $\epsilon(\mathbf{r},n,\omega) = \epsilon_0 [1 - \omega_{pl}^2(\mathbf{r},n,\omega)/\omega^2]$, and

$$\omega_{pl}^2(\mathbf{r},n) = [8\pi^2 cs/(\epsilon_0 \Phi_0)] J_{n,n+1}(\mathbf{r}) \cos\, \theta^0_{n,n+1}(\mathbf{r}). \tag{8}$$

Equation (7) is equivalent to the form previously discussed by Bulaevskii and co-workers.[5] This equation is also formally equivalent to the Schrödinger equation for a particle in a two-dimensional random potential proportional to $\omega_{pl}^2(\mathbf{r},n)$. In general, $\omega_{pl}^2(\mathbf{r},n)$ depends on both transverse position $\mathbf{r}$ and n, and is also a thermodynamically fluctuating quantity, since both $J_{n,n+1}(\mathbf{r},T)$ and $\cos\, \theta_{n,n+1}(\mathbf{r},T)$ have these dependencies. We will estimate it by replacing $\omega_{pl}^2(\mathbf{r},n)$ by $\langle \omega_{pl}^2(\mathbf{r},n) \rangle \equiv \omega_{pl}^2$, where $\langle \cdots \rangle$ denotes both a statistical average in the canonical ensemble and an average over $\mathbf{r}$ and n. This may be reasonable at high magnetic fields, where the vortex cores overlap to such an extent that the effects of the randomness in the potential are small.

At sufficiently high fields, it is adequate to expand $\psi_n(\mathbf{r})$ in LLL states of the nth layer.[7,9,11–13] With the gauge choice $\mathbf{A} = -By\hat{x}$, this expansion may be written

$$\psi_n(\mathbf{r}) = \left(\frac{\sqrt{3} a_H^2(T)}{4b^2} \right)^{1/4} \sum_k c_{k,n} \exp\left(ikx - \frac{(y-kl)^2}{2l^2} \right). \tag{9}$$

Here $l = \sqrt{\Phi_0/(2\pi B)}$, $a_H(T) = a(T)[1 - B/H_{c2}(T)]$, $H_{c2}(T)$ is defined by $a(T) + \hbar e^* H_{c2}(T)/(2m_{ab}c) = 0$, and $b = 2\pi\kappa^2 [e^* \hbar/(m_{ab}c)]^2$, $\kappa = \lambda_{ab}^2(0)/\xi_{ab}^2(0)$ [where $\lambda_{ab}(0)$ and $\xi_{ab}(0)$ are the zero-temperature penetration depth and coherence length], and finally, T_{c0} is the mean-field transition temperature for $\mathbf{B} = 0$. The index $k = 2\pi p/L_x$ labels the different LLL's in a given layer, with $0 \leq p \leq N_\phi - 1$, where $N_\phi \equiv N_x N_y = L_x L_y/(2\pi l^2)$ is the number of vortices in each layer, and $L_x = (4\pi/\sqrt{3})^{1/2} N_x l$, $L_y = (\sqrt{3}\pi)^{1/2} N_y l$ are the linear dimensions of the system in the x and y directions. The coefficients c_{kn} are the fluctuating amplitudes of the various LLL's in different layers.

With this choice of basis, the final expression for ω_{pl}^2 is

$$\omega_{pl}^2 = [\omega_{pl}^{MF}]^2 \frac{\beta_A}{2} \frac{N_x}{N_\phi N_z} \left\langle \sum_{k,n} (c_{kn} c^*_{k,n+1} + \text{c.c.}) \right\rangle_T, \tag{10}$$

where $\langle \cdots \rangle_T$ denotes a thermal average in the canonical ensemble, $\omega_{pl}^{MF} = \sqrt{[(H_{c2}(T) - B)ce^*]/[\epsilon_0 \kappa^2 \gamma^2 \hbar \beta_A]}$ is the mean-field Josephson plasmon frequency, and $\beta_A = 1.159\,595\ldots$ is the Abrikosov ratio.

III. RESULTS

A. Numerical results from Monte Carlo simulations

We evaluate ω_{pl} in an $N_x \times N_y \times N_z$ system with periodic boundary conditions, using $N_z = 8$ layers, and choosing $N_x = N_y$ to accommodate exactly $N_\phi = 64$ vortex pancakes in a triangular lattice. We use the following parameters for $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$: $T_{c0} = 85$ K, $dH_{c2}(T)/dT = -5.2 \times 10^3$ Oe/K, $s = 15$ Å, $d = 4$ Å, $\gamma = 150$, and $\kappa = 100$. We carry out the required average for various B and T both above and below the melting temperature $T_M(B)$.[14] To carry out the simulation, for each T and B, we average over 3×10^5 Monte Carlo (MC) sweeps through the lattice, after discarding the first 2×10^4 sweeps for equilibration. At each field, we gradually warm up from an initial triangular lattice, using the final configuration for a lower T as the initial one for the next higher T. This procedure may correspond to the ''field-cooled'' measurements of Ref. 3. By contrast, a ''zero-field-cooled'' measurement (i.e., one in which the system is first cooled to the desired temperature in zero field, after which the field is turned on at fixed T), might well give different, possibly nonequilibrium, results.

Figure 1(a) shows the calculated quantity $\sqrt{\epsilon_0}\omega_{pl}(B,T)$ (which is independent of ϵ_0) for $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$. At all

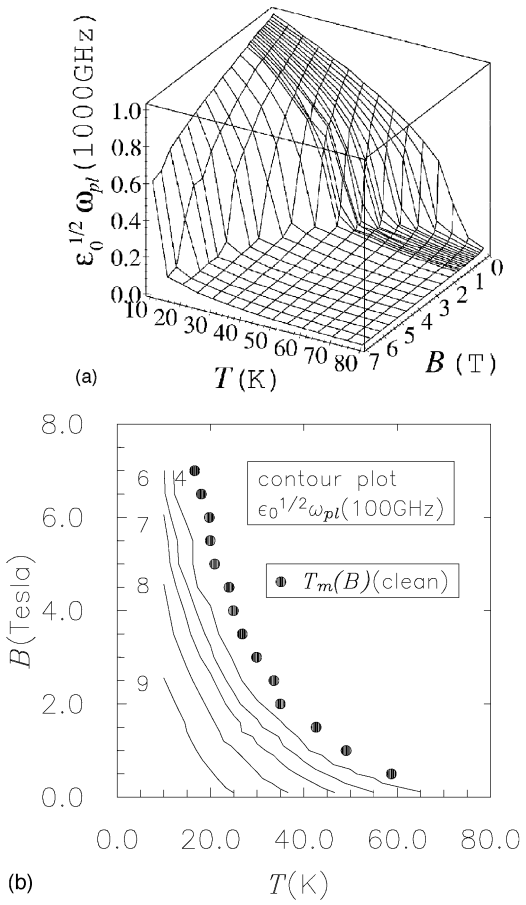

FIG. 1. (a) Perspective plot of $\sqrt{\epsilon_0}\omega_{pl}(B,T)$ for clean $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$ vs temperature T for a range of magnetic inductions B. ω_{pl} is the c-axis plasmon frequency and ϵ_0 the background interlayer dielectric constant. Curves at constant B are straight-line interpolations between calculated points. (b) Calculated lines of constant $\sqrt{\epsilon_0}\omega_{pl}(B,T)$ in the B-T plane, for clean $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$ (in units of 100 GHz). Points denote the calculated melting curve.

fields, $\omega_{pl}(B,T)$ drops discontinuously at $T_M(B)$,[7] but ω_{pl} *remains finite even above* T_M, as already predicted by several calculations[6] and also reported experimentally.[15] This finite ω_{pl} is due to the persistence of interlayer short range phase order even in the flux liquid state. If we assume $\epsilon_0 \sim 20-50$, then $\omega_{pl} \sim 100$–200 GHz at very low T (only weakly dependent on B), which agrees with the values in the range 105–220 GHz found at $B=0$ by Walkenhorst *et al.*[15] Our calculated $\omega_{pl}(B,T) \sim 10$–50 GHz near and above $T_M(B)$ (for an $\epsilon_0 \sim 20$–50). In Fig. 1(b), we show calculated lines of constant ω_{pl} in the B-T plane, which are directly measured in experiments. Qualitatively, our curves somewhat resemble those shown in Refs. 1–3, but in contrast to those, never cross the measured irreversibility line. Note that because the LLL approximation is not reliable at low B and low T, neither the melting curve nor the lines of constant ω_{pl} are accurately calculated at low B and low T.

Our calculated $\omega_{pl}(B,T)$ also drops discontinuously at $T_M(B)$. This discontinuity, which is not reported experimentally, is probably due to the first-order melting transition in this model, which produces discontinuities in measurable quantities across the melting line. One way to remove the discontinuity is to include quenched disorder in the LLL model. Such disorder should, of course, be present in most real samples, and is expected to convert the first-order melting line into a continuous transition. For example, random point disorder should lead to a continuous vortex-glass transition,[19] while randomly distributed columnar defects parallel to the c axis[16,17] are thought to produce a continuous transition in the Bose-glass universality class.[18] In our model calculations, we choose to consider random columnar defects, simply because, at comparable strengths of pinning potential, these produce a much larger shift in the melting temperature than do point defects, and hence are easier to study numerically. However, random point defects presumably have a similar effect in converting $\omega_{pl}(B,T)$ from discontinuous to continuous across the melting curve.

To describe these defects, we add to the free energy (1) an additional term,

$$F_d[\psi] = -k_B T_{c0} \sum_{n,\mathbf{R}_n} \Delta_{\mathbf{R}_n} \delta(\mathbf{r}-\mathbf{R}_n) |\psi_n(\mathbf{r})|^2. \qquad (11)$$

Here $\mathbf{R}_n$ is the position of a pinning center at the nth layer, and $\Delta_{\mathbf{R}_n}$ is its strength. For columnar defects, the $\mathbf{R}_n$'s and $\Delta_{\mathbf{R}_n}$'s are independent of n. We chose $\Delta_{\mathbf{R}}$ to equal 1.1 and 0.9×10^{-10} cm^2 at random, and considered three different densities D: 4, 8, and $16\times10^{+10}$ cm^{-2}. At $B=2$ T, there are 26, 52, and 105 columnar defects in the $N_\phi=64$ system.

In Fig. 2, we compare the helicity modulus[7] $\gamma_{zz}(B,T)$ and $\sqrt{\epsilon_0}\omega_{pl}(B,T)$ for the clean system and one with columnar disorder at $B=2$ T. The latter are averaged over 10 disorder realizations at each temperature; the error bars represent root-mean-square deviations with respect to these realizations. Columnar disorder eliminates the obvious discontinuities in $\gamma_{zz}(B,T)$ that exist in the clean system, and it *raises* $T_M(H)$ (here usually called the irreversibility line), as expected for columnar defects.[18,20] As in the clean system, ω_{pl} is clearly *nonzero* in the liquid phase.[21] Moreover, for any field and defect density, $\sqrt{\epsilon_0}\omega_{pl}$ appears to be slightly reduced at low temperatures, but substantially enhanced at high temperatures, relative to the clean material at the same temperature. There is no obviously special behavior at the ''matching field'' $D=B/\Phi_0$, perhaps because our statistics are not adequate to see such an effect at these small system sizes.

In Fig. 3, we show calculated lines of constant $\sqrt{\epsilon_0}\omega_{pl}$ in the B-T plane with defect density $D=4\times10^{10}$ cm^2. Once again, as in the clean case, our curves look quite similar to the measured curves in Refs. 16 and 17, but in contrast to those, appear not to cross the measured irreversibility line. This is the same discrepancy noted earlier in our results for the clean sample. Because of the error bars in the dirty calculation, however, the absence of a crossing is not so unequivocal as in the calculations for the clean material.

B. An analytical result for ω_{pl}^2 in the liquid phase

Next, we obtain an expression for ω_{pl} in the liquid state, which is valid *beyond* the LLL approximation. Our result is based on the fact that γ_{zz} vanishes in the liquid. We start with Eq. (8), the average of which gives the ω_{pl}^2 in either the vortex solid or the vortex liquid phase. This expression, being independent of the LLL approximation, is applicable, in

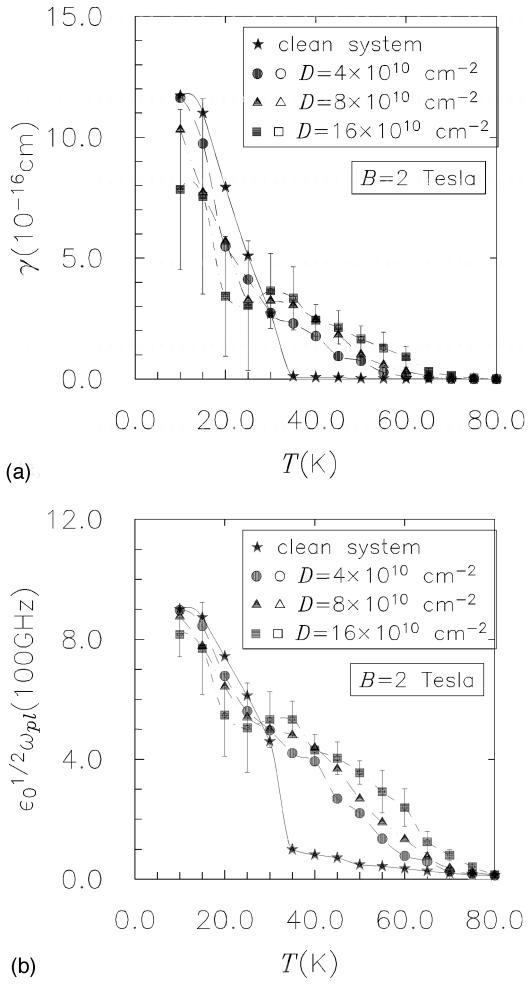

FIG. 2. (a) Calculated helicity modulus $\gamma_{zz}(B,T)$ (in units of 10^{-16} cm), and (b) the product $\sqrt{\epsilon_0}\omega_{pl}(B,T)$ (in units of 100 GHz) as functions of T at $B=2$ T, in clean $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$ and with various densities D of randomly distributed columnar defects parallel to the c axis. For each defect density, the calculated points are averages over ten disorder realizations; the error bars are rms deviations over the disorder. For the clean system, the lines are quasi-Hermite spline fits; for the dirty systems, straight-line interpolations.

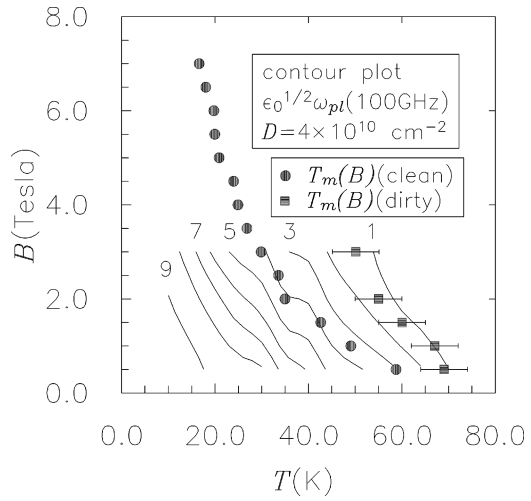

FIG. 3. Calculated lines of constant $\sqrt{\epsilon_0}\omega_{pl}(B,T)$ in the B-T plane, for $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$ with a density $D=4\times10^{10}$ cm^{-2} of columnar defects, averaged over 10 disorder realizations. Circles are melting points for clean system. Squares denote estimated glass transition temperatures for dirty systems; error bars represent computational uncertainty in these temperatures.

principle, at both low fields as well as high fields, and in disordered as well as ordered systems.

To obtain our result, we use a general formula for γ_{zz}[22] to cast the average of Eq. (8) into a different form. Using the vanishing of γ_{zz} in the liquid state, together with Eq. (8) of Ref. 22, we find that in the flux liquid state, $\omega_{pl}^2(B,T)$ can be expressed in the form

$$\omega_{pl}^2(B,T)=\frac{4\pi\langle J_z^2\rangle_T}{\epsilon_0 V k_B T}. \tag{12}$$

Here $J_z\equiv\int d^3x j_z(\mathbf{x})$ is the volume integral of the supercurrent density $j_z(\mathbf{x})$ in the c direction, and V is the system volume. To obtain this expression, we have used the fact that $J_z=J_{n,n+1}(T)\Sigma_n\int \sin\theta_{n,n+1}(\mathbf{r})\mathbf{dr}$. Equation (8) of Ref. 22 can then be applied directly once this integral is discretized on a suitable lattice, provided that thermal *amplitude* fluctuations in $J_{n,n+1}(\mathbf{r},T)$ can be neglected.

Equation (12) may be further simplified if we make the approximation that $\langle j_z(\mathbf{x})j_z(\mathbf{x}')\rangle$, in the liquid, depends only on $\mathbf{x}-\mathbf{x}'$. (In fact, this assumption is exact in the clean system, but not in the dirty system, where impurities break the translational invariance.) This assumption leads to the form

$$\omega_{pl}^2(B,T)=\frac{4\pi}{\epsilon_0 k_B T}\int\langle j_z(0)j_z(\boldsymbol{\xi})\rangle d^3\xi. \tag{13}$$

To estimate $\omega_{pl}^2(B,T)$ from Eq. (13), we assume that the integrand equals its zero-$\boldsymbol{\xi}$ value, $\langle j_z^2(0)\rangle$, whenever $\boldsymbol{\xi}$ lies within a correlation volume v_c, and vanishes everywhere else. Deep into the liquid phase, a reasonable estimate is $v_c\sim s l_0^2$, where $l_0=\sqrt{\Phi_0/B}$ is the average intervortex spacing. With these approximations, we obtain

$$\omega_{pl}^2(B,T)\sim\frac{4\pi\Phi_0 s}{\epsilon_0 B k_B T}\langle j_z^2(0)\rangle. \tag{14}$$

Equation (14) is similar to Koshelev's expression,[6] obtained from a high-temperature expansion in the liquid state of a clean material, but differs slightly because we have estimated the relevant correlation function using different approximations.

According to Eq. (14), ω_{pl} should vary as $1/\sqrt{B}$ at fixed temperature everywhere in the flux liquid state. This is precisely the field dependence that is observed experimentally in both the liquid and the solid phases.[4] We do not have a similar argument for the behavior of ω_{pl}^2 in the solid phase, however.

IV. DISCUSSION AND CONCLUSIONS

Using a Monte Carlo method, we have calculated $\omega_{pl}^2(B,T)$ for a model of $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$ in the LLL approximation both with and without the columnar defects, in both the flux solid and flux liquid state. In both cases, we

find, as expected, that ω_{pl}^2 is nonzero in the liquid phase. Randomly distributed columnar defects are found to raise the melting temperature above that of the clean system $T_M(B)$. They also appear to convert melting from a first-order phase transition to a continuous transition, as expected from previous work.[18] Likewise, in the presence of columnar disorder, $\omega_{pl}^2(B,T)$ appears to vary continuously with temperature at fixed B, while without it, ω_{pl}^2 is discontinuous across the melting line.

Besides our numerical results, we have also obtained a general analytical expression for ω_{pl}^2 in the flux liquid state, in terms of thermal fluctuations of current density. The expression is valid in both clean and dirty systems, and is not limited to the LLL approximation. It strongly suggests that ω_{pl}^2 is nonzero everywhere in the liquid state. If we further approximate this exact result, we obtain an expression very similar to Koshelev's result from a high-temperature expansion.[6] This expression predicts, in agreement with experiment,[4] that $\omega_{pl}(B,T) \sim 1/\sqrt{B}$ in the flux liquid state.

Our numerical results agree qualitatively with experiment in some respects, but not all. In the absence of disorder, our calculated $\omega_{pl}^2(B,T)$ decreases monotonically with T, in agreement with field-cooled experiments.[3] But in contrast to experiment, the calculated ω_{pl}^2 for clean systems is discontinuous across the melting curve, and the contours of constant ω_{pl}^2 do not cross the melting curve. When disorder is introduced (in our case, random columnar disorder), $\omega_{pl}^2(B,T)$ varies continuously with T across the glass transition. Presumably, the experimental curves are continuous because the experimentally studied materials undergo a continuous glass transition rather than the first order melting transition expected for the clean system. Even in our systems with quenched columnar disorder, the calculated contours of constant ω_{pl}^2 appear not to cross the glass transition line.

Quenched point disorder should produce similar continuous behavior of ω_{pl}^2 across the melting line, since such disorder is believed to convert the first-order melting curve to a continuous vortex-glass transition.[19] We have not studied this case numerically purely for convenience: since random columnar disorder produces a much more dramatic effect on the melting curve, it hence is easier to study numerically.

A recent paper by Glazman and Koshelev[23] suggests that, in a highly anisotropic superconductor such as $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$, there are actually *two* transitions in the clean limit: a lower melting temperature where the phase stiffness γ_{zz} vanishes, and an upper one where the layers become decoupled. Although the lower temperature corresponds to a sharp phase transition, the upper one is probably a crossover rather than a sharp phase transition. In the presence of point disorder, these transitions exhibit more complex behavior,[24] while the nature of the upper transition in the presence of line disorder is unclear.

Our results suggest that, in both clean samples and ones with line disorder, there is indeed a melting transition, followed at higher temperatures by a decoupling transition. The phase stiffness in the c direction vanishes at the lower transition, while ω_{pl}^2 becomes very small at the upper transition, which might conceivably be identified with the Glazman-Koshelev decoupling transition in either the ordered or line-disordered system. We believe that the upper transition, at least in our LLL approximation, is a crossover rather than a sharp phase transition in both the ordered and line-disordered samples.

The Glazman-Koshelev theory leads to a so-called decoupling field, which is in the range of 1 kOe for $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$. Near that field, the melting curve could exhibit some anomalies, even in the clean system. However, such behavior would be very difficult to detect in our present LLL approximation, which is appropriate for high fields, probably above 1 T. For the same reason, we are unable to use the LLL approximation to study the behavior of the melting curve in the clean system at low magnetic fields. (The melting curve of clean $Bi_2Sr_2Ca_2Cu_2O_{8+\delta}$, as calculated in the LLL, has been further discussed in Ref. 25.)

Finally, at low magnetic fields, $\omega_{pl}(B,T)$ has been reported to vary as $1/\sqrt{B}$. Our exact expression for ω_{pl}^2 in the flux liquid state does lead to such a variation when certain further approximations are made [cf. Eq. (14)]. This expression is applicable even to the moderately low-field regime where such behavior is reported.[4]

The present work leaves open a number of future problems. For example there should be a *spread* in ω_{pl}^2, rather than a single sharp line, arising from fluctuations in local magnetic field; this spread should be included in the calculations. There will be further damping from the normal interlayer current in parallel with the supercurrent. Also, it is possible that the electrodynamics underlying our calculation for ω_{pl}^2 are oversimplified (neglecting, for example, the z dependence of E_z). Finally, of course, it will be important to carry out calculations beyond the LLL approximation, and to connect ω_{pl}^2 more completely to the details of the phase diagram in the H-T plane. We hope to return to some of these points in future work.

V. ACKNOWLEDGMENTS

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