Hysteresis in $\pm J$ Ising square lattices

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The magnetic hysteresis of $\pm J$ Ising lattices is analyzed performing a zero-temperature random-walk minimizing energy. A steplike structure presenting a loop divided in four sections is observed. It is shown by Monte Carlo calculations that this structure is rounded off as temperature increases until a thin S shape is obtained, which is in general agreement with experimental results. A simple explanation for this form of hysteresis is given supporting universality and size independence. [S0163-1829(99)02202-X]

It is well known that spin glasses exhibit hysteresis.[1–4] Their behavior is related to that of site-diluted antiferromagnets in the sense that their original or virgin state tends to have a zero magnetization.[5] Ising models with random local fields evidenced hysteresis curves similar to experimental ones.[6]

In the present paper we report hysteresis for $\pm J$ Ising spin glasses in two dimensions. It is accepted that three-dimensional $\pm J$ lattices behave as spin glasses under a temperature $T_g^{(3)}=1.175$ (in units of J).[7] Two-dimensional (2D) systems show no overall order at any temperature if enough time is given, which leads us to think that $T_g^{(2)}=0.0$.[8] However, such 2D systems behave close to spin glasses when looking, e.g., at the universality breakdown.[9] Additionally, some physical properties studied by numerical methods are better fit with $T_g^{(2)}>0$.[10] Moreover, the unfrustrated portion of the lattice presents nontrivial percolation properties pointing to spin-glass domains.[11,12] A basic explanation of the hysteresis of $\pm J$ lattices would be interesting, comparing such theoretical result to experiments for spin glasses.

Let us consider a 2D lattice with N Ising spins S_i occupying every site i, spanning a square array. The Hamiltonian can be written as

$$\mathcal{H}(B)=\frac{1}{2}\sum_i F_iS_i-\sum_i BS_i\,;\quad F_i=\sum_j J_{ij}S_j\,, \tag{1}$$

where F_i denotes the *exchange field* (EF) acting on spin S_i due to nearest neighbors. B is the uniform and constant magnetic field measured in units of J. Samples are prepared randomly, with an equal amount of antiferromagnetic (AF) and ferromagnetic (F) interactions ($J_{ij}=\pm J$) that remain fixed always; periodic boundary conditions are assumed.

There are previous studies on Ising systems with frustration that find the exact ground state for each value of the field[13] exhibiting a steplike variation in the magnetization as the field is varied. Such a behavior is presented for some unfrustrated systems where the ground state is directly attained.[14] In $\pm J$ Ising systems a ground state is not always reached for each B, showing a steplike hysteresis. To our knowledge, such a phenomenon has not been characterized, except for some general comments in the literature.[11]

A state α is a set of the N spin orientations S_i^α. Total and reduced magnetization are given by $M^\alpha(B)=\Sigma S_i^\alpha$ and $m^\alpha(B)=M^\alpha(B)/N$, respectively. Let us call local ensemble of ground states (LEG's) all ground states interconnected by single spin flips without raising energy.[15] At extremely low temperatures, the system evolves with field lowering or conserving the energy in a random way that usually does not lead to a ground state, being trapped in a metastable state. All metastable states connected by one-spin flips without raising energy belong to a local ensemble of metastable states (LEM's).

We begin by presenting numerical results on one example to illustrate this special form of hysteresis. In parallel, a probabilistic analysis is done to explain size independence and other features of this phenomenon. We follow the evolution of the system starting from saturation and slowly decreasing B. Two different numerical methods are used. On one hand, we do a zero-temperature walk (ZTW), on the other hand we perform a Monte Carlo calculation (MCC) with a Metropolis algorithm.[16] We apply the former to a

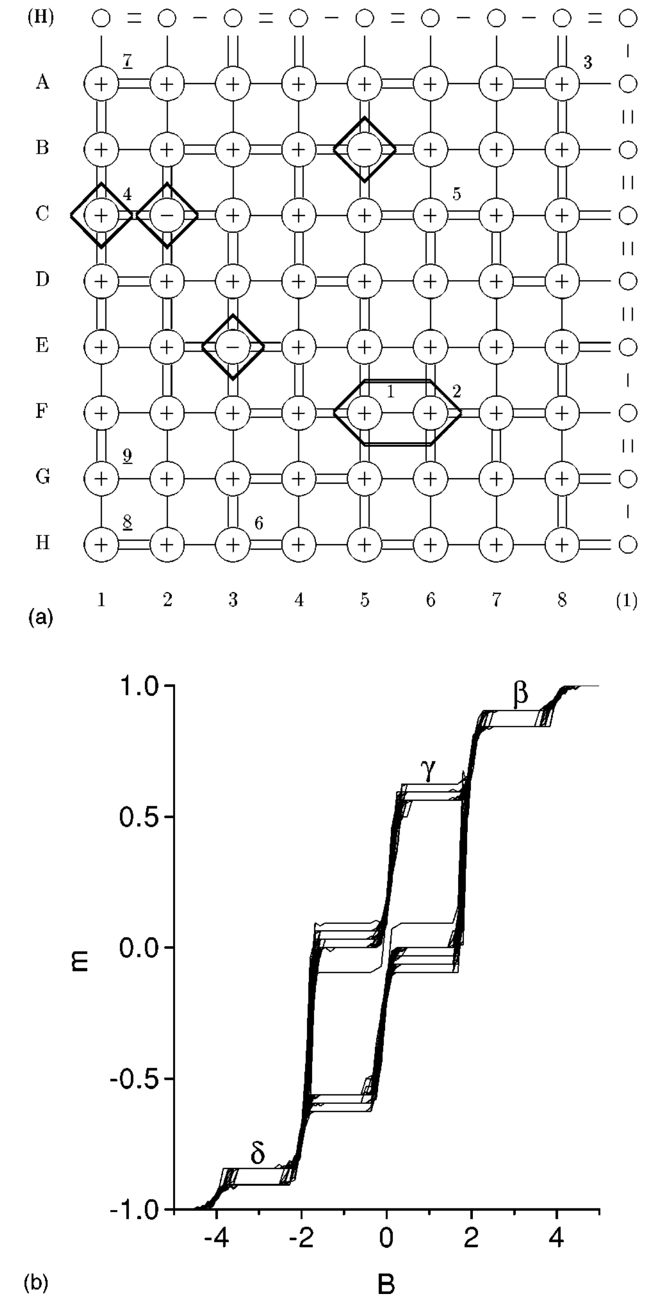

FIG. 1. (a) One particular 8×8 sample. A double (single) line represents a(n) AF (F) bond. Rhomboids mark stars; other symbols are discussed in the text. Spin orientations give the state β used in the example for a field below 4. (b) 100 low-temperature hysteresis loops obtained by MCC for this sample.

single large sample, while the latter is used to measure 500 samples at different temperatures. Finally previous results are compared to experiments.

General presentation and probabilistic analysis. One 8×8 sample is presented in Fig. 1(a). In Fig. 1(b) we show 100 hysteresis cycles done on this sample. Let us call *star* an individual site surrounded by four AF interactions, like sites $B5$ and $E3$ in Fig. 1(a) (code for site positions is chess-board like). We call neighboring stars those that share one AF bond, like sites $C1$ and $C2$. Finally, we call double (multiple) star a couple (sector) of sites joined by F bonds and surrounded by AF bonds, like $F5$ and $F6$.

For $T\rightarrow0$, we begin the analysis at $B\gtrsim4$. (We follow the cycle down to $B\lesssim-4$, returning back). Upon reaching $B=4.0$, all spins subject to an EF of four become unstable and are able to flip at no energy cost. If the field is slightly diminished (denoted 4^-), single stars and some sites in neighboring stars definitely overturn. This is illustrated in Fig. 1(a), where spins $E3$ and $B5(C1$ or $C2)$ can reverse, giving $m^{\beta}=58/64=0.906$. This value is depicted in Fig. 1(b) for $2.0<B<4.0$. Notice that either $C1$ or $C2$ can turn, but not both of them simultaneously. At $B=4^-$, one of the two states is pinned at random; we assume it is the one illustrated in Fig. 1(a), called state β from now on.

The probability of having a F or AF bond is 0.5 in the present case, denoted by u. Then the chance of having a star is u^4. Once corrections coming from neighboring stars are partly considered, the most probable value for the magnetization in this range is given by

$$m_p(4^-)\approx1-2u^4(1-2u^3)=0.906, \tag{2}$$

where we assume an infinite reservoir of bonds. [In this case, $m_p(4^-)$ coincidentally agrees with m^{β}].

The system reaches $B=2.0$ frozen in state β, evolving from there until it lands in one of the several possible LEM's available, depending on previous state and the sequence of the spins flipped in the unstable condition. Reversing one spin changes the local field on neighboring sites, inhibiting some flippings and allowing others. Within one LEM the evolution goes on states that maximize (minimize) magnetization for $B>0$ $(B<0)$. Magnetization is not unique, as shown by the different cycles in Fig. 1(b). We go back to our example for $B=2^-$. State β is abandoned searching a lower energy by flipping the following six spins $F5$, $F6$,$A8$,$C1$,$C6$,$H3$ in any sequence. We arbitrarily choose this order to continue our example marking these sites 1 through 6, as shown in Fig. 1(a). But the process does not stop there since after flipping spin at site $A8$, the spin at site $A1$ becomes also subject to an EF of -2, further flipping spins at $H1$ and $G1$, in a sequence marked as $\underline{7}$, $\underline{8}$, and $\underline{9}$, like in a domino effect. (This sites are underlined to show that their flipping depends on a previous flip). We designate by γ this last state [not displayed in Fig. 1(a)]. Adding the three previously flipped spins we get $m^{\gamma}=40/64=0.625$. This particular value corresponds to the one marked by γ in Fig. 1(b).

From the viewpoint of the probabilistic analysis, we need to count the sites with EF=2; this occurs when three AF bonds plus one F bond converge over the same spin. The probability for such site is $4u^4$, where the factor comes from the four different places the F bond can occupy. However, a neighboring star inhibits the flipping. Additionally, two of these objects sharing an AF bond cannot simultaneously flip. Using the same arguments leading to Eq. (2), we find that $m_p(2^-)\approx0.625$, where the domino effect has been ignored. It is important to realize that inhibitions on sites of EF $=2$ are very important, so about half the spins on these sites cannot flip. This effect can be appreciated in Fig. 1(a), where nine such inhibitions can be recognized.

Usually, many LEM's are possible at $B=0.0$. Moreover, each LEM can have a huge degeneracy because the energy is larger than the corresponding one for a LEG at zero field. At

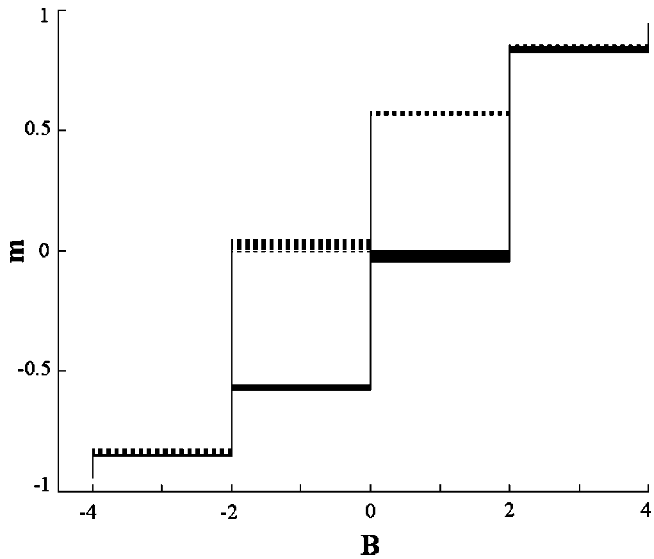

FIG. 2. Many possible hysteresis loops for a 70×70 sample using the technique of zero-temperature walk. The range for the different possible magnetizations are indicated by the width of the horizontal portions of the cycles.

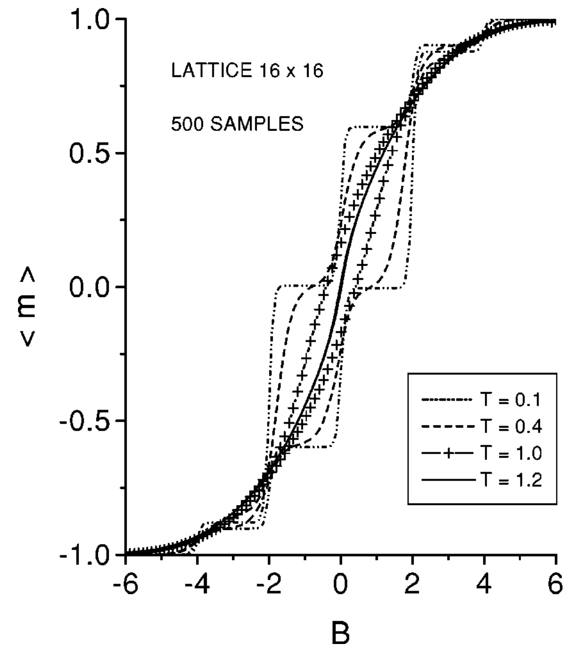

FIG. 3. Average hysteresis curves for 500 samples of size 16 ×16, obtained by MCC at different temperatures.

$B=0^-$, the single spin-flip dynamics evolves to the state of least magnetization within the same LEM. This bottom magnetization varies for a particular sample [as shown in Fig. 1(b)] and from sample to sample, leading to zero average value.

At $B=0$, all sites with the original EF of 4 and 2 have flipped (5/16 of all sites). From the sites with zero field (6/16) chances are that about half such spins (3/16) would overturn when $B=0^-$, while the other half will remain inhibited. Then $m_p(0^-)\approx 0.0$.

When the field reaches -2.0 from above, a new unstable condition arises. As the field turns more negative, all spins point downwards except those inhibited because their local field is -4, as it happens for stars. In double stars, only one spin can flip. In the example under consideration, five spins are left upwards in state δ, leading to a unique possible magnetization of $m^{\delta}=-0.844$, as illustrated in Fig. 1(b) for the interval $-4.0<B<-2.0$. For this same range we find $m_p(-2^-)\approx -0.883$, using probabilistic analysis and considering the very small correction coming from double stars. Of course, when $B<-4.0$ we face forced ferromagnetic orientation in the negative direction. From this position, the magnetic field can be reversed to obtain the lower part of the hysteresis curve in an exactly symmetric way to the one discussed so far.

The previous discussion is general, valid for samples of any size, showing that hysteresis belongs to the systems studied here. Due to topology, hysteresis loop breaks into a number of sectors that depends on connectivity. Thus, there are four sectors for square lattices, six for triangular lattices, and three for honeycomb lattices.

Hysteresis for a large sample (ZTW). Figure 2 shows the superposition of many hysteresis cycles for one 70×70 sample. For this sample one exact ground state at $B=0$ was calculated by De Simone *et al.*[17] At each field, one spin S_i is randomly selected and tested for flipping. If $B\leqslant F_i$, that particular spin flips. This procedure is continued until no new state can be generated from any of the previously stored states with the same energy. Our idea is to visit many LEM's, exhausting all states within each LEM for each value of the field. Degeneracy can be very high for $B=$EF. We start again at $B>4.0$. As we go under 4.0, all stars and some neighboring stars reverse their spins. In a large sample this can be done over states with different magnetizations as can be appreciated from the width of the dashed portion of the cycle between $4.0>B>2.0$. The curve is wide in the range $0.0>B>-2.0$, due to the larger degeneracy at $B=0.0$, which means that states with several different magnetizations can be pinned down. The rest of the hysteresis cycle is similar to that one already discussed.

In a large lattice the phenomenon prevails, retaining exactly the features observed in the small sample. Moreover, the decrease of the width of the curve in the normalized scale used for the magnetization axis is an indication for self-averaging. Therefore we expect these results to be present in the thermodynamic limit, too.

Average hysteresis for a set of samples (MCC). In previous studies on small samples it was found that reliable average values for physical magnitudes can be reported after considering 500 samples.[18] We consider here 16×16 lattices, insisting on averaging over 500 samples, although this can be more than necessary. The average magnetization for each field is obtained considering one full cycle per sample thus weighting the most frequently transited magnetic trajectories in the overall statistics. In Fig. 3 we present the average hysteresis cycle for these systems at four temperatures. The curve for $T=0.1$ is basically the same obtained for the two single samples already analyzed, showing that this curve is a self-averaging phenomenon.

We find that the low-temperature average magnetization values, namely: $\langle m(4^-)\rangle=0.902$, $\langle m(2^-)\rangle=0.598$, $\langle m(0^-)\rangle=-0.005$, and $\langle m(-2^-)\rangle=-0.880$, are in good agreement with the corresponding values m_p obtained by probabilistic analysis.

As T grows the steplike hysteresis curve becomes less abrupt. At $T \approx 0.8$ (not shown) the different sectors in the curve disappear. Then the normal hysteresis curve for most spin glasses is obtained up to $T \approx 1.2$, where a thin S-shape magnetization curve is obtained.

Comparison with experiments. The usual hysteresis curve for a spin glass consists of a single loop that gets thinner as temperature grows.[1,2] From this respect, the intermediate temperature curves of Fig. 3 represent qualitatively the phenomenon of such spin glasses.

For some particular spin glasses, the derivative of the low-temperature $m(B)$ curve varies in a pronounced way,[2,4] representing the onset of the steplike cycles obtained in our low-temperature simulations. In particular, measurements for $Ni_{79}Mn_{21}$ presented in Fig. 5 of Senoussi,[2] shows a hysteresis loop that clearly opens in two sectors for low temperatures (around $T=4$ K) merging in one modulated loop at around $T=9$ K, in a similar way to our results of Fig. 3 for $T=0.1$ and $T=0.4$, respectively. Also $Ni_{79}Mn_{21}$, at very low temperatures, has a remanent magnetization very close to zero (as our low-temperature simulations) establishing a clear difference with other hysteresis curves. Such remanent magnetization grows initially with temperature, as shown in Fig. 3. Another remarkable similarity with some experimental results[2] is that the virgin curve (not shown in the figures for simplicity) does not lay within the hysteresis cycle for some samples. The main difference with experimental results is the fact that the low-temperature hysteresis curves present a loop divided in four sectors, rather than two sectors or even just one loop as shown by experiments.

We conclude that $\pm J$ Ising lattices exhibit hysteresis that resembles experimental curves for some known spin glasses. The main reason for this is that the evolution of the system goes through LEM's that cannot be connected by one spin dynamics. The transfer from one LEM to another happens when the flipping of one spin implies the reversal of other spins in one of several or many possible sequences. Independently from the details of the model we have presented here, it can be stated that the steplike structures of the hysteresis curves at low temperatures are physically originated by the interplay between the existence of isolated metastable valleys (i.e., LEM's in our systems) and the dynamics of evolution which is inherently driven by one-element processes. The rounding off effects at higher temperatures are evident.

Moreover, it is shown for the $\pm J$ Ising lattices, that magnetization can vary within the spectrum of extremal values for the corresponding LEM's. However, its average value over cycles in different samples $\langle m \rangle$, is very stable. It does not vary with size and can be approximately predicted by means of basic probability theory. This means that such a hysteresis curve is universal and should reflect the behavior in the thermodynamic limit. The area under the curve at 0 K, representing the energy loss per cycle, takes the approximate value 2.48.

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