Pauli blocking effects in quantum wells

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We derive generalized analytical expressions of changes in the oscillator strength of an α-dimensional exciton ($2 \leqslant \alpha \leqslant 3$) due to Pauli blocking mechanisms in quantum wells. The effect of dimensionality, temperature, and density of charge carriers on the exciton binding energy are studied in the light of known experimental results in $GaAs/Al_xGa_{1-x}As$ and $ZnSe/Cd_xZn_{1-x}Se$ quantum wells. Our results show that Pauli blocking effects reach minimum levels at critical well widths depending on the material composition. The quantitative study of α provides an improvement on works which employ exact two-dimensional exciton models. [S0163-1829(99)10803-8]

I. INTRODUCTION

Experimental works[1–5] performed using modulation-doped quasi-two-dimensional structures have shown that excitons in the lowest band levels can exist simultaneously with free carriers up to sheet carrier densities of about 4×10^{11} cm^{-2}. The exciton oscillator strength decreases gradually (exciton bleaching) as the carrier density is increased from zero. At a high enough carrier density, an abrupt transition from excitonic to free-electron-hole recombination occurs.[1] Such excitonic nonlinearities originate from a combination of many-body-related effects[6,7] like Pauli blocking mechanisms, short-range exchange interaction, and long-range Coulomb interaction among charge carriers (Coulomb screening).

So far, the well-known theories[7] of Pauli blocking mechanisms in quantum wells have been developed within the strictly two-dimensional limit, even though experiments have been performed in quantum wells with widths comparable to the size of excitons. While the theories were able to explain some experimental results,[2,8] they contradict other theoretical calculations[9] and experimental work done on II-VI quantum wells.[5]

It is well known that the relative motion of the electron-hole pair which constitutes an exciton can never be considered exactly two or three dimensional in quantum wells. A more appropriate approach to such systems is to treat excitons as intermediate between an exact two- and three-dimensional system by utilizing the concept of a fractional dimensional space.[10] The problem of Wannier-Mott excitons placed in strongly anisotropic media (semiconductor quantum wells, wires, and superlattices) was first popularized by He (Ref. 11) and Lefebvre, Christol, and Mathieu[12] who used a metric space with a noninteger dimension to obtain exact solutions for the excitonic energies and wave functions. The advantage of using such an approach is that only a single parameter, known as the degree of dimensionality (denoted by α), is needed to incorporate the effect of change in the widths of the well or barrier regions on the strength of interaction. α increases from 1 in an exact one-dimensional system (e.g., ideal quantum wires) to 3 in an exact three-dimensional system (e.g., bulk crystals).

In this paper, we present a systematic study of the effect of a continuous transition from an exact two-dimensional system to an exact three-dimensional system on Pauli blocking mechanisms in confined systems. We do not attempt to investigate many-body effects like band-gap renormalization and band filling,[13] but focus only on a single effect—Pauli blocking effects in quantum wells. We utilize the concept of fractional dimensionality to analyze the qualitative effects of Pauli blocking mechanisms for any arbitrary value of the dimension of a confined excitonic system ($2 \leqslant \alpha \leqslant 3$). The fractional dimensional approach is expected to interpret experimental results correctly and to clarify aspects of the theory of excitonic optical nonlinearities in low-dimensional heterostructures.

This paper is organized as follows. In Sec. II, we derive generalized analytical expressions of saturation densities due to Pauli blocking mechanisms in fractional dimensional space. We also analyze the separate contributions of an exciton gas and an electron-hole plasma in Sec. II. In Sec. III, we study the dimensionality and temperature dependence of Pauli blocking effects. We present the conclusion of this work in Sec. IV.

II. PAULI BLOCKING IN FRACTIONAL DIMENSIONAL SPACE

As soon as an exciton is generated, a Fermi sphere is formed which reduces the **k** space available for single-particle states. Since the single-particle states contribute to the bound electron-hole state, the Fermi sphere forming process, also known as Pauli blocking, directly causes a reduction of the exciton oscillator strength denoted by (f_α) in an α dimensional space. Thus a decrease in the exciton oscillator strength due to Pauli's exclusion principle is mainly due to the reduction in the number of single-particle states that contribute to the exciton state.

The change in the exciton oscillator strength due to Pauli blocking effect in an α-dimensional space is expressed as[7]

$$\frac{\Delta f_\alpha}{f_\alpha} = -\frac{N}{N_s^\alpha} = -\sum_{\mathbf{k}} \left[f_e(k) + f_h(k)\right] \frac{U_{1s}(k)}{U_{1s}(k=0)}, \quad (1)$$

where N is the density of electron-hole pairs and N_s^α is the α-dimensional total saturation density, which is determined

using the forms assumed for $f_e(k)$ and $f_h(k)$, the respective electron and hole distribution functions. $U_{1s}(k)$ in Eq. (1) denotes the Fourier transform of electron-hole relative motion orbital wave function, $U_{1s}(r)$, and N is the exciton density. Equation (1) is interpreted as the fractional change that occurs in the exciton oscillator strength, f_α, due to effects arising from the presence of neighboring bound and unbound electron-hole pairs. Note that we have generalized Eq. (1) by introducing a noninteger dimension α, so that it reduces to well-established forms for both $\alpha=2$ and 3.

The forms for $f_e(k)$ and $f_h(k)$ for charge carriers in a nondegenerate plasma at equilibrium are expressed by the Boltzmann distribution

$$f_i(k)=\frac{N\hbar^2\pi}{m_i k_B T}\exp\left(\frac{-\hbar^2k^2}{2m_i k_B T}\right), \tag{2}$$

where $m_i, i=e(h)$ is the electron (hole) effective mass, and T is the temperature. On the other hand, the distribution function for a system of correlated electron-hole pairs (exciton gas) is expressed as[7]

$$f_e(k)=f_h(k)=\frac{N}{2}|U_{1s}(k)|^2, \tag{3}$$

where $U_{1s}(k)$ is the Fourier transform of the electron-hole relative motion orbital wave function $U_{1s}(r)$, and N is the exciton density. Equation (3) means that the creation of a single exciton is associated with the occupation probability $|U_{1s}(k)|^2$ in fermion space. The factor $\frac{1}{2}$ appears as result of sharing between spin-up and -down particles.

In order to obtain an accurate form of the wave function for the electron-hole relative motion, a Schrödinger equation incorporating all the known many-body effects has to be solved. However, this is not a trivial procedure and an exact solution is therefore not available. In order to simplify calculations, we use the form of $U_{1s}(r)$ for an isolated exciton in an α-dimensional space[14] to evaluate $U_{1s}(k)$:

$$U_{1s}(r)=F(\alpha)\exp\left[-\frac{2}{\alpha-1}\frac{r}{a_B}\right], \tag{4}$$

where a_B is the three-dimensional Bohr radius of the exciton, and $F(\alpha)$ is

$$F(\alpha)=\left[\frac{2^{\alpha+1}\pi^{(1-\alpha)/2}}{\Gamma\left[\frac{\alpha-1}{2}\right](\alpha-1)^{\alpha+1}}\frac{1}{a_B^\alpha}\right]^{1/2}. \tag{5}$$

Using the relation[15,16]

$$\int_{\alpha D}d\mathbf{r}e^{-2\pi|\mathbf{r}|c}e^{-2\pi i\mathbf{q}\cdot\mathbf{r}}=\frac{\Gamma\left[\frac{\alpha+1}{2}\right]}{\pi^{(\alpha+1)/2}}\frac{c}{(c^2+q^2)^{(\alpha+1)/2}} \tag{6}$$

and the hydrogenic form of the wave function U_{1s} in α-dimensional space given in Eqs. (4) and (5), $U_{1s}(k)$ in an α-dimensional space is evaluated as

$$U_{1s}(k)=(4\pi)^{(\alpha/4)-(1/4)}(\alpha-1)^{(\alpha/2)+(1/2)}\times\sqrt{\Gamma\left[\frac{\alpha-1}{2}\right]}\frac{a_B^{\alpha/2}}{\left(1+\left[\frac{\alpha-1}{2}ka_B\right]^2\right)^{(\alpha+1)/2}}. \tag{7}$$

Equation (7) yields the expected forms in the exact three- and two-dimensional limits.[7]

In deriving Eq. (7), we have made use of the spatial integral relation in α-dimensional space:[11]

$$\int_{\alpha D}dr=\frac{2\pi^{(\alpha-1)/2}}{\Gamma\left[\frac{\alpha-1}{2}\right]}\int_0^\infty r^{\alpha-1}dr\int_0^\pi d\theta\sin^{\alpha-2}\theta. \tag{8}$$

A. Exciton gas

Pauli blocking due to an exciton gas is determined using Eq. (1) and the form of $f_i(k)$ given by Eqs. (3) and (7) as:

$$\frac{\Delta f_\alpha}{f_\alpha}=-NG(\alpha)\frac{2^{3\alpha-2}(\alpha-1)}{\sqrt{\pi}}V_\alpha(a_B), \tag{9}$$

where $V_\alpha(a_B)$ is the volume of an α-dimensional sphere[10] with radius a_B:

$$V_\alpha(a_B)=\frac{\pi^{\alpha/2}}{\Gamma\left[1+\frac{\alpha}{2}\right]}a_B^\alpha, \tag{10}$$

and $G(\alpha)$ is given by

$$G(\alpha)=\frac{\Gamma\left[\frac{\alpha-1}{2}\right]\Gamma\left[1+\frac{\alpha}{2}\right]\Gamma\left[\frac{\alpha}{2}\right]\Gamma\left[\alpha+\frac{3}{2}\right]}{\Gamma\left[\frac{3}{2}(\alpha+1)\right]}. \tag{11}$$

The inverse of the α-dimensional saturation density corresponding to the exciton gas, N_g^α, is obtained, using Eqs. (1) and (9), as

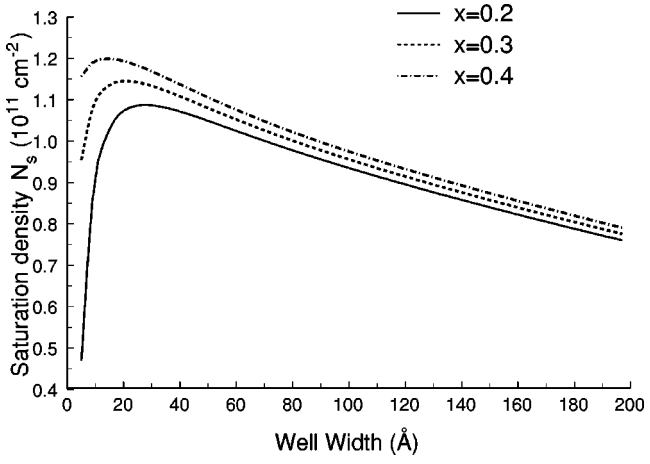

FIG. 1. Total saturation density N_s^α [Eq. (15)] as a function of the well width in GaAs/Al_xGa_{1-x}As quantum wells.

$$\frac{1}{N_g^\alpha}=G(\alpha)\frac{2^{3\alpha-2}(\alpha-1)}{\sqrt{\pi}}V_\alpha(a_B). \tag{12}$$

Thus $1/N_g^2=\frac{32}{7}\pi a_B^2$, which is the same as in an earlier calculation[7] obtained for the exact two-dimensional exciton model. Equation (12) means that an α-dimensional volume $\Lambda_\alpha=1/N_g^\alpha$ is formed as a blocking space around the newly created exciton. Any extra exciton cannot be formed within this space. It should be noted that the dimensionless volume $[\Lambda_\alpha/V_\alpha(a_B)]$ of the Pauli blocking space increases with dimensionality. This means that the possibility of a newly created exciton sustaining more excitons is highest when the dimensionality α of a confined exciton is at its minimum. It also explains why excitonic optical spectra is significantly stronger in quantum wells than in bulk crystals as the exciton oscillator strength is increased by the contributions of a larger number of electron-hole states at a lower dimension.

B. Electron-hole plasma

An analytical expression for the dimensionless volume, $[\Lambda_\alpha/V_\alpha(a_B)]=[1/N_c^\alpha V_\alpha(a_B)]$, of Pauli-blocked space due to free electron and holes is evaluated using Eqs. (1), (2), and (7) as

$$\frac{\Lambda_\alpha}{V_\alpha(a_B)}=\sum_{i=e,h}\gamma_i 2^{2\alpha-1}\Gamma\left[1+\frac{\alpha}{2}\right]\pi^{(3-\alpha)/2}\left\{\frac{\Gamma\left[\frac{\alpha}{2}\right]}{2\Gamma\left[\frac{\alpha+1}{2}\right]}\Phi\left[\frac{\alpha}{2},\frac{1}{2};\gamma_i\right]-\sqrt{\gamma_i}\Phi\left[\frac{\alpha+1}{2},\frac{3}{2};\gamma_i\right]\right\} \tag{13}$$

where $\gamma_i=2\hbar^2/m_i k_B T a_B^2$ with $i=e$ or h, and $\phi[a,b;c]$ denotes a confluent hypergeometric function.[17]

In deriving Eq. (13), we have made use of the integral relation[17]

$$\int_0^\infty \frac{x^c\exp(-ax^2)}{(1+bx^2)^{\frac{c}{2}+1}}dx=\frac{\sqrt{\pi}}{b^{\frac{c+1}{2}}}\left\{\frac{\Gamma\left[\frac{1+c}{2}\right]}{2\Gamma\left[\frac{c}{2}+1\right]}\Phi\left[\frac{1+c}{2},\frac{1}{2};\frac{a}{b}\right]-\frac{1}{\sqrt{b}}\Phi\left[\frac{c+2}{2},\frac{3}{2};\frac{a}{b}\right]\right\}. \tag{14}$$

Equations (12) and (13) are then used to evaluate the total saturation density, N_s^α for a given value of α:

$$\frac{1}{N_s^\alpha}=\frac{1}{N_c^\alpha}+\frac{1}{N_g^\alpha}. \tag{15}$$

It is important to note that the temperature dependence of Pauli blocking effects in Eq. (13) is due to the form of the Fermi function in Eq. (2). Experimental results[18] of the carrier effects on the excitonic absorption in quantum wells have demonstrated that phase-space-filling effects depend more on temperature through the Fermi function than on screening or other many-body effects. In this sense, the mechanism of Pauli blocking by an electron-hole plasma is distinguished by its dependence on temperature. Further discussion of this important aspect is given in Sec. III. Unlike in Eq. (2), the distribution of exciton gas in Eq. (3) is independent of temperature. This is because the excitonic wave function remains unaffected by temperature although it changes with the well width, as confirmed by absorption measurements[19] of excitonic resonances in the temperature range 8–300 K of quantum-well systems.

III. EFFECT OF CARRIER DENSITY ON DIMENSIONALITY α

A. Low carrier densities

At zero or very low carrier densities, reliable values of the dimensionality α of an exciton in a quantum well of a known well width is obtained by comparing experimental values with a well-known expression of the exciton binding energy,[12]

$$E_b=\frac{R_y}{\left(n+\frac{\alpha-3}{2}\right)^2}, \tag{16}$$

where R_y is the exciton Rydberg, and $n=1,2,\ldots$ is the principal quantum number state. The ideal confinement at which $\alpha=2$ is never reached due to the spreading of electron and hole wave functions into the barrier regions in quantum wells. This spreading effect becomes increasingly significant at critical well widths.[20,21] Using Eqs. (15) and (16) and experimental exciton binding energies[22] in $GaAs/Al_xGa_{1-x}As$ quantum wells, we have plotted the total saturation density N_s^α as a function of the well width for various aluminum concentration in $GaAs/Al_xGa_{1-x}As$ quantum wells in Fig. 1. Pauli blocking effects reach minimum levels at critical well widths, depending on the material composition. This is due to the gradual decrease of critical well widths as well as α with the aluminum concentration, i.e., with the depth of the quantum well.

B. High carrier densities

It is generally difficult to obtain an accurate relation between α and the exciton binding energy when the carrier density becomes high. However, by studying the effect of an

increasing carrier density on the exciton binding energy, one can estimate the change of α with the carrier density. In order to do this, we use the approximation[18] that the perturbation on an exciton by free carriers is small enough so that the excitonic wave function of the form given in Eq. (4) is still valid. The effects of the surrounding charge carriers are incorporated in the exciton Bohr radius a_B, which changes with the carrier density N. The wave equation for an exciton surrounded by charge carriers is described by an effective-mass Schrodinger equation

$$\left[-\frac{\hbar^2}{2\mu r^{\alpha-1}}\frac{\partial}{\partial r}r^{\alpha-1}\frac{\partial}{\partial r}-E\right]U_{1s}(r)=(1-\eta)\frac{e^2}{\epsilon r}U_{1s}(r), \tag{17}$$

where μ is the reduced mass of the exciton and the correction term, η in an α-dimensional space is derived as

$$\eta(r)U_{1s}(r)=\int_0^{k_F}dk\left(\frac{k}{2\pi}\right)^{\alpha-1}U_{1s}(k)J_{(\alpha-2)/2}(kr), \tag{18}$$

where $k_F^2=2\pi N$. For long-range screening at which $a_Bk_F \ll 1, \eta(r)$ in Eq. (18) is evaluated for small r, using Eq. (7), as

$$\eta \approx S(\alpha)(a_Bk_F)^2, \tag{19}$$

where $S(\alpha)\approx\frac{1}{8}$ for $2\leqslant\alpha\leqslant 3$.

Using Eqs. (17)–(19), the change in exciton binding energy due to the presence of surrounding charge carriers is obtained as

$$\frac{E_b(N)}{E_b(0)}=\left(1-\frac{N}{2N_s^\alpha}\right)^2, \tag{20}$$

where N_s^α for a given α is given in Eq. (15). Using Eq. (20), the effect of an increasing density of charge carriers on α is determined as

$$\Delta\alpha=\left(\frac{\alpha-1}{\frac{2N_s^\alpha}{N}-1}\right). \tag{21}$$

The above equation shows that the change in dimensionality $\Delta\alpha$ vanishes when the charge density N is reduced to zero, but gradually increases as N is increased. This is attributed to the enlargement in excitonic Bohr radius with N, so that the degree of exciton confinement, as measured by α, is increased. While α is almost unchanged at low densities, determination of α at larger N values becomes complicated due to the rapidly vanishing exciton binding energy brought about by many-body-related effects.[6]

The effect of increasing the density of charge carriers, N, on the oscillator strength f_α, is determined using

$$\frac{f_\alpha(N)}{f_\alpha(0)}=\frac{1}{1+\frac{N}{N_s^\alpha}}. \tag{22}$$

In Fig. 2, the decrease of N_s^α with α in CdSe quantum wells at 10 K is plotted, using[23,24] $m_e^*=0.13, m_h^*=0.45$, and $\varepsilon=9.3$ and Eqs. (12), (13), and (15). The normalized oscillator strength $[f_\alpha(N)/f_\alpha(0)]$ [Eq. (22)], as a function of sheet carrier density, N is compared in Fig. 3 with experimental data[25] in $ZnSe/Cd_{0.1}Zn_{0.9}Se$ quantum wells of width 30 Å. For the purpose of comparison, we have assumed $V_\alpha(a_B)\simeq\pi a_B^2$. The dimensionality parameter α for excitons in 30-Å CdSe well materials is calculated[12] as $\alpha=3-\exp(-L_w/2a_B)\approx 2.4$. From Fig. 2, we obtain $N_c^{2.4}\sim 2.17\times10^{11}$ cm^{-2}, $N_g^{2.4}\sim 1.75\times10^{11}$ cm^{-2}, and $N_s^{2.4}\sim 0.97\times10^{11}$cm^{-2}. While the theoretical fit agrees well with the experimental data at lower carrier densities, the noticeable difference at higher carrier densities is possibly due to increasing importance of effects like band-gap shrinkage and exchange interactions.

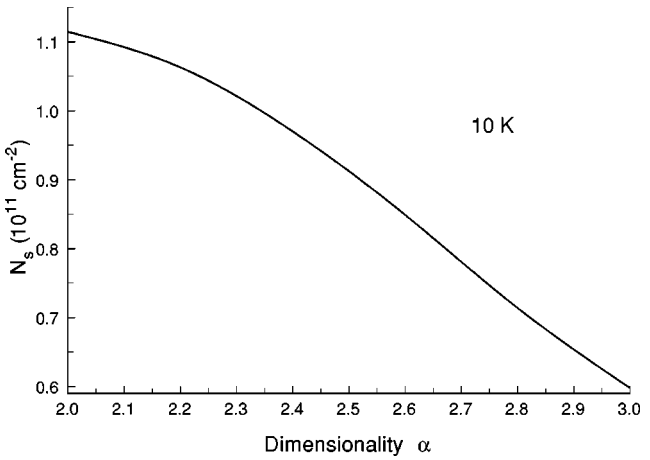

FIG. 2. Total saturation density, N_s^α as a function of α in $ZnSe/Cd_xZn_{1-x}Se$ quantum wells at 10 K.

In Fig. 4, we compare theoretical results calculated using Eq. (20) with experimental data[25] of the normalized binding energy $[E_b(N)/E_b(0)]$ [Eq. (20)] as a function of sheet carrier density N. The experiment[25] was performed on $ZnSe/Cd_{0.1}Zn_{0.9}Se$ quantum wells of width 30 Å at 10 K.

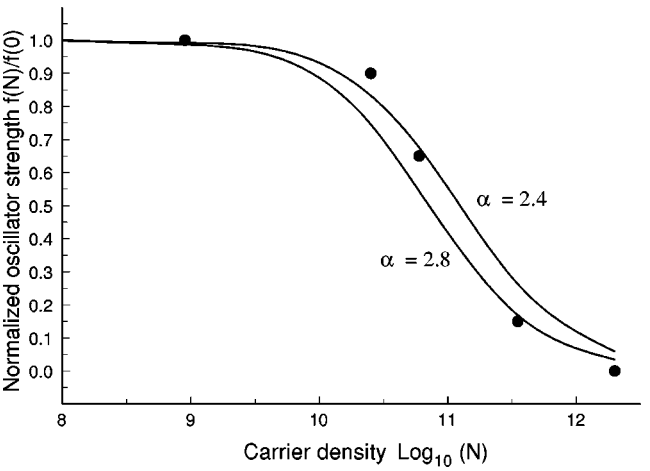

FIG. 3. Dots denote experimental values (Ref. 25) of the normalized oscillator strength $[f_\alpha(N)/f_\alpha(0)]$ as a function of sheet carrier density, $N(\text{cm}^{-2})$, in $ZnSe/Cd_{0.1}Zn_{0.9}Se$ quantum wells of width 30 Å. Solid lines at $\alpha=2.4$ and 2.8 are obtained using Eq. (22).

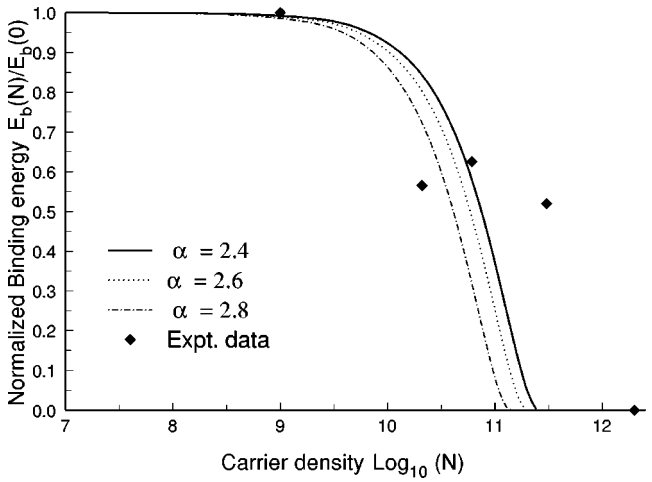

FIG. 4. Diamond-shaped points denote experimental data (Ref. 25) of the normalized binding energy $[E_b(N)/E_b(0)]$ [Eq. (20)] as a function of sheet carrier density, $N(\mathrm{cm}^{-2})$ in $ZnSe/Cd_{0.1}Zn_{0.9}Se$ quantum wells of width 30 Å at 10 K. Solid ($\alpha=2.4$), dotted ($\alpha=2.6$), and dashed-dotted ($\alpha=2.8$) lines are obtained using Eq. (20).

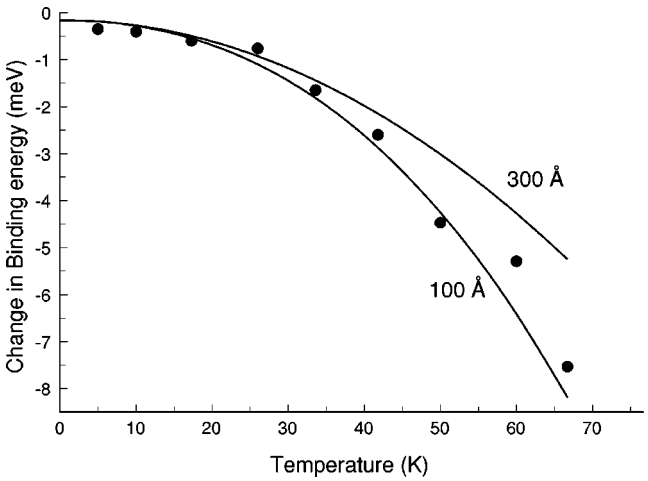

FIG. 6. Dots denote experimental values (Ref. 18) of the change in exciton binding energies with temperature in an *n*-type modulation-doped $GaAs/Al_{0.3}Ga_{0.7}As$ quantum well of width, 100 Å and an electron density of 1.5×10^{11} cm^{-2}. Lines at well widths 100 and 300 Å are obtained using Eqs. (12), (13), (15), and (20).

While Eq. (20) remains valid up to carrier densities $\sim1\times10^{11}$ cm^{-2}, it is no longer applicable at larger densities due to the assumptions used in its derivation and underestimated values of α [Eq. (21)]. Nevertheless, it is interesting to note the gradual decrease in critical carrier densities at which the exciton becomes unbound, with an increase in α.

IV. TEMPERATURE DEPENDENCE OF PAULI BLOCKING EFFECTS

In Fig. 5, we plot the individual dimensionless volume, $\Lambda_\alpha/[V_\alpha(a_B)]$, [Eqs. (12) and (13)], as functions of dimensionality, α at 20 and 200 K in $GaAs/Al_{0.3}Ga_{0.7}As$ quantum wells. We have used $m_e=0.067m_o$ and $m_h=0.15m_o$, where m_o is the free-electron mass. At low temperatures (~20 K) and dimensionality range ($\alpha\leqslant2.32$), Pauli blocking effects due to an uncorrelated electron-hole plasma are larger than effects due to an exciton gas. This trend is reversed at higher temperatures and larger α due to changes in the overlap of excitonic wave functions.

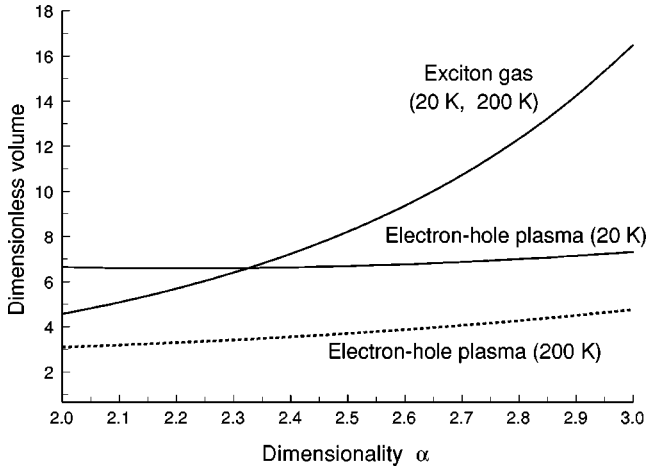

FIG. 5. Dimensionless volume, $\Lambda_\alpha/[V_\alpha(a_B)]$ [Eqs. (12) and (13)] as a function of dimensionality, α at 20 and 200 K in $GaAs/Al_{0.3}Ga_{0.7}As$ quantum wells.

In Fig. 6, Eqs. (12), (13), (15), and (20) substituted with the values of $\alpha=2.44$ for well widths of 100 Å and $\alpha=2.85$ for well widths of 300 Å obtained from Fig. 1, are used to compare our theoretical results with experimental data.[18] The experiment setup[18] was designed to investigate the change in heavy-hole binding energies with temperature in *n*-type modulation-doped $GaAs/Al_{0.3}Ga_{0.7}As$ multiple quantum wells of well widths of 100 Å and an electron density of 1.5×10^{11} cm^{-2}. The figure shows the good agreement with experimental results at well widths of 100 Å, with a smaller change in binding energies at larger well widths. Since heavy-hole (HH) excitons have smaller binding energies than light-hole (LH) excitons, α_{HH} is greater than α_{LH} at a fixed well width. This means that HH excitons quench faster than LH excitons, as reported by experimental observations[18] of carrier effects on excitonic absorption in $GaAs/Al_xGa_{1-x}As$ quantum wells.

It is to be noted that II-VI materials with high exciton binding energies, such as those of $ZnSe/Cd_{0.1}Zn_{0.9}Se$ and $ZnTe/Cd_xZn_{1-x}Te$ quantum wells, display quenching characteristics different from excitons with smaller binding energies, for instance in $GaAs/Al_xGa_{1-x}As$ quantum wells. Due to the comparable exciton binding and thermal energy (kT_L) in $ZnTe/Cd_xZn_{1-x}Te$ and related II-VI materials, Pauli blocking effects due to highly confined excitons [Eqs. (12) and (13)] become more important than those due to a cool electron-hole plasma. Conversely, in materials where excitons have lower binding energies, Pauli blocking effects are dominated by uncorrelated electron-hole pairs. Thus the fine interplay between Pauli blocking effects due to neutral excitons and electron-hole plasma depends on the exciton dimensionality α and thermal kinetics of the material system under study. Such an approach would resolve some of the disagreement between earlier theoretical works,[7] employing an exact two-dimensional exciton model, and experimental results.[3,5]

It is to be noted that studies[26,27] of the temperature varia-

tion of excitonic transition energies indicate the strong temperature dependence of the band gap of the constituent material in the well region. Furthermore, an analysis[28] of the exciton resonance spectra points to the vital role of localized excitons at lower temperatures (below 80 K) and that of free excitons at high temperatures. These factors therefore have to be included in order to make a further quantitative analysis of excitonic optical spectra obtained via experiments.

V. CONCLUSION

We have derived generalized analytical expressions which describe changes in the oscillator strength of an α-dimensional exciton ($2 \leqslant \alpha \leqslant 3$) due to Pauli blocking mechanisms in quantum wells. Our results, which agree well with experimental data for carrier densities $\leqslant 1 \times 10^{11}$ cm^{-2}, show that the quenching characteristics of excitons are critically dependent on their dimensionality and the thermal kinetics of carriers in the material system under study. These factors are critical in resolving disagreements between theoretical works which employ strictly two-dimensional exciton models and experimental results of exciton absorption spectra in quantum wells. It is expected that our results will be of importance in a quantitative understanding of Pauli blocking processes in low-dimensional systems.

[1] P. A. Folkes, M. Dutta, S. Rudin, H. Shen, W. Zhou, D. D. Smith, M. Taysing-Lara, P. Newman, and M. Cole, Phys. Rev. Lett. **71**, 3379 (1993).

[2] K. H. Schlaad, C. Weber, J. Cunningham, C. V. Hoof, G. Borghs, G. Weimann, W. Schlapp, H. Nickel, and C. Klingshirn, Phys. Rev. B **43**, 4268 (1991).

[3] M. Kemerink, P. M. Koenraad, P. C. M. Christianen, R. van Schaijk, J. C. Maan, and J. H. Wolter, Phys. Rev. B **56**, 4853 (1997).

[4] S. Hunsche, K. Leo, H. Kurz, and K. Kohler, Phys. Rev. B **49**, 16 565 (1994).

[5] P. C. Becker, D. Lee, A. M. Johnson, A. G. Prosser, R. D. Feldman, R. F. Austin, and R. E. Behringer, Phys. Rev. Lett. **68**, 1876 (1992).

[6] H. Haug and S. W. Koch, *Quantum Theory of the Optical and Electronic Properties of Semiconductors* (World Scientific, Singapore, 1994).

[7] S. Schmitt-Rink, D. S. Chemla, and D. A. B. Miller, Phys. Rev. B **32**, 6601 (1985).

[8] K. Tai, J. Hegarty, and W. T. Tsang, Appl. Phys. Lett. **51**, 86 (1987).

[9] R. Zimmermann, Phys. Status Solidi B **146**, 371 (1988).

[10] F. H. Stillinger, J. Math. Phys. **18**, 1224 (1977).

[11] X. F. He, Phys. Rev. B **43**, 2063 (1991).

[12] P. Lefebvre, P. Christol, and H. Mathieu, Phys. Rev. B **48**, 17 308 (1993).

[13] C. Ell, R. Blank, S. Brenner, and H. Haug, J. Opt. Soc. Am. B **6**, 2006 (1989).

[14] A. Thilagam, Phys. Rev. B **56**, 9798 (1997).

[15] S. Bochner and K. Chandrasekharan, *Fourier Transforms*, Annals of Mathematical Studies No. 19 (Princeton University Press, Princeton, 1949).

[16] E. M. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Space* (Princeton University Press, Princeton, 1975), p. 7.

[17] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, Boston, 1994).

[18] D. Huang, J. Chyi, and H. Morkoc, Phys. Rev. B **42**, 5147 (1990).

[19] C. Arena, L. Tarricone, F. Genova, and C. Rigo, Mater. Sci. Eng., B **21**, 189 (1993).

[20] V. Voliotis, R. Grousson, P. Lavallard, and R. Planel, Phys. Rev. B **52**, 10 725 (1995).

[21] Z. L. Yuan, Z. Y. Xu, Weikun Ge, J. Z. Xu, and B. Z. Zheng, J. Appl. Phys. **79**, 424 (1996).

[22] G. Oelgart, M. Proctor, D. Martin, F. M. Genaud, F. K. Reinhart, B. Orschel, L. C. Andreani, and H. Rhan, Phys. Rev. B **49**, 10 456 (1994).

[23] E. Kartheuser, *Polarons in Ionic Crystals and Polar Semiconductors*, edited by J. T. Devreese (North-Holland, Amsterdam, 1972), p. 717.

[24] H. J. Lozykowski and V. K. Shastri, J. Appl. Phys. **69**, 3235 (1991).

[25] L. Calcagnile, R. Rinaldi, P. Prete, C. J. Stevens, R. Cingolani, L. Vanzetti, L. Sorba, and A. Franciosi, Phys. Rev. B **52**, 17 248 (1995).

[26] C. F. Li, D. Y. Lin, Y. S. Huang, Y. F. Chen, and K. K. Tiong, J. Appl. Phys. **81**, 400 (1997).

[27] Y. G. Zhao, R. A. Masut, J. L. Brebner, C. A. Tran, and J. T. Graham, J. Appl. Phys. **76**, 5921 (1994).

[28] W. Z. Shen, S. C. Shen, W. G. Tang, Y. Chang, Y. Zhao, and A. Z. Li, J. Phys.: Condens. Matter **8**, 4751 (1996).