Tunneling of heavy holes in semiconductor microstructures

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We use a six-band effective-mass theory to calculate the rate of tunneling of heavy holes in two kinds of GaAs/AlAs semiconductor microstructure: a multiple-quantum-well system with alternating wide and narrow quantum wells and, for comparison, a double-barrier structure. Our results take account of valence-band mixing induced by confinement of the carriers and for the two structures considered are obtained from effective-mass generalizations of, respectively, Bardeen's transfer-Hamiltonian method and Gamow's method for resonant states. After correcting for uncertainties in the band-structure parameters, we find good agreement with time-resolved photoluminescence experiments on the multiple-quantum-well structure. [S0163-1829(99)06304-3]

I. INTRODUCTION

The fabrication of semiconductor quantum-well structures provides opportunities not only to create a range of novel device structures, but also to study fundamental physical phenomena such as quantum-mechanical tunneling. Fairly direct measurements of tunneling rates can be made for particular systems, e.g., by time-resolved photoluminescence measurements[1] and by excitation-correlation methods.[2] A notable result of such experiments is that the tunneling of heavy holes proceeds much faster than would be expected on the basis of the combined effects of their relatively large effective mass and high potential barriers. The reason for this was given some time ago:[3] quantum confinement leads to mixing of light- and heavy-hole states, so that penetration by heavy holes into the classically forbidden region is, crudely speaking, determined by the mass of the *light* hole.

In this paper we calculate heavy-hole tunneling rates and their dependence on temperature and barrier width in two kinds of system, illustrated in Fig. 1. In the double-barrier system of Fig. 1(a), carriers in the GaAs well region can tunnel through the AlAs barriers into states of bulk GaAs. Figure 1(b) shows part of a GaAs/AlAs mixed type-I and type-II multiple-quantum-well structure in which wide wells alternate with narrow wells. Carriers can be generated optically in the narrow quantum wells, and the electrons transfer rapidly to the wide wells by scattering via the *X* state in the barrier region.[1] The much slower rate of the transfer of holes can be determined by measuring the decay of the photoluminescence from the wide quantum well, where the holes recombine after tunneling. Hole transfer rates obtained by this method vary exponentially with the barrier width, so that we may be confident that the method of transfer is tunneling.

It might at first be supposed that tunneling would proceed at similar rates in double-barrier and multiple-quantum-well systems, provided that the barrier widths and heights were the same in each case. This, however, would ignore the part played by the final state in tunneling. For systems whose interfaces are not rough and in the absence of other scattering mechanisms, a hole may tunnel only if a final state is available with the same energy and wave vector as the initial state. As a result, there may be relatively few states from which holes can tunnel, and this can lead to a strong temperature dependence of the tunnelling rate via the thermal occupation probability of the initial state.

Because of the results' expected sensitivity to the precise shape of the hole subbands, it is important to use an accurate model for the valence band. For this reason, rather than use the simple four-band model chosen by Yu, Jackson, and McGill,[3] we use a six-component effective-mass theory, discussed briefly in Sec. II below. Rates for tunneling out of a double-barrier system are calculated in Sec. III using an effective-mass generalization of Gamow's theory[4] of resonant states. In Sec. IV we calculate transfer rates for holes tunneling between quantum wells; this requires an extension of Bardeen's transfer-Hamiltonian method[5] to the six-band effective-mass theory. Our results are compared with experiment in Sec. V.

II. EFFECTIVE-MASS THEORY

We use Foreman's six-band effective-mass theory based on zone-center heavy-hole, light-hole, and spin-orbit split-off states.[6] In effective-mass theory the time-dependent Schrödinger equation takes the form

$$i\hbar \frac{\partial F}{\partial t} = \hat{H} F, \tag{1}$$

where F is a six-component column vector of envelope functions. The 6×6 matrix Hamiltonian has the general Hermitian form

$$\hat{H} = \frac{\partial}{\partial z} A \frac{\partial}{\partial z} + \frac{\partial}{\partial z} B - B^{\dagger} \frac{\partial}{\partial z} + C, \tag{2}$$

where the matrices A, B, and C depend on position z along the growth direction (via the material-dependent band parameters), and B and C also depend on the wave vector $\mathbf{k}$ in the plane of the quantum well; A and C are Hermitian matrices, but B is not equal to $B^{\dagger}$ in general. The perturbative method of Burt[7] provides a definite ordering of differential operators with respect to the band parameters of the materi-

als of the microstructure and, hence, the precise form of the matrix B. Integration of Schrödinger's equation across an interface then gives the boundary conditions on the wave function and its derivatives, namely, continuity of the vectors F and $A\partial F/\partial z + BF$. Although the approximations leading to these boundary conditions have been questioned,[8] extensive calculations by Meney, Gonul, and O'Reilly[9] show unequivocally that Burt's boundary conditions should be preferred to the symmetrized conditions used, e.g., by Eppenga, Schuurmans, and Colak.[10]

It is easy to show from Schrödinger's equation and the corresponding equation for $F^\dagger$ that the probability density $\rho = F^\dagger F$ satisfies a continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial J}{\partial z} = 0;$$

the method of derivation follows precisely the usual derivation of the particle current in quantum mechanics.[11] The current density of holes, J, is given by

$$J = \frac{i}{\hbar}\left\{ F^\dagger A \frac{\partial F}{\partial z} - \frac{\partial F^\dagger}{\partial z} A F + F^\dagger (B - B^\dagger) F \right\}; \qquad (3)$$

it is, of course, continuous at any interface by virtue of the boundary conditions on F and $\partial F/\partial z$. This expression for the current will be used below to calculate the rate of tunneling out of a double-barrier system.

In our numerical work we use mainly the band parameters quoted in Ref. 12, but for consistency with Burt's method[7] (which uses the same basis of periodic functions in each material) we assume a constant value of the Kane[13] matrix element, $P = 9.90\ \text{eV Å}$, appropriate to GaAs. Generally, we consider tunneling of holes with energies close to E_v(GaAs), the valence-band edge of the well material, and we have adjusted the AlAs Luttinger parameters to allow for this as follows. An expression such as

$$\gamma_{1,\text{expt}} = \gamma_{1,\text{res}} + 2mP^2/(3\hbar^2 E_g),$$

derived from $\mathbf{k}\cdot\mathbf{p}$ perturbation theory, is used to separate the experimentally determined Luttinger parameter $\gamma_{1,\text{expt}}$ into a part due to the nearest conduction-band states at the Γ point and a residual contribution $\gamma_{1,\text{res}}$ from the more remote bands. The values of γ(AlAs) used in our effective-mass Hamiltonian are then obtained by replacing $E_g = E_c$(AlAs) $- E_v$(AlAs), the AlAs zone-center band gap, by E_c(AlAs) $- E_v$(GaAs). For both materials we have taken account of the temperature variation of the Luttinger parameters via the decrease of the band gaps with increasing temperature. In this, we take P and the residual Luttinger parameters to be independent of temperature, but in practice the correction has a relatively small effect on our tunneling results. Uncertainty in the ratio of conduction- to valence-band offsets might be expected to have a significant effect on the results; we have assumed the value 67:33, independent of temperature.

We note what we consider to be a significant limitation on the applicability of the expression (3) for the current. To treat a narrower quantum well or a material with a small band gap, it might appear convenient to use a more accurate *energy-dependent* six-band effective-mass Hamiltonian, such as the one developed by Eppenga, Schuurmans, and Colak.[10]

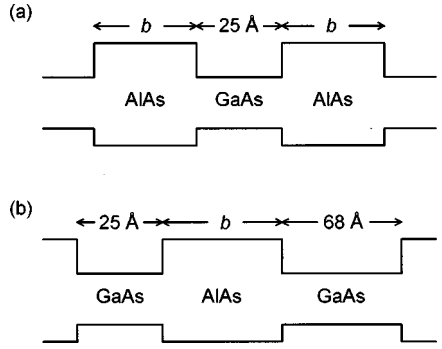

FIG. 1. Schematic diagram showing conduction- and valence-band edges for two GaAs/AlAs microstructures: (a) a double-barrier structure and (b) one period of a multiple-quantum-well structure in which wide and narrow wells alternate, separated by barriers of width b.

But away from the valence-band edge, the probability density ρ is not simply $F^\dagger F$, based on the valence-band envelope functions, but should include a contribution $F_c^\dagger F_c$ due to states F_c admixed from the conduction band. This state-dependent contribution to ρ is of relative order $\Delta E/E_g$, where ΔE is the confinement energy, and can be expected to be important whenever the energy-dependent effective-mass theory provides a significant correction to the valence subband structure. Fortunately, the ratio $\Delta E/E_g$ never exceeds 7% in our applications; so any state-dependent corrections to the density and current are expected to be small. Nevertheless, rather than use an energy-dependent theory in conjunction with Eq. (3), we note that the general methods of this paper apply equally well to calculations based on an energy-independent 14-band Hamiltonian.

III. DOUBLE BARRIER: RESONANT DECAY

The structures considered theoretically in this paper are illustrated in Fig. 1. We assume idealized, planar interfaces between different materials and neglect coupling to impurities, phonons, and other carriers, so that the holes tunnel through barriers without change of energy or in-plane wave vector.

A hole confined in a GaAs quantum well by two thick AlAs barriers can persist for a relatively long time in a quasistationary state before it tunnels to the outer GaAs region. A state of this kind shows up as a resonance in scattering or as an approximately Lorentzian peak in the transmission probability as a function of the energy of a hole incident on the structure. This fact has been used by Yu, Jackson, and McGill,[3] who estimate carrier tunneling rates out of a double-barrier structure by measuring the widths of the peaks in the calculated transmission coefficient. Such an approach is impractical, however, for the wide-barrier systems considered here: the widths of the resonances are too small.

Instead, we calculate the lifetime of the resonance by adapting to effective-mass theory a well-known technique due to Gamow.[4,14] The method is accurate only to first order in the tunneling probability, but this is sufficient for systems with wide barriers. We regard the normalized bound states of the isolated quantum well as a first approximation to the resonant state. At the interface with an outer GaAs region,

the wave function is exponentially small and can be matched to outgoing plane waves,[15] which correspond physically to escape of the hole from the double-barrier structure. To satisfy the boundary conditions at the interfaces, we must also introduce *increasing* exponentials within the barrier regions. In general, there may be six of these, $\exp[\kappa_i z]$, but their coefficients are small, at most of order $\exp[-\kappa_{\min} b]$ (b is the barrier thickness), the least rapidly decreasing contribution to the bound state wave function. Any correction to the wave function inside the well region is therefore even smaller, of order $\exp[-2\kappa_{\min} b]$, and may be neglected without inconsistency.

The outgoing current density J, given by Eq. (3), may be calculated at any point near the interface of the AlAs barrier region with the outer GaAs region. For a bound state normalized in unit area of the quantum well, the probability per unit time of escape from the structure will be given by $2J$, the factor of 2 taking account of the two possible directions of tunneling out of the well. Our approach automatically includes the current due to holes in all outgoing valence-band channels, as in the multichannel scattering theory presented by Morifuji and Hamaguchi.[16] In our calculations the carriers emerge in both heavy- and light-hole states.

The tunneling rate calculated in this way is a function of the in-plane wave vector $\mathbf{k}=(k_x,k_y)$ and the subband index and must be averaged over the distribution of holes. The tunneling time τ is then defined as the inverse tunneling rate

$$1/\tau = 2\langle J \rangle;$$

it determines the decay of the population of trapped holes, whose number will varies as $\exp[-t/\tau]$, provided τ is large compared with the time for the holes to reach thermal equilibrium within the well and that the carrier density is small enough for Boltzmann statistics to apply.

Results calculated in this way are shown in Fig. 2; they have been corrected by the method discussed later in Sec. V. At low temperatures the tunneling time varies with temperature roughly as $1/T$, a fact due to the valence-band mixing,[3] as we now explain. At sufficiently low temperatures only heavy holes are present in the quantum well, and these will mostly have small energies, of order $k_B T$. It might be imagined that the tunneling rate would be dominated by its value for $k=0$, but at the zone center the heavy-hole band is in fact not coupled to the light-hole and spin-orbit split-off states, so that the tunneling rate is anomalously small: the rate of decay of the zone-center heavy-hole wave function in the barrier is greater than that of the light hole, $\kappa_{\rm hh} > \kappa_{\rm lh}$, leading to $\exp[-2\kappa_{\rm hh} b] \ll \exp[-2\kappa_{\rm lh} b]$ for a wide barrier. To lowest order the effects of band mixing appear in the Hamiltonian (2) via B and $B^\dagger$, which are proportional to k. Regarding these terms as a perturbation to the zone-center heavy-hole confined state shows the light-hole amplitude to be proportional to k and the corresponding contribution to the heavy-hole tunneling rate to vary as $k^2 \exp[-2\kappa_{\rm lh} b]$. After thermal averaging the contribution to the tunneling rate is roughly proportional to $T \exp[-2\kappa_{\rm lh} b]$ at low temperature, and it forms the major part of the tunneling rate, except at unattainably low temperatures, where pure heavy-hole tunneling should be dominant. This general conclusion is confirmed by our detailed calculations. At higher temperatures ($T > 20$ K), the tunneling rate levels off rapidly, an effect due mainly to saturation of the light-hole contribution to the wave function as k increases.

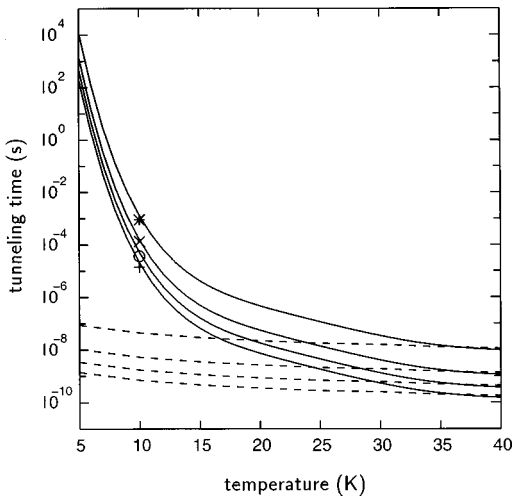

FIG. 2. Experimentally determined tunneling times τ at 10 K for the multiple-quantum-well structure of Fig. 1(b) with barrier widths $b=72$ Å [+], 80 Å [○], 90 Å [×], and 109 Å [*]. Results for this system (with the same barrier widths) calculated by the Bardeen method of Sec. IV (solid lines) and results for the double-barrier system of Fig. 1(a) calculated by the wave-function-matching method of Sec. III (dashed lines). All results have been corrected by the method explained in Sec. V. The experimental value of α is 0.113 Å^{-1}. Calculated values are 0.272 Å^{-1} for the multiple-quantum-well and 0.260 Å^{-1} for the double-barrier structures. The calculated values differ from one another because they correspond to tunneling by holes with different values of the in-plane wave vector.

IV. DOUBLE WELL: BARDEEN FORMALISM

To calculate the rate of tunneling between two quantum wells, we use the transfer-Hamiltonian method introduced by Bardeen.[5] Although this may also be used to calculate the rate of tunneling out of a double-barrier structure,[17] the wave-function matching technique described in Sec. III is considerably simpler in that case. Like the wave-function matching technique, Bardeen's method is accurate only to lowest order in the tunneling probability.

We consider tunneling of holes out of the narrow, left-hand well of Fig. 1(b). Following Bardeen, we expand the wave function in terms of confined states of the left- and right-hand wells, F_L and F_R (with energies E_L and E_R), each calculated as if the other well were absent. After forming the time-dependent superposition

$$F(\mathbf{r},t) = c_L(t) F_L(\mathbf{r}) e^{-iE_L t/\hbar} + \sum_R c_R(t) F_R(\mathbf{r}) e^{-iE_R t/\hbar}$$

and using the initial conditions $c_L(0)=1$ and $c_R(0)=0$ corresponding to a hole initially in the left-hand well, the rate of transition $W(L \to R)$ from a particular state L to any final state R is found by the normal methods of time-dependent perturbation theory,[11] giving

$$W(L\rightarrow R)\simeq\frac{2\pi}{\hbar}|M_{RL}|^2\delta(E_L-E_R). \tag{4}$$

In Eq. (4) the tunneling matrix element is given by

$$M_{RL}=\int_{z>z_0}F_R^\dagger(\hat{H}-E_L)F_L d^3\mathbf{r}$$
$$=\int_{z>z_0}[F_R^\dagger\hat{H}F_L-(\hat{H}F_R)^\dagger F_L]d^3\mathbf{r}.$$

The integration can be restricted to the region right of a plane $z=z_0$ within the barrier: F_L solves Schrödinger's equation throughout the left-hand well and the barrier region, so that the integrand is zero for $z\leqslant z_0$. Use of the condition $E_L=E_R$ implied by the δ function in Eq. (4) allows us to replace $E_LF_R^\dagger$ by $(\hat{H}F_R)^\dagger$ in the second line.

If we now use the expression (2) for $\hat{H}$ and integrate by parts with respect to z, we obtain

$$M_{RL}=\int_{z=z_0}\left\{\frac{\partial F_R^\dagger}{\partial z}AF_L-F_R^\dagger A\frac{\partial F_L}{\partial z}+F_R^\dagger(B^\dagger-B)F_L\right\}dx\,dy,$$

which differs from Bardeen's result only in the use of an effective-mass expression for the matrix element of the current operator. For systems with ideal planar interfaces, the wave functions contain plane-wave factors $\exp[i(k_xx+k_yy)]$, and the integrations with respect to x and y lead to a δ function expressing conservation of the wave vector. We write the final result in the form $M_{RL}=M_{rl}\delta(\mathbf{k}_R,\mathbf{k}_L)$, where r and l label the quantum well subbands and $\delta(\mathbf{k}_R,\mathbf{k}_L)$ denotes the Kronecker delta.

For the case of the optically excited mixed type-I and type-II microstructures, the holes recombine quickly ($\sim$1 ns) after tunneling so that the final states may be assumed to be empty. After averaging with respect to the thermal distribution of holes, the tunneling rate per particle is

$$\frac{1}{\tau}=2\sum_{\substack{\mathbf{k}\\l,r}}\frac{2\pi}{\hbar}|M_{rl}|^2f(E_l(\mathbf{k}))$$
$$\times\delta(E_l(\mathbf{k})-E_r(\mathbf{k}))\Big/\sum_{\mathbf{k},l}f(E_l(\mathbf{k})), \tag{5}$$

where the summations run over the in-plane wave vector $\mathbf{k}$ and the two quantum-well subband indices l and r, which must include the Kramers-degenerate pairs of states. As in Sec. III the factor of 2 allows for the escape of holes to either the left or the right. In Eq. (5), $f(E)$ denotes the Fermi-Dirac distribution function, which in general depends on the density of carriers via the chemical potential. At the relatively low densities relevant to our experiments, $f(E)$ reduces to the Boltzmann distribution, so that the result for the tunneling rate is independent of density.

Because of the constraints of energy and wave-vector conservation contained in Eq. (5), tunneling can occur only when energy subbands of the two quantum wells intersect one another. This situation arises for 25- and 68-Å wells, as shown in Fig. 3. Figure 4 shows the closed curve of intersection in the (k_x,k_y) plane. The wave-vector summation in Eq. (5) reduces to a line integral,

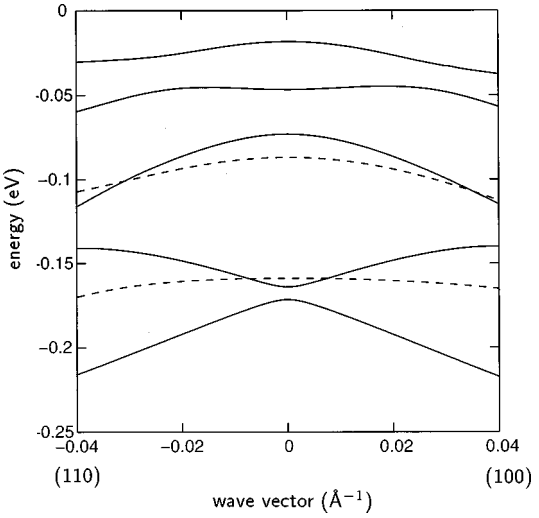

FIG. 3. Valence subbands for 25- and 68-Å GaAs/AlAs single quantum wells (dashed and solid lines, respectively). Holes in the first subband of the narrow well may tunnel to states of the third subband of the wide well at points where the subbands intersect. All energies are measured from the valence-band edge of GaAs.

$$\sum_{\mathbf{k}}\delta(E_l(\mathbf{k})-E_r(\mathbf{k}))\rightarrow\frac{S}{(2\pi)^2}\oint\frac{dk_\parallel}{|\partial(E_l-E_r)/\partial k_\perp|}, \tag{6}$$

where $dk_\parallel$ denotes the element of arc length along the contour, $\partial/\partial k_\perp$ the derivative normal to the contour, and S the area of the quantum well in the plane perpendicular to z.

The two conservation laws lead to a much stronger temperature dependence of the tunneling results than in the double-barrier case, where there were always final states available, regardless of the initial state of the hole. In the present case, only heavy holes with energy in the neighborhood of $\Delta=13$ meV above the subband minimum of the

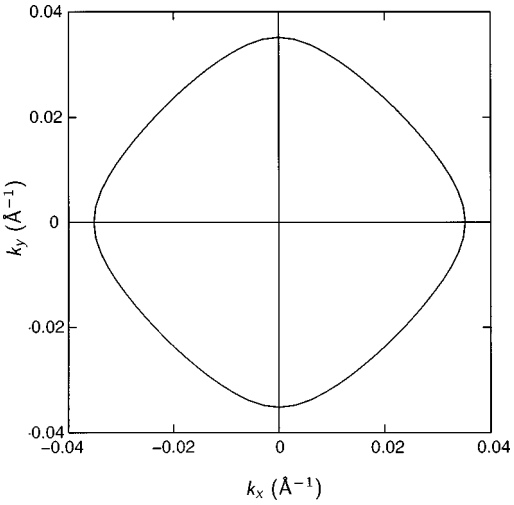

FIG. 4. Contour of integration for the Bardeen calculation of Sec. IV. It is the curve of intersection of the first valence subband of the 25-Å well with the third valence subband of the 68-Å well.

25-Å well can tunnel elastically to the 68-Å well. At low temperature this leads to the characteristic behavior $\tau \propto T \exp[\Delta/k_B T]$ for the tunneling time. Inevitably, our results can be expected to be sensitive to the errors of order a few meV that are typical of any effective-mass calculation of a subband structure. An error of 1 meV would alter the results by about a factor of 3 at 10 K and by less at higher temperatures. This should be borne in mind when making comparisons with experiment.

V. COMPARISON WITH EXPERIMENT

Tunneling calculations are remarkably sensitive to the precise values of the band parameters of the barrier material, much more so than calculations of the subband structure on which they depend. In both theory and experiment, the tunneling time varies with barrier width as $\tau(b) = \tau_0 \exp[\alpha b]$. But the calculated value of α, $\alpha_{\mathrm{calc}} \simeq 2\kappa_{\mathrm{lh}}$, is a function of the valence-band parameters and the energy of the tunneling holes, and errors in these lead to errors in α_{calc}. After exponentiation, the discrepancy between theoretically and experimentally determined tunneling times can be several orders of magnitude: it is essential to make some correction for this. We have chosen to multiply all of our calculated results by $\exp[(\alpha_{\mathrm{expt}} - \alpha_{\mathrm{calc}})b]$, where α_{expt} is obtained by a least-squares fit to the experimental data at 10 K and is the *only* fitted parameter. Note, however, that the overall magnitude of the results (rather than their variation with barrier width) is *not* a fitted parameter, so that it is meaningful to compare tunneling times calculated by different methods. This overall magnitude, determined by the factor τ_0, is relatively insensitive to the experimental uncertainty in the material parameters, but does depend on the effective-mass model used, including the choice of boundary conditions.

The comparison with experiment is made in Fig. 2. It shows clearly that tunneling between quantum wells is a much slower process at low temperature than tunneling out of a double-barrier structure with the same widths of barrier and initial well. This was expected from the constraints of energy and wave-vector conservation, as described above. Our numerical results for tunneling between quantum wells agree reasonably well with experiment, which gives us confidence that the tunneling process is correctly described. Note that a discrepancy of even one order of magnitude in the tunneling rate could be accounted for by an error of 2 meV in the calculated alignment of subbands in the two wells, so that the level of agreement we obtain is actually slightly fortuitous. On the other hand, results calculated for the double-barrier system disagree with the experimental values from the multiple-quantum-well system by more than four orders of magnitude at low temperature. This fact may provide reassurance on the validity of the method used to correct our results for comparison with experiment and also shows that the Bardeen method applied to tunneling in these structures is a much better starting point for discussing experiments than the somewhat simpler theory for the double-barrier structure.

Theoretical results for the multiple-quantum-well system have previously been reported in Ref. 1. They were obtained by extrapolation of results for the double-barrier structure calculated by Yu, Jackson, and McGill.[3] The extrapolated results fail to show the observed dependence on barrier width, so that the agreement with experiment now appears fortuitous. We find that if the extrapolated results are corrected in the manner discussed above, they agree (within a factor of about 2) with our new results for the double-barrier structure, calculated by the method described in Sec. III.

Finally, we suggest that our general conclusions would be unaltered by inclusion of the coupling of light- and heavy-hole states that results from the microscopic structure of the GaAs/AlAs interface. Pseudopotential[18] and tight-binding[19] calculations have shown that this mixing occurs even at the center of the two-dimensional (2D) Brillouin zone, and the effect has recently been brought within the framework of effective-mass theory.[20] The resulting additive, δ-function terms in the Hamiltonian[21] affect only the boundary conditions on the effective-mass wave function, so that the formalism developed in Secs. III and IV remains unchanged.

First, we note that the mixing terms should have only a small effect on our results for tunneling between wells of different widths, because the states between which holes can tunnel have large in-plane wave vectors and so are already subject to a large mixing due to the $\mathbf{k}\cdot\mathbf{p}$ interaction. On the other hand, the effect on our results for the double-barrier structure may be much greater, as in our simple model the heavy holes acquire some light-hole character only through thermal excitation away from $\mathbf{k}=\mathbf{0}$. The magnitude of the band-mixing terms proposed in Ref. 20 is somewhat uncertain, but any additional band mixing will tend to reduce the tunneling time for heavy holes near the zone center, and so will further increase the large discrepancy between the double-barrier calculation and experiment illustrated in Fig. 2. We therefore expect the introduction of interface terms to strengthen our earlier conclusion that tunneling in the multiple-quantum-well structure cannot be modeled by a theory of tunneling through a barrier into bulk GaAs.

VI. SUMMARY

We have shown how to use Gamow's theory to calculate the lifetime of a heavy hole confined in a double-barrier structure; our results, which neglect important band-mixing effects due to the interface, vary approximately as $1/T$ at low temperature. Our method is direct and starts from the effective-mass wave function for a hole in an isolated quantum well; in particular, it does not require the calculation of a transmission probability over a range of energies, a method previously used,[3] though like that method it does allow for emission of the hole into both light- and heavy-hole final states.

By adapting Bardeen's transfer-Hamiltonian method, we have calculated the rate at which holes tunnel between quantum wells of different widths. In this case our results are highly sensitive to the shape of the valence subbands in both wells, τ varying as $T \exp[\Delta/k_B T]$ at low temperature for our particular choice of well widths. The resulting temperature dependence of the tunneling rate may provide a way of distinguishing experimentally between different proposed tunneling mechanisms, e.g., elastic versus inelastic tunneling, though we have not investigated this possibility here.

The methods used in this work can readily be extended to effective-mass models based on more than six bands and also to include band-mixing effects due to the interfaces. The former may be needed to describe tunneling in structures with narrower quantum wells or those using materials with smaller band gaps. Nevertheless, when introducing more complex models with a greater number of experimentally determined band-structure parameters, it should be kept in mind that uncertainty in these affects the decay rate of the wave function in the barrier; this sensitivity is magnified in the calculated tunneling rate itself, especially if the barriers are wide.

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[1] I. Galbraith, P. Dawson, and C. T. Foxon, Phys. Rev. B **45**, 13 499 (1992).

[2] M. K. Jackson, M. B. Johnson, D. H. Chow, T. C. McGill, and C. W. Nieh, Appl. Phys. Lett. **54**, 552 (1989).

[3] E. T. Yu, M. K. Jackson, and T. C. McGill, Appl. Phys. Lett. **55**, 744 (1989).

[4] G. Gamow, Z. Phys. **51**, 204 (1928); G. Gamow and C. L. Critchfield, *Theory of Atomic Nucleus and Nuclear Energy-Sources* (Clarendon, Oxford, 1949), Chap. VI, Sec. 3; E. C. Kemble, *Fundamental Principles of Quantum Mechanics* (McGraw-Hill, New York, 1937), Sec. 31f.

[5] J. Bardeen, Phys. Rev. Lett. **6**, 57 (1961).

[6] B. A. Foreman, Phys. Rev. B **48**, 4964 (1993).

[7] M. G. Burt, J. Phys.: Condens. Matter **4**, 6651 (1992).

[8] Use of these boundary conditions leads, in particular, to the retention of spurious, non-normalizable solutions of an eight-band effective-mass Hamiltonian, even when describing the bound states of a carrier in a quantum well: M. J. Godfrey and A. M. Malik, Phys. Rev. B **53**, 16 504 (1996).

[9] A. T. Meney, B. Gonul, and E. P. O'Reilly, Phys. Rev. B **50**, 10 893 (1994).

[10] R. Eppenga, M. F. H. Schuurmans, and S. Colak, Phys. Rev. B **36**, 1554 (1987).

[11] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, 3rd ed. (Pergamon, Oxford, 1977).

[12] *Physics of Group IV Elements and III–V Compounds*, edited by O. Madelung, Landolt-Börnstein, New Series, Group III, Vol. 17, Pt. a (Springer, Berlin, 1982).

[13] E. O. Kane, in *Handbook on Semiconductors*, edited by W. Paul (North-Holland, Amsterdam, 1982), Vol. 1, p. 193.

[14] The method was developed as part of Gamow's explanation of α decay. A similar technique is used in Ref. 11 to calculate the disintegration probability of a hydrogen atom in an electric field.

[15] These ''outgoing plane waves'' may include decreasing exponentials when the latter are also present at the energy of interest. In our calculations there are always evanescent waves associated with the spin-orbit split-off band.

[16] M. Morifuji and C. Hamaguchi, Phys. Rev. B **52**, 14 131 (1995).

[17] The Bardeen formalism and a single-band effective-mass approximation have been used by M. C. Payne to investigate resonant tunneling through a double-barrier structure: J. Phys. C **19**, 1145 (1986).

[18] G. Edwards and J. C. Inkson, Solid State Commun. **89**, 595 (1994).

[19] J. N. Schulman and Y.-C. Chang, Phys. Rev. B **31**, 2056 (1985).

[20] E. L. Ivchenko, A. Yu. Kaminski, and U. Rössler, Phys. Rev. B **54**, 5852 (1996).

[21] A derivation of such interface terms has been given by B. A. Foreman, who starts from Burt's envelope-function theory (Ref. 7): Phys. Rev. Lett. **81**, 425 (1998).