Static properties of stacked Josephson junctions: Comparison of experiments with the inductively coupled sine-Gordon model

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We report on the static properties of twofold stacks of long Josephson junctions fabricated with the Nb/Al-AlO_x/Nb technology. The current-voltage characteristics of these structures typically show three different values of the critical current for a constant external magnetic field. This property can be explained by the inductive coupling model. Numerical calculations based on a system of coupled sine-Gordon equations show that the effective magnetic field in one of the junctions depends on whether the other junction is in the zero voltage state or not. This leads to different critical currents for each junction. This phenomenon was observed for the annular and straight overlap junctions with various parameters. [S0163-1829(99)03902-8]

Following many years of systematic investigation of the static and dynamic properties of single Josephson junctions, now the interest in vertically stacked junctions is growing. Several aspects converge to stimulate this field of research. In fact, stacking turns out to be a promising technique for Josephson oscillators. As demonstrated by Koshelets *et al.*,[1] long Josephson junctions can work as local oscillators in integrated submillimeter-wave receivers. For a suitable coupling of stacked Josephson junctions such devices could provide larger output power and smaller linewidth than single junctions (depending on the number of coupled junctions), since the fluxon oscillations can phase-lock, as already observed in experiment.[2] In comparison to planar arrays, the mutual inductive coupling in the stacked system can be by several orders of magnitude stronger and can be easily controlled by the thickness of the intermediate superconducting electrode. Another application that could benefit from junction stacking is the Josephson voltage standard.[3] The model that we assume to describe the static and the dynamic behavior of stacked Josephson junctions is the coupled sine-Gordon system.[4] We emphasize that also the intrinsic Josephson effect in high-T_c superconductors[5] by now seems to be well described by this model, and therefore the low-T_c artificial stacks can well serve as model systems for natural high-T_c stacks.

In this paper we consider the effect of an external magnetic field applied in-plane to the barriers of twofold Josephson junction stacks, a configuration which plays a crucial role for the use of stacked systems. After introducing the model and the numerical results, the sample preparation and the experiments will follow and a brief discussion will conclude the paper.

We make use of the inductive coupling model by Sakai, Bodin, and Pedersen,[4] which well accounts for many dynamical phenomena observed in long stacked junctions. They wrote a set of equations (for arbitrary barrier geometry and different parameters for each layer) in which the interaction between the junctions depends strongly on the spatial nonuniformity of the local magnetic field. We apply the equations to a twofold stack with annular geometry in the presence of an external magnetic field. Grønbech-Jensen *et al.* suggested[6] that a uniform magnetic field acts on a ring-shaped junction as a sinklike potential and it is accounted by a sinusoidal bias term in the perturbed sine-Gordon equation. Considering the critical current density for each barrier as the only different parameter between the junctions ($\Delta J = J_1/J_2$) the equations are

$$\varphi_{tt}=\varphi_{xx}+S\psi_{xx}-\alpha\varphi_t-\sin\varphi-\eta k^2\sin(kx)+\gamma,$$

$$\psi_{tt}=\psi_{xx}+S\varphi_{xx}-\alpha\psi_t-\frac{1}{\Delta J}\sin\psi-k_\eta\eta k^2\sin(kx)+\gamma, \tag{1}$$

where φ and ψ are the gauge invariant phase differences across the two barriers which have periodic boundary conditions. Length and time are normalized as usual[6] and the damping constant α is defined as for the single junction, S is

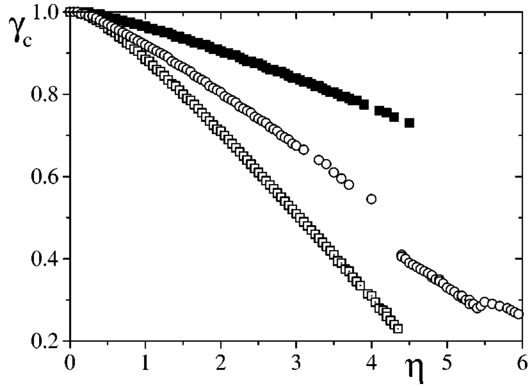

FIG. 1. Magnetic field dependence of the critical currents of a twofold annular stack obtained from the numerical simulation using Eqs. (1). Parameters: $L=12$, $\alpha=0.05$, $S=0.5$, $k_\eta=1.2$, $\Delta J=1$.

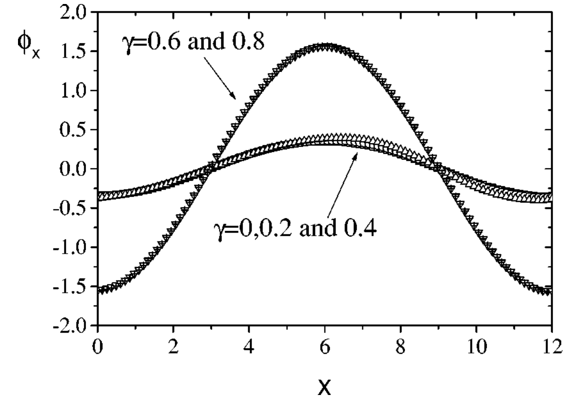

FIG. 2. Numerically calculated spatial phase derivative along an annular Josephson junction for different values of the bias γ. Parameters: $L=12$, $\alpha=0.05$, $\eta=3.0$.

the coupling term depending on the physical and geometrical parameters of the system,[4] and it can vary from 0 (no coupling) to 1 (infinite coupling), η and γ stand for the normalized flux coupled into the junction and the uniform bias current normalized to the maximum junction Josephson current, respectively, k_η is the ratio between the effective external fields acting on either junction which can be calculated from the original equations[4] and can be ascribed to the different thickness of the upper and lower electrode. The term k accounts for an annular junction and it is defined as $k=2\pi/L$, where L is the circumference of the junction.[6,7] In the following, A is the junction whose phase is φ, and B is the other one. Recently, it has been verified for single-barrier annular junctions[7–9] that the field-induced term $[\propto \sin(2\pi x/L)]$ well accounts for their experimentally observed behavior.

In Fig. 1 we show a calculated dependence of the critical current versus the external field calculated for a stack of two long annular junctions. The parameters were chosen to be consistent with the experiments reported below. The three patterns correspond to the switching from the zero voltage state: (i) lower pattern, both junctions switch together from the zero voltage state to the normal state, (ii) middle pattern, critical current of junction B while junction A is in the normal state, (iii) higher pattern, critical current of junction A while junction B is in the normal state.

Hence, the different states, static or dynamic, of one junction while the other is still in the zero-voltage state are responsible for the different critical current patterns. This phenomenon, which has been also experimentally observed earlier,[10,11] can be visualized considering the spatial modulation of the phase originated by the magnetic field and can be explained in the framework of the model in Eq. (1). In order to clarify this point we focus on the behavior of a single junction. Figure 2 shows the numerically calculated space derivative of the phase in a single annular junction embedded in a small magnetic field ($\eta=3$) for different values of the normalized bias current. The curves belong clearly to two families corresponding to a zero voltage state (the lower amplitude family) and a nonzero voltage state (the higher amplitude family). This result can be easily interpreted by observing that when the junction is in the zero voltage state it is in a Meissner-like state and a screening supercurrent prevents the field from entering the barrier completely. On the contrary, when the junction is in the whirling (normal) state, the external field can freely enter the barrier and modulate the phase. In a stack such behavior influence the equation of the other junction through the term proportional to φ_{xx}, originating a less perturbed state when the first junction is still static (lower pattern) and a more perturbed state when the first junction has switched to a dynamic state. A detailed analysis of this behavior will appear elsewhere.

To check the numerical results we prepared the samples using a modified selective niobium anodizatip process (SNAP) Nb/Al-AlO$_x$/Nb technology.[12,13] All films were dc sputtered and the oxide barrier was defined by thermal oxidation under well controlled pressure and temperature. Standard twofold stacks were fabricated by replacing the single trilayer sputtering with an *in situ* sputtered multilayer. Before using the anodic oxidation for passivating the junctions, the upper Nb/Al-AlO$_x$/Nb films were etched by reactive ion etching (RIE). Other type of samples, with a superconducting contact to the middle Nb electrode, was produced using a different process. After the first trilayer was deposited on the substrate, an additional lithographic step for defining the access to the middle electrode was introduced and only after that the second trilayer was sputtered *ex situ*. This preparation procedure leads to a larger parameter spread between the stacked junctions when compared to the *in situ* process. Two different geometries were prepared and investigated. Long overlap junctions were fabricated (with and without access to the middle electrode), and annular stacks. The normalized length of these junctions varied between 5 and 12 for the overlap geometry and was 14, 20, or 28 for the annular ones. The intermediate Nb layer was 90 nm thick, which results in a coupling parameter S of about 0.5.

The measurements were performed at 4.2 K. I-V curves and I_c-H patterns were recorded with a PC using a LABVIEW interface. The dc currents for the junction and the solenoid supplying the external magnetic field were provided by battery powered sources. Figure 3 shows the I-V characteristics of a twofold stack with an applied field of about 1.5 Oe. The two different critical currents $I_{C\text{base}}$ and $I_{C\text{top}}$ at the first gap voltage at about 2.6 mV are clearly distinguished. Comparing the slightly different voltages of the two branches (see inset in Fig. 3) with the measurement of the single junction characteristics, we can identify the corresponding junction I_C's. The magnetic field dependence of these three values for

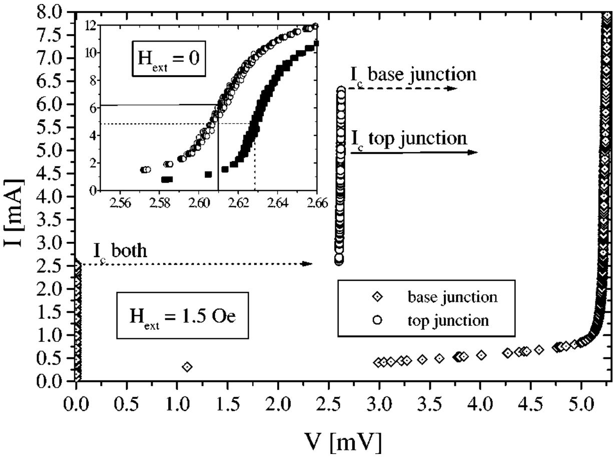

FIG. 3. Measured *I-V* curve for a twofold annular stack (dimension: diameter 122 μm, width 10 μm) at $H_{\rm ext}=1.5$ Oe. Inset: individually measured junctions at $H_{\rm ext}=0$.

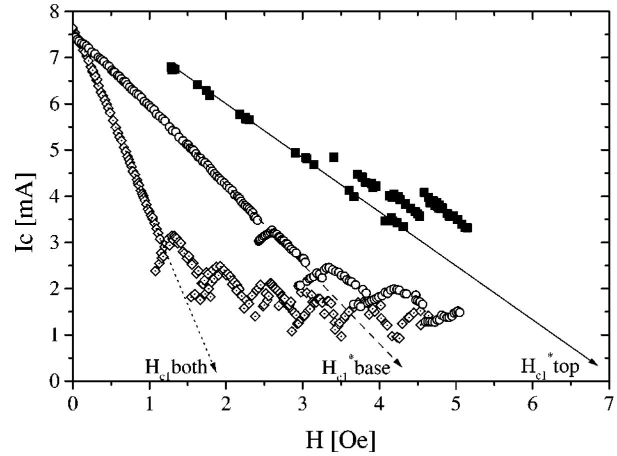

FIG. 4. Measured magnetic field dependence of the critical currents for the twofold annular stack (see Fig. 3) biased in series.

the stack biased in series is shown in Fig. 4. In good agreement with the numerical results, we can see three patterns with different values for the first critical field (H_{c1}). Tests of individual junctions using the access to the middle electrode have proved that the critical field for each junction changes depending on whether the second junction is in the zero-voltage state or in the whirling state. When the second junction is in zero-voltage state, the critical field is small. While biasing the second junction at the gap voltage, the critical field increases. This effect is also seen in numerical simulations, that predict different sensitivity to the magnetic field for a junction depending on the state of its neighbor. We carried out similar measurements on twofold stacks having straight overlap geometry, which showed qualitatively similar but less pronounced difference in the critical currents.

To summarize, the static properties of twofold stacked Josephson junctions were studied in numerical simulations and experiments. The numerical simulations using the coupled sine-Gordon equations predicted different critical currents at a given field value for the fields different from zero. Very similar dependencies were also measured in experiments. Biasing both junctions in series leads to the ''current locking'' effect[14] at zero voltage. The critical current here is not equal to the critical current of the single junction, due to the inductive coupling between the junctions. Indeed, it can be shown analytically[14] that a whirling state of the second junction simply cancels any influence of its outer electrode on the properties of the first junction; the latter junction in this case behaves identically to a single junction with a thin electrode (middle electrode of the stack). Measuring the critical currents from the *I-V* characteristics of the stack at the first gap, i.e., where one of the junctions is biased at the gap voltage, we are able to distinguish between the different critical currents of the junctions in the external magnetic field. These measurements do not show any anomaly in their temperature dependence.

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