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Effects of energy-level quantization on the supercurrent decay of Josephson junctions

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We present measurements of the supercurrent decay in high quality Nb/AlO$_x$/Nb Josephson junctions which show evidence of energy-level quantization within the washboard potential describing the junction. The effect of discrete energy levels leads to oscillations in the rate of escape from the zero voltage state as a function of the bias current, observed at different temperatures. The oscillation spacing fits that expected from the energy level quantization. Moreover, a fit with the quantum theory allows us to derive interesting conclusions about the effective dissipation of the system at low temperatures. [S0163-1829(99)08701-9]

The observation of quantum effects in macroscopic systems is an interesting and difficult task to which many experimentalists have devoted effort in the last decade. The question of whether quantum mechanics survives at the macroscopic level is not yet resolved due to the extreme difficulty in realizing a good ''probe.''[1] Many authors are presently analyzing these effects by studying collective phenomena based on the Josephson effect. This is because the Josephson phase can show effects on a macroscopic scale due to the coherent addition of quantum states whose behavior can be described by quantum mechanics in the presence of dissipation. The most ambitious goal is to obtain an experimental evidence for the coherent superposition of macroscopically distinct states [macroscopic quantum coherence (MQC)].[2,3] However, the experiments reported in the literature give evidence only for the incoherent superposition of macroscopic quantum states, as macroscopic quantum tunneling (MQT),[4] energy level quantization (ELQ),[4,5] and related effects.[6] Previously, the observation of quantum effects in Josephson systems was evidenced by a ''saturation'' at low temperatures of the measured escape rate Γ, observed both in single junctions[4,7] and in superconducting quantum interference devices (SQUID's),[8] due to macroscopic quantum tunneling, and a slight oscillatory behavior of Γ in the presence of microwave irradiation,[4] due to the presence of quantized energy levels.[9] Moreover, in an experiment performed on a SQUID, Rouse, Han, and Lukens[10] reported the first observation of resonant tunneling (RMQT)[11] between macroscopically distinct quantum levels. This experiment required extremely low dissipation. Recently we have observed energy level quantization in Josephson junctions at temperatures well above the crossover temperature T_0 between quantum and thermal effects.[5] This was achieved by measuring the switching current distribution using an external bias with a high sweep frequency, in order to enhance nonstationary conditions in the occupation probabilities of the energy levels.[12] Under nonstationary conditions the ''crossover temperature'' loses the absolute meaning of crossover between escape dominated by macroscopic quantum tunneling $(T<T_0)$ and thermal activation TA $(T>T_0)$, and we can only state that above T_0, the dominant mechanism of escape can be either TA or tunneling from excited levels.[5,12] The energy difference ΔE between the first excited energy level and the ground state also sets an energy scale and temperature ($\Delta E/k$ can be defined as a quantum temperature T_q, so that $T_0 \cong T_q/2\pi$).

In this paper we extend earlier escape measurements to lower temperatures. The data reported support our recent results[5] and add information. The most important element is evidence of energy quantization obtained at a temperature smaller than the level spacing $T<T_q$ and in ''stationary'' conditions. Quantum levels can be observed with our technique only at temperatures above the bias dependent crossover temperature T_0. Below T_0 tunneling can only occur from the ground state and just one ''peak'' is seen in the switching current distribution, as typically observed in previous MQT experiments. Energy levels are observed for $T_0<T<T_q$ even under quasistationary conditions (low sweep frequency) since the transition probabilities between levels become smaller with decreasing temperature. This is

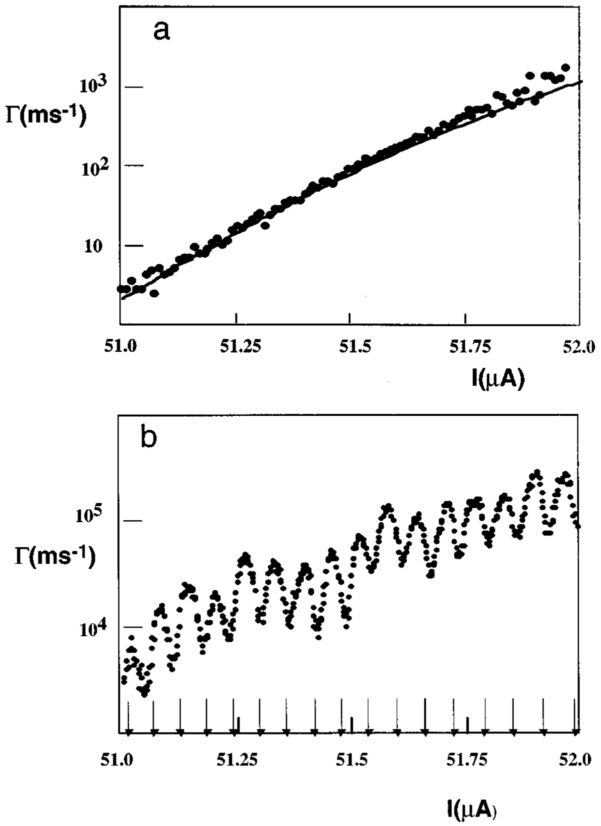

FIG. 1. Experimental data (dots) and theoretical predictions (solid line) for the rate of escape from the $V=0$ state as a function of the bias current, Γ vs I, at two different sweep frequencies $\omega=1/I_c\ (dI/dt)$ and at a temperature of $T=1.4$ K. (a) Data obtained from the stationary distribution taken at a low sweeping frequency, $\omega=4.7\times10^2$ sec^{-1}. The full curve is the theoretical prediction within the classical Kramers theory (Ref. 14) with the following junction parameters: $I_c=52.8$ μA, $C=5.5$ pF, $T=1.4$ K, and $R=20$ kΩ (Ref. 15). (b) Data obtained from the nonstationary distribution taken at a higher sweeping frequency, $\omega=4.7\times10^5$ sec^{-1}. The arrows correspond to energy levels following the theory reported in Ref. 9.

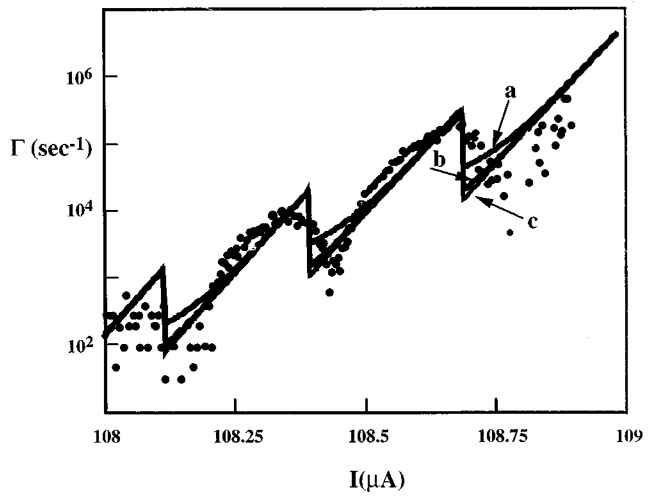

FIG. 2. Experimental data (dots) and theoretical predictions (solid lines) for the rate of escape from the $V=0$ state as a function of the bias current, Γ vs I, at a sweep frequency $\omega=10^3$ sec^{-1}. The theoretical predictions are calculated within the quantum theory.[9] Here $I_c=110$ μA, $T=0.75$ K, $C=0.8$ pF, and the effective dissipation resistance R is (a) $R=100$ kΩ, (b) $R=500$ kΩ, and (c) $R=1$ MΩ.

consistent with the quantum picture of the junction,[9] assuming very low effective dissipation. Using the junction resistance R as a fitting parameter, we fit our data with the theoretical curves based on the stationary solutions of the Larkin-Ovchinnikov kinetic equation.[9] Because stationary conditions are never completely fulfilled in the experiments, there is some uncertainty in the determination of R, and we do not give an exact value for R. However, the observation of energy levels under quasistationary conditions is by itself evidence of extremely low dissipation. This conclusion is important because the intrinsic dissipation of a Josephson junction represents the final barrier to the possibility of maintaining the coherence of SQUID states for a time long enough to observe MQC or to use such devices as elements of quantum computers.[13]

The macroscopic quantum variable describing the junction dynamics is the phase difference φ between the macroscopic wave functions of the two superconductors forming the junction. Within the RSJ model the system dissipation is described by an effective resistance R. In this way the junction behavior can be understood in analogy with the motion of a particle in a washboard potential: $U(\varphi)=-U_0(\alpha\varphi+\cos\varphi)$, where α is the bias current normalized to the critical one, $\alpha=I/I_c$ and $U_0=\hbar I_c/2e$. For $\alpha<1$, the potential $U(\varphi)$ shows an infinite series of minima separated by energy barrier of heights $E_0/U_0=-\pi\alpha+2[\alpha\sin^{-1}\alpha+(1-\alpha^2)^{1/2}]$. The Josephson state ($V=0$) corresponds to the particle trapped in one of the minima. The barrier decreases with increasing current, and when E_0 is small enough the system can escape from the potential well by thermal activation or by quantum tunneling. In this case we observe a switching from the $V=0$ state to the $V\neq0$ state in the junction. The crossover temperature between thermal and quantum effects is typically defined as $T_0=\hbar\omega_j/2\pi k$, where ω_j is the plasma frequency of the junction, $\omega_j=(2\pi I_c/\phi_0C)^{1/2}(1-\alpha^2)^{1/4}$, k is the Boltzmann constant, ϕ_0 is the magnetic flux quantum, and C is the junction capacitance. In a quantum picture we must consider the presence of energy levels in the potential wells of the washboard potential. In the weak friction limit the energy levels E_j are sharp and well separated inside the potential well, and the dynamics of the escape process must be described by the kinetic equation for the probabilities ρ_j of finding the particle at the jth energy level:[9]

$$\frac{\partial\rho_j}{\partial t}=\sum_i (w_{ji}\rho_i-w_{ij}\rho_j)-\gamma_j\rho_j,\quad j=0,\ldots,N \tag{1}$$

where w_{ji} is the transition rate from the ith into the jth level due to the interaction with the thermal bath, and γ_j is the tunneling rate through the barrier, which strongly depends on the energy level position E_j. In the experiments, the bias current is increased with time at a certain rate dI/dt until a transition from the $V=0$ state is observed at a value of I smaller than I_c. This switching current value is a random variable whose probability distribution $P(I)$ is measured by repeating the observation many times. The process is described in statistical terms by Eq. (1). Initially the total probability $\rho=\Sigma_j\rho_j$ of finding the system in one of the energy levels inside the potential well is $\rho(t=0)=1$. However, $\rho(t)$

is a decreasing function of time due to the escape process. The rate of escape from the metastable state is $\Gamma(I) = -d \ln \rho/dt$.

The Josephson junctions used in the experiments are high quality Nb/AlO$_x$/Nb junctions. Nonstationary conditions are obtained by a fast sweep of the external bias I in measuring the switching current distribution $P(I)$ and the related escape rate $\Gamma(I)$. The experiment setup for this kind of measurement is described in detail in Ref. 5. We have measured several junctions with different critical current densities and, as a consequence, different level spacing. As an example, we show in Fig. 1 the escape rate Γ vs I measured for two sweep frequencies $\omega = 1/I_c\ (dI/dt)$ at $T = 1.4$ K. Here the relevant junction parameters are $I_c = 52.8\ \mu$A, $C = 5.5$ pF, and $A = 10\times 10\ \mu\mathrm{m}^2$. As is clear from the Fig. 1(a), the measurement at low sweep frequency ($\omega = 4.7\times 10^2\ \mathrm{sec}^{-1}$) is well fitted with Kramers' theory.[14,15] In contrast, if the sweep rate is fast ($\omega = 4.7\times 10^5\ \mathrm{sec}^{-1}$) enough with respect to the transition rates between levels [$w_{ji} < (dN/dI)\ (dI/dt)$, where N is the total number of levels inside the well] the escape process is nonstationary, leading to clear evidence of ELQ, as an oscillatory behavior of the escape rate, Γ vs I, shown in Fig. 1(b). Decreasing the temperature, we expect the transition elements w_{ji} to decrease due to a direct temperature effect (the w_{ji} elements are temperature dependent)[9] as well as because the intrinsic dissipation of the junction (related to the presence of thermally activated quasiparticles) depends exponentially on temperature. In the w_{ij} elements are very small the effect of ELQ may be evident also at low sweeping frequency. In fact Eq. (1) predicts a nonmonotonic behavior even in quasistationary conditions (namely, $d\rho_j/dt = 0$ for $j = 0, \ldots, N-2$). To test this idea we extend the measurements to lower temperature in a ^{3}He-^{4}He dilution refrigerator.

The dilution refrigerator is a Leiden Cryogenics refrigerator having a base temperature of approximately 10 mK. The refrigerator is placed inside a system of three concentric μ-metal shields giving an attenuation for dc magnetic fields greater than 60 dB. The silicon chip with the junction is glued on a fiberglass substrate and inserted on a chip holder into the mixing chamber of the ^{3}He-^{4}He refrigerator. This mounting assures that the junction is always in contact with the cold mixture. All of the wires coming from the junction are filtered at the output of the mixing chamber by means of two pole Tchebychev CLCL filters with a cutoff frequency of 5 MHz. The wires to room temperature are two couples of coaxial superconducting cables with a characteristic impedance of 50 Ω. The sweep frequency ω ranges between 3 sec^{-1} up to a maximum of $10^4\ \mathrm{sec}^{-1}$. The voltage across the junction is amplified and sent to a discriminator to provide the trigger of a NBMIO16X National Instruments 16bit ADC converter. Each measurement is repeated for 5000–20 000 times depending on the value of the sweep frequency and the temperature stability required by the measure.

In Fig. 2, we show the low temperature Γ vs I dependence measured on a junction fabricated at IESS-CNR whose relevant junction parameters at $I_c = 110\ \mu$A, $C = 0.8$ pF, and $A = 4\times 4\ \mu\mathrm{m}^2$.[16] The measured subgap resistance in the junction at very low bias is $R_{qp} = 1\ \mathrm{M}\Omega$ for $T < 1$ K. Data shown refer to a sweep frequency $\omega = 1/I_c\ (dI/dt) = 10^3$ sec^{-1} but the oscillations of the escape rate are evident at any frequency. To fit data with theory we have solved the Eq. (1) in the quasistationary limit, following the technique reported in Ref. 9. Since the effective resistance R is the only free fitting parameter, we just plot the theoretical curves for different R, to compare data and theory. The theoretical predictions, assuming very small values of dissipation, present humps due to the presence of ELQ. The agreement between data and theory in terms of level spacing, amplitude of humps and shape of the oscillations is quite convincing. This evidence induces us to conclude that the dissipation in our system is extremely low, in spite of the fact that the relevant effective dissipation is often assumed to be the impedance of the system (junction, bias circuitry, and environment) at the plasma frequency[17] estimated, for our systems, to be a few hundred of Ω or less.[18] However, other similar experiments performed in Josephson junctions also find that the dissipation corresponds to resistance much higher than a few hundred of Ω.[5,10,15,19–21]

In conclusion, we have observed oscillations in the rate of escape from the $V = 0$ state as a function of the bias current, which are interpreted as due to the presence of energy level quantization in the Josephson junction. The data fit well with the theory of Ref. 9, assuming very low dissipation. We stress that such low dissipation, if confirmed, is encouraging with regard to the possible observation of MQC in Josephson systems. Work is in progress to obtain another independent measurement of the effective system dissipation.

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