Electron localization and anisotropic magnetoconductivity in GaAs-AlAs superlattices

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We have performed magnetoresistance measurements in two GaAs-AlAs superlattices at low magnetic fields and various *B*-field orientations. Due to the multilayer structure of the samples, the magnetic correction $\Delta \sigma(B)$ to the conductivity depends on the magnetic field orientation and is largest when *B* is perpendicular to the layers. It decreases slightly in the wide miniband superlattice [quasi-three-dimensional (3D) system] whereas it decreases strongly in the narrow miniband superlattice (quasi-2D system) when the magnetic field is tilted. In fact, these microstructures show $k_F\lambda$ not much larger than unity such that weak localization theories cannot be strictly applied. We show that a self consistent model recently proposed by Bryksin and Kleinert accounts properly for the magnetoconductivity anisotropy. [S0163-1829(99)00723-7]

I. INTRODUCTION

Negative magnetoresistance is observed at low magnetic field (B) in various compounds and is generally interpreted by electron weak localization theories accounting for the dimensionality of the system. We focus here on short period semiconductor superlattices which behave as a structure intermediate between three-dimensional (3D) and 2D systems. Weak localization (WL) describes quantum interference effects in the electronic transport at low temperatures in the high density limit. These constructive interferences arise from an enhanced backscattering and lead to correction to Drude conductivity. Anderson¹ demonstrated that, for randomly distributed scattering centers, the phases of the backscattered electron waves are coherent and interfere constructively. Therefore, the probability that an electron returns to its initial position is enhanced, resulting in localization. Applying a magnetic field causes an additional phase shift proportional to the magnetic flux enclosed by the interfering paths. The phase coherence is broken, leading to a decreased resistance so-called negative magnetoresistance effect (NMR).^{2,3} Many theoretical models have been proposed in the last decade in the frame of the weak localization concept, i.e., in the limit of weak disorder, the formalization depending on the system dimensionality. $4-6$

In this paper, we are concerned by anisotropic systems consisting of semiconducting GaAs-AlAs superlattices. Depending on the periodicity of the GaAs-AlAs sequence, electrons are conducting in the GaAs wells $(e.g., quasi-2D)$ conduction in multiple quantum well structures) or in the whole sample $(e.g., quasi-3D$ conduction in short period superlattices showing wide minibands). The NMR magnitude depends strongly on the magnetic field orientation and is vanishing in pure 2D systems when the magnetic *B* field is parallel to the conducting plane (e.g., Si inversion layers⁵ and GaAs-GaAlAs heterostructures⁶). Thus, almost all authors studying the NMR effect and applying WL theories set the magnetic field perpendicular to the layers to obtain the largest effect. In this study, we performed magnetoresistance measurements at various magnetic field orientations and we apply a theory of the electron localization recently proposed by Bryksin and Kleinert^{\prime} in anisotropic materials near the metal to insulator transition (MIT) .

II. EXPERIMENT

Two microstructures are investigated. The first sample denoted in the following as WSL (wide miniband superlattice) behaves as a quasi-3D anisotropic semiconductor because of its very short period and large bandwidth (see Table I). The microstructure is delta-doped with silicon in the GaAs layers to have a high conductivity at low temperatures since the conductivity is controlled by the so-called *DX* center associated with Si deep donor in GaAs. The second sample denoted as NSL (narrow miniband superlattice) or quasi-2D

TABLE I. m_{\parallel} and $m_{\tilde{z}}$ are the superlattice longitudinal and perpendicular effective masses, respectively. $2w$ denotes the superlattice bandwith and E_F the Fermi energy in respect with the conduction miniband edge. μ_{Hall} is the Hall mobility measured at 4.2 K. τ_{ph} is the phase breaking time derived at 4.2 K from the BK modelization (Refs. 9,10).

sample	m_{\parallel}/m_{Ω}	m_z/m_0	2w (meV)	E_{F} (meV)	$\mu_{\rm Hall}$ $\text{(cm}^2/\text{V s})$	$k_F\lambda_0$	$\tau_{\rm ph}$ (ps)
WSL	0.071	0.10	127	8	875	1.42	1.91
NSL	0.046	0.21	1.66	4.6	529	1.01	0.08

FIG. 1. The magnetoconductivity at 4.2 K in the GaAs-AlAs pseudoalloy (sample WSL) and in an AlGaAs alloy for comparison at two *B*-field orientations (B_{\parallel} and B_{\perp} , respectively). The solid lines correspond to the theory (self-consistent model).

structure has a narrow bandwidth and the Fermi energy E_F $\geq 2w$ at low temperature. It is δ doped with silicon in the AlAs layers and exhibits a low conductivity at low temperatures. However, the conductivity increases strongly after illumination of the sample (persistent photoconductivity effect of the Si - DX deep donor in AlAs).

The energy of the conduction miniband is calculated in the frame of the Kronig-Penney picture. The effective masses $\lceil m_{\parallel} \rceil$ for the effective mass in the (x, y) plane and m_{τ} for the effective mass perpendicular to the layer, respectively] are calculated using the formulas given by Einevoll and Hemmer⁸ for superlattices (see Table I). The temperature dependence of both the conductivity $\sigma(T)$ and the magnetic correction $\Delta \sigma(B,T)$ of these microstructures have been checked experimentally and interpreted elsewhere⁹ in the framework of WL models. When fitting to the experimental data,¹⁰ we have determined the phase coherence time $\tau_{\rm ph}$ vs temperature in both samples.

Figure 1 shows the magnetic correction in the 3D pseudoalloy (sample WSL) and in an AlGaAs alloy for comparison. Measurements are performed for two magnetic field orientations (parallel and perpendicular to the layers respectively). Figure 2 shows the magnetoconductivity in sample NSL at various magnetic field orientations.

III. INTERPRETATION AND DISCUSSION

In the 1980's, Kawabata proposed an expression for the low magnetic field correction to the conductivity based on the Feynman diagrammatic picture. Within the assumptions $\omega_c \tau_e \ll 1$ and $E_F \tau_e \gg \hbar$ (ω_c is the cyclotron frequency, E_F is the Fermi energy and τ_e is the elastic scattering time), he found for a 3D system⁴

$$
\Delta \sigma(B,T) = \alpha \frac{e^2}{2\pi^2 \hbar L_B} F(\delta)
$$
 (1)

FIG. 2. The magnetoconductivity at 4.2 K in the quasi-2D sample NSL at various *B*-field orientations. The solid lines correspond to the theory (self-consistent model).

with $\delta = \hbar/4eD_{3D}B\tau_{ph}$. *L_B* denotes the magnetic length and $F(\delta)$ is the Kawabata function. The prefactor α describes the effective mass anisotropy of the system and D_{3D} denotes the 3D diffusion constant.

Considering the superlattice as a stacked structure of independent 2D layers has been found to be unadequate in treating the magnetoconductivity. Therefore, Szott *et al.* developed in a series of papers 11^{-14} a theory of weak localization in superlattices in the propagative Fermi surface approximation $\left[(w\tau_e)/\hbar > 1 \right]$. The model is based on the anisotropy (D_{\parallel}, D_{z}) of the diffusion constant and they found the D_z coefficient to be strongly dependent on the miniband width (2*w*) of the superlattice. With condition $E_F > 2w$, the magnetic correction in the current perpendicular magnetic *B*-field configuration can be expressed as (1) with the anisotropic factor defined as $\alpha = \alpha_D = (D_{\parallel}/D_{z})^{1/2}$ where D_{\parallel} $= v_{\parallel}^2(E_F) \tau_e/2$ and $D_z = (wa/\hbar)^2 \tau_e/2$ are the in-plane (x, y) and perpendicular diffusion constant, respectively, a denoting the superlattice period. In the case of low electron mobility, the propagative conduction approximation is no longer valid. Therefore, Cassam-Chenai *et al.*¹⁵ reconsidered the WL theory in quasi-2D systems in the case of a diffusive Fermi surface.

In fact, weak disorder implies the condition $k_F \lambda \ge 1$, where λ is the mean free path and k_F is the Fermi wave vector. This condition is not always verified in anisotropic structures having a low electron mobility such as that measured in our samples (see values of the Hall mobility and values of $k_F\lambda$ given in Table I). Thus, the perturbative approach of the electron localization has to be replaced by a self-consistent one. Bryksin and Kleinert⁷ (BK) proposed such a description of the NMR effect in anisotropic systems under the influence of a weak magnetic field with an arbitrary direction θ . Generalizing the diffusion equation for the Cooperon propagator to anisotropic systems in tilted magnetic fields, they obtain a self-consistent equation for the renormalized elastic scattering time:

$$
\frac{\tau_e}{\tau_0} = 1 - \frac{1}{4\pi^2 k_F \lambda} \frac{1}{A} \int_0^\infty \frac{dt}{\cosh(ht)} \exp\left(-\frac{\tilde{s}t}{A}\right) F(t, \theta), \tag{2}
$$

with

$$
F(t,\theta) = \int_{-1}^{+1} du \int_0^{2\pi} dv \left[\frac{1}{2g^{3/2}} \left[\sqrt{\pi} \operatorname{erf}(\sqrt{g}) - 2\sqrt{g} e^{-g} \right] \right]
$$

and

$$
g = \frac{\tanh(ht)}{h} \frac{u^2(\gamma^2 - 1) + 1}{A} + \frac{\gamma^2}{A^3} \left(t - \frac{\tanh(ht)}{h} \right)
$$

$$
\times \left(u \cos(\theta) + \frac{1}{\alpha \gamma} \sin(\theta) \sin(\nu) \sqrt{1 - u^2} \right)^2,
$$

where

$$
\widetilde{s} = \frac{2 \tau_0}{\tau_e \tau_{\text{ph}}}
$$

and

$$
h = \frac{2}{\left(L_B \kappa_{\parallel}\right)^2}
$$

is the dimensionless magnetic field. *A* stands for

$$
\left[\cos^2(\theta) + \frac{1}{\alpha^2} \sin^2(\theta)\right]^{1/2}.
$$

At an arbitrary field orientation, two parameters are necessary to account for the coupling between parallel and perpendicular diffusive transport induced by the magnetic field. These two parameters have to be determined before fitting the experimental results. The first anisotropic parameter is defined as $\alpha = (D_{\parallel}/D_{z})^{1/2}$. The second anisotropic parameter appearing in $F(t, \theta)$ is given by $\gamma = (1/\alpha)(\kappa / \kappa)$ in the particular case of the perpendicular *B*-field configuration (κ _z corresponds to a momentum cutoff parameter of the wave vector $k_z = 1/v_F \tau_e$, so that one recovers the one-parameter scaling theory. Thus the magnetoconductivity can be calculated, the relevant parameter in fitting the magnetic correction $\Delta \sigma(B,T)$ being the phase coherence time $\tau_{\rm ph}$. The model is applicable only in the case of low magnetic fields since the theory assumes scaling limits (sample size *L* $\ll L_B$, *L* being replaced by L_{ph} at temperatures different from $0 K$ as mentioned by Kleinert and Bryksin in their microscopic theory of Anderson localization.¹⁶ Note that the model reduces to the classical 3D WL model when $k_F\lambda \ge 1$ and to the pure-2D model when $\alpha^{-1} \rightarrow 0$. It can be then applied to quasi-3D structures (sample WSL) as well as to quasi-2D structures (sample NSL).

In interpreting the magnetoconductivity in these microstructures, we have previously applied the Kawabata's model and the SLWL model of Szott *et al.* which differ only in the anisotropic factor (α_m, α_D) .¹⁷ A fairly good fit was obtained for the two magnetic field orientations (B_{\parallel}, B_{z}) but the low mobility measured in the samples leads to experimental values of the mean free path no much larger than the Fermi wave length so that the interpretation in terms of WL theories is questionable. As mentioned above, the new selfconsistent approach by BK works with relaxed conditions on

 $k_F\lambda$ in comparison to WL models. Moreover, Bryksin and Kleinert developed their modelization at an arbitrary orientation of the magnetic field such that theoretical predictions can be verified by experiment. When fitting to the data at various temperatures with the different models,^{9,10} we have determinated the phase coherence time $\tau_{ph} \infty T^{-1}$, denoting that the dominant inelastic process is the electron-electron scattering in the dirty limit. Keeping for $\tau_{\rm ph}$ the values found at the helium liquid temperature (i.e., $\tau_{\text{ph}} = 0.08 \text{ ps}$ in sample NSL and τ_{ph} =1.9 ps in sample WSL as determined as from the self-consistent modelization¹⁰), we are able to match to the data at arbitrary magnetic field orientation as shown in Fig. 1 and 2. The BK modelization renormalizes the elastic scattering time τ_e introducing the field orientation θ and two anisotropic coefficients (α and γ , respectively). We take α $= \alpha_m = 1.2$ and $\gamma = 1$, respectively, in sample WSL since the anisotropy of the diffusion constant (D_{\parallel}, D_{z}) reduces to the effective mass anisotropy $(m_{\parallel}, m_{\tau})$ in that quasi-3D superlattice. As shown in Fig. 1, the magnetoconductivity is slightly decreased when the *B* field is rotated. Consequently, the sample WSL behaves as an AlGaAs pseudoalloy. Note that the magnetoconductivity of a AlGaAs alloy was measured for comparison and did not exhibit any anisotropy.

Figure 2 shows the magnetoconductivity in sample NSL at various magnetic field orientations. The solid lines correspond to the BK modelization keeping $\alpha = \alpha_D = 23.3$ and γ $=0.28$, respectively. These values are calculated introducing the anisotropy of the diffusion constant $(D_{\parallel}, D_{\tau})$ which depends on the bandwidth 2*w* of the superlattice $\alpha = \alpha_D$ $= \hbar v_F / w_a$ and $\gamma = \pi w \tau_0 / \hbar$. This sample exhibits a strong quasi-2D character since the magnetoconductivity is vanishing when the magnetic field is rotated as already observed in pure-2D systems.

In conclusion, we have applied a new self-consistent approach of the electron localization in anisotropic systems, i.e., in short period superlattices showing $k_F\lambda$ no much larger than unity. The magnetoconductivity is satisfactorily interpreted both in weakly anisotropic (quasi-3D microstructures) and in anisotropic samples (quasi-2D microstructures). This model works without drastic conditions when the MIT is approached from the metallic side in contrast to classical WL transport theories. When the magnetic field deviates from the parallel or perpendicular orientations, two anisotropic coefficients (α and γ) have to be considered in the modelization. They account for the superlattice microstructure (period, miniband width, and diffusion anisotropy). The anisotropic coefficients being calculated from the sample characteristics, the dependence of the magnetic correction $\Delta \sigma$ with the magnetic field orientation is interpreted taking for the phase coherence time the value previously determined in modeling the experimental data $\Delta \sigma(B,T)$ within the same formalization.

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