Self-consistent approach to Coulombic effects on the quantum magnetotransport in a nondegenerate two-dimensional electron liquid

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To eliminate locally a strong quasiuniform many-electron field and to restore the applicability of the self-consistent approximation, we propose the transcription into frames moving ultrafast along with each orbit center. This allows us to find new formulas for the magnetoconductivity and collision broadening of Landau levels affected by strong Coulomb interaction. As a test for the new theory, in the same experiment, we have measured the magnetoconductivity and cyclotron resonance linewidth of surface electrons on liquid helium. Obtained data are in good agreement with the theoretical concept. [S0163-1829(99)04524-5]

Electrons trapped at the surface of liquid helium[1] form a remarkable two-dimensional (2D) electron liquid with practically unscreened Coulomb interaction V_C, which is at average tens or even a hundred times larger than the mean kinetic energy k_BT. Such extreme coupling conditions with regard to V_C are unique among electron systems studied. In the presence of a strong magnetic field **B** oriented normally ($\hbar\omega_c \gg k_BT$), the system represents a quantum nondegenerate Coulomb liquid that exhibits interesting magnetotransport properties.[2–4] When the effect of strong Coulomb interaction is considered for such a system, it is difficult to avoid naturally arisen questions: (1) To what extent can we rely on the discrete Landau spectrum $\varepsilon_n = \hbar\omega_c(n+1/2)$, if $V_C > \hbar\omega_c$? (2) How do internal forces affect the Landau-level width, or, is it Coulomb broadening or Coulomb narrowing that is caused by mutual interactions?

Historically, the fact that electron-electron interaction broadens the Landau levels and affects the conductivity was noted in Ref. 5. The important insight into the problem of the many-electron magnetotransport was made by Dykman and Khazan[6] (DK). According to them, the many-electron fluctuational field E_f can be considered approximately as a quasiuniform electric field which makes the electron spectrum continuous $\varepsilon_{n,X} = \hbar\omega_c(n+1/2) - eE_fX$ (here X is the coordinate of the center of the cyclotron orbit along the field). This affects the electron scattering rate at vapor atoms (a sort of short-range impurities) or ripplons (capillary wave quanta). In Ref. 6, no collision broadening of Landau levels is introduced, which means that the energy range where an electron can be scattered $eE_f(X'-X) = eE_fq_yl^2 \sim \Delta_f \equiv eE_fl$ should be much larger than the Landau-level broadening $\Gamma_{se} = \hbar\sqrt{(2/\pi)\omega_c\nu_0}$ defined by the self-consistent Born approximation[7] (SCBA) (here $\hbar\mathbf{q}$ is the momentum exchange, $l=\sqrt{\hbar/m\omega_c}$ is the magnetic length and ν_0 is the collision frequency for $B=0$). With the increase of the magnetic field B, the system eventually enters the opposite regime ($\Delta_f < \Gamma_{se}$), since $\Gamma_{se} \propto \sqrt{B}$ and $\Delta_f \propto 1/\sqrt{B}$. Therefore, a more extended theory is necessary to describe electron transport properties in the ultraquantum limit.

In Refs. 3 and 8, a qualitative self-consistent procedure involving Γ_{se}, Δ_f and the Einstein diffusion equation for the conductivity was introduced to describe the transition from the extreme many-electron theory[6] to the extended single-electron SCBA.[2] Two equations proposed in this way[3,8] differ from each other by approximately 30%, which is too much for the quantitative comparison with an experiment. In Refs. 4 and 9, the same ideas were analyzed phenomenologically. The Coulomb correction to the magnetoconductivity was described by means of the replacement $\Gamma_{se} \to \sqrt{\Gamma_{se}^2 + b^2\Delta_f^2}$ ($b \sim 1$) in the electron-density structure factor (DSF) $S(q,0) \propto 1/\Gamma_{se}$ defined for the center-of-mass frame. To reproduce the results of Ref. 6, the numerical parameter b should be fixed to $2^{3/2}/\sqrt{\pi}$. In this treatment the "Coulomb broadening" represents the energy range where an electron can be scattered due to the quasiuniform fluctuational field $eE_fq_yl^2$, which depends on **q** and therefore can be considered only as an effective broadening for a particular frame. It is clear that an uniform electric field cannot really broaden Landau levels, since it can be eliminated by a proper choice of the reference frame $\mathbf{E}_f' = \mathbf{E}_f - 1/c\mathbf{B}\times\mathbf{u}_f = 0$ (in our case, it is the frame that moves along with the orbit center with the velocity $\mathbf{u}_f$).

The intrinsic problem of combining the DK theory with the SCBA is that the notions of the continuous spectrum and the discrete spectrum broadened due to collisions are in contradiction with each other, and to be logically consistent we should keep to only one of them. In this paper, we report a many-electron theory of quantum magnetotransport in a 2D electron liquid that avoids the introduction of the continuous electron spectrum as well as Coulomb broadening and naturally includes the many-electron fluctuational field into the SCBA. The theory proposed is valid for any relation between Γ_{se} and Δ_f, and reproduces the results of the DK theory and SCBA as extreme limiting cases ($\Gamma_{se} \ll \Delta_f$ and $\Gamma_{se} \gg \Delta_f$). In this treatment, the Coulomb reduction of the electron magnetoconductivity comes rather from the sharp frequency dependence of the single-electron DSF $S_{1e}(q,\omega)$ than from the change of the real broadening of the density of states. We show that the narrowing of the Landau-level width produced by the ultrafast drift velocity of the orbit center acts in the opposite way, reducing the many-electron effect in the ultraquantum limit. At weak magnetic fields, the transition to the Drude conductivity behavior occurs without smearing of Landau levels. The experimental data for the magnetoconductivity and the cyclotron resonance (CR) linewidth presented here are in agreement (even quantitative) with the theory.

We confine ourselves to the electron-vapor atom-scattering regime, and assume the fluctuational field acting on an electron when it encounters scatterers to be nearly uniform. Each electron feels its own fluctuational field produced by other electrons $\mathbf{E}_f^{(i)}$, the direction of which is randomly distributed along the isotropic liquid. As for the absolute values E_f, the Gaussian distribution can be used. Then the transport of strongly interacting electrons can be formulated as the transport of independent electrons exposed to the fields $\mathbf{E}_f^{(i)}$. First, we note that the discrete Landau spectrum of a 2D electron in a magnetic field and the conditions of the SCBA can be locally restored by the transcription into a proper frame moving with the drift velocity $u_f^{(i)}=cE_f^{(i)}/B$, where $\mathbf{E}_f^{(i)\prime}=0$. This frame can be considered as a frame of local equilibrium for a particular electron.

In the extended SCBA,[2] the conductivity tensor is found from the quantum-mechanical momentum balance equation, which includes the total kinetic friction acting on electrons by vapor atoms $\mathbf{F}_{fr}$. Since the momentum exchange at a collision $\hbar\mathbf{q}$ and the probability of scattering are independent of the frame considered (we employ the nonrelativistic limit), in our case, $\mathbf{F}_{fr}$ can be found as a sum of average momentum loss of each electron per unit time evaluated in the moving frames where the spectrum of a particular electron is purely discrete. Indeed, the kinetic friction of an electron depends only on the relative velocity of the electron and the scatterers. Then the main difference from the extended SCBA is that the scatterers are moving as a whole with the velocity $-\mathbf{u}_f^{(i)}-\mathbf{u}_{dr}$ with regard to the local frames (here $\mathbf{u}_{dr}$ is the drift velocity of the whole electron liquid in crossed external fields; we assume $u_{dr}\ll u_f$). It should be noted that in moving frames the electron energy is not preserved at a collision with an impurity (vapor atoms), since in these frames an impurity hits the electron with the velocity $-\mathbf{u}_f^{(i)}$. This affects significantly the energy exchange, due to the usual correction $\hbar\mathbf{q}\cdot\mathbf{u}_f\sim\Delta_f$, and reduces the probability of electron scattering, if $\Delta_f\gtrsim\Gamma_{\rm se}$.

In the linear theory, $\mathbf{F}_{fr}$ is proportional to $\mathbf{u}_{dr}$: $\mathbf{F}_{fr}=N_e m\nu(B,\{u_f\})\mathbf{u}_{dr}$. The effective collision frequency ν is affected by $\mathbf{u}_f$ distributed accordingly and can be found in terms of the electron DSF $S(q,\omega)$. Mathematically, the approximation used here means that in the laboratory frame, the single-electron DSF defined by the one-particle density operator $n_{\mathbf{q}}^{(1e)}=\exp(-i\mathbf{q}\cdot\mathbf{r})$ can be written as $S_L(\mathbf{q},\omega)=S_{1e}[q,\omega-\mathbf{q}\cdot(\mathbf{u}_f+\mathbf{u}_{dr})]$, where $S_{1e}(q,\omega)$ is the single-electron DSF in the absence of electric fields. Following the way proposed for the extended SCBA (Ref. 2) and cold nonlinear transport,[9,10] we can find

$$\nu_{me}(B)=\frac{\nu_0\hbar^3}{2m^2k_BT}\sum_{\mathbf{q}} q_x^2\langle S_{1e}(q,-\mathbf{q}\cdot\mathbf{u}_f)\rangle_f, \qquad (1)$$

here $\langle\rangle_f$ means an average over the fluctuational field $\mathbf{E}_f$. As usual, the frequency argument of the DSF of Eq. (1) reflects the energy exchange at a collision. In the elementary equations for the conductivity tensor or in the resistivity equation ($\rho_{xx}=m\nu/ne^2$), the effective collision frequency of Eq. (1) should be employed instead of $\nu_{se}(B)$ of the extended SCBA.[2]

Thus, in the presence of a strong fluctuational field the many-electron DSF can be approximated as $S_{me}(q,\omega)=\langle S_{1e}(q,\omega-\mathbf{q}\cdot\mathbf{u}_f)\rangle_f$. In the ultraquantum limit, employing the Gaussian shape of the Landau levels and averaging over the direction of $\mathbf{u}_f$ gives: $S_{me}(q,0)/S_{1e}(q,0)=\langle(\Gamma_{se}/\Gamma_0)\exp(-\lambda_f^2)I_0(\lambda_f^2)\rangle_f$, where Γ_0 is the real broadening of the ground Landau level affected by u_f, $I_0(z)$ is the Bessel function, and $\lambda_f=\hbar q u_f/\sqrt{2}\Gamma_0$. In the limit $\lambda_f\gg 1$, the new equation for the DSF reproduces the DSF of extreme many-electron theory.[11]

In the general case, $S_{1e}(q,\omega)$ can be represented as a double sum of integrals $\int d\varepsilon f(\varepsilon)\,\mathrm{Im}\,G_n(\varepsilon)\,\mathrm{Im}\,G_{n'}(\varepsilon+\hbar\omega)$ with proper coefficients,[9] where $G_n(\varepsilon)$ is the single-electron Green's function, and $f(\varepsilon)$ is the distribution function. The diagonal terms ($n=n'$), which correspond to electron transitions within Landau levels, decrease with the energy exchange $\mathbf{q}\cdot\mathbf{u}_f$. The off-diagonal terms are very small if the Landau-level broadening $\Gamma_n\ll\hbar\omega_c$ and $\Delta_f\ll\hbar\omega_c$. If Δ_f becomes comparable with the level separation $\hbar\omega_c$, these terms describe the transition to the Drude conductivity behavior due to electron scattering between different Landau levels.

Aiming mostly at describing the quantum-transport regime, we evaluate exactly the term $n=n'=0$. For the off-diagonal terms ($n'>0$) we use the approximation $\Gamma_n\to 0$, valid at $\hbar\omega_c\gg\Gamma_n$. It should be noted that for nondegenerate electrons, both semielliptic and Gaussian shapes of $\mathrm{Im}\,G_0(\varepsilon)$ give numerically very close results if $\Gamma_0\ll k_BT$. Here we employ the Gaussian shape, since it allows us to present most of our results analytically. After averaging over the directions of $\mathbf{u}_f$, the final result can be written in the form

$$\nu_{me}(B)/\nu_{se}(B)=\langle\Gamma_{se}(\Gamma_0^2+\Delta_f^2)/[\Gamma_0^2+2\Delta_f^2]^{3/2}\rangle_f + \langle\Gamma_{se}D_0(x_f)/(\sqrt{2}\Delta_f)\rangle_f. \qquad (2)$$

Here we introduced the following notations:

$$D_0(x)=\frac{1}{\sqrt{\pi}}\sum_{n=1}^{\infty}\frac{1}{n!}e^{-x\cdot n^2}\int_0^\infty(t+xn^2)^{n+1}e^{-t}\frac{dt}{\sqrt{t}},$$

$x_f=\frac{1}{2}(\hbar\omega_c/\Delta_f)^2$. $D_0(x)$ can be analytically approximated as $D_0(x)\simeq e^{-x}(x^2+x-0.05+3.5/x)$.

The first term of Eq. (2) describes the reduction of the electron magnetoconductivity or resistivity due to the many-electron effect. The second term restores the Drude conductivity behavior at weak-magnetic fields. It is important to note that the strong many-electron reduction of ν_{me} and the transition to the result of the DK theory exist even for the unchanged Landau-level broadening $\Gamma_0=\Gamma_{se}$. The proper

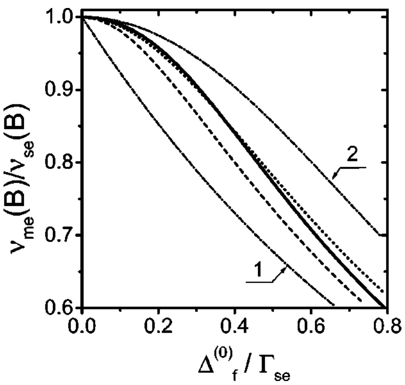

FIG. 1. The normalized effective collision frequency vs $\Delta_f^{(0)}/\Gamma_{se}$: the many-electron theory employing Γ_0 defined by Eq. (4) (solid) and Γ_0 fixed to Γ_{se} (dash); previously used approximations of Ref. 3 (curve 1) and Ref. 8 (curve 2); the phenomenological treatment of Ref. 9 with b fixed as described in the text (dot).

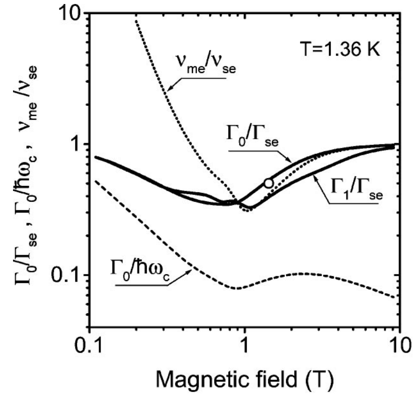

FIG. 2. The magnetic-field dependence of normalized Landau-level broadening (solid and dash) and effective collision frequency (dot) for $n=10^8$ cm^{-2} and $T=1.36$ K. A circle is the CR linewidth w datum normalized to $\sqrt{2}\Gamma_{se}$.

graph is shown in Fig. 1 as a dash curve. We use the Gaussian distribution of E_f with $\sqrt{\langle E_f^2\rangle}=E_f^{(0)} \equiv 0.84\sqrt{4\pi k_B T_e n_s^{3/2}}$,[8] and $\Delta_f^{(0)}=eE_f^{(0)}l$; here n_s is the electron density.

The self-consistent equation for the broadening of the ground Landau level $\Gamma_0=-2\,\text{Im}\,\Sigma_0(\varepsilon_0)$ [here $\Sigma_n(\varepsilon)$ is the electron self-energy] affected by the ultrafast drift velocity of scatterers $-\mathbf{u}_f$ can be written as[9]

$$\Gamma_0^2/\Gamma_{se}^2=\sum_{n=0}^{\infty}\frac{1}{n!}\int_0^{2\pi}\frac{d\varphi}{2\pi}\int_0^{\infty}dx x^n e^{-x} \times \text{Im}\, G_n[\varepsilon_0-\sqrt{2x}\Delta_f\cos(\varphi)]/\text{Im}\, G_0(\varepsilon_0). \quad (3)$$

Contrary to the conventional SCBA, here the argument of the electron Green's function is affected by the Doppler shift correction.

The equation for $G_0(\varepsilon)$ is actually an integral equation and the shape of $\text{Im}\, G_0(\varepsilon)$ differs from the semielliptic function of the SCBA. Since the strict solution of the self-consistent equation is a very difficult problem and the correction to ν_{me} due to the dependence $\Gamma_0(u_f)$ is relatively small compared with the effect described above, we employ the Gaussian shape for solving Eq. (3). We assume also that the mixing of Landau levels becomes important ($x_f \lesssim 1$) at $\Delta_f \gg \Gamma_{se}$, since $\Gamma_{se} \ll \hbar\omega_c$. Then Eq. (3) yields

$$\Gamma_0^2=\sqrt{[1+C_0(x_f)]^2\cdot\Gamma_{se}^4+4\Delta_f^4}-2\Delta_f^2, \quad (4)$$

where

$$C_0(x)=\frac{1}{\sqrt{\pi}}\sum_{n=1}^{\infty}\frac{1}{n!}e^{-x\cdot n^2}\int_0^{\infty}(t+xn^2)^n e^{-t}\frac{dt}{\sqrt{t}}$$

represents the effect of mixing of Landau levels caused by the energy exchange $\hbar\mathbf{q}\cdot\mathbf{u}_f$ and can be analytically approximated as $C_0(x)\simeq e^{-x}(x-0.6+3/\sqrt{\pi x})$.

Typical magnetic-field dependences of Γ_0 normalized to both Γ_{se} (solid) and $\hbar\omega_c$ (dash) are shown in Fig. 2. With the decrease of B, the Coulomb effect first reduces the Landau-level broadening due to $\Delta_f \gtrsim \Gamma_{se}$; then, at $x_f \lesssim 1$, the effect of mixing of Landau levels starts to increase the collision broadening. In the high-field region, the Coulomb effect makes the level broadening strongly dependent on the level number n, while in the weak-field limit this dependence is practically vanishing. According to the dashed curve, the Landau levels are well defined ($\Gamma_0/\hbar\omega_c \ll 1$) at least down to $B\sim 0.2$ T, and the strong increase of the collision frequency (dotted curve), restoring the Drude conductivity behavior, occurs without smearing of Landau levels.

The inclusion of Eq. (4) into Eq. (2) reduces partly the many-electron effect on $\nu_{me}(B)/\nu_{se}(B)$ as is shown in Fig. 1 (solid curve). This curve become very close to the dependence $\Gamma_{se}/\sqrt{\Gamma_{se}^2+b^2\Delta_f^2}$ (dot curve) proposed in Ref. 9 phenomenologically, if the numerical parameter b is fixed to $2^{3/2}/\sqrt{\pi}$. At the same time, the solid curve lies between the curves 1 and 2 representing the equations proposed in Refs. 3 and 8. The difference between the solid and both of these curves ranges within 15%.

We measured σ_{xx} employing the usual Corbino technique described elsewhere (Ref. 8). The conductivity data are obtained by means of the conventional transmission line model. To conduct CR measurements in the same experiment, the Corbino electrodes were placed inside a cavity in the form of an upright cylinder resonating in the TE_{011} mode at 40 GHz. The details of the CR experiment and the complete CR data are presented in a separate publication.[12] The experimental data for the magnetoconductivity are shown in Fig. 3 together with different theoretical models. The agreement between the data and the theory (solid curve) is quite convinc-

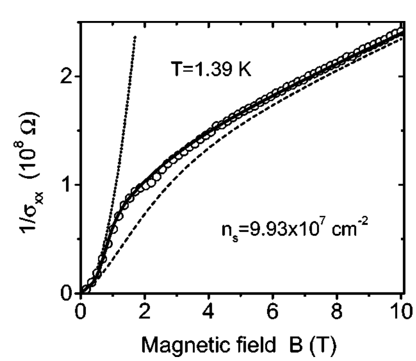

FIG. 3. $1/\sigma_{xx}$ vs B: data (circles), the new many-electron theory (solid), extended SCBA (dash), and Drude equation (dotted).

ing. The CR condition $B=1.4$ T corresponds to the quantum regime. At such conditions, according to Ref. 13, the CR linewidth is determined by the Landau-level width itself: the Gaussian width parameter $w=\sqrt{\Gamma_0^2+\Gamma_1^2}$. In the wide density range $10^7\ \mathrm{cm}^{-2}<n_s<2\times10^8\ \mathrm{cm}^{-2}$, our CR linewidth data reported in Ref. 12 behave in accordance with Γ_0 and Γ_1 determined by the theory. As an example, the linewidth datum (normalized as $w/\sqrt{2}\Gamma_{se}$) that corresponds to the condition of Fig. 2 is shown there as a circle point. As it should be, the latter is placed in between the solid curves for Γ_0/Γ_{se} and Γ_1/Γ_{se}.

Concluding, we feel that our paper gives answers to the basic questions mentioned in the introduction. As far as the internal forces can be considered as quasiuniform ones, the discrete Landau spectrum is well defined in the frames moving along with the electron orbit center, in spite of the strong interaction. Moreover, the ultrafast drift velocity of the electron orbit center caused by the Coulomb interaction produces a *narrowing* of the Landau-level width. In this regime, according to the theory, the main Coulomb reduction of σ_{xx} is not due to any change of the Landau-level broadening but due to the conflict of the energy exchange at a collision $\hbar\mathbf{q}\cdot\mathbf{u}_f$ (with regard to the moving frames) and the level broadening (a sort of inelastic effect). We performed conductivity measurements in which the Landau-level broadening was controlled separately by a CR experiment. Obtained σ_{xx} and CR linewidth data are in good agreement with the new theory, if $\Delta_f^{(0)}\ll k_BT$, which is just the criterion[6] for the internal forces to be uniform. Beyond this criterion the CR linewidth increases with electron density faster than according to the effect of mixing of Landau levels described above, which might be attributed to a nonuniform internal field correction.

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[1] *Two-Dimensional Electron Systems on Helium and Other Substrates*, edited by E. Andrei (Kluwer, New York, 1997).
[2] P.J.M. Peters, P. Scheuzger, M.J. Lea, Yu.P. Monarkha, P.K.H. Sommerfeld, and R.W. van der Heijden, Phys. Rev. B **50**, 11 570 (1994).
[3] M.J. Lea, P. Fozooni, P.J. Richardson, and A. Blackburn, Phys. Rev. Lett. **73**, 1142 (1994).
[4] Yu.P. Monarkha, S. Ito, K. Shirahama, and K. Kono, Phys. Rev. Lett. **78**, 2445 (1997).
[5] V.I. Ryzhii, Fiz. Tekh. Poluprovodn. **3**, 1704 (1969) [Sov. Phys. Semicond. **3**, 1432 (1969)].
[6] M.I. Dykman and L.S. Khazan, Zh. Éksp. Teor. Fiz. **77**, 1488 (1979) [Sov. Phys. JETP **50**, 747 (1979)].
[7] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **36**, 959 (1974).
[8] M.J. Lea, P. Fozooni, A. Kristensen, P.J. Richardson, K. Djerfi, M.I. Dykman, C. Fang-Yen, and A. Blackburn, Phys. Rev. B **55**, 16 280 (1997).
[9] Yu.P. Monarkha, K. Shirahama, K. Kono, and F.M. Peeters, Phys. Rev. B **58**, 3762 (1998).
[10] Yu.P. Monarkha and F.M. Peeters, Europhys. Lett. **34**, 611 (1996).
[11] M.I. Dykman, M.J. Lea, P. Fozooni, and J. Frost, Phys. Rev. Lett. **70**, 3975 (1993).
[12] E. Teske, Yu.P. Monarkha, M. Seck, and P. Wyder, Phys. Rev. Lett. **82**, 2772 (1999).
[13] T. Ando, J. Phys. Soc. Jpn. **38**, 989 (1975).