Self-organization of vortices in type-II superconductors during magnetic relaxation

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(Received 9 November 1998)

We revise applicability of the theory of self-organized criticality (SOC) to the process of magnetic relaxation in type-II superconductors. The driving parameter of self-organization of vortices is the energy barrier for flux creep and not the current density. The power spectrum of the magnetic noise due to vortex avalanches is calculated and is predicted to vary with time during relaxation. [S0163-1829(99)07221-5]

I. INTRODUCTION

The magnetic response of hard type-II superconductors, in particular magnetic flux creep, is an issue of contemporary research (see for review Refs. 1–3). In the 1960's a very useful model of the critical state was developed to describe the magnetic behavior of type-II superconductors.[4–6,8] One of the distinguishing features of this behavior, observed experimentally, is that the density of flux lines varies across the whole sample. This model of the critical state remains in use, even though a significant progress has been made in understanding the particular mechanisms of a magnetization and creep in type-II superconductors.[1–3,6] It has also been noted that the magnetic flux distribution in type-II superconductors is, in many aspects, similar to a sandpile formed when, for example, sand is poured onto a stage.[6,8,7] When a steady state is reached the slope of such a pile is analogous to the critical current density j_c of a superconductor. The study of the dynamics (i.e., sand avalanches) of such strongly correlated many-particle systems has led to the development of a new concept called self-organized criticality (SOC), proposed originally by Bak and co-workers.[9] Tang first analyzed the direct application of SOC to type-II superconductors.[7] Later numerous studies significantly elaborated on this topic.[10–14]

In practice, especially in high-T_c superconductors, persistent current density j in the experiment is much lower than the critical current density j_c due to ''giant'' flux creep.[3] The classical concept of SOC is strictly applied to the vicinity of the critical state $j=j_c$, and it describes the system dynamics *towards* the critical state. Nevertheless, it is tempting to analyze the magnetic flux creep in type-II superconductors when the system moves *away* from the critical state. The dynamics of the avalanches, triggered by thermal activation, can be described by the modified theory of self-organized criticality.[12–14] However, it was found that modifications of the relaxation law due to vortex avalanches are minor and can hardly be reliably distinguished in the analysis of experimental data. Furthermore, flux creep *universality* has been analytically demonstrated in the elegant paper by Vinokur *et al.*[10] Universality of the spatial distribution of the electric field during flux creep has also been found by Gurevich and Brandt.[15] The direct application of SOC to the problem of magnetic flux creep thus meets a number of serious general difficulties. It is clear that critical scaling (power laws for vortex-avalanche lifetimes and size distributions) observed in the vicinity of the critical state must change during later stages of relaxation due to a time-dependent (or current-dependent) balance of the Lorentz and pinning forces.

In this paper we propose a physical picture of self-organization in a vortex matter during magnetic flux creep in type-II superconductors. In this approach the driving parameter is the energy barrier for magnetic flux creep rather than the current density. We show that notwithstanding its minor influence on the relaxation rate, self-organized behavior may be observed by measuring magnetic noise during flux creep.

II. BARRIER FOR MAGNETIC FLUX CREEP AS THE DRIVING PARAMETER OF SELF-ORGANIZATION

We consider a long superconducting slab infinite in the y and z directions and having width $2d$ in the x direction. The magnetic field is directed along the z axis. In this geometry, the flux distribution is one-dimensional, i.e., $\mathbf{B}(\mathbf{r},t)=[0,0,B(x,t)]$. As a mathematical tool for our analysis we use a well known differential equation for flux creep:[1,2,5]

$$\frac{\partial B}{\partial t}=-\frac{\partial}{\partial x}\{Bv_0\exp[-U(B,T,j)/T]\}. \quad (1)$$

Here B is the magnetic induction, $v=v_0\exp[-U(B,T,j)/T]$ is the mean velocity of vortices in the x direction and $U(B,T,j)$ is the effective barrier for flux creep. Note that we adopt units with $k_B=1$, thus energy is measured in K. Since in our geometry $4\pi M=\int_V(B-H)dV$ we get for the mean volume magnetization $m=M/V$ from Eq. (1)

$$\frac{\partial m}{\partial t}=-A\exp[-U(H,T,j)/T], \quad (2)$$

where $A\equiv Hv_0/4\pi d$.

It is important to emphasize that we do not modify the pre-exponent factor Bv_0 of Eq. (1) or A of Eq. (2), as suggested by previous works on SOC (see, e.g., Ref. 7). Such modifications result only in logarithmic corrections to the effective activation energy, and may be omitted in a flux creep regime.[12–14] Instead, we concentrate on the details of the spatial behavior of flux creep barrier $U(x)$, as analyzed

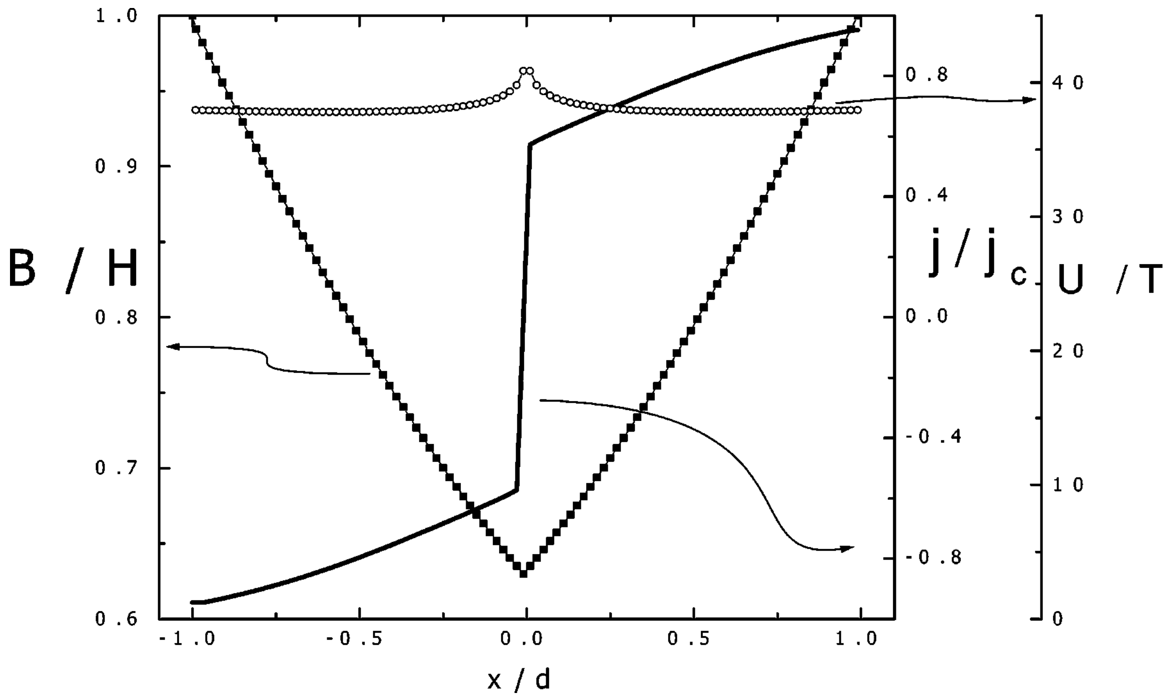

FIG. 1. Results of numerical solution of Eq. (1) for $U(j)=U_0(B/B_0)^5(j_c/j-1)$ at $j<j_c$. Spatial distribution of magnetic induction $B(x)/H$ (filled squares), corresponding profile of the normalized current density (solid line), and the corresponding profile of the effective barrier for flux creep $U(x)/T$ (open circles).

in detail in our previous work.[16] In that work Eq. (1) was solved numerically and semianalytically for different situations. We emphasize that, in general, the barrier for flux creep depends on magnetic field B, and persistent current density $j(x)$ is not uniform across the sample (see Fig. 1). Thus, j cannot be used as a driving parameter for a SOC model. Instead the relevant parameter is U, which stays constant across the sample. Also, since experiments on magnetic relaxation are usually carried out at constant temperature and at high magnetic field, we can assume $U(B,T,j)=U(j)$. The central results of Ref. 16 are shown in Fig. 1 using a ''collective creep-type'' dependence $U(j)=U_0(B/B_0)^5(j_c/j-1)$ as an example , see also Eq. (5) below (other models are analyzed in Ref. 16 as well and produce essentially similar results). Filled squares in Fig. 1 represent the distribution of the magnetic induction $B(x)/H$ at some late stage of relaxation (so that $j<j_c$), the solid line represents the normalized current density profile (note that j_c is constant across the sample), and open circles show the profile of the effective barrier for flux creep $U(x)/T$. All quantities are calculated numerically from Eq. (1). The important thing to note is that the energy barrier $U(x)$ is nearly independent of x, so that its maximum variation δU is of order of T. As also shown from general arguments,[16] such behavior means that the fluxon system organizes itself to maintain a uniform distribution of the barrier U across the sample.

The vortex avalanches are introduced in an integral way. An avalanche of size s causes a change in the total magnetic moment $\delta M \equiv s$. This change is equivalent to a change of the average current density $\delta j=\delta M \gamma=\gamma s$, where $\gamma=2c/dV$. If the barrier for flux creep is $U(j)$, then the variation of current δj leads to a variation of the energy barrier

$$\delta U=\left|\frac{\partial U}{\partial j}\right|\delta j=\gamma\left|\frac{\partial U}{\partial j}\right|s. \tag{3}$$

As mentioned above, maximum fluctuation in the energy barrier $|\delta U|_{\max}$ is of order of T in the creep regime ($\delta U \ll U$). Any fluctuation δU larger than T is suppressed before it arrives to the sample edge due to exponential feedback of the local relaxation rate, which is proportional to $\exp(-U/T)$ [Eq. (1)]. This means that only fluctuations $\delta U \leqslant T$ can be observed in global measurements of the sample magnetic moment. Thus,

$$s_m=\frac{T}{\gamma|\partial U/\partial j|}\propto VT, \tag{4}$$

where we denote the maximum possible avalanche as s_m, which depends on time via $\partial U/\partial j$. It is worth noting that Eq. (4) gives the correct dependence of s_m on the system size and on temperature. It is clear that in a finite system the largest possible avalanche must be proportional to the system volume. Since it is thermally activated, it is proportional to temperature T, consistent with our derivation. The characteristic time-dependent upper cutoff of the avalanche size was experimentally observed by Field *et al.*[12] who studied magnetic noise spectra at different magnetic field sweep rates, i.e., at different time windows of the experiment.

Our central idea is that in the vicinity of j_c the system of fluxons, indeed, exhibits self-organized *critical* behavior, as initially proposed by Tang.[7] During flux creep, it maintains itself in a self-organized, however, *not critical* state in the sense that it cannot be described by the critical scaling. The self-organization manifests itself by maintaining almost spatially constant energy barrier U. Avalanches do not vanish, but there is a constraint on the largest possible avalanche, see Eq. (4). Importantly, s_m depends upon current density and, as we show below, decreases with decrease of current (or with increase of time), so their relative importance vanishes.

In order to calculate physically measured quantities let us derive the time dependence of s_m assuming a very useful generic form of the barrier for flux creep, introduced by Griessen[19]

$$U(j)=\frac{U_0}{\alpha}\left[\left(\frac{j_c}{j}\right)^{\alpha}-1\right]. \tag{5}$$

This formula describes all widely known functional forms of $U(j)$ if the exponent α attains both negative and positive values. For $\alpha=-1$ Eq. (5) describes the Anderson-Kim barrier;[5] for $\alpha=-1/2$ the barrier for plastic creep[20] is obtained. Positive α describes collective creep barriers.[1] In the limit $\alpha\rightarrow 0$ this formula reproduces exactly logarithmic barrier.[18] An activation energy written in the form of Eq. (5) results in an ''interpolation formula'' for flux creep[1] if the logarithmic solution of the creep equation $U(j)=T\ln(t/t_0)$ is applied[17] (for $\alpha\neq 0$):

$$j(t)=j_c\left[1+\frac{\alpha T}{U_0}\ln\left(\frac{t}{t_0}\right)\right]^{-1/\alpha}. \tag{6}$$

For $\alpha=0$, a power-law decay is obtained $j(t)=j_c(t_0/t)^n$, where $n=T/U_0$.

Using this general form of the current dependence of the activation energy barrier, we obtain from Eq. (4)

$$s_m(j)=\frac{Tj}{\gamma U_0}\left(\frac{j}{j_c}\right)^{\alpha} \tag{7}$$

and

$$s_m(t)=\frac{Tj_c}{\gamma U_0}\left[1+\frac{\alpha T}{U_0}\ln\left(\frac{t}{t_0}\right)\right]^{-(1+1/\alpha)}. \tag{8}$$

As we see, the upper limit for the avalanche size decreases with the decrease of current density or with the increase of time for all $\alpha>-1$. For $\alpha<-1$ the curvature

$$\frac{\partial^2 U}{\partial j^2}=\frac{(\alpha+1)}{j^2}U_0\left(\frac{j_c}{j}\right)^{\alpha} \tag{9}$$

is negative and largest avalanche does not change with current, but is limited by its value at criticality. In this case, self-organized *criticality* describes the system dynamics down to very low currents. On the other hand the Kim-Anderson barrier must be always relevant when $j\rightarrow j_c$,[1] thus our model produces a correct transition to a self-organized critical state at $j=j_c$. In practice, most of the observed cases obey $\alpha\geqslant -1$ and s_m decreases with decrease of current density (due to flux creep).

III. AVALANCHE DISTRIBUTIONS AND THE POWER SPECTRUM

Before starting with calculation of the power spectrum of the magnetic flux noise due to flux avalanches, let us stress that the time dependence of s_m is very weak [logarithmic, see Eq. (8)]. This allows us to treat the process of the flux creep as quasistationary, which means that during a short time, as required for the sampling of the power spectrum, current density is assumed to be constant. In more sophisticated experiments[12] the external field can be swept with the constant rate, which insures that the current density does not change, although $j<j_c$. Actually, constant sweep rate fixes a certain time window of the experiment $t/t_0\propto 1/(\partial H/\partial t)$. Thus, decreasing the sweep rate allows the noise spectra to be studied at effectively later stages of the relaxation.

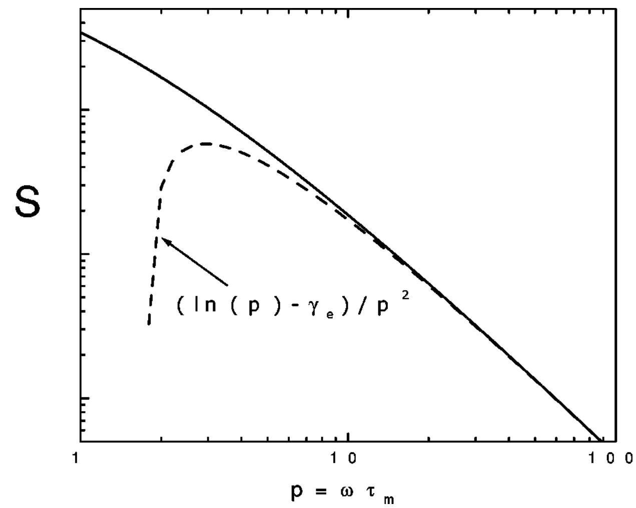

FIG. 2. The power spectrum $S(\omega,t)$ described by Eq. (13) (solid line) and the approximate asymptotic solution (dashed line), Eq. (14).

Once an avalanche is triggered by a thermal fluctuation, its subsequent dynamics is governed only by interactions between vortices for which motion is *not* due to thermal fluctuations. Thus, we expect same relationship between the avalanche lifetime τ and its size s as in the case of a sandpile, $\tau(t)\propto s^{\sigma}(t)$ and $\tau_m(t)\propto s_m^{\sigma}(t)$, respectively. Using the simplified version of the distribution of lifetimes estimated for a superconductor in a creep regime from computer simulations by Pan and Doniach[13]

$$\rho(\tau)\propto\exp(-\tau/\tau_m), \tag{10}$$

and assuming that avalanches of size s and lifetime τ contribute the Lorentzian spectrum

$$L(\omega,\tau)\propto\frac{\tau}{1+(\omega\tau)^2}, \tag{11}$$

the total power spectrum of magnetic noise during flux creep is

$$S(\omega)=\int_0^{\infty}\rho(\tau)L(\omega,\tau)d\tau. \tag{12}$$

Using Eq. (10) we find

$$S(p)\propto\frac{1}{2p^2}\left\{\cos\left(\frac{1}{p}\right)\mathrm{Re}\left[\mathrm{Ei}\left(\frac{i}{p}\right)\right]-\sin\left(\frac{1}{p}\right)\mathrm{Im}\left[\mathrm{Ei}\left(\frac{i}{p}\right)\right]\right\}. \tag{13}$$

Here $p\equiv\omega\tau_m(t)$ and $\mathrm{Ei}(x)=\int_0^{\infty}e^{-x\eta}/\eta d\eta$ is the exponential integral. The power spectrum $S(\omega,t)$ described by Eq. (13) is plotted in Fig. 2 using a solid line. Since there an upper cutoff for the avalanche lifetime at τ_m, the lowest frequency which makes sense is $2\pi/\tau_m$. Thus, only frequency domain

$2\pi/\tau_m<\omega$ $(p>1)$ is important. In the limit of large p, the spectral density of Eq. (13) has a simple asymptote:

$$S(\omega)\propto\frac{\ln(p)-\gamma_e}{p^2}, \tag{14}$$

where $\gamma_e\approx0.577\ldots$ is Euler's constant. This simplified power spectrum is shown in Fig. 2 by a dashed line. For $p>10$ this approximation is quite reasonable. The usual way to analyze the power spectrum is to present it in a form $S(\omega)\propto 1/\omega^\nu$ and extract the exponent ν simply as $\nu=-\partial\ln(S)/\partial\ln(\omega)$. In our case the parameter $p=\omega\tau_m$ is a reduced frequency, so the exponent ν can be estimated as

$$\nu=-\frac{\partial\ln(S)}{\partial\ln(p)}=2-\frac{1}{\ln(p)-\gamma_e}. \tag{15}$$

This result is very important, since it fits quite well the experimentally observed values of ν which were found to vary between 1 and 2.[12,21] As seen from Fig. 2, it is impossible to distinguish between real $1/\omega^\nu$ dependence and that predicted by Eq. (13) at large enough frequencies. Remarkably, in many experiments the power spectrum was found to deviate significantly from the $1/\omega^\nu$ behavior at lower frequencies, which fits, however, Eq. (13).

Using Eqs. (13) or (14) one can find the temperature, magnetic field and time dependence of the power spectrum substituting $p=\omega\tau_m=\omega s_m^\sigma$ and using values of $s_m(H,T,t)$ derived in the previous section. Specifically, from Eq. (8) we obtain that at any given frequency, the amplitude of a power spectrum increases with time in the collective creep regime, but saturates in the case of the logarithmic barrier and remains constant in the case of the Kim-Anderson barrier.

In general, we emphasize that the power spectrum of the magnetic noise during flux creep depends on time. Since parameter p decreases with the increase of time, the exponent ν becomes closer to 1 during flux creep. At these later stages of relaxation the effect of the avalanches is negligible and magnetic noise is mostly determined by thermally activated jumps of vortices with the usual (noncorrelated) $1/\omega$ power spectrum. Thus, the manifestation of the avalanche-driven dynamics during flux creep is noise spectra with $1/\omega^\nu$ and decreasing $\nu(t)$ when sampled at different times during relaxation. This explains the experimental results obtained by Field *et al.*,[12] who measured directly vortex avalanches at different sweep rates. Those found that the exponent ν decreased from a relatively large value of 2 at a large sweep rate of $20\,G/\mathrm{sec}$ to a smaller value of 1.5 for a sweep rate of $1\,G/\mathrm{sec}$. This is in good agreement with our model.

IV. CONCLUSIONS

In conclusion, self-organization of vortices in hard type-II superconductors during magnetic flux creep was analyzed. Using results of a numerical solution[16] of the differential equation for flux creep, it was argued that the self-organized *criticality* describes the system dynamics at $j=j_c$. During flux creep, the vortex system remains *self-organized*, but there is *no criticality* in the sense that there are no simple power laws for distributions of the avalanche size, lifetime, and for the power spectrum. The driving parameter of the self-organized dynamics is the energy barrier $U(B,j)$ and not the current density j, as proposed by previous work. Using a simple model the power spectrum $S(\omega)$ of the magnetic noise is predicted to depend on time. Namely, fitting $S(\omega)$ to a $1/\omega^\nu$ behavior will result in a time-dependent exponent $\nu(t)$ decreasing in the interval between 2 and 1.

ACKNOWLEDGMENTS

We acknowledge fruitful discussions with L. Burlachkov and B. Shapiro. We thank F. Nori for critical remarks, D.G. acknowledges support from the Clore Foundations. This work was partially supported by the National Science Foundation (DMR 91-20000) through the Science and Technology Center for Superconductivity and by DOE Grant No. DEFG 02-91-ER45439.

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