Influence of the Hall force on the vortex dynamics in type-II superconductors

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The effect of the Hall force on the pinning of vortices in type-II superconductors is considered. A field theoretic formulation of the pinning problem allows a nonperturbative treatment of the influence of quenched disorder. A self-consistent theory is constructed using the diagrammatic functional method for the effective action, and an expression for the pinning force for independent vortices, as well as vortex lattices, is obtained. We find that the pinning force for a single vortex is suppressed by the Hall force at low temperatures while it is increased at high temperatures. The effect of the Hall force is more pronounced on a single vortex than on a vortex lattice. The results of the self-consistent theory are shown to be in good agreement with numerical simulations. [S0163-1829(99)02621-1]

The advent of high-temperature superconductors has led to a renewed interest in vortex dynamics. We shall consider the influence of quenched disorder on the vortex dynamics in type-II superconductors in the presence of a Hall force. The description of the vortex dynamics will be based on the phenomenological Langevin equation[1,2]

$$m\ddot{\mathbf{u}}_{\mathbf{R}t}+\eta\dot{\mathbf{u}}_{\mathbf{R}t}+\sum_{\mathbf{R}'}\Phi_{\mathbf{R}\mathbf{R}'}\mathbf{u}_{\mathbf{R}'t}=\alpha\dot{\mathbf{u}}_{\mathbf{R}t}\times\hat{\mathbf{n}}-\nabla V(\mathbf{R}+\mathbf{u}_{\mathbf{R}t})+\mathbf{F}_{\mathbf{R}t}+\boldsymbol{\xi}_{\mathbf{R}t}, \tag{1}$$

where $\mathbf{u}_{\mathbf{R}t}$ is the displacement at time t of the vortex that initially has equilibrium position $\mathbf{R}$, η is the friction coefficient, and m is a possible mass (per unit length) of the vortex. The dynamic matrix $\Phi_{\mathbf{R}\mathbf{R}'}$ of the hexagonal Abrikosov vortex lattice describes the interaction between the vortices in the harmonic approximation. Having a thin superconducting film in mind, the system is two dimensional (normal to $\hat{\mathbf{n}}$) and the dynamic matrix is specified within the continuum theory of elastic media[3] by the compression modulus c_{11} and the shear modulus c_{66},

$$\Phi_{\mathbf{q}}=\frac{\phi_0}{B}\begin{pmatrix} c_{11}q_x^2+c_{66}q_y^2 & (c_{11}-c_{66})q_xq_y \\ (c_{11}-c_{66})q_xq_y & c_{66}q_x^2+c_{11}q_y^2\end{pmatrix}, \tag{2}$$

where ϕ_0/B is equal to the area of the unit cell of the vortex lattice, and $\phi_0=h/2e$ is the flux quantum. The force (per unit length) on the right-hand side of Eq. (1) consists of the Hall force characterized by the parameter α, and $\mathbf{F}_{\mathbf{R}t}=\phi_0\,\mathbf{j}(\mathbf{R},t)\times\hat{\mathbf{n}}$ is the Lorentz force due to the transport current density $\mathbf{j}$, and the thermal white-noise stochastic force $\boldsymbol{\xi}_{\mathbf{R}t}$ is specified according to the fluctuation-dissipation theorem $\langle\xi^{\alpha}_{\mathbf{R}t}\xi^{\beta}_{\mathbf{R}'t'}\rangle=2\eta k_BT\delta(t-t')\delta_{\alpha\beta}\delta_{\mathbf{R}\mathbf{R}'}$, and V is the pinning potential due to quenched disorder. The pinning is described by a Gaussian distributed stochastic potential with zero mean, and thus characterized by its correlation function (where now the brackets denote averaging with respect to the quenched disorder) $\langle V(\mathbf{x})V(\mathbf{x}')\rangle=\nu(\mathbf{x}-\mathbf{x}')=\nu_0/(4\pi a^2)\exp[-|\mathbf{x}-\mathbf{x}'|^2/(4a^2)]$, taken to be a Gaussian function with range a and strength ν_0.

Upon averaging with respect to the quenched disorder, the average restoring force $\mathbf{F}_R=-\Sigma_{\mathbf{R}'}\Phi_{\mathbf{R}\mathbf{R}'}\langle\langle\mathbf{u}_{\mathbf{R}'t}\rangle\rangle$ of the lattice vanishes. On the average, corresponding to the lattice reaching a steady-state velocity $\mathbf{v}=\langle\langle\dot{\mathbf{u}}\rangle\rangle$, there will be a balance, $\mathbf{F}+\mathbf{F}_f+\mathbf{F}_H+\mathbf{F}_p=\mathbf{0}$, between the Lorentz force $\mathbf{F}$, the friction force $\mathbf{F}_f=-\eta\mathbf{v}$, the Hall force $\mathbf{F}_H=\alpha\mathbf{v}\times\hat{\mathbf{n}}$, and the pinning force $\mathbf{F}_p=-\langle\langle\nabla V\rangle\rangle$. The pinning force is due to time-reversal symmetry invariant under reversal of the direction of the magnetic field, and is therefore antiparallel to the velocity.[1] Thus, the pinning yields a renormalization of the friction coefficient in terms of a velocity-dependent effective friction coefficient, $\mathbf{F}_f+\mathbf{F}_p\equiv-\eta_{\mathrm{eff}}(v)\mathbf{v}$, which reduces in the absence of disorder to the bare friction coefficient η, and has previously only been determined to lowest order in the disorder.[4] The relationship between the average vortex velocity and the induced electric field, $\mathbf{E}=\mathbf{v}\times\mathbf{B}$, leads to the expressions for the resistivity tensor and Hall angle,

$$\rho=\frac{\phi_0B}{\eta_{\mathrm{eff}}^2+\alpha^2}\begin{pmatrix}\eta_{\mathrm{eff}} & \alpha \\ -\alpha & \eta_{\mathrm{eff}}\end{pmatrix},\qquad \theta=\arctan\frac{\alpha}{\eta_{\mathrm{eff}}}. \tag{3}$$

The average vortex motion is conveniently described by reformulating the stochastic Langevin problem in terms of a path integral. The probability functional for a realization $\{\mathbf{u}_{\mathbf{R}t}\}_{\mathbf{R}}$ of the motion of the vortex lattice may be expressed, using the equation of motion, through a functional integral over a set of auxiliary variables $\{\tilde{\mathbf{u}}_{\mathbf{R}t}\}_{\mathbf{R}}$, and we are led to consider the generating functional[5,6]

$$\mathcal{Z}[\mathbf{F},\mathbf{J}]=\int\prod_{\mathbf{R}}\mathcal{D}\mathbf{u}_{\mathbf{R}t}\int\prod_{\mathbf{R}'}\mathcal{D}\tilde{\mathbf{u}}_{\mathbf{R}'t'}\mathcal{J}e^{iS[\mathbf{u},\tilde{\mathbf{u}}]}, \tag{4}$$

where in the action $S[\mathbf{u},\tilde{\mathbf{u}}]=\tilde{\mathbf{u}}[(D^R)^{-1}\mathbf{u}+\mathbf{F}-\nabla V+\boldsymbol{\xi}]+\mathbf{J}\mathbf{u}$, we have introduced a source field $\mathbf{J}$ coupling to the vortex positions $\mathbf{u}$, and used matrix notation in order to suppress the integrations over time and summations over vortex positions and Cartesian indices. The retarded Green's operator is given by $-(D^R)^{-1}\mathbf{u}\equiv m\ddot{\mathbf{u}}_{\mathbf{R}t}+\eta\dot{\mathbf{u}}_{\mathbf{R}t}+\Sigma_{\mathbf{R}'}\Phi_{\mathbf{R}\mathbf{R}'}\mathbf{u}_{\mathbf{R}'t}+\alpha\mathbf{n}\times\dot{\mathbf{u}}_{\mathbf{R}t}$, and its Fourier transform is

$$(D^R)^{-1}_{\mathbf{q}\omega} = \begin{pmatrix} m\omega^2 + i\eta\omega & -i\alpha\omega \\ i\alpha\omega & m\omega^2 + i\eta\omega \end{pmatrix} - \Phi_{\mathbf{q}}. \quad (5)$$

In order to immediately be able to perform the average with respect to both the Langevin noise and the disorder, we have chosen a nonzero mass, $m \neq 0$, leaving the Jacobian $\mathcal{J}$ an irrelevant constant[6,7] (in final expressions the mass can be set to zero, and will, in fact, for the values chosen not affect the obtained numerical results) and we obtain the averaged functional

$$Z[f] \equiv \langle\langle \mathcal{Z} \rangle\rangle = \int \mathcal{D}\phi \, e^{iS[\phi] + if\phi}, \quad (6)$$

which generates, for example, the average position and correlations

$$i\langle\langle \mathbf{u}_{\mathbf{R}t} \rangle\rangle = \frac{\delta Z}{\delta \mathbf{J}_{\mathbf{R}t}}\bigg|_{\mathbf{J}=\mathbf{0}}, \quad \langle\langle \mathbf{u}_{\mathbf{R}t}\mathbf{u}_{\mathbf{R}'t'} \rangle\rangle = \frac{i^2 \delta^2 Z}{\delta \mathbf{J}_{\mathbf{R}t}\delta \mathbf{J}_{\mathbf{R}'t'}}\bigg|_{\mathbf{J}=\mathbf{0}}. \quad (7)$$

We have introduced the notation $\phi = (\tilde{\mathbf{u}}, \mathbf{u})$ and $f = (\mathbf{F}, \mathbf{J})$, and the action $S = S_0 + S_V$ consists of a quadratic term, $S_0[\phi] = \phi D^{-1}\phi/2$, where the matrix D^{-1} in addition is a matrix in Cartesian indices, and time and vortex positions $[\delta^{tt'}_{\mathbf{RR}'} \equiv \delta_{\mathbf{RR}'}\delta(t - t')]$,

$$D^{-1} = \begin{pmatrix} 2i\eta k_B T \delta_{\alpha\beta}\delta^{tt'}_{\mathbf{RR}'} & (D^R)^{-1} \\ (D^A)^{-1} & 0 \end{pmatrix}, \quad (8)$$

and a term originating from the disorder

$$iS_V[\phi] = \frac{1}{2}\sum_{\mathbf{RR}'\alpha\beta} \int dt \int dt' \, \tilde{u}^\alpha_{\mathbf{R}t} \partial_\alpha \partial_\beta \nu(\mathbf{u}_{\mathbf{R}t} - \mathbf{u}_{\mathbf{R}'t'}) \tilde{u}^\beta_{\mathbf{R}'t'}. \quad (9)$$

This reformulation of the stochastic problem in terms of a field theory is equivalent to the formalism of Martin, Siggia, and Rose,[8] as noted previously.[9]

Our aim is to express the effective action in terms of all two-particle irreducible vacuum diagrams, and we therefore add a two-particle source term to the generating functional

$$Z[f,K] = \int \mathcal{D}\phi \, e^{iS[\phi] + if\phi + (i/2)\phi K\phi}. \quad (10)$$

The generator of connected Green's functions, $W[f,K] \equiv -i \ln Z[f,K]$, has accordingly derivatives given by (the overbar consequently denotes the average with respect to the action $S[\phi] + f\phi + \phi K\phi/2$)

$$\frac{\delta W}{\delta f^\alpha_{\mathbf{R}t}} = \bar{\phi}^\alpha_{\mathbf{R}t}, \quad \frac{\delta W}{\delta K^{\alpha\beta}_{\mathbf{R}t\mathbf{R}'t'}} = \frac{1}{2}[\bar{\phi}^\alpha_{\mathbf{R}t}\bar{\phi}^\beta_{\mathbf{R}'t'} + iG_{\alpha\beta}(\mathbf{R}t, \mathbf{R}'t')], \quad (11)$$

where G is the full connected Green's function of the theory. The quantity of interest is the effective action $\Gamma[\bar{\phi}, G] = W[f,K] - f\bar{\phi} - \bar{\phi}K\bar{\phi}/2 - iGK/2$, the Legendre transform which satisfies the equations

$$\frac{\delta\Gamma}{\delta\bar{\phi}} = -f - K\bar{\phi}, \quad \frac{\delta\Gamma}{\delta G} = -\frac{i}{2}K. \quad (12)$$

In the physical problem of interest the sources K and $\mathbf{J}$ are absent, $K = 0$ and $\mathbf{J} = \mathbf{0}$, and the full matrix Green's function has, due to the normalization of the generating functional, $Z[\mathbf{F}, \mathbf{J} = \mathbf{0}, K = \mathbf{0}] = 1$, the structure

$$G_{ij} = \begin{pmatrix} 0 & G^A \\ G^R & G^K \end{pmatrix} = -i\begin{pmatrix} 0 & \langle\langle \delta\tilde{u}^\alpha \delta u^\beta \rangle\rangle \\ \langle\langle \delta u^\alpha \delta\tilde{u}^\beta \rangle\rangle & \langle\langle \delta u^\alpha \delta u^\beta \rangle\rangle \end{pmatrix}, \quad (13)$$

where $\delta\mathbf{u} = \mathbf{u} - \langle\langle \mathbf{u} \rangle\rangle$ and $\delta\tilde{\mathbf{u}} = \tilde{\mathbf{u}} - \langle\langle \tilde{\mathbf{u}} \rangle\rangle$. The retarded Green's function $G^R_{\alpha\beta}$ gives the linear response to the force F_β, and $G^K_{\alpha\beta}$ is the correlation function (both matrices in Cartesian indices as indicated).

According to Cornwall *et al.*,[10] the effective action can be written on the form $\Gamma[\bar{\phi}, G] = S[\bar{\phi}] + (i/2)\mathrm{Tr}[(D_S^{-1} - \ln D^{-1})G - 1] - i \ln\langle e^{iS_{\mathrm{int}}}\rangle^{2\mathrm{PI}}_G$, where $D_S^{-1} = \delta^2 S[\bar{\phi}]/\delta\bar{\phi}\delta\bar{\phi}$, and $S_{\mathrm{int}}[\psi, \bar{\phi}]$ is the part of $S[\bar{\phi} + \psi]$, which is higher than second order in ψ in the expansion around $\bar{\phi}$, and Tr denotes the trace over all variables. The superscript "2PI" on the last term indicates that only the two-particle irreducible vacuum diagrams should be included in the interaction part of the effective action, and the subscript that propagator lines represent G, i.e., the brackets with subscript G denote the average $\langle F[\psi]\rangle_G = (\det G)^{-1/2}\int \mathcal{D}\psi \, e^{i\psi G^{-1}\psi/2} F[\psi]$, for an arbitrary functional F. We now expand the exponential and keep only the first-order term in S_{int} and obtain

$$-i \ln\langle e^{iS_{\mathrm{int}}[\psi,\bar{\phi}]}\rangle^{2\mathrm{PI}}_G = \langle S_V[\bar{\phi} + \psi]\rangle^{2\mathrm{PI}}_G. \quad (14)$$

For the physical problem of interest the two-particle source K vanishes, and the last of the equations in Eq. (12) therefore yields the Dyson equation, $G^{-1} = D^{-1} - \Sigma[\bar{\phi}, G]$, with the matrix self-energy given by

$$\Sigma_{ij} = \begin{pmatrix} \Sigma^K & \Sigma^R \\ \Sigma^A & 0 \end{pmatrix} = 2i \frac{\delta\langle S_V[\bar{\phi} + \psi]\rangle^{2\mathrm{PI}}_G}{\delta G_{ij}}\bigg|_{K=0, \mathbf{J}=\mathbf{0}}. \quad (15)$$

The Dyson equation and Eq. (15) constitute a set of self-consistent equations for the Green's functions and the self-energies. The average field occurring in Eq. (15) is given by $\bar{\phi} = (\langle\langle \tilde{\mathbf{u}} \rangle\rangle, \langle\langle \mathbf{u}_{\mathbf{R}t} \rangle\rangle) = (\mathbf{0}, \mathbf{v}t)$, as the expectation value of the auxiliary field vanishes, $\langle\langle \tilde{\mathbf{u}} \rangle\rangle = -iZ^{-1}\delta Z/\delta\mathbf{F}|_{\mathbf{J}=\mathbf{0}, K=0} = \mathbf{0}$, due to the normalization of the generating functional. The matrix self-energy has two independent components, Σ^R and Σ^K (as $\Sigma^A_{\beta\alpha}(\mathbf{R}t, \mathbf{R}'t') = [\Sigma^R_{\alpha\beta}(\mathbf{R}'t', \mathbf{R}t)]$), and for N vortices we have according to Eq. (15), $\Sigma^R_{\mathbf{q}\omega} = \sigma^R_{\mathbf{q}\omega} - \sigma^R_{\mathbf{q}=\mathbf{0}, \omega=0}$, where $\sigma^R_{\alpha\beta}(\mathbf{R}t\mathbf{R}'t') = 1/N\Sigma_{\mathbf{k}}\nu(\mathbf{k})k_\alpha k_\beta[\mathbf{k}G^R(\mathbf{R}t\mathbf{R}'t')\mathbf{k}]e^{i\varphi_{\mathbf{k}}}$, and $\Sigma^K_{\alpha\beta}(\mathbf{R}t, \mathbf{R}'t') = -i/N\Sigma_{\mathbf{k}}\nu(\mathbf{k})k_\alpha k_\beta e^{i\varphi_{\mathbf{k}}}$. The influence of thermal and disorder induced fluctuations is described by the phase $\varphi_{\mathbf{k}} = i\mathbf{k}M\mathbf{k} + \mathbf{k}\cdot(\mathbf{R} - \mathbf{R}' + \mathbf{u}_{\mathbf{R}t} - \mathbf{u}_{\mathbf{R}'t'})$, specified by the Cartesian matrix $M_{\alpha\beta}(\mathbf{R}t, \mathbf{R}'t') = i[G^K_{\alpha\beta}(\mathbf{R}t, \mathbf{R}t) - G^K_{\alpha\beta}(\mathbf{R}t, \mathbf{R}'t')]$. Using the Langevin equation and the first equation in Eq. (12) we obtain for the pinning force

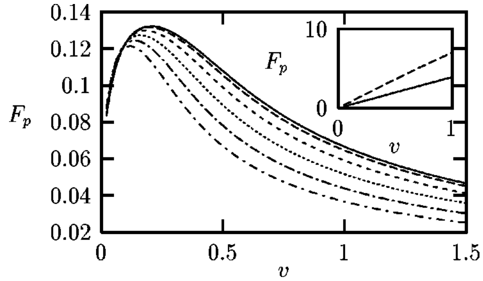

FIG. 1. Pinning force (in units of $\nu_0^{1/2}a^{-2}$) as a function of velocity (in units of $\eta^{-1}a^{-2}\nu_0^{1/2}$) for a single vortex for various strengths of the Hall force. The curves correspond to $\alpha/\eta = 0,0.2,0.4,0.6,0.8,1$, where the uppermost curve corresponds to $\alpha = 0$. The mass is $m = 0.1\eta^2 a^3 \nu_0^{-1/2}$ and the temperature is $T = 0.1\nu_0^{1/2}/(k_B a)$. Inset: Pinning force (in units of $10^{-4}\nu_0^{1/2}a^{-2}$) as a function of velocity (in units of $\eta^{-1}a^{-2}\nu_0^{1/2}$) according to the self-consistent theory at high temperature, $k_B T a/\nu_0^{1/2} = 10$. The upper curve corresponds to $\alpha = \eta$, the lower to $\alpha = 0$. The mass is $m = 0.01\eta^2 a^3 \nu_0^{-1/2}$.

$$\mathbf{F}_p = \frac{i}{N}\sum_{\mathbf{R}'}\int dt' \sum_{\mathbf{k}} \mathbf{k}\,\nu(\mathbf{k})[\mathbf{k}G^R(\mathbf{R}t\mathbf{R}'t')\mathbf{k}]e^{i\varphi_{\mathbf{k}}}. \tag{16}$$

We first consider the case of noninteracting vortices. This is appropriate for low magnetic fields where the vortices are so widely separated that the interaction between them can be neglected. We have solved the above set of self-consistent equations by numerical iteration. In Fig. 1 the resulting pinning force as a function of the velocity is shown for a set of different strengths of the Hall force in the low-temperature regime, i.e., $T \ll \nu_0^{1/2}/(k_B a)$. The Hall force is seen to reduce the pinning force in this temperature regime except, of course, at low velocities. The high velocity behavior, $v \gg \sqrt{\nu_0}/(\eta a^2)$, can be compared with the second-order perturbation expression, which is obtained by replacing the full Green's function in Eq. (16) by the free Green's function, and omitting M in the exponent (the mass term can be neglected assuming $m \ll \eta^2 a^3/\sqrt{\nu_0}$)

$$\mathbf{F}_p = -\frac{\eta\nu_0}{4\pi(\eta^2+\alpha^2)a^4 v^2}\ \mathbf{v}. \tag{17}$$

According to Fig. 2, there is good agreement between the self-consistent and perturbation theory in the reduction of the pinning force due to the Hall force at high velocities.

At high temperatures, $T \gg \nu_0^{1/2}/(k_B a)$, and moderate velocities, $v < \sqrt{\nu_0}/(\eta a^2)$, the Hall force has the opposite effect on the pinning force. According to Eq. (16) we obtain (for $m \ll \eta^2 a^3/\sqrt{\nu_0}$)

$$\mathbf{F}_p = -\frac{\nu_0(\eta^2+\alpha^2)}{8\pi\eta(k_B T)^2 a^2}\mathbf{v}. \tag{18}$$

In this high-temperature limit (which can be realized in high-temperature superconductors) we observe that the self-consistent theory yields a pinning force that has a linear velocity dependence and that the Hall force yields an increase of the pinning force, as shown in the inset in Fig. 1.

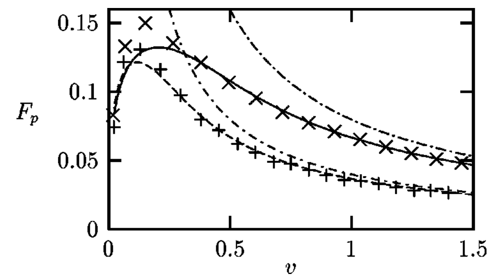

FIG. 2. Comparison of the simulation results for the pinning force and the results of the self-consistent and second-order perturbation theory for a single vortex for the case of no Hall force ($\alpha = 0$) and a moderately strong Hall force ($\alpha = \eta$). The solid line represents the self-consistent result and the crosses represent the simulation result, while the uppermost dashed-dotted line represents the perturbation theory result, all for the case $\alpha = 0$. The dashed line and the plus symbols represent the self-consistent and simulation results, while the lowest dashed-dotted line represents the perturbation theory, all for $\alpha = \eta$. The mass is $m = 0.1\eta^2 a^3 \nu_0^{-1/2}$ and the temperature is $T = 0.1\nu_0^{1/2}/(k_B a)$. The units of the pinning force and velocity are chosen as in Fig. 1.

In order to fully test the validity of the self-consistent theory its results are also compared to numerical simulations as shown in Fig. 2. The agreement between the self-consistent theory and the simulations is good except around the maximum of the pinning force. In this region the relative velocity fluctuations are large and the self-consistent theory predicts that the relative fluctuations are diverging at zero velocity even at $T = 0$. The self-consistent equations (as well as the numerical simulations) can therefore be expected to yield the largest errors at low velocities.

The Hall angle is from the self-consistent theory found to increase monotonically from zero at low velocities to the disorder independent value $\theta_0 = \arctan(\alpha/\eta)$ at high velocities, as shown in Fig. 3 for the single vortex case. The agreement between the self-consistent theory and the numerical simulations is seen to be good, testifying to the validity of the approximation made in Eq. (14). As shown in Fig. 3 we find that increasing the temperature increases the Hall angle at low velocities and that this feature vanishes at high velocities.

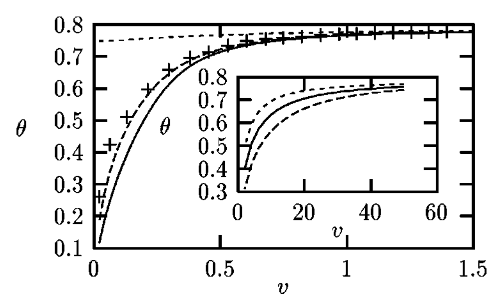

FIG. 3. Hall angle for a single vortex as a function of velocity. The curves represent the self-consistent results for the temperatures $k_B T a \nu_0^{-1/2} = 0,0.1,1$, where the uppermost curve corresponds to the highest temperature. The plus symbols represent the simulation result for $k_B T a \nu_0^{-1/2} = 0.1$. The parameter α/η is unity and the mass is $0.1\eta^2 a^3 \nu_0^{1/2}$. Inset: Hall angle for a vortex lattice as a function of velocity in descending order of lattice stiffnesses $Ac_{66} = 200\nu_0^{1/2}$, $100\nu_0^{1/2}$, $50\nu_0^{1/2}$. The unit of the velocity occurring in the figures is chosen as in Fig. 1.

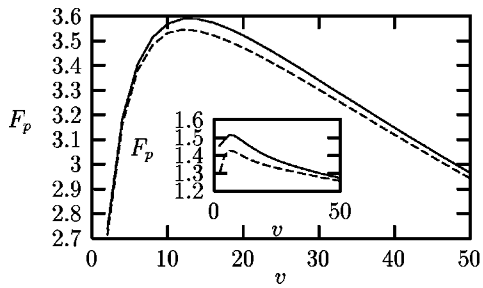

FIG. 4. Pinning force (in units of $\nu_0^{1/2}A^{-1/2}$) as a function of velocity (in units of $\eta^{-1}\nu_0^{1/2}A^{-1/2}$) for a vortex lattice of size 16×16. The range of the disorder correlator a is chosen to be $0.1A^{1/2}$, where A is the unit cell area. The solid curve corresponds to $\alpha=0$, while the dashed curve corresponds to $\alpha=\eta$. The temperature and mass are both set to zero. The elastic constants are $Ac_{11}=10^4\nu_0^{1/2}$ and $Ac_{66}=100\nu_0^{1/2}$. Inset: Pinning force as a function of velocity for $\alpha=0$ and $\alpha=\eta$, respectively. Here, $Ac_{66}=300\nu_0^{1/2}$ and the other parameters are chosen as above.

Finally, we consider a vortex lattice treating the interaction between the vortices in the harmonic approximation. The pinning force obtained from the self-consistent theory for the case of zero temperature is shown in Fig. 4. As expected there is no influence of the Hall force on the pinning force at low velocities, but we find a suppression at intermediate velocities, and at high velocities, $v\gg c_{11}a/\eta$, we recover the high velocity limit of the single vortex result, i.e., Eq. (17). By comparison of Fig. 1 and Fig. 4, we find that the Hall force has a much weaker influence at intermediate velocities on the pinning of an interacting vortex lattice than on a system of noninteracting vortices. The influence of the Hall force on the pinning force is more pronounced for a stiff than a soft lattice as seen from the inset in Fig. 4, and is similarly reflected in the Hall angle dependence on the stiffness of the lattice as seen from the inset in Fig. 3; the stiffest lattice has the greatest Hall angle.

A possible experimental verification of the obtained results would be to measure the Hall angle and pinning force of a type-II superconductor, and thereby obtain the value of α of the particular material according to Eq. (3). The parameters characterizing the disorder, a and ν_0, may, e.g., be determined by both measuring the velocity dependence of the pinning force at high vortex velocities and at high temperature at moderate velocities. The self-consistent theory can then be compared to the experimental results for pinning forces and Hall angles using the experimentally obtained parameters as input.

In conclusion, we have studied analytically as well as through simulations the vortex dynamics in type-II superconductors in the presence of a Hall force and quenched disorder. For the case of a single vortex we find that the Hall force reduces the pinning force in the high-velocity regime where the influence of fluctuations is negligible and the only effect of the Hall force is through the response function. The situation at high temperatures is the opposite since then the thermal fluctuations are dominating over the influence through the response function, and the Hall force thus increases the pinning force because it suppresses the fluctuations. The influence of the Hall force on a vortex lattice is found to be weaker than on a single vortex.

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