Observing the Berry phase in diffusive conductors: Necessary conditions for adiabaticity

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We investigate Berry phase effects in the magnetoconductance of diffusive systems, and determine the precise criterion for adiabaticity within the weak-localization formalism. We show that the exact solution of the Cooperon propagator for the special case of a cylindrically symmetric texture agrees with the adiabatic approximation in the adiabatic limit characterized by $\tau \gg (1/d)(l^2/L^2)$. We point out that orientational inhomogeneities in the magnetic field induce dephasing effects that can mask the Berry phase (and any other phase-coherent phenomena) for certain parameter values of system and field. [S0163-1829(99)09119-5]

I. INTRODUCTION

The Berry phase[1] remains a fascinating subject with many consequences in a variety of physical systems.[2] Some time ago we proposed[3–7] a number of scenarios in condensed-matter settings where the Berry phase manifests itself in the phase-coherent quantum dynamics of a particle carrying a spin and moving through orientationally inhomogeneous magnetic fields $\mathbf{B}(\mathbf{x})$. Such manifestations of the Berry phase can occur, e.g., in semiconductors or metals in the form of persistent currents[3–6] or oscillations of the magnetoconductance or universal conductance fluctuations.[4,7] As recognized early on,[4] all these effects share the common feature that the orbital motion of the particle is modified by the Berry phase in very much the same way as it is in well-known phase-coherent phenomena based on the Aharonov-Bohm effect.

The first experimental evidence for such a Berry phase effect was recently found in semiconductors,[8] in which a local effective magnetic field is produced via the Rashba effect. However, whereas Aharonov-Bohm effects occur regardless of the strength B of the field, Berry phase effects appear only in the adiabatic limit, i.e., for sufficiently large magnetic fields. This limit requires that—roughly speaking—the typical orbital frequency of the particle carrying the spin through the field is much smaller than the precession frequency of the spin around the local-field direction. In this limit, the spin will remain in its instantaneous eigenstate, i.e., will continuously align itself along the *local* field direction $\mathbf{B}(\mathbf{x})$ as it moves through the magnetic-field texture. If, in addition, the particle trajectory is closed, the spin will acquire a Berry phase, which is purely geometric in character. As spin and orbital motion couple via the inhomogeneity of the field, the Berry phase can ultimately enter the orbital part of the effective Hamiltonian in the same way that the Aharonov-Bohm phase does.

There seems to be general agreement that once the adiabatic limit is reached, the results found previously[3–7] are correct. The central question then is: What is the proper criterion for the adiabatic regime? Again, there is no issue of contention in ballistic rings, e.g., for which adiabaticity is reached when $\omega_B t_o \gg 1$, where ω_B is the Bohr frequency (to be defined below), and t_o is the typical time it takes the particle to go around the ring once. This situation occurs, e.g., in clean semiconductors.

But what about diffusive systems, such as normal metal rings? It is this question that we have previously addressed in detail,[7] and that was recently reconsidered by van Langen *et al.*,[9] who reported a rather pessimistic conclusion about the observability of the Berry phase effect—in contrast to earlier findings.[7] Since the range of applicability of the adiabatic approximation is of central importance for experimental investigations, it seems worthwhile to reanalyze the question of adiabaticity from an alternative point of view, and to demonstrate explicitly the validity of our earlier results for the special case of a cylindrically symmetric textures. To this end, we first state the problem of adiabaticity in this section again, and then provide in the following sections a general discussion on the issue of dephasing induced by inhomogeneous magnetic fields. This discussion is then followed by explicit examples that demonstrate the observability of Berry phase effects in diffusive systems of immediate experimental interest.

Now, in the context of weak-localization physics we have advanced detailed physical and technical arguments[7] that adiabaticity is reached more easily in diffusive than in ballistic systems (all other parameters being equal). The physical explanation for this is simple: In diffusive motion around, say, a ring, the particle spends on the average much more time in a given region of field direction than it would do in purely ballistic motion. Thus there is more time for the spin to execute precessions around a given field direction, and thus the spin will have a higher probability of aligning itself along the local-field direction than it would in purely ballistic motion. Translating this picture into more concrete terms for an electron diffusing around a d-dimensional ring of circumference L with static random disorder, adiabaticity is reached if the Zeeman energy $\hbar\omega_B = g\mu_B B/2$ exceeds the Thouless energy $E_{\mathrm{Th}} = hD/L^2$. Here g is the electron g factor, μ_B is the Bohr magneton, $D=v_F^2\tau/d$ is the diffusion constant with v_F being Fermi velocity, $\tau=l/v_F$ is the elastic mean free time, and l is the elastic mean free path. More generally, we can also allow for the case in which the field reorients f times as the particle goes around once the ring. Whereas the case of $f=1$ is physically realizable,[5] it seems very difficult to implement cases with $f>1$ experimentally. Still, as some recent conclusions[9] are based on the case $f=5$, we shall include this possibility, and the criterion for adiabaticity as found in Ref. 7 then reads

$$\omega_B\tau \gg \frac{f}{d}\,\frac{l^2}{L^2}\sqrt{1-|\mathbf{N}|}. \tag{1}$$

Here the texture-dependent vector $\mathbf{N}$ is some average of the direction of the magnetic field.[5] The factor $\sqrt{1-|\mathbf{N}|}$ accounts for nonuniformity in the direction of the magnetic field, and encodes the fact that the adiabatic approximation becomes exact, regardless of ω_B, in the limit of a homogeneous field, for which $|\mathbf{N}|=1$. In the following discussion, however, we shall—for the sake of simplicity—omit this factor, noting that its inclusion would render the criterion even less stringent.[10] As in metals, one typically has τ on the order of 10^{-14} s, $g=2$, and $l=10^{-8}$ m, we should have, for a ring of circumference $L=10^{-6}$ m, magnetic fields at least of the order of $100-1000$ G to be within the adiabatic regime. Note that without the diffusive factor $(l/L)^2=10^{-4}$, the required fields would be too large to be attainable experimentally (i.e., on the order of $100-1000$ T).

The regime of adiabaticity defined in Eq. (1) follows from a detailed derivation of the Cooperon and diffuson propagator based on weak-localization techniques and an adiabatic approximation scheme.[7] This adiabatic approximation is performed in the path integral representation for the Cooperon (diffuson). As emphasized in an analogous discussion of the imaginary-time propagator in the context of persistent currents,[5] the adiabatic approximation can contain additional angle-dependent terms that are different from the Berry phase, and these terms can mask the Berry phase in certain physical observables. (For an explicit example of such a case, see Sec. VI F of Ref. 5.) The origin of this additonal term can be traced back to quantum fluctuations of the particle trajectory, which induce nonsmooth variations of the magnetic field (and thereby violate the ''smooth variation'' assumption that underlies the adiabatic approximation).[5] An alternative way to express this point is to say that in certain cases the Berry phase can be masked by dephasing effects—in very much the same way that the Aharonov-Bohm phase can become unobservable if dephasing influences become too large. Such dephasing effects are difficult to calculate for a general texture, but can sometimes be obtained in special cases for which an exact solution is available (see Ref. 5 and below). As suggested in Ref. 7, it is possible to extend the exact solution for a propagator containing a single spin-$\frac{1}{2}$ particle[5] to the one containing two spin-$\frac{1}{2}$ particles. Indeed, by following this suggestion van Langen *et al.*[9] recalculated the magnetoconductance for a cylindrically symmetrical texture, and found deviations from our adiabatic solution.[7] [As we shall show, these deviations do *not* occur within the range of validity of the weak-localization (semiclassical) regime.] From this observation it is concluded[9] that the exact solution does not contain the Berry phase effect, and thus that the regime of adiabaticity, given in Eq. (1), is invalid. Instead, adopting a suggestion made first by Stern,[11] it is argued[9] that it is necessary for the much more stringent conditon,

$$\omega_B\tau \gg 1, \tag{2}$$

to be satisfied in diffusive systems before adiabaticity is reached, and thus before the Berry phase effect can become observable in the magnetoconductance. However, in contrast to this we will find that the adiabaticity criterion given above [Eq. (1)], is indeed appropriate for diffusive systems, and that the observability or nonobservability of the Berry phase crucially depends on the choice of physical parameters [in the adiabatic regime given by Eq. (1)]. In particular, in the unrealistic situation that the field winds five times around the ring (i.e., $f=5$), and as dephasing effects grow strongly with f (as f^2; see below), it is not surprising that Berry phase oscillations are not discernible in this extreme case. However, upon choosing $f=1$—the physically most relevant case—not only do Berry phase effects show up in the exact solution, but also they agree well with previously obtained adiabatic predictions.[7]

The issue of adiabaticity has also been studied in terms of Boltzmann equations.[9] Due to the coupling of the magnetic field to the orbital motion of the charged electron, these Boltzmann equations are valid in the diffusive regime defined by $\omega_c\tau \ll 1$, where ω_c is the cyclotron frequency. As ω_c and ω_B are typically of the same order of magnitude in metals, the regime $\omega_B\tau \gg 1$ lies *outside* the physical regime to which Boltzmann equations can legitimately be applied. Still, even if we ignore such orbital effects (i.e., set the electron charge to zero), the regime $\omega_B\tau \gg 1$ is problematic for an additional reason[7]. If $\omega_B\tau \gg 1$, the Zeeman rate ω_B is large compared to the elastic collision rate $1/\tau$. In this case we expect the Zeeman interaction to have a strong dephasing influence on the orbital motion (for inhomogeneous fields), especially when $f \gg 1$, and the system lies outside the semiclassical regime in the sense of weak localization theory (see, e.g., Secs. 4 and 10 of Ref. 12 and below).[13]

II. BERRY PHASE AND MAGNETOCONDUCTANCE

A. Exact solution and adiabatic approximation

We consider a quasi-one-dimensional ring of circumference L, embedded in a magnetic field texture given by $\mathbf{B} = B\mathbf{n} = B[\sin\eta\cos(2\pi f x/L), \sin\eta\sin(2\pi f x/L), \cos\eta]$, where x is the location on the ring, η is the tilt angle of the magnetic field, and $f(=1,2,3,\ldots)$ is the winding of the magnetic field along the propagation direction. The magnitude B and, in particular, the tilt angle η are assumed to be constant. It is this special case that can be solved exactly (as pointed out in Ref. 7) along the same lines as discussed in Ref. 5 for a single-spin propagator. Van Langen *et al.*[9] were the first to write down this solution explicitly for a two-spin propagator.

The magnetoconductance resulting from weak localization corrections and in the presence of the field texture $\mathbf{B}$ derived in Ref. 7, and reads

$$\delta g=-\frac{e^2}{\pi\hbar}\frac{L}{(2\pi)^2}\sum_{\alpha,\beta=\pm1}\langle x,\alpha,\beta|\frac{1}{\gamma-h}|x,\beta,\alpha\rangle, \quad (3)$$

where the effective (non-Hermitian) Hamiltonian h is given by

$$h=\frac{L^2}{(2\pi)^2}\frac{\partial^2}{\partial x^2}+i\kappa\,\mathbf{n}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2), \quad (4)$$

where $\boldsymbol{\sigma}_i$ (with $i=1,2$) are spin-$\frac{1}{2}$ Pauli matrices, and where

$$\kappa=\frac{\omega_B}{D}\frac{L^2}{(2\pi)^2}=\omega_B\tau d\frac{L^2}{(2\pi l)^2} \quad (5)$$

is the dimensionless adiabaticity parameter [see Eq. (1)]. The factor $\gamma=(L/2\pi L_\phi)^2$ is a damping constant expressed in terms of the dephasing length L_ϕ (which is specified in more detail below). Note that γ is introduced here in a phenomenological way with the particular *ad hoc* choice that it be a c number and diagonal in spin space.

We now evaluate δg explicitly, but instead of using the exact eigenstates,[9] we use an alternative approach in terms of unitary gauge transformations, which has the virtue of making the emergence of the Berry phase immediately transparent. For this purpose we define unitary transformations U and V of the forms

$$U=V\,e^{(i\pi f/L)x(\sigma_{1z}+\sigma_{2z})}, \quad V=e^{(i/2)\eta(\sigma_{1y}+\sigma_{2y})}, \quad (6)$$

with the property that

$$\mathbf{n}\cdot(\sigma_1-\sigma_2)=U^\dagger(\sigma_{1z}-\sigma_{2z})U. \quad (7)$$

By noting that $U(-i\partial/\partial x)U^\dagger=-i\partial/\partial x-iU\partial U^\dagger/\partial x$, we find

$$UhU^\dagger=-\left(-i\frac{L}{2\pi}\frac{\partial}{\partial x}-\frac{f}{2}[(\sigma_{1z}+\sigma_{2z})\cos\eta-(\sigma_{1x}+\sigma_{2x})\sin\eta]\right)^2+i\kappa(\sigma_{1z}-\sigma_{2z}). \quad (8)$$

Next we rewrite the matrix elements occurring in δg:

$$\langle x,\alpha,\beta|U^\dagger\frac{1}{\gamma-UhU^\dagger}U|x,\beta,\alpha\rangle=\langle x,\alpha,\beta|V^\dagger\frac{1}{\gamma-h_{\alpha\beta}}\Pi_{12}V|x,\alpha,\beta\rangle, \quad (9)$$

where $h_{\alpha\beta}=UhU^\dagger[-i(L/2\pi)(\partial/\partial x)\to-i(L/2\pi)(\partial/\partial x)+(f/2)(\alpha+\beta)]$ and $\Pi_{12}|\alpha\beta\rangle=|\beta\alpha\rangle$. The effective Hamiltonian $h_{\alpha\beta}$ is now diagonal in the angular momentum eigenstates $\langle x|j\rangle=e^{i(2\pi/L)xj}/\sqrt{L}$, with $j=0,\pm1,\pm2,\ldots$ (imposing periodic boundary conditions), and we find

$$\delta g=-\frac{e^2}{\pi\hbar}\frac{1}{(2\pi)^2}\sum_j \mathrm{Tr}_{12}\frac{1}{\gamma-h(j)}\Pi_{12}, \quad (10)$$

where Tr_{12} is the trace in spin space, and

$$h(j)=-\left(j-\frac{f}{2}(\sigma_{1z}+\sigma_{2z})\cos\eta\right)^2-\frac{f^2}{2}(1+\sigma_{1x}\sigma_{2x})\sin^2\eta-jf(\sigma_{1x}+\sigma_{2x})\sin\eta$$
$$+\frac{f^2}{4}(\sigma_{1x}\sigma_{2z}+\sigma_{2x}\sigma_{1z})\sin2\eta+i\kappa(\sigma_{1z}-\sigma_{2z}). \quad (11)$$

Here we have absorbed the integer $f(\alpha+\beta)/2$ into j. Note that two of the eigenvalues of $(f/2)(\sigma_{1z}+\sigma_{2z})\cos\eta$ are given by the (geometric) Berry phase $\pm\Phi^g=\pm f\cos\eta$ for an effectively integral spin.[14] The term $(f^2/2)(1+\sigma_{1x}\sigma_{2x})\sin^2\eta$ provides a source of dephasing that can mask the Berry phase—and more generally the Aharonov-Bohm effect (see Sec. II B below). All the other off-diagonal terms turn out to be irrelevant in the adiabatic limit (see Sec. II E). To proceed, we express the above operators in the σ_z basis $\{|1,1\rangle,|1,-1\rangle,|-1,1\rangle,|-1,-1\rangle\}$. The Hamiltonian $h(j)$ then has matrix elements

$$\langle\alpha',\beta'|h(j)|\alpha,\beta\rangle=-\begin{pmatrix}(j-f\cos\eta)^2+a & jf\sin\eta-b & jf\sin\eta-b & a\\ jf\sin\eta-b & j^2+a-i2\kappa & a & jf\sin\eta+b\\ jf\sin\eta-b & a & j^2+a+i2\kappa & jf\sin\eta+b\\ a & jf\sin\eta+b & jf\sin\eta+b & (j+f\cos\eta)^2+a\end{pmatrix}, \quad (12)$$

where $a=(f^2/2)\sin^2\eta$ and $b=(f^2/4)\sin2\eta$. Finding the inverse of $\gamma-h(j)$ is then straightforward, and we finally obtain for the magnetoconductance

$$\delta g=-\frac{e^2}{\pi\hbar}\frac{1}{2\pi^2}\sum_{j=-\infty}^{+\infty}\left\{(\gamma+m^2+f^2)(\gamma+m^2)^2+4\kappa^2\left(\gamma+m^2+f^2\cos^2\eta+\frac{f^2}{2}\sin^2\eta\right)\right\}([\gamma+(m-f)^2][\gamma+(m+f)^2](\gamma$$
$$+m^2)^2+4\kappa^2\{[\gamma+(m-f\cos\eta)^2][\gamma+(m+f\cos\eta)^2]+f^2\sin^2\eta(\gamma+m^2+f^2\cos^2\eta)\})^{-1}, \quad (13)$$

where $m=j-\Phi$, i.e., we have allowed for an Aharonov-Bohm flux $\Phi=2\phi/\phi_0$, with $\phi_0=h/e$ being the flux quantum. The foregoing result is exact and is seen to be identical to the one obtained by van Langen *et al.*[9] (for their choice $d=2$). However, our alternative derivation has led us to a form in which the Berry phase contribution is made manifest in the terms of the form $(m\pm f\cos\eta)^2$.

Next we go over to the adiabatic limit, defined here[10] by $\kappa\gg 1/(2\pi)^2$, which, for $f=1$, is equivalent to $\omega_B\tau\gg l^2/(L^2d)$ [see Eq. (1)]. (Below, in Sec. II D, we give explicit numerical values of κ for which adiabaticity is reached.) In this limit we may drop the terms independent of κ in Eq. (13) (this is justified as terms with large j give a negligible contribution to δg). Thus, in the adiabatic limit we finally obtain

$$\delta g^{\mathrm{Ad}}=-\frac{e^2}{\pi\hbar}\frac{1}{(2\pi)^2}\sum_{\alpha=\pm1}\sum_{j=-\infty}^{+\infty}\frac{[\gamma+(m+\alpha f\cos\eta)^2]+(f^2/2)\sin^2\eta}{[\gamma+(m-\alpha f\cos\eta)^2][\gamma+(m+\alpha f\cos\eta)^2]+(\gamma+m^2+f^2\cos^2\eta)f^2\sin^2\eta},\tag{14}$$

where the sum over $\alpha=\pm1$ has been introduced artificially for later convenience. Note that the Berry phase $\Phi^g=f\cos\eta$ couples to the momentum like the Aharonov-Bohm phase does, i.e., via $j-\Phi-\alpha\Phi^g$. We note that the remaining η dependence cannot be accounted for by this type of coupling to the momentum. We particularly emphasize that (apart from the flux appearing in $m=j-\Phi$) the adiabatic limit of the magnetoconductance δg^{Ad} is independent of the field amplitude B; thus increasing the field further, say up to $\omega_B\tau\gg1$ [cf. Eq. (2)], has no effect.

It is now instructive to compare Eq. (14) with the one previously derived[7] for arbitrary textures and in the adiabatic approximation scheme for the Berry phase. The latter result reads (the superscript LSG refers to Ref. 7)

$$\delta g^{\mathrm{LSG}}=-\frac{e^2}{\pi\hbar}\frac{L'_\phi}{2L}\sum_{\alpha=\pm1}\frac{\sinh(L/L'_\phi)}{\cosh(L/L'_\phi)-\cos[2\pi(\Phi+\alpha f\cos\eta)]}=-\frac{e^2}{\pi\hbar}\frac{1}{(2\pi)^2}\sum_{\alpha}\sum_{j=-\infty}^{+\infty}\frac{1}{\gamma'+(m-\alpha f\cos\eta)^2}$$

$$=-\frac{e^2}{\pi\hbar}\frac{1}{(2\pi)^2}\sum_{\alpha}\sum_{j=-\infty}^{+\infty}\frac{\gamma'+(m+\alpha f\cos\eta)^2}{[\gamma'+(m-\alpha f\cos\eta)^2][\gamma'+(m+\alpha f\cos\eta)^2]},\tag{15}$$

where, again, $m=j-\Phi$, and $\gamma'=(L/2\pi L'_\phi)^2$, and where we have used some identities to facilitate comparison. Note that in general $\gamma\neq\gamma'$ (see below). The virtue of δg^{LSG} is that it is valid for arbitrary field textures (with the appropriate Berry phase[7]). It is thus important to understand its relation to the special but exactly solvable case.

Now, by comparing δg^{LSG} with δg^{Ad} we see that the two expressions have the same structure with respect to the Berry phase, $\Phi^g=f\cos\eta$, but differ in additional η- and f-dependent terms. (From now on we put the Aharonov-Bohm flux Φ to zero but shall return to nonzero flux later.) Particularly important is the additional term in the denominator of δg^{Ad}, i.e., $f^4\sin^2\eta\cos^2\eta$ (the physical origin of such additional terms is discussed below in Sec. II E). It is this term that acts as a *dephasing source* for certain tilt angles and windings f by suppressing the ''resonance peaks'' that would occur at integral values of the Berry phase $\Phi^g=f\cos\eta$ (for small enough γ'). For $f>1$ the suppression due to this term is so strong that all resonances except the ones at $\eta=0,\pi/2,\pi$ become masked, i.e., these resonances due to the Berry phase are no longer visible in graphs of δg^{Ad} versus η, whereas they do show up in δg^{LSG} *provided* one chooses γ' to be independent of the tilt angle η (and sufficiently small). This choice for f and γ' has been adopted by van Langen *et al.*,[9] in particular, $f=5$ and a constant $\gamma=0.4053$. As in this case δg^{LSG} and δg^{Ad} behave differently for $\gamma=\gamma'$ (see Fig. 3 of Ref. 9), it is concluded[9] that δg^{Ad} does not show adiabatic behavior and, thus, that the adiabaticity criterion [Eq. (1)], is not valid. However, it is premature to draw such a conclusion since there is additional dephasing induced by the inhomogeneity of the magnetic field which for $f>1$ is so strong that the semiclassical approximation on which the derivation of the Cooperon propagator rests breaks down. We now proceed to explain this point in detail, and then present explicit physical examples to illustrate the general discussion.

B. Dephasing due to magnetic fields

The *ad hoc* choice[9] of putting $\gamma=\gamma'$ and choosing them to be independent of η means that δg^{Ad} and δg^{LSG} do *not* describe the same physical situation. This is so for the following reason. First we note again that the dephasing parameters γ, and γ' are ''put in by hand'' into the Cooperon to account for dephasing in a phenomenological way (this is just dictated by the complexity of the involved many-body problem and by our inability to address this issue in a more systematic way in general). In the derivation of δg^{LSG} dephasing due to the field is only taken into account *a posteriori* in terms of a phenomenological parameter γ', while the exact solution [Eq. (13)] not only includes the Berry phase but simultaneously also those dephasing effects that are caused by the field through the Zeeman coupling. The remaining dephasing effects in δg or δg^{Ad} are then included via the phenomenological parameter γ. Obviously, γ and γ' are in general different for the same physical situation.

Next, it is a well-known fact in the context of weak-localization phenomena[15] that dephasing in general depends on the magnetic field **B** penetrating the sample (as we must allow for there to be any Zeeman interaction at all). Most importantly, γ' not only depends, in general, on the magnitude B of the field but also on its tilt angle η that the field makes with the z axis perpendicular to the ring plane. (This is already so even without Zeeman terms; see, e.g., Sec. 2 of Ref. 15. Such an angle dependence becomes even more pronounced in the presence of our *non*uniform Zeeman interaction). Now, the various dephasing effects are accounted for phenomenologically in terms of dephasing lengths,[15] $1/L_\phi^2 = 1/(L_\phi^0)^2 + 1/(L_\phi^B)^2$, where the dephasing length L_ϕ^0 contains all field-independent contributions, such as the one coming from inelastic collisons of the diffusing electron with, say, phonons, $L_\phi^{\rm in} = \sqrt{D\tau_{\rm in}}$, where the dephasing time $\tau_{\rm in}$ is some inelastic scattering time. The magnetic length L_ϕ^B contains all effects coming from the field penetrating the sample.

If now $L_\phi^B \ll L$ for some field configurations, we no longer expect to see phase coherence in general. As a matter of fact, in Sec. IV of Ref. 7 we have estimated the upper bound of the dephasing length (due to the inhomogeneous Zeeman interaction) in metallic films to be given by the characteristic field-reorientation length[7] $l_B = |\boldsymbol{\nabla}(\mathbf{B}/B)|^{-1}$. This estimate follows from the observation that quantum corrections begin to be eliminated when the largest phase-coherent paths enclose roughly one quantum of Berry flux. For the symmetric texture considered here, we find $l_B = L/(2\pi f|\sin\eta|)$. Obviously, for certain tilt angles and for $f \gg 1$ this upper bound on the dephasing length quickly becomes smaller than L. Translated into a dephasing parameter $\gamma = (L/2\pi L_\phi^B)^2$, this estimate reads

$$\gamma > f^2 \sin^2\eta, \tag{16}$$

i.e., we see that the dephasing becomes explicitly η dependent and grows like f^2.

Thus, as expected, the exact solution confirms this general property, in the sense that explicit dephasing terms are present in δg that are field dependent and which can become so large, for *particular* field inhomogeneities, that they completely suppress the resonances in the magnetoconductance, Eq. (13), with respect to the Berry phase,[16] no matter how large ω_B is. Of course, as implied by above discussion leading to Eq. (16), such a dephasing effect must also be accounted for explicitly in $\delta g^{\rm LSG}$ [Eq. (15)] by an appropriate choice for the phenomenological damping parameter γ'. In particular, in view of the estimate given in Eq. (16), it is reasonable to make the ansatz $\gamma' = f^2\sin^2(2\eta)$.[17] Then, choosing the dephasing parameter of $\delta g^{\rm Ad}$ to be constant (i.e., η independent) and much smaller than unity, say $\gamma = 10^{-2}$, we see that the qualitative discrepancy between $\delta g^{\rm LSG}$ and $\delta g^{\rm Ad}$ disappears: Both expressions show no resonances (away from $\Phi^g = 0$ and 1). (We note that as γ and γ' are introduced phenomenologically anyway, there is no need to obtain quantitative agreement, and it suffices to find the same qualitative suppression of the resonances for $f > 1$ in both $\delta g^{\rm LSG}$ and $\delta g^{\rm Ad}$. We shall not be making any further use of this ansatz for γ'.)

The suppression of the Cooperon due to homogeneous fields is standard;[15] the discussion above shows that additional dephasing is induced by the field inhomogeneity. The advantage of having the exact solution for δg [Eq. (13)] at hand is that we can now calculate the field dependence of such dephasing terms explicitly; this allows us to make more precise statements than before[7] about the regime in which one can expect to observe consequences of the Berry phase (see Sec. II D below).

C. Self-consistency of the semiclassical approximation

The magnetoconductance correction δg is expressed in terms of the Cooperon propagator. The derivation of the Cooperon is, in turn, performed within the *semiclassical limit*. In particular, this means that ''back-reaction effects,'' i.e., non-phase-coherent dynamical effects of the field-dependent Zeeman term on the *orbital motion*, are assumed to be negligibly small throughout. This is a fundamental assumption in weak-localization theory,[12] and it was explicitly adopted in the derivation of the Cooperon and of $\delta g^{\rm LSG}$ as well. (This is emphasized, e.g., in Appendix A of Ref. 7.) Evidently, dephasing effects such as the ones discussed in Sec. II B are nothing but such back-reaction effects. Thus, if dephasing becomes so large (as turns out to be the case in the adiabatic limit and for $f > 1$) that phase coherence is completely suppressed in the orbital part, the semiclassical approximation breaks down and the self-consistency of the entire treatment is lost.[18] Consequently, the expressions for the magnetoconductance are no longer reliable in the case of complete dephasing, and no weight should be put on conclusions drawn under such circumstances. Obviously, semiclassical and adiabatic approximations are interconnected issues, in the sense that the semiclassical approximation might break down in the adiabatic limit and for certain field configurations. In other words, adiabaticity alone is not a sufficient criterion for the observability of Berry phase effects, in addition the system must be in the mesoscopic regime characterized by phase-coherence.

To summarize our conclusions so far, we have seen that the adiabaticity criterion [Eq. (1)] is sufficient for reaching the adiabatic limit involving the Berry phase [cf. Eqs. (14) and (15)]. However, the criterion does not guarantee (and one should not expect it) that the Berry phase will be observable under all circumstances. Indeed, it can happen that the phase coherence, which is necessary for observing such quantum phase phenomena, can be destroyed by a variety of dephasing sources, in particular also by magnetic fields penetrating the sample. If dephasing becomes so strong in the adiabatic regime that quantum phase effects of the orbital motion are completely washed out, the semiclassical approximation underlying the derivation of the Cooperon breaks down, and results based on it (such as δg) are no longer reliable.

In the light of above discussion it should now be clear that the only conclusion one can draw from the observation[9] of the absence of Berry phase effects for $f = 5$ within the semiclassical theory is that field textures with $f > 1$ suppress phase coherence very efficiently, and thus such extreme textures cannot serve as a general test case for the existence or nonexistence of the Berry phase and the associated adiabaticity regime—at least not within the semiclassical regime to which the results [Eqs. (13)–(15)] are confined.

D. Observability of Berry phase effects for $f = 1$

Up to now we have mainly concentrated on regimes where $f > 1$. Such regimes, however, are of little experimental interest (quite apart from the difficulty of how to produce them) since the Berry phase effect would be masked by the strong dephasing effect of the field. The situation, however, is entirely different for the case where the magnetic field

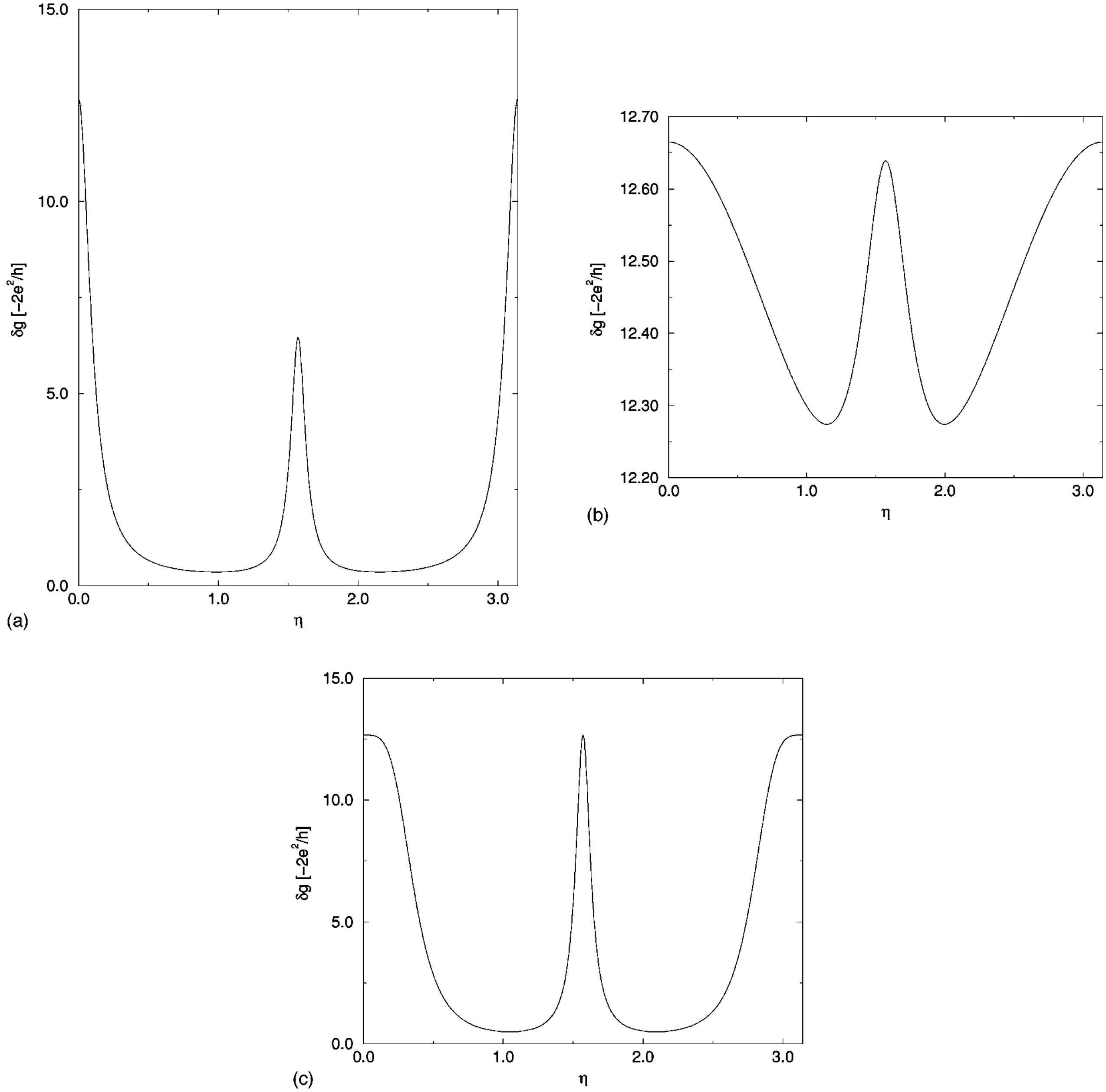

FIG. 1. The (dimensionless) magnetoconductance $\delta g/(-e^2/\pi\hbar)$ [Eq. (13)] as a function of the tilt angle $0 \leqslant \eta \leqslant \pi$. (a) shows δg in the adiabatic limit (i.e., $\kappa=1$); (b) shows δg outside the adiabatic limit (i.e., $\kappa=0.01$), with a strongly reduced amplitude. The remaining parameter values are $f=1,\ \gamma=0.4053/100$, and $\Phi=0$. (c) shows the adiabatic result $\delta g^{\mathrm{LSG}}/(-e^2/\pi\hbar)$ [Eq. (15)] as function of tilt angle $0 \leqslant \eta \leqslant \pi$, with $f=1,\ \gamma'=0.4053/100$ and $\Phi=0$. Note that (a) and (c) agree very well qualitatively, and show pronounced resonances at integral values of the Berry phase $\Phi^g=0,1,\ldots$.

winds only once around the ring, i.e., when $f=1$ (such field textures can be produced experimentally[5]). Indeed, we shall see now that for $f=1$ the dephasing is sufficiently small and the Berry phase has observable consequences within an experimentally accessible regime. We shall illustrate this with two specific examples: First we discuss resonances in the magnetoconductance due to the Berry phase (for vanishing Aharonov-Bohm flux Φ); then we discuss phase shifts in the Aharonov-Bohm oscillations induced by the Berry phase.

First we consider the magnetoconductance as a function of the Berry phase in the absence of an Aharonov-Bohm flux, i.e., $\Phi=0$. We make the realistic assumption that the dephasing length independent of the tilt angle can be made to exceed L, say, $L_\phi=2.5L$, giving $\gamma=4.053\times10^{-3}$ (this value for γ is 100 times smaller than the one chosen in Sec. II A). In Fig. 1, we plot the magnetoconductance δg [Eq. (13)] as a function of the tilt angle η in the adiabatic regime, $\kappa=1$, and find pronounced resonance peaks at the Berry phase values $\Phi^g=0$ and 1 —in very good qualitative agreement with the general result δg^{LSG}, given in Eq. (15), even if we simply choose $\gamma'=\gamma$. For comparison, we also plot (see Fig. 1) the magnetoconductance δg outside the adiabatic regime, i.e., for $\kappa=0.01$, where the resonances are (nearly) absent—demonstrating that adiabaticity is needed for the emergence of the Berry phase. We note that the above choice for the adiabatic parameter (i.e., $\kappa_0=1$) corresponds to $\omega_{B_0}\tau=(2\pi)^2 l^2/(L^2 d) \gg l^2/L^2 d$. In particular, if we follow van Langen *et al.*[9] and choose $L/l=500$ (i.e., a typical ratio for a mesoscopic metal ring), we see that $\kappa_0=1$ is equivalent to $\omega_{B_0}\tau=1.57\times10^{-4}/d$. Note that we are orders of magnitude below the regime of Eq. (2), where $\omega_B\tau\gg1$. Translated into magnetic fields, $\kappa_0=1$ corresponds to

$$B_0=\frac{2(2\pi)^2}{gd}\,\frac{v_F l\hbar}{\mu_B L^2}=\frac{4\pi}{g\mu_B}\,\frac{hD}{L^2}, \tag{17}$$

which, for $g=2$ and $d=3$[19], gives

$$B_0=1.5\times10^{-6}\frac{v_F l}{L^2}[\mathrm{G\,s}]=4.5\times10^{-6}\frac{D}{L^2}[\mathrm{G\,s}]. \tag{18}$$

To illustrate this with concrete numbers we assume the Fermi velocity $v_F=10^6\,\mathrm{ms}^{-1}$ and the ring circumference $L=7\,\mu$m, and again $L/l=500$. We then find that the field corresponding to $\kappa=1$ is about 400 G. The resonance structure due to the Berry phase starts to emerge for κ at around 0.1, i.e., for fields of the order of 40 G. Finally, we note that when the tilt angle η is varied, then typically there will be a concommittant change of the Aharonov-Bohm flux Φ. This flux, however, can easily be compensated for by applying a field perpendicular to the ring such that Φ again becomes an integral multiple of the flux quantum. Note that the maximal fields required for this compensation are about 10 G, or so for a ring of $L=7\,\mu$m. Thus such fields would have a negligible effect on the inhomogeneous field required for adiabaticity, except if η is very close to $\pi/2$.

A further experimentally interesting scenario is that of the *phase shift* in the Aharonov-Bohm oscillation induced by the Berry phase. In particular, this effect is most pronounced for half-integral Berry phases, $\Phi^g=\pm\frac{1}{2}$ (i.e., $\eta=\pi/3$ or $2\pi/3$), for which we expect [see Eqs. (14) and (15)] to obtain a phase shift in the Aharonov-Bohm oscillation of the magnetoconductance by the flux value $\frac{1}{2}$ (i.e., by one quarter of the flux quantum h/e). Note that in this case the sign of the oscillation slope (e.g., at $\Phi=0$) is reversed with respect to the case without Berry phase. This sign reversal is reminiscent of similar effects induced by spin-orbit scattering;[15] it is actually not unexpected, as the Zeeman term induces an effective spin-orbit coupling due to the inhomogeneity of the magnetic field.[3,5] This phase shift is shown in Figs. 2(a) and 2(b), which show δg as a function of the Aharonov-Bohm flux Φ for Berry phases $\Phi^g=0$ and $\frac{1}{2}$, both in the adiabatic limit (i.e., $\kappa=1$), and with the choice $\gamma=0.1$ (i.e., $L=2L_\phi$). For the sake of comparison, in Fig. 2(d) we also show a nonadiabatic case, $\kappa=0.1$, for which the phase shift is absent. The phase shift remains discernible down to about $\kappa=0.7$ before disappearing. The adiabatic limit is fully reached at about $\kappa=10$, by which not only the phase shift (which is the important feature) but also the amplitude becomes identical to δg^{Ad} given in Eq. (14). The amplitude at $\kappa=1$ increases about by 20% upon increasing the field to $\kappa=10$.[19]

To obtain realistic estimates for some physical parameters, we now concentrate on a Au ring and use the material parameters recently determined in Ref. 20 (see sample Au-1 in their Table I). The relevant values are $D=9\times10^{-3}\,\mathrm{m}^2\,\mathrm{s}^{-1}$ and $\tau_\phi^0=3.41\times10^{-9}$ s (at a temperature of 11 mK), which for the dephasing length give $L_\phi^0=\sqrt{D\tau_\phi^0}=5.54\,\mu$m. Thus the above choice $L=2L_\phi^0$ requires a ring of circumference $L=11\,\mu$m. In this case, the field corresponding to $\kappa_0=1$ becomes $B_0=335$ G, and the limiting case $\kappa=0.7$ at which the phase shift emerges, corresponds to $B=235$ G.[21]

Precisely the same phase shift occurs in δg^{LSG} [Eq. (15)] as shown in Fig. 2. To obtain roughly the same amplitudes as in δg we must account for the η-dependent dephasing in δg^{LSG}. To this end we choose an effective $\gamma'=\gamma=0.1$ (for $\eta=\pi/2$) and $\gamma'=5\gamma=0.5$ (for $\eta=\pi/3$). This phenomenological choice is not vital for the qualitative behavior of δg^{LSG}, but it does allow us to estimate an effective dephasing length L_ϕ', as we now explain. First we note that the (peak-to-peak) amplitude of the magnetoconductance δg for $\Phi^g=\frac{1}{2}$ is considerably reduced (by about a factor of 25) with respect to that for $\Phi^g=0$. As is clear by now, this is due to the η-dependent dephasing terms. Now, without such dephasing the Aharonov-Bohm amplitudes for $\Phi^g=0$ and $\Phi^g=1/2$ would be equal [see, e.g., Eq. (15), with a γ' that is η independent]. Thus the reduction of the Aharonov-Bohm amplitude at $\eta=\pi/3$ (relative to that at $\eta=0$) serves as a quantitative measure of the η-dependent dephasing. Expressed in terms of an effective dephasing length $L_\phi'=L/2\pi\sqrt{\gamma'}$, we find $L_\phi'=2.5\mu$m for the particular values chosen above (i.e., $\gamma'=0.5$, and $L=11\,\mu$m). This dephasing length should be compared with above value $L_\phi=L/2=5.5\,\mu$m (corresponding to $\gamma=0.1$ and $L=11\,\mu$m).

Finally, there is also the usual (spin-independent) dephasing arising from the field B_z penetrating a ring of finite width a. On the one hand, we need a sufficiently large field so as to reach adiabaticity, and on the other hand such a field can induce dephasing. Thus, to satisfy these conflicting requirements in an optimal way we should consider rings with a width a as small as possible. To obtain a rough estimate of such a width, we take $B_z=B\cos\eta$, for the field and insert this into the standard formula,[15] $L_\phi^{B_z}=\sqrt{3}\phi_0/2\pi aB_z$. We now require that this dephasing length should not become (much) smaller than L_ϕ^0, so we choose $L_\phi^{B_z}=L_\phi^0=5.5\,\mu$m. On the other hand, the field required for adiabaticity is about $B=200$ G, and, together with $L_\phi^{B_z}=5.5\,\mu$m and $\eta=\pi/3$, this corresponds to a ring width a of the order of 20 nm. Note that as the effective dephasing length is obtained via $1/(L_\phi^0)^2+1/(L_\phi^{B_z})^2$, the dephasing effect due to B_z penetrating the sample increases γ by a factor of 2 (i.e., $\gamma=0.2$). As seen from Fig. 2, the cases $\gamma=0.1$ and 0.2 behave in the same way, i.e., with phase shift, but the amplitude of δg for $\eta=\pi/3$ and $\gamma=0.2$ is now reduced by a factor of 52 compared with δg for $\eta=0$ and $\gamma=0.1$. (Note that for $\eta=0$ the magnetic field for the Aharonov-Bohm oscillations can be chosen to be very small, so that L_ϕ^0 dominates over $L_\phi^{B_z}$ and thus $\gamma=0.1$.) Finally, we note that the field component $B_z=B\cos\eta$ gives rise to an Aharonov-Bohm phase $\Phi_z=L^2B_z/4\pi$ that is, in general, not equal to $n\phi_0$ (with n integral). Therefore, this offset flux Φ_z must be accounted for in order to assign the above phase shift unambiguously to the Berry phase $\Phi^g=\frac{1}{2}$. For instance, for $L=11\,\mu$m, we need $B_z^0=4.2$ G in order to generate one flux quantum $\phi_0=h/e$ through the ring. Now consider $\eta=\pi/3$, and, say, $B=200$ G, i.e., $B_z=100$ G. To compensate for the offset Φ_z,

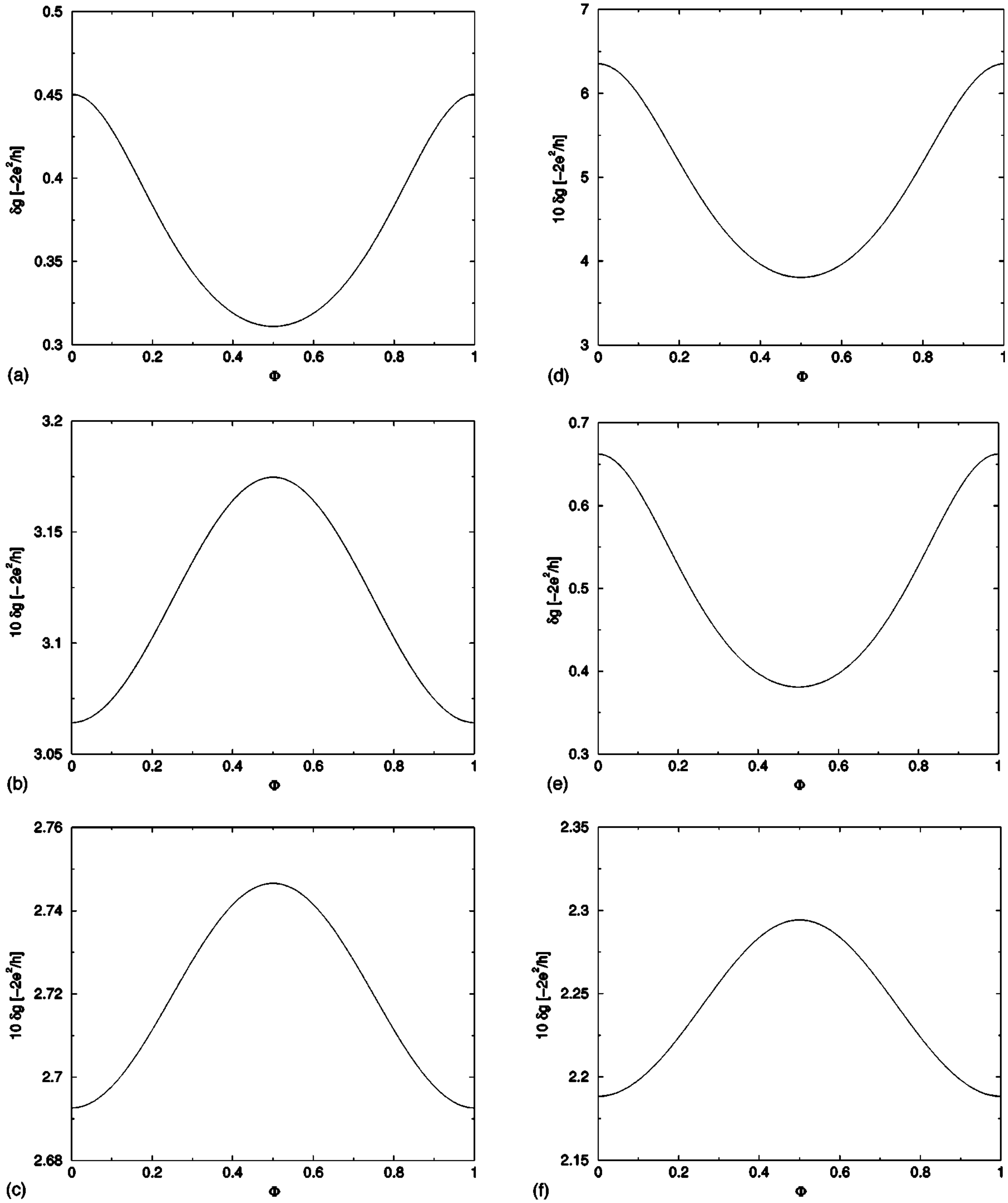

FIG. 2. The (dimensionless) magnetoconductance $\delta g/(-e^2/\pi\hbar)$ [Eq. (13)] as a function of Aharonov-Bohm flux $0\leqslant\Phi=2\phi/\phi_0\leqslant 1$. (a) shows δg at a vanishing Berry phase in the adiabatic regime (i.e., $\eta=\pi/2$), and with parameter values $\kappa=1$ and $\gamma=0.1$; (b) shows $10\delta g$ with a Berry phase of $\frac{1}{2}$ in the adiabatic regime, i.e., $\eta=\pi/3$, $f=1$, $\kappa=1$, and $\gamma=0.1$. Note the phase shift (due to the Berry phase) by the amount $\Phi=1/2$ between (a) and (b). (c) shows the same as (b), except that here $\gamma=0.2$ (this accounts for the dephasing due to B_z; see the text). (d) shows $\delta g/(-e^2/\pi\hbar)$ as a function of the Aharonov-Bohm flux $\Phi=2\phi/\phi_0$, but outside the adiabatic regime: $10\delta g$ at the Berry phase $\Phi^g=\frac{1}{2}$, i.e., $\eta=\pi/3$, $f=1$, $\kappa=0.1$, and $\gamma=0.1$. Note that there is no phase shift, which shows that the Berry phase is not yet in effect. (e) and (f) show the magnetoconductance in the adiabatic limit, $\delta g^{\mathrm{LSG}}/(-e^2/\pi\hbar)$ [Eq. (15)] as a function of the Aharonov-Bohm flux $\Phi=2\phi/\phi_0$. (e) shows δg^{LSG} with a vanishing Berry phase, i.e., $\eta=\pi/2$, and $\gamma'=0.1$; (f) shows $10\delta g^{\mathrm{LSG}}$ with a Berry phase of $\frac{1}{2}$, i.e., $\eta=\pi/3$, $f=1$, and $\gamma'=5\times 0.1$ (the increased γ' accounts for the η-dependent dephasing; see the text). Again there is a phase shift by $\Phi=1/2$ between (e) and (f), in full agreement with the adiabatic limit of δg as shown in (a) and (b).

we need to increase B_z by, say, 5 G to $B_z=105$ G, in which case $B_z/B_z^0=\Phi_z/\phi_0$ becomes an integer ($=25$).

The amplitude-reduction mentioned above demands sufficient experimental resolution, which we now estimate. For the parameter values given above for a Au ring and for $\eta=\pi/3$, we find [cf. Fig. 2(c)] that the peak-to-peak amplitude of δg is about $5.3\times 10^{-3}\times(e^2/\pi\hbar)$ for an effective $\gamma=0.2$. The relative ratio $\delta g/g\propto\delta R/R$ thus becomes of the order of

10^{-4} for a ring resistance $R \propto 1/g$ of the order of 30 $\times(L/\mu\mathrm{m})$ Ω,[20] and $L=11$ μm. Such sensitivity, as well as all the parameters estimated above, appear to be within present-day experimental reach. Further scenarios for the Berry phases in transport can be easily worked out (see also Ref. 7).

It should be obvious by now that the explicit agreement between δg in the adiabatic limit and δg^{LSG} demonstrates (and reinforces the general points made in the previous subsections) that the adiabaticity criterion [Eq. (1)] is sufficient for the existence of the Berry phase, and that, moreover, there exist physical regimes where this Berry phase can be observed in magnetoconductance oscillations (and other quantities). By contrast, the far more stringent criterion Eq. (2) is certainly not necessary, and therefore sets unwarranted demands on experimental searches for Berry phase effects.

E. Physical interpretation of the dephasing terms

We now briefly return to the issue of the source of dephasing in the Hamiltonian $h(j)$ given in Eq. (11), as well as its physical interpretation. For this purpose we assume from the outset that we are in the adiabatic regime, $\kappa \gg 1/(2\pi)^2$, and simply retain the leading contributions when finding the inverse of $\gamma - h(j)$. This allows us to identify those terms in the Hamiltonian h that are responsible for the dephasing.

From the matrix representation (12) of $h(j)$, it is straightforward to see that only those matrix elements are important in the adiabatic limit that are simultaneously either diagonal or off-diagonal in both spin subspaces. No other matrix elements contribute at the leading order κ^2 for the determinant or subdeterminants of $\gamma - h(j)$ that are necessary to calculate the inverse. Thus we can replace $h(j)$ by the matrix

$$-\begin{pmatrix} (j-f\cos\eta)^2+a & 0 & 0 & a \\ 0 & j^2+a-i2\kappa & 0 & 0 \\ 0 & 0 & j^2+a+i2\kappa & 0 \\ a & 0 & 0 & (j+f\cos\eta)^2+a \end{pmatrix}, \tag{19}$$

and we see that it is only the term $(f^2/2)(1+\sigma_{1x}\sigma_{2x})\sin^2\eta$ in $h(j)$ that causes dephasing and leads to those η-dependent terms in δg^{Ad} that are absent in δg^{LSG} (apart from the differences in γ and γ'). Now the first term, $(f^2/2)\sin^2\eta$, has already been identified in the discussion of the exact solution (for $f=1$) for a propagator containing only a single spin $\frac{1}{2}$.[5] In a general path-integral approach, this term has been interpreted as a consequence of quantum fluctuations: The particle trajectory fluctuates around its classical path and these fluctuations in turn lead to a fluctuating local magnetic field. Such fluctuations, however, violate the standard assumption underlying the adiabatic approximation that the field should vary smoothly as a function of its parameters [in the present case the parameter is given by the position $x(t)$ of the particle on the ring]. We have pointed out previously (see Sec. VI F in Ref. 5) that this term might lead to deviations from the adiabatic approximation, which is valid only for smooth variations.

The second term, $(f^2/2)\sigma_{1x}\sigma_{2x}\sin^2\eta$, is new, and describes an effective spin-spin interaction induced by the inhomogeneity of the magnetic field (i.e., in the Cooperon, the path and its time-reversed partner are interacting with each other via their respective spins). This interaction between spin 1 and spin 2 is transmitted via the orbital motion, and in this sense involves a back-reaction of the Zeeman term on the orbital motion. However, as pointed out in Sec. II C, such back-reactions that act to suppress the phase coherence are consistently assumed to be negligible in the semiclassical treatment. Thus, in Ref. 7 we performed an adiabatic approximation on propagators for the path and for its time-reversed partner separately and independently, and all possible dephasing effects are included phenomenologically in terms of γ' at the end. This finally explains the apparent discrepancy between δg^{LSG} and δg^{Ad}. However, as shown in previous sections, this discrepancy vanishes when allowing for η-dependent dephasing terms γ' in δg^{LSG}.

III. CONCLUSION

By using the exact solution for the Cooperon, we have shown that the Berry phase leads to observable effects in the magnetoconductance oscillation within the adiabatic regime defined by Eq. (1). This is in full agreement with previous findings,[7] and in contrast to recent claims.[9] We have pointed out the role of dephasing and emphasized its angle and winding dependences. We have illustrated the general discussion with explicit examples which support an optimistic outlook for the experimental search of the Berry phase in diffusive metallic samples.

ACKNOWLEDGMENTS

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[13] Finally, none of the effects discussed in terms of Boltzmann equations (Ref. 9) have been shown explicitly to be related to the Berry phase. Without such information at hand, it is not possible to decide whether the obtained field effects are of dynamical (non-phase-coherent) or geometrical (Berry phase) origin, as both of them can occur in an adiabatic approximation to the quantum dynamics. As we are interested in the Berry phase effect associated with phase coherence and occurring in physical observables, we shall not comment any further on the Boltzmann equation approach, and instead shall concentrate on the magnetoconductance only as expressed in terms of the Cooperon propagator (Ref. 7).
[14] Note that the complete Berry phase is usually defined as $f(1-\cos\eta)s$, with s the spin value and for constant tilt angle (cf. Ref. 7). For two spin-$\frac{1}{2}$ particles the total spin s is integral. The Berry phase is then equivalent to $\pm f(n-\cos\eta)$ (with $n=0,\pm1,\pm2,\ldots$) [see, e.g., Eq. (15)], and we shall simply refer to $\Phi^g=f\cos\eta$ as the Berry phase. In particular, $\Phi^g=0$ and $\Phi^g=1$ are equivalent in their effect, but they belong to different field configurations.
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[17] The additional factor of 2 in $\sin 2\eta$ is intended to provide a phenomenological account of the smaller period with respect to the Berry phase in the Cooperon resulting from the interference of the path with its time-reversed partner. For instance, when $\eta=\pi/2$, one can easily convince oneself that the fluctuations in the individual Berry fluxes of spin 1 and spin 2 must exactly cancel each other as the trajectories that the spins describe on the unit sphere in spin space are related by time-reversal symmetry. Thus the dephasing must be reduced at $\eta=\pi/2$. While certainly plausible, this argument is only of suggestive value, as it ignores the effective interaction between the paths (see Sec. II E). Also, we ignore for the moment the fact that for $f>1$ such large γ's actually violate the self-consistency of the semiclassical approximation (see Sec. II C).
[18] Note that if the suppression of phase coherence is only *partial*, the treatment is of course still self-consistent within the semiclassical regime. This is, e.g., the case for $f=1$ and $\eta=\pi/3$, for which the Berry phase is still visible albeit with a reduced amplitude (see Sec. II D).
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