Fractional exclusion statistics and the universal quantum of thermal conductance: A unifying approach

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We introduce a generalized approach to one-dimensional (1D) conduction based on Haldane's [Phys. Rev. Lett. **67**, 937 (1991)] concept of fractional exclusion statistics (FES) and the Landauer formulation [IBM J. Res. Dev. **1**, 223 (1957); Phys. Lett. **85A**, 91 (1981)] of transport theory. We show that the 1D ballistic thermal conductance is independent of the statistics obeyed by the carriers and is governed by the universal quantum $\kappa^{univ}=(\pi^2/3)(k_B^2T/h)$ in the degenerate regime. By contrast, the electrical conductance of FES systems is statistics dependent. This work unifies previous theories of electron and phonon systems, and explains an interesting commonality in their behavior. [S0163-1829(99)08119-9]

I. INTRODUCTION

Recent theoretical investigations of quantum transport have revealed an intriguing commonality in the behavior of some apparently very dissimilar systems: It has been predicted that in one dimension the low-temperature ballistic thermal conductances of ideal electron gases,[1,2] phonons,[3] and interacting electrons that form chiral[4] or normal[5] Luttinger liquids should all be quantized in integer multiples of a universal quantum $\kappa^{univ}=(\pi^2/3)(k_B^2T/h)$, where T is the temperature, k_B the Boltzmann constant, and h is Planck's constant. That is, a one-dimensional (1D) band populated with bosons described by a Planck distribution (phonon modes) has been predicted to transport the *same* amount of heat as one populated by fermions (the ideal electron gas) or a Luttinger liquid. Also, experimental evidence has been reported that ropes of single-walled nanotubes conduct heat in amounts proportional to κ^{univ}.[6] However each of these systems was studied separately using a different theoretical approach. Thus it has been unclear whether the convergence of the results that have been obtained is simply a coincidence or whether it has a deeper significance and broad ramifications. The purpose of this work is to resolve this question with the help of the concept of fractional exclusion statistics (FES), proposed by Haldane,[7] that allows one to discuss the behavior of bosons, fermions, and particles having fractional statistical properties, all on the same footing. Besides the universal thermal conductance, from this theory we also naturally obtain the quantized electrical conductance for ballistic electrons in 1D quantum wires and in the fractional quantum Hall (FQH) regime.

FES (Ref. 7) extends the concept of anyons,[8] i.e., particles with fractional statistics, from two dimensions to arbitrary spatial dimensions by introducing a generalization of the Pauli exclusion principle, and has yielded insights into fractional quantum Hall systems,[7,9] spinons in antiferromagnetic spin chains,[7] systems of interacting electrons in 2D quantum dots,[10] and the Calogero-Sutherland model.[11] In Haldane's sense the statistics of a system composed of different species of particles (or quasiparticles) is defined by the relation $\Delta d_i=-\Sigma_j g_{ij}\Delta N_j$, where N_i is the number of particles of species i, and d_i is the dimension of the N_i-particle Hilbert space, holding the coordinates of the N_i-1 particles fixed. The parameter g_{ij} is the statistical interaction. For a system of identical particles, g is a scalar quantity, with $g=1$ (0) for fermions (bosons). Wu[12] used this definition of FES to establish the statistical distribution function for an ideal gas of particles with fractional statistics. It has been proposed that such ideal FES gases provide an accurate representation of the physics of a number of interacting electron systems.[10,11]

While much attention has been given to the thermodynamic properties of FES systems,[11–16] their transport properties have not received the same consideration. In this paper we use the Landauer formulation of transport theory[17] to study conduction in ideal one-dimensional FES systems. Remarkably, we find that their low-temperature thermal conductance is quantized in integer multiples of the universal quantum $\kappa^{univ}=(\pi^2/3)(k_B^2T/h)$, *irrespective of the value of the statistical parameter* g_{ij}. Thus we demonstrate that the quantization of thermal conductance and the associated quantum are statistics independent and truly universal. By contrast we find the electrical conductances of FES systems to be statistics dependent.

II. SINGLE SPECIES

Consider a two-terminal transport experiment where two infinite reservoirs are adiabatically connected to each other by a one-dimensional channel. Each reservoir is characterized by a temperature (T) and a chemical potential (μ), considered to be independent variables. In the case of reservoirs with charged particles, μ can be redefined as the electrochemical potential, that is, a combination of the chemical potential and an electrostatic particle energy governed by an external field. In terms of T and μ the electric (I) and energy ($\dot{U}$) currents in the linear response regime are

$$\delta I=\frac{\partial I}{\partial\mu}\bigg|_T\delta\mu+\frac{\partial I}{\partial T}\bigg|_\mu\delta T, \tag{1}$$

$$\delta\dot{U}=\frac{\partial\dot{U}}{\partial\mu}\bigg|_T\delta\mu+\frac{\partial\dot{U}}{\partial T}\bigg|_\mu\delta T, \tag{2}$$

where $\delta T = T_R - T_L$ and $\delta\mu = \mu_R - \mu_L$, with R (L) representing the right (left) reservoir.

Using Landauer theory we write the fluxes between the two reservoirs as

$$I = \sum_n q \int_0^\infty \frac{dk}{2\pi} v_n(k)[\eta_R - \eta_L]\zeta_n(k), \tag{3}$$

$$\dot{U} = \sum_n \int_0^\infty \frac{dk}{2\pi} \varepsilon_n(k) v_n(k)[\eta_R - \eta_L]\zeta_n(k). \tag{4}$$

The sum over n takes into account the independent propagating modes admitted by the channel. $\varepsilon_n(k)$ and $v_n(k)$ are the energy and velocity of the particle with wave vector k, $\zeta_n(k)$ is the particle transmission probability through the channel, η_i represents the statistical distribution functions in the reservoirs, and q is the particle charge. In one dimension the particle velocity $v_n(k) = \hbar^{-1}(\partial\varepsilon_n/\partial k)$, is canceled by the 1D density of states $\mathcal{D}(\varepsilon_n) = \partial k/\partial\varepsilon_n$, and the fluxes become independent of the dispersion:

$$I = \frac{q}{h} \sum_n \int_{\varepsilon_n(0)}^\infty d\varepsilon [\eta_R - \eta_L]\zeta_n(\varepsilon), \tag{5}$$

$$\dot{U} = \frac{1}{h} \sum_n \int_{\varepsilon_n(0)}^\infty d\varepsilon\, \varepsilon [\eta_R - \eta_L]\zeta_n(\varepsilon). \tag{6}$$

Throughout the remainder of this paper we will assume $\zeta_n(\varepsilon) = 1$, which corresponds to ballistic transport and a perfectly adiabatic coupling between the reservoirs and the 1D system. This assumption can be considered realistic in view of the present stage of the mesoscopics technology. Substitution of expressions (5) and (6) for the fluxes into Eqs. (1) and (2), while taking the limit $\delta T \to 0$ and $\delta\mu \to 0$, gives us the transport coefficients.

Having introduced the model, we consider systems of generalized statistics, which can be investigated within FES theory. Initially we concentrate on identical particle systems and the distribution function derived by Wu[12] for an ideal gas of particles obeying FES,

$$\eta_g = \frac{1}{\mathcal{W}(x,g) + g}, \tag{7}$$

with $x \equiv \beta(\varepsilon - \mu)$, $\beta \equiv 1/(k_B T)$, and $\mathcal{W}(x,g)$ given by the implicit equation

$$\mathcal{W}^g(x,g)[1 + \mathcal{W}(x,g)]^{1-g} = e^x. \tag{8}$$

Making $g = 0$ or 1, Eq. (7) becomes the Bose-Einstein or Fermi-Dirac distribution function, respectively.

For a system of generalized statistics, the transport coefficients are

$$L_{11} = \left.\frac{\partial I}{\partial \mu}\right|_T = \frac{q}{h} \sum_n \int_{x_{0n}}^\infty dx F(x,g), \tag{9}$$

$$L_{12} = \left.\frac{\partial I}{\partial T}\right|_\mu = \frac{q}{h} k_B \sum_n \int_{x_{0n}}^\infty dx\, x F(x,g), \tag{10}$$

$$L_{21} = \left.\frac{\partial \dot{U}}{\partial \mu}\right|_T = \frac{1}{h\beta} \sum_n \int_{x_{0n}}^\infty dx (x + \mu\beta) F(x,g), \tag{11}$$

$$L_{22} = \left.\frac{\partial \dot{U}}{\partial T}\right|_\mu = \frac{k_B}{h\beta} \sum_n \int_{x_{0n}}^\infty dx (x^2 + x\mu\beta) F(x,g), \tag{12}$$

with $x_{0n} \equiv \beta(\varepsilon_n(0) - \mu)$ and

$$F(x,g) = \frac{\mathcal{W}(x,g)[\mathcal{W}(x,g) + 1]}{[\mathcal{W}(x,g) + g]^3}. \tag{13}$$

Fermions and bosons are the special cases of the theory; however, our interest is to develop a formalism able to treat all FES systems. Analytic solutions of Eq. (8) can also be obtained for the special cases $g = \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 2, 3$, and 4, but for general g the approach of analytically solving the equation for $\mathcal{W}$ is not possible. We now present a comprehensive method to treat this problem. Initially, solve Eq. (8) for $x = \beta(\varepsilon - \mu)$

$$x(\mathcal{W},g) = \ln(\mathcal{W}+1) + [\ln(\mathcal{W}) - \ln(\mathcal{W}+1)]g. \tag{14}$$

We notice that $\lim_{\mathcal{W}\to 0} x = -\infty$ for $g \neq 0$, which corresponds to the lowest energy for the degenerate nonbosons. Moreover, $\lim_{\mathcal{W}\to 0} x = 0$ when $g = 0$, which corresponds to the lowest energy modes of bosons described by the Planck distribution. On the other hand, $\lim_{\mathcal{W}\to\infty} x = \infty$ for any $g \geqslant 0$. This shows that x can be supplanted by $\mathcal{W}$ as the variable of integration in our general FES expressions for L_{ij}, with $\mathcal{W}$ ranging from 0 to infinity. Notice that no other specification on the functional form of the particle spectra is made in this derivation. Then, using Eq. (14), we can write

$$F(x,g)dx = \frac{d\mathcal{W}}{(\mathcal{W}+g)^2}. \tag{15}$$

The transport coefficients L_{ij} can then be evaluated analytically for arbitrary g. When $g > 0$,

$$L_{11} = M \frac{q}{h} \int_0^\infty \frac{d\mathcal{W}}{(\mathcal{W}+g)^2} = M \frac{q}{h} \frac{1}{g}, \tag{16}$$

$$L_{12} = M \frac{q}{h} k_B \int_0^\infty d\mathcal{W} \frac{x(\mathcal{W},g)}{(\mathcal{W}+g)^2} = 0, \tag{17}$$

$$L_{21} = M \frac{1}{h\beta} \int_0^\infty d\mathcal{W} \frac{x(\mathcal{W},g) + \mu\beta}{(\mathcal{W}+g)^2} = M \frac{\mu}{h} \frac{1}{g}, \tag{18}$$

and, for all $g \geqslant 0$,

$$L_{22} = M \frac{k_B^2 T}{h} \int_0^\infty d\mathcal{W} \frac{x^2(\mathcal{W},g) + \mu\beta x(\mathcal{W},g)}{(\mathcal{W}+g)^2} = M \frac{k_B^2 T}{h} \frac{\pi^2}{3}, \tag{19}$$

with $x(\mathcal{W},g)$ given by Eq. (14). Here M is an integer number that takes into account the number of occupied modes (assuming a degenerate population in each one of them). Therefore, the transport equations for a system of identical particles of generalized statistics are

$$\delta I = \frac{1}{g}\frac{q}{h} M\,\delta\mu, \tag{20}$$

$$\delta \dot{U} = \frac{1}{g}\frac{\mu}{h} M\,\delta\mu + \frac{\pi^2}{3}\frac{k_B^2 T}{h} M\,\delta T. \tag{21}$$

One important result that we obtain with this formalism is the universal thermal conductance, *valid for all ballistic FES systems*. Since the electrochemical potential is an independent variable in this model, we can set $\delta\mu=0$, so that no electric current flows between the reservoirs. This also eliminates the energy flow that is due to a net flux of particles between the two reservoirs, leaving us with only the coefficient L_{22}. In this case the energy current is equal to the heat current that is generated by δT, and so the 1D universal thermal conductance is

$$\kappa^{univ} = \frac{\pi^2}{3}\frac{k_B^2 T}{h}. \tag{22}$$

Therefore a 1D subband populated with bosons described by the Planck distribution transports the same amount of heat as one populated by fermions, despite the fact these systems have very different statistical behaviors. The thermopower vanishes because of the assumptions made: degenerate systems and unitary transmission coefficients independent of the energy. For Planck bosons, μ is not a parameter describing the system; therefore, only L_{22} is present, and result (22) is recovered.

We note that, in contrast to the thermal conductance, the 1D ballistic electrical conductance is *not* statistics independent, since g appears explicitly in Eq. (20) for the electric current. For instance, when $\delta T=0$, the fermion case is readily obtained by setting $g=1$, and we obtain the well-known 1D electrical conductance $G=(e^2/h)M$ for ballistic electrons.

Equations (20) and (21) should also describe the transport properties of the Laughlin states of the FQH effect, for which the Landau-level filling fraction $\nu=1/(2m+1)$ with m integer. We use the composite fermion (CF) picture[18] to derive the statistical interaction parameter g of these particles. Integrating the expression $\Delta d=-g\Delta N$ for $g=g^{CF}$, we obtain $d_N=d_0^{CF}-g^{CF}(N-1)$, where d_N is the dimension of the one particle Hilbert space when N composite fermions exist in the system, whereas d_0^{CF} is its analog in the absence of CF. The term d_0^{CF} is the degeneracy of the CF Landau level, that can be written in terms of the CF density as $d_0^{CF}=(eB/hc)-2mN$, with B representing the external magnetic field. Using this relation in the expression for d_N along with the fact that the CF's behave like fermions ($g^{CF}=1$), we obtain $d_N=(eB/hc)-(2m+1)N+1$. This means that, from the perspective of the FES theory, the transport properties of these states are due to particles of charge $|q|=e$, and a fractional exclusion rule given by $g=(2m+1)$ in the thermodynamic limit. In other words, each electron added to the system excludes $2m+1$ single-particle states. Returning to the transport equations, substitution of $g=(2m+1)$ in Eq. (20) immediately gives us the well-known values of the conductance plateaus of the Laughlin states, $G=[1/(2m+1)](e^2/h)$. Since on the FQH plateaus the two-terminal conductance is equal to the quantized Hall conductance, this result is in agreement with experimental data[20] on FQH devices. It is important to mention that, since we are concerned with transport, our analysis applies to the electrons themselves as distinct from the quasiparticle excitations studied in Ref. 19, which was concerned with thermodynamics.

The universality presented by the degenerate 1D systems at finite T can be physically understood if we consider the total energy flux for a single band,

$$\dot{U} = \frac{\mu^2}{2gh} + \frac{\pi^2}{6}\frac{(k_B T)^2}{h} = \dot{U}_{pot} + \dot{U}_{thermal}, \tag{23}$$

which shows that the energy current flowing through the one-dimensional system can be divided into two independent components: one due exclusively to the flux of particles and carrying no heat ($\dot{U}_{pot}$), and the other entirely determined by the temperature of the emitting reservoir irrespective of the number of particles ($\dot{U}_{thermal}$). The last term gives rise to the thermal conductance being the same for Planck bosons and all other FES particles. This division is possible because of the cancellation of the density of states by the particle velocity in the 1D system along with the degenerate condition of the system. On the other hand, the electric current for degenerate systems depends only on the number of particles regardless their temperature, which leads to $L_{12}=0$.

III. GENERALIZED EXCLUSION AND FQH EQUATION

In Sec. II we have shown that the generalized exclusion approach for a system of identical particles leads naturally to the transport coefficients of the Laughlin fractions of the FQH effect. In the remainder of this paper we extend this formalism to treat systems composed of multiple species with a mutual statistical interaction acting among them.

A. Exclusion statistics for various species

In its most general form the occupation numbers η_i of each species that assembles into an ideal gas of FES particles are given by

$$\mathcal{W}_i = \frac{1}{\eta_i} - \sum_{j=1}^{S} g_{ij}\frac{\eta_j}{\eta_i} \tag{24}$$

and

$$(1+\mathcal{W}_i)\prod_{j=1}^{S}\left(\frac{\mathcal{W}_j}{\mathcal{W}_j+1}\right)^{g_{ji}} = e^{x_i}, \tag{25}$$

where $x_i=\beta_i(\epsilon_i-\mu_i)$, and S is the number of species. Details of this derivation can be found in Ref. 12.

To proceed with the construction of the statistics of the model, it is convenient to introduce the actual values of g_{ij}. To do so we use Jain's composite fermion picture[18] and the generalized exclusion principle

$$G_{eff,i} = G_i - \sum_j g_{ij}(N_j-\delta_{ij}). \tag{26}$$

The main properties of the FQH effect can be understood if we attach an even number of fictitious flux quanta to each single electron by a Chern-Simons gauge transformation.[21] In this picture a dressed particle is formed which has the same charge as well as the statistical properties of the electron. In the mean-field approximation the CF's form a Fermi liquid. From this perspective the FQH effect is then seen as the integer quantum Hall effect (IQHE) of the CF particles, which experience an *effective* magnetic field that depends on the density of carriers $B_{eff}=B-2mN\Phi_0$, where B is the external magnetic field, $\Phi_0=ch/e$ is the quantum of magnetic flux, and N is the total density of CFs in the mean-field approximation (the same as the electronic density). Therefore, according to this picture, the quasi-Landau-levels (qLL's) occupied by the CF have a degeneracy that is

$$G^{CF}=B_{eff}/\Phi_0=B/\Phi_0-2m\sum_{j=1}^{S} N_j, \tag{27}$$

with the index j representing the qLL index. Moreover, because CF's are fermions, we have $g_{ij}^{CF}=\delta_{ij}$, and expression (26) can be written as

$$G_{eff,i}=G_i^{CF}-\sum_j g_{i,j}^{CF}(N_j-\delta_{ij})$$

$$=\frac{B}{\Phi_0}-\sum_j (2m+\delta_{ij})N_j+1. \tag{28}$$

Regrouping the elements according to the densities N_j of each qLL, we obtain $g_{ii}=2m+1$ and $g_{ij}=2m$, for $i\neq j$. In the FES theory the diagonal terms are the self-interaction parameters and rule the exclusion properties among particles of the same species, whereas the nondiagonal terms are the statistical mutual interaction parameters which describe the exclusion relations among particles of different species. In this case the population of each qLL is viewed as a distinct species. What expression (28) shows is that we can incorporate the physics of Jain's fractions $\nu=p/(2mp+1)$ (m and p integers) into the generalized exclusion principle of particles in the FES theory. These particles have the same charge as the electron ($|q|=e$), but their exclusion statistics is governed by g_{ij}.

The knowledge of g_{ij} allows us to solve Eq. (24) to obtain the occupation functions η_i for each species. For a system composed of a number S of species that obey the exclusion rules derived above, we have

$$\eta_i=\frac{\prod_{j\neq i}^{S} (\mathcal{W}_j+1)}{\prod_{j=1}^{S} (\mathcal{W}_j+2m+1)-\Lambda}, \tag{29}$$

with

$$\mathcal{W}_i\prod_{j=1}^{S}\left(\frac{\mathcal{W}_j}{\mathcal{W}_j+1}\right)^{2m}=e^{x_i}. \tag{30}$$

The quantity Λ is given by the series

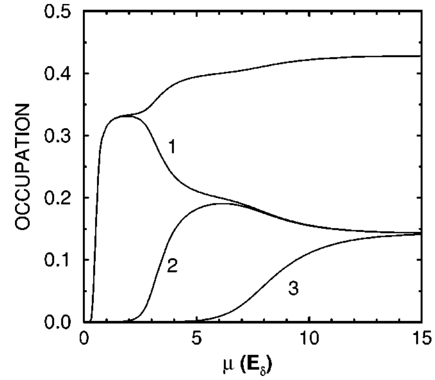

FIG. 1. Occupation values of the three lowest qLL's as a function of the generalized chemical potential. The indices indicate the occupation due to each level, and the highest curve is the total occupation. The energy of the qLL's are $E_1=\frac{1}{2}E_\delta$, $E_2=\frac{3}{2}E_\delta$ and $E_3=\frac{5}{2}E_\delta$. The occupation and E_δ are obtained self-consistently for $m=1$.

$$\Lambda=\lambda_0+\lambda_1\sum_{j=1}^{S} \mathcal{W}_j+\lambda_2\sum_j \sum_{k<j} \mathcal{W}_j\mathcal{W}_k+\cdots, \tag{31}$$

whose coefficients are ($l=0,1,2,\ldots,S-1$)

$$\lambda_l=(2m+1)^{S-l}-[2(S-l)m+1]. \tag{32}$$

According to the equations above, $\lim_{\mathcal{W}_i\to\infty}\eta_i=0$, leaving us with $S-1$ species in the system. In this case, Eqs. (29) and (30) will automatically converge to represent a system with the shortage of one species. However, due to the energy structure of the CF's there is a hierarchy in the values of $\mathcal{W}_i$. The ratio $\mathcal{W}_{i+1}/\mathcal{W}_i$ can be obtained from Eq. (30),

$$\frac{\mathcal{W}_{i+1}}{\mathcal{W}_i}=e^{x_{i+1}-x_i}=e^{\beta[\varepsilon_{i+1}(k)-\varepsilon_i(k)]}, \tag{33}$$

where we have assumed a common temperature and chemical potential for all species. Therefore, we see that if $\varepsilon_i(k)$ is the energy of the ith qLL, then $\mathcal{W}_i<\mathcal{W}_{i+1}$.

If this model is intended to reproduce the behavior of CF's some caution is necessary because the gap energies depend on the total density $N(\vec{x})$ self-consistently:

$$E_\delta=\hbar\omega_{eff}=\frac{\hbar|e|}{m^*c}B\left[1-2m\sum_j N_j(\vec{x})\right]. \tag{34}$$

In Fig. 1 we represent the occupation values of the three lowest qLL's as a function of the generalized chemical potential μ of the FES particles. The parameter $m=1$ and the density is assumed to be homogeneous. The occupation values were calculated self-consistently using Eqs. (29), (30), and (34). The generalized chemical potential is given in units of E_δ, and the temperature is defined by $\beta E_\delta^0=30$, where E_δ^0 is given by Eq. (34) when $N_j(\vec{x})=0$. We see that the occupation values have plateaus that correspond to the fractions $1/(2pm+1)$. These become poorly defined as the chemical potential rises, since E_δ decreases with a increasing density.

Having defined the statistical properties of these particles, we procced with a calculation of the transport coefficients in Sec. III B.

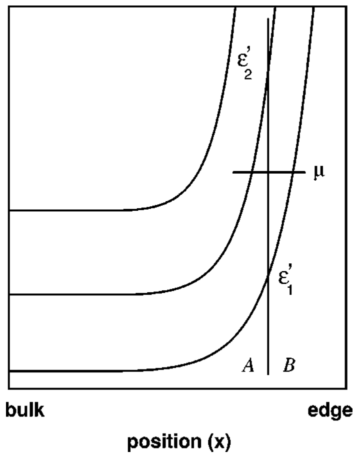

FIG. 2. This scheme of the band structure shows the energy dispersion of the three lowest bands as a function of position, from the bulk toward the right edge of the sample. The generalized chemical potential is indicated by the horizontal line. The vertical line is a reference. The energy values ϵ_2' and ϵ_1' indicate the points where this reference line crosses the bands.

B. Transport coefficients

The transport coefficients L_{ij} for any filling factor $\nu = p/(2pm+1)$ can be obtained numerically using the distributions η_i, these being calculated self-consistently with the gap energy. Nonetheless, for degenerate conditions analytical solutions are possible.

The result of Sec. II for the Laughlin fractions can be obtained from the general formalism above when we make $\mathcal{W}_i \to \infty$ for all $i \neq 1$. In this limiting case,

$$\eta_1 = \frac{1}{\mathcal{W}_1 + 2m + 1}, \tag{35}$$

with $g = 2m+1$.

We now consider the situation in which two qLL's are populated. This means that $\eta_i = 0$ for $i > 2$ and

$$\eta_1 = \frac{\mathcal{W}_2 + 1}{(\mathcal{W}_1 + 2m + 1)(\mathcal{W}_2 + 2m + 1) - 4m^2}, \tag{36}$$

$$\eta_2 = \frac{\mathcal{W}_1 + 1}{(\mathcal{W}_1 + 2m + 1)(\mathcal{W}_2 + 2m + 1) - 4m^2}, \tag{37}$$

where η_1 describes the lowest band and η_2 the highest one. Relation (33) is now

$$\mathcal{W}_1 = \mathcal{W}_2 e^{-\beta E_\delta(k)}, \tag{38}$$

where $E_\delta(k)$ is the wave-vector-dependent spectral gap between the bands

$$E_\delta(k) = \epsilon_2(k) - \epsilon_1(k). \tag{39}$$

For positive $E_\delta(k)$, at low temperatures, $\beta E_\delta(k) \gg 1$, which leads to the other important relation $\mathcal{W}_1 \ll \mathcal{W}_2$.

In Fig. 2, a schematic band structure shows the energy dispersion of the three lowest bands as a function of position, from the bulk toward the right edge of the sample. The generalized chemical potential is indicated by the horizontal line. The vertical line is a reference, it divides the band structure into regions $\mathcal{A}$ and $\mathcal{B}$, whose meaning will be discussed bellow. The energy values ϵ_2' and ϵ_1' indicate the points where this reference line crosses the bands. We assume that the system is degenerate, so that $\beta\epsilon_1' \ll \beta\mu$ and $\beta\epsilon_2' \gg \beta\mu$. Notice that the generalized chemical potential (μ) is above the lowest unoccupied state and lies over the third qLL band. This does not mean, however, that this level is populated. It occurs because the statistical mutual interaction modifies the electronic chemical potential.

Because of the mutual statistical interaction, the occupation functions η_i depend on both $\epsilon_2(k)$ and $\epsilon_1(k)$. We take advantage of the degenerate condition of the system to circumvent this difficulty. Since composite fermions at the same position in space have the same k, if $k_B T \ll (\epsilon_2' - \epsilon_1')$ each $\mathcal{W}_i$ should go from 0 to ∞ at different values of k, although around the same energy (μ). In other words, the transitions of $\mathcal{W}_i$ from 0 to ∞ are decoupled. We have *region* $\mathcal{A}$, where $\mathcal{W}_1 \ll 1$, while $\mathcal{W}_2$ makes the transition $0 \to \infty$; and *region* $\mathcal{B}$, where $\mathcal{W}_2 \gg 1$, while $\mathcal{W}_1$ makes the transition $0 \to \infty$.

Therefore, the following approximations are possible. Focusing initially on the highest band, in region $\mathcal{A}$,

$$\eta_2(k) = \frac{1}{(2m+1)\mathcal{W}_2 + 4m + 1}, \tag{40}$$

$$\frac{\mathcal{W}_2^{4m+1}}{(\mathcal{W}_2 + 1)^{2m}} = e^{x_2(k) + 2m\beta E_\delta(k)}, \tag{41}$$

while $\eta_2 = 0$ in region $\mathcal{B}$. Thus integration over region $\mathcal{A}$ is enough to give us I_2 and $\dot{U}_2$ for the highest edge state.

For the lowest band the entire energy dispersion ought to be considered. In region $\mathcal{A}$,

$$\eta_1^{\mathcal{A}}(k) = \frac{\mathcal{W}_2 + 1}{(2m+1)\mathcal{W}_2 + 4m + 1}, \tag{42}$$

with $\mathcal{W}_2(k)$ given by Eq. (41). In region $\mathcal{B}$,

$$\eta_1^{\mathcal{B}}(k) = \frac{1}{\mathcal{W}_1 + 2m + 1}, \tag{43}$$

$$\frac{\mathcal{W}_1^{2m+1}}{(\mathcal{W}_1 + 1)^{2m}} = e^{x_1(k)}. \tag{44}$$

Therefore, for this band we have $I_1 = I_1^{\mathcal{A}} + I_1^{\mathcal{B}}$ and $\dot{U}_1 = \dot{U}_1^{\mathcal{A}} + \dot{U}_1^{\mathcal{B}}$.

Consider the current due to the highest band,

$$I_2 = q \int_0^\infty \frac{dk}{2\pi} v_2(k)\, \eta_2(k), \tag{45}$$

with η_2 given by Eq. (40). It is convenient to define the variable $E_2(k) = \epsilon_2(k) + 2mE_\delta(k)$, in terms of which we write the velocity

$$v_2(k) = \frac{1}{\hbar}\frac{d\epsilon_2}{dk} = \frac{1}{\hbar}\left(\frac{dE_2}{dk} - 2m\frac{dE_\delta}{dk}\right). \tag{46}$$

Using E_2 as the variable of integration, I_2 is rewritten as

$$I_2=\frac{q}{h}\int_{E_2^0}^{\infty}\left[1-2m\frac{dE_\delta}{dE_2}\right]\eta_2(E_2)dE_2. \tag{47}$$

The lower limit of integration $E_2^0=\epsilon_2^0+2mE_2^{bulk}$ is a constant, with E_2^{bulk} representing the value of the gap in the bulk of the sample.

Another transformation eliminates the explicit dependence on the energy. From Eq. (41), we obtain E_2 as a function of $\mathcal{W}_2$,

$$\frac{dE_2}{d\mathcal{W}_2}=\frac{1}{\beta F}\frac{dF}{d\mathcal{W}_2}, \tag{48}$$

with

$$F(\mathcal{W}_2)\equiv\frac{\mathcal{W}_2^{4m+1}}{(\mathcal{W}_2+1)^{2m}}. \tag{49}$$

The gap E_δ can also be written in terms of $\mathcal{W}_2$. With this purpose we return to the CF picture that gives us $E_\delta=\hbar\omega_{eff}=\hbar(eB_{eff}/m^*c)$. The effective magnetic field depends on the local density of particles and so does the gap. Therefore,

$$E_\delta(k)=\frac{\hbar|q|}{m^*c}B[1-2m\nu] \tag{50}$$

$$=E_\delta^0\{1-2m[\eta_1(k)+\eta_2(k)]\}, \tag{51}$$

where we have defined $E_\delta^0=(\hbar|q|B/m^*c)$. In the limit $\mathcal{W}_1\ll 1$, which characterizes region $\mathcal{A}$,

$$\eta_1+\eta_2\approx\frac{\mathcal{W}_2+2}{(2m+1)\mathcal{W}_2+4m+1}. \tag{52}$$

We are now able to write I_2 in a form that is independent of the details of the particle spectra. Substituting expressions (48)–(52) into Eq. (47), we obtain

$$I_2=\frac{q}{h\beta}\lim_{\mathcal{W}_2^0\to 0}\int_{\mathcal{W}_2^0}^{\infty}\left[\frac{1}{F}\frac{dF}{d\mathcal{W}_2}+a_m\frac{d(\eta_1+\eta_2)}{d\mathcal{W}_2}\right]\eta_2(\mathcal{W}_2)d\mathcal{W}_2, \tag{53}$$

where $a_m=4m^2\beta E_\delta^0$. Separating this expression, the integration of the first part gives us

$$\frac{q}{h\beta}\int_{\mathcal{W}_2^0}^{\infty}\frac{\eta_2}{F}\frac{dF}{d\mathcal{W}_2}d\mathcal{W}_2=\frac{q}{h\beta}[\ln(\mathcal{W}_2^0+1)-\ln(\mathcal{W}_2^0)], \tag{54}$$

whereas the integration of the second term produces a quantity independent of temperature and chemical potential when we make $\mathcal{W}_2^0=0$, which will make no contribution to the transport coefficients. The limit $\mathcal{W}_2^0\to 0$ of expression (54) can be obtained from Eq. (41), which gives us

$$\mathcal{W}_2^0=e^{\beta(E_2^0-\mu)/(4m+1)}(\mathcal{W}_2^0+1)^{2m/(4m+1)}. \tag{55}$$

Substituting this result into Eq. (54), the limit $\mathcal{W}_2^0\to 0$ of I_2 can be easily obtained. It depends only on the chemical potential and, therefore, the coefficients that describe the electrical current for the highest band are

$$\frac{\partial I_2}{\partial\mu}=\frac{1}{4m+1}\frac{q}{h},\quad \frac{\partial I_2}{\partial T}=0. \tag{56}$$

The electrical current I_1 due to the lowest band is obtained by the same approach. We perform the integration dividing the whole domain in two regions (see Fig. 2). Along the first portion, designated by $\mathcal{A}$, $\mathcal{W}_1\ll 1$ throughout the range, whereas $\mathcal{W}_2$ increases from $\mathcal{W}_2\ll 1$ to $\mathcal{W}_2\gg 1$. In the second part, represented by $\mathcal{B}$, $\mathcal{W}_2$ remains very large while $\mathcal{W}_1$ makes the transition $\mathcal{W}_1\ll 1$ to $\mathcal{W}_1\gg 1$.

Along region $\mathcal{A}$ the occupation η_1 is given by Eq. (42) and the group velocity for this band is

$$v_1(k)=\frac{1}{\hbar}\frac{d\epsilon_1}{dk}=\frac{1}{\hbar}\left(\frac{dE_2}{dk}-(2m+1)\frac{dE_\delta}{dk}\right). \tag{57}$$

Using Eq. (57) along with the identities (48)–(52), we are able to write

$$I_1^{\mathcal{A}}=\frac{q}{h}\int_{\mathcal{A}}\left[1-(2m+1)\frac{dE_\delta}{dE_2}\right]\eta_1^{\mathcal{A}}(E_2)dE_2 \tag{58}$$

$$=\frac{q}{h\beta}\lim_{\mathcal{W}_2^0\to 0}\int_{\mathcal{W}_2^0}^{\mathcal{W}_2'}\left[\frac{1}{F}\frac{dF}{d\mathcal{W}_2}+b_m\frac{d(\eta_1+\eta_2)}{d\mathcal{W}_2}\right]\eta_1^{\mathcal{A}}(\mathcal{W}_2)d\mathcal{W}_2, \tag{59}$$

where $b_m=2m(2m+1)\beta E_\delta^0$, and $\mathcal{W}_2'=\mathcal{W}(\varepsilon_2')$ is independent of the thermodynamical parameters for degenerate systems.

Once again the second term in the integrand will produce a quantity that is independent of temperature and chemical potential, and therefore irrelevant for the transport coefficients. On the other hand, the first term is

$$\frac{q}{h\beta}\int_{\mathcal{W}_2^0}^{\mathcal{W}_2'}\frac{\eta_1^{\mathcal{A}}}{F}\frac{dF}{d\mathcal{W}_2}d\mathcal{W}_2=\frac{q}{h\beta}[\ln(\mathcal{W}_2')-\ln(\mathcal{W}_2^0)]. \tag{60}$$

Then, using Eq. (55), we obtain

$$\frac{\partial I_1^{\mathcal{A}}}{\partial\mu}=\frac{1}{4m+1}\frac{q}{h},\quad \frac{\partial I_1^{\mathcal{A}}}{\partial T}=0. \tag{61}$$

As for region $\mathcal{B}$, its contribution to the current due to the lowest band is

$$I_1^{\mathcal{B}}=\frac{q}{h\beta}\int_{\mathcal{W}_1'}^{\infty}\frac{\eta_1^{\mathcal{B}}}{H}\frac{dH}{d\mathcal{W}_1}d\mathcal{W}_1, \tag{62}$$

with

$$H(\mathcal{W}_1)=\frac{\mathcal{W}_1^{2m+1}}{(\mathcal{W}_1+1)^{2m}}. \tag{63}$$

The lower limit $\mathcal{W}_1'=\mathcal{W}_1(\varepsilon_1')$ corresponds to a continuation from $\mathcal{W}_2'$ (see Fig. 2). The differentiation of Eq. (62) with respect to the thermodynamical parameters vanishes.

Therefore, we sum up the results of this calculation as

$$L_{11}=\frac{\partial(I_1+I_2)}{\partial\mu}=\frac{2}{4m+1}\frac{q}{h}, \tag{64}$$

$$L_{12}=0. \tag{65}$$

The calculations leading to the coefficients L_{21} and L_{22} are not presented since they follow the same formalism. However, we write the final results

$$\delta I=\frac{2}{4m+1}\frac{q}{h}\delta\mu \tag{66}$$

$$\delta\dot{U}=\left\{\frac{2}{4m+1}\frac{\mu}{h}+C_m\frac{E_\delta}{h}\right\}\delta\mu+2\frac{\pi^2}{3}\frac{k_B^2T}{h}\delta T, \tag{67}$$

where C_m is a nonuniversal coefficient that depends on m. Although μ differs from the electrochemical potential $\delta\mu$ does not when the system is in a FQH plateau, as shown by Eq. (66). Despite the statistical coupling between the two CF quasi-Landau-levels, once again the universal quantum of thermal conductance is obtained, this time multiplied by 2, which reflects the presence of the two modes of propagation. The two-terminal electrical conductance $G=2/(4m+1)\times(e^2/h)$ is also obtained for this family of states, in agreement with experiment.

IV. CONCLUSIONS

In conclusion, we have presented a generalized theory of transport of 1D systems. We have shown that the ballistic thermal conductance of one-dimensional systems is statistics independent and thus truly universal: $\kappa^{univ}=(\pi^2/3)(k_B^2T/h)$. This result is valid in the degenerate regime for systems of particles obeying fractional exclusion statistics, whether they present a Fermi surface or are described by a Planck distribution. Electrical conductances for ballistic electrons in 1D quantum wires and in the FQH regime, although not universal (in the sense that they depend on statistics), also follow naturally from this theory.

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