Noise in multiterminal diffusive conductors: Universality, nonlocality, and exchange effects

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We study noise and transport in multiterminal diffusive conductors. Using a Boltzmann-Langevin equation approach we reduce the calculation of shot-noise correlators to the solution of diffusion equations. Within this approach we prove the universality of shot noise in multiterminal diffusive conductors of arbitrary shape and dimension for purely elastic scattering as well as for hot electrons. We show that shot noise in multiterminal conductors is a nonlocal quantity and that exchange effects can occur in the absence of quantum phase coherence even at zero electron temperature. It is also shown that the exchange effect measured in one contact is always negative — in agreement with the Pauli principle. We discuss a new phenomenon in which current noise is induced by thermal transport. We propose a possible experiment to measure locally the effective noise temperature. Concrete numbers for shot noise are given that can be tested experimentally. [S0163-1829(99)06019-1]

I. INTRODUCTION

Shot noise is a nonequilibrium fluctuation of the current in mesoscopic conductors caused by random flow of the charge. It can be thought of as an uncorrelated Poisson process[1] giving rise to a simple formula for the spectral density of the shot noise, $S^c = eI$, where I is the current through the conductor and e is the electron charge. Being the result of charge quantization, the shot noise is an interesting and highly nontrivial physical phenomenon.[2] In contrast to the thermal fluctuations of the current, the shot noise provides important information about microscopic transport properties of the conductors beyond the linear response coefficients such as the conductance. For instance, the shot noise serves as a sensitive tool to study correlations in conductors: while shot noise assumes the Poissonian value in the absence of correlations, it becomes suppressed when correlations set in as, e.g., imposed by the Pauli principle.[3–7] In particular, the shot noise is completely suppressed in ballistic conductors,[8] and it appears thus only in the presence of a disorder.

In diffusive mesoscopic two-terminal conductors where the inelastic scattering lengths exceed the system size the shot-noise suppression factor for ''cold'' electrons (i.e., for vanishing electron temperature) was predicted[9–14] to be 1/3. The suppression of shot noise in diffusive conductors is now experimentally confirmed.[15–19] While some derivations are based on a scattering matrix approach[9,11] or conventional Green's function technique[12,13] and thus *a priori* include quantum phase coherence, no such effects are included in the semiclassical Boltzmann-Langevin equation approach, which nevertheless leads to the same result.[10,14] However, while in the quantum approach for a two-terminal conductor the factor 1/3 was even shown to be universal,[12] the semiclassical derivations given so far[10,2] are restricted to quasi-one-dimensional conductors. Thus, although phase coherence is believed not to be essential for the suppression of shot noise,[20] the equivalence of different approaches for calculating noise in mesoscopic conductors is not evident. In the regime of hot electrons the noise suppression factor was found[21,22] to be $\sqrt{3}/4$. Again, this result, which is based on a Boltzmann-Langevin equation approach, is restricted to quasi-one-dimensional conductors. The generalization of these results to the case of arbitrary multiterminal conductors is not obvious.

We present here the systematic study of transport and noise in multiterminal diffusive conductors. This problem has been recently addressed by Blanter and Büttiker in Ref. 23, where they use the scattering-matrix formulation followed by an impurity averaging procedure. Having the advantage of including quantum phase coherence, this approach is somewhat cumbersome to generalize to an arbitrary geometry and arbitrary disorder. In contrast to this, our approach is based on semiclassical Boltzmann-Langevin equation, which greatly simplifies the calculations.

We consider a multiterminal mesoscopic diffusive conductor [see Fig. 1(a)] connected to an arbitrary number N of perfect metallic reservoirs at the contact surfaces L_n, $n=1,\dots,N$, where the voltages V_n or outgoing currents I_n are measured. The reservoirs are maintained at equilibrium and have in general different lattice temperatures T_n. Unless specified otherwise the conductor has an arbitrary three-dimensional (3D) or 2D geometry with an arbitrary disorder distribution. Our goal is to calculate the multiterminal spectral densities of current fluctuations $\delta I_n(t)$ at zero frequency $\omega=0$,

$$S^c_{nm} = \int_{-\infty}^{\infty} dt \langle \delta I_n(t)\delta I_m(0)\rangle, \tag{1.1}$$

where the brackets $\langle \dots \rangle$ indicate an ensemble average. We consider the effects of purely elastic scattering and those of energy relaxation due to electron-electron and electron-phonon scattering on the same basis.

Starting our analysis with a brief summary of the Boltzmann-Langevin kinetic equation approach,[24,25] we then apply the standard diffusion approximation and reduce the problem of evaluating Eq. (1.1) to the solution of a diffusion equation. First, we solve the diffusion equation for the distribution function to obtain the multiterminal conductance matrix and energy-transport coefficients in terms of well-

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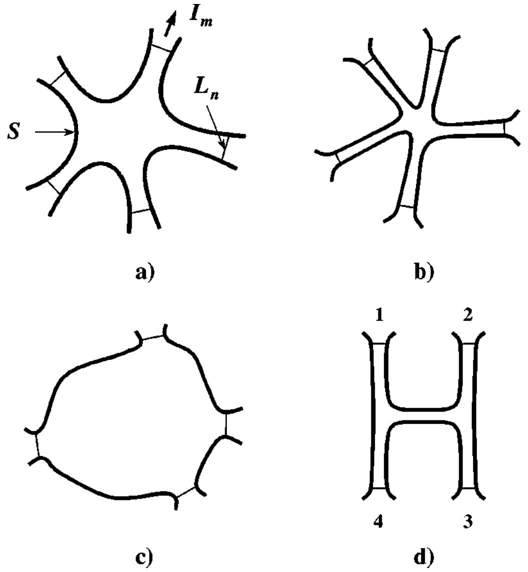

FIG. 1. (a) Multiterminal diffusive conductor of arbitrary 2D or 3D shape and with arbitrary impurity distribution. There are N leads with metallic contacts of area L_n, and I_m is the mth outgoing current. S denotes the remaining surface of the conductor where no current can pass through. (b) Conductor of a star geometry with N long leads, which join each other at a small crossing region. The resistance of this region is assumed to be much smaller than the resistance of the leads. (c) Wide conductor: the contacts are connected through a wide region, so that the resistance of the conductor comes mainly from the regions near the contacts, while the resistance of the wide region is negligible. (d) H-shaped conductor with four leads of equal conductances, $G/4$, connected by a wire in the middle of conductance G_0.

defined ''characteristic potentials.''[26] We formulate the Wiedemann-Franz law for the case of a multiterminal conductor. Then we turn to the calculation of the noise spectrum. We derive the exact general formula (3.16) for the multiterminal spectral density of the noise, which together with Eqs. (4.3), (5.2), (6.2), and (6.5) is the central result of our paper. Using this formula we demonstrate that the shot-noise suppression factor of 1/3 is *universal* also in the semiclassical Boltzmann-Langevin approach, in the sense that it holds for a multiterminal diffusive conductor of arbitrary shape, electron spectrum, and disorder distribution. We first prove this for cold electrons and then for the case of hot electrons where the suppression factor is $\sqrt{3}/4$. Thereby we extend previous semiclassical investigations[21,22] for two-terminal conductors to an arbitrary multiterminal geometry. This allows us then to compare our semiclassical approach with the scattering-matrix approach for multiterminal conductors,[7,27,28] in particular with some explicit results recently obtained for diffusive conductors.[23] The universality of shot noise proven here gives further support to the suggestion[29] that phase coherence is not essential for the suppression of shot noise in diffusive conductors.[30]

Another remarkable property of shot noise in mesoscopic conductors is the exchange effect introduced by Büttiker.[27] Although this effect is generally believed to be phase sensitive, we will show that this need not be so. Indeed, for the particular case of an H-shaped conductor [see Fig. 1(d)] we show that exchange effects can be of the same order as the shot noise itself even in the framework of the semiclassical Boltzmann approach. We prove that while the exchange effect measured in different contacts (cross-correlations) can change the sign, it is always negative when measured in the same contact (autocorrelations). Thus, the autocorrelations are always suppressed, in agreement with the Pauli principle. Formally, these exchange effects are shown to come from a nonlinear dependence on the local distribution function. Similarly we show that the same nonlinearities are responsible for nonlocal effects such as the suppression of shot noise by open leads even at zero electron temperature.

Finally, we discuss a new phenomenon, namely the current noise in multiterminal diffusive conductors induced by thermal transport. We consider the cases of hot and cold electrons and prove the universality of noise in the presence of thermal transport. We also propose a possible experiment which would allow one to measure locally the effective noise temperature. Throughout the paper we illustrate the general formalism introduced here by concrete numbers for various conductor shapes that are of direct experimental interest. We note that some of the results of the present paper has been published in Ref. 31 in less general form. Here we present the details of the derivation of these results and generalize them to a finite temperature and an arbitrary electron spectrum (band structure).

II. BOLTZMANN-LANGEVIN EQUATION: DIFFUSIVE REGIME

To calculate the spectral density of current fluctuations we use the Boltzmann-Langevin kinetic equation[24,25] for the fluctuating distribution function $F(\mathbf{p},\mathbf{r},t)=f(\mathbf{p},\mathbf{r})+\delta f(\mathbf{p},\mathbf{r},t)$, which depends on the momentum $\mathbf{p}$, position $\mathbf{r}$, and time t,

$$(\partial_t+\mathbf{v}\cdot\partial_{\mathbf{r}}+e\mathbf{E}\cdot\partial_{\mathbf{p}})F-I[F]-I_{im}[F]=\delta F^s, \qquad (2.1)$$

where $\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r})+\delta\mathbf{E}(\mathbf{r},t)$ is the fluctuating electric field, $\mathbf{v}=\nabla_{\mathbf{p}}\varepsilon$ is the velocity of the electron, and ε is its kinetic energy. $I[F]=I_{ee}[F]+I_{e-ph}[F]$ contains the electron-electron and electron-phonon collision integrals, respectively (we do not need to specify them here), and $I_{im}[F]$ is the impurity collision integral,

$$I_{im}[F]=\sum_{\mathbf{p}'}(J_{\mathbf{p}'\mathbf{p}}-J_{\mathbf{p}\mathbf{p}'}),$$

$$J_{\mathbf{p}\mathbf{p}'}(\mathbf{r},t)=W_{\mathbf{p}\mathbf{p}'}(\mathbf{r})F(\mathbf{p},\mathbf{r},t)[1-F(\mathbf{p}',\mathbf{r},t)], \qquad (2.2)$$

where the elastic scattering rate from $\mathbf{p}$ into $\mathbf{p}'$, $W_{\mathbf{p}\mathbf{p}'}(\mathbf{r})$, depends on the position $\mathbf{r}$ in the case of disorder considered here.

The Langevin source of fluctuations $\delta F^s(\mathbf{p},\mathbf{r},t)$ is induced by the random (stochastic) process of the electron scattering, which is also responsible for the momentum relaxation of the electron gas. On the other hand, electron-electron scattering conserves total momentum of the electron gas, and therefore does not contribute to δF^s. Furthermore, we neglect the momentum relaxation due to electron-phonon scattering and electron-electron Umklapp processes, assuming that they are weak compared to the scattering by impurities in diffusive conductors (phonon induced shot noise in ballistic wires has been studied in Ref. 32). In other words, we assume that the collision integrals $I_{ee}[F]$ and $I_{e-ph}[F]$ describe only energy relaxation process in the electron gas, but it is only impurity scattering that gives rise to momentum relaxation and to the shot noise in diffusive conductors.

To describe the fluctuations δF^s we make use of the Langevin formulation introduced by Kogan and Shul'man.[25] In this approach there are two contributions to the fluctuations of the impurity collision integral. First, there is the contribution $I_{im}[\delta f]$ due to the fluctuations of the distribution function, which has already been included in Eq. (2.2). The second contribution, $\delta I_{im}[f]$, stems from the random character of the electron scattering, which is the extra source of fluctuation δF^s occurring on the right-hand side of Eq. (2.1), i.e.,

$$\delta F^s=\sum_{\mathbf{p}'}\,(\delta J_{\mathbf{p}'\mathbf{p}}-\delta J_{\mathbf{p}\mathbf{p}'}),\tag{2.3}$$

where the random variables $\delta J_{\mathbf{p}\mathbf{p}'}$ are intrinsic fluctuations of the incoming and outgoing fluxes $J_{\mathbf{p}\mathbf{p}'}$.

Assuming now that the flow of electrons, say, from state $\mathbf{p}$ to state $\mathbf{p}'$ is described by a Poisson process we can write[25]

$$\langle\delta J_{\mathbf{p}\mathbf{p}'}(\mathbf{r},t)\,\delta J_{\mathbf{p}_1\mathbf{p}_1'}(\mathbf{r}_1,t_1)\rangle=\delta(t-t_1)\,\delta(\mathbf{r}-\mathbf{r}_1)\,\delta_{\mathbf{p}\mathbf{p}_1}\delta_{\mathbf{p}'\mathbf{p}_1'}\langle J_{\mathbf{p}\mathbf{p}'}(\mathbf{r},t)\rangle,\tag{2.4}$$

where

$$\langle J_{\mathbf{p}\mathbf{p}'}(\mathbf{r},t)\rangle=W_{\mathbf{p}\mathbf{p}'}(\mathbf{r})f(\mathbf{p},\mathbf{r})[1-f(\mathbf{p}',\mathbf{r})].\tag{2.5}$$

Using the preceding two equations together with Eq. (2.3), we obtain the correlator of the Langevin sources,

$$\langle\delta F^s(\mathbf{p},\mathbf{r},t)\,\delta F^s(\mathbf{p}',\mathbf{r}',t')\rangle=\delta(t-t')\,\delta(\mathbf{r}-\mathbf{r}')\sum_{\mathbf{p}''}\,(\delta_{\mathbf{p}\mathbf{p}'}-\delta_{\mathbf{p}''\mathbf{p}'})W_{\mathbf{p}\mathbf{p}''}[f(1-f'')+f''(1-f)],\tag{2.6}$$

with $f''\equiv f(\mathbf{p}'',\mathbf{r})$, and $W_{\mathbf{p}\mathbf{p}''}=W_{\mathbf{p}''\mathbf{p}}$.

Next, we consider the left-hand side of Eq. (2.1). Since we are only interested in the $\omega=0$ limit of the spectral density (the effect of screening on frequency dependent shot noise in quasi-one-dimensional diffusive conductors has been studied recently in Refs. 33 and 34), we may drop the first term $\partial F/\partial t$ in Eq. (2.1). The term $e\mathbf{E}\cdot\partial_{\mathbf{p}}F$ can be rewritten as follows: $e\mathbf{E}\cdot\partial_{\mathbf{p}_F}F+e\mathbf{v}\cdot\mathbf{E}\partial_\varepsilon F$, where $\mathbf{p}_F$ is the momentum at the Fermi surface. From this we see that the electric field $\mathbf{E}$ induced by an applied voltage plays a twofold role: it effects the trajectories and changes the energy of electrons. The first effect, $e\mathbf{E}\cdot\partial_{\mathbf{p}_F}F\sim eE/p_F$, is weak compared to $\mathbf{v}\cdot\partial_{\mathbf{r}}F\sim v_F/L$ (L is the size of the conductor) and gives contribution of order eV/ε_F, which can be neglected.[35] The second effect can be taken into account by the replacement $\varepsilon\to\varepsilon-eV(\mathbf{r},t)$ in the argument of the distribution function F, so that ε now is the total (kinetic + potential) energy of the electron. Then, the two terms $\mathbf{v}\cdot\partial_{\mathbf{r}}F+e\mathbf{E}\cdot\partial_{\mathbf{p}}F$ in Eq. (2.1) can be replaced by the total derivative $\mathbf{v}\cdot\nabla F$.

In a next step we apply the standard diffusion approximation to the kinetic equation[36] where the distribution function is split into two parts,

$$F(\mathbf{p},\mathbf{r},t)=F_0(\varepsilon,\mathbf{r},t)+\mathbf{l}(\mathbf{p}_F,\mathbf{r})\cdot\mathbf{F}_1(\varepsilon,\mathbf{r},t),\tag{2.7}$$

where the vector $\mathbf{l}$ obeys the equation,

$$\sum_{\mathbf{p}'}W_{\mathbf{p}\mathbf{p}'}(\mathbf{r})[\mathbf{l}(\mathbf{p}_F,\mathbf{r})-\mathbf{l}(\mathbf{p}_F',\mathbf{r})]=\mathbf{v}.\tag{2.8}$$

The choice of the distribution function F in the form (2.7) is dictated by the fact that the impurity collision integral $I_{im}[F]$ does not affect the energy dependence of the distribution function. Inserting this ansatz into Eq. (2.1) and averaging subsequently over the momentum first weighted with one and then with $\mathbf{l}$, we arrive at

$$\nabla\cdot\hat{D}\mathbf{F}_1-\overline{I[F]}=0,\tag{2.9}$$

$$\hat{D}(\nabla F_0+\mathbf{F}_1)=\overline{\mathbf{l}\delta F^s}.\tag{2.10}$$

Here the overbar means averaging over $\mathbf{p}_F$ at the Fermi surface inside the Brillouin zone, $\overline{(\,\ldots)}=\int d\mathbf{p}_F v_F^{-1}(\,\ldots)/\int d\mathbf{p}_F v_F^{-1}$, and we introduced the diffusion tensor,

$$\hat{D}(\mathbf{r})\equiv D_{\alpha\beta}(\mathbf{r})=\overline{v_\alpha l_\beta(\mathbf{p}_F,\mathbf{r})}.\tag{2.11}$$

We also used $\overline{\delta F^s}=0$, which follows from Eq. (2.6), and which reflects the conservation of the number of electrons in the scattering process.

Using the distribution function (2.7) we can calculate the current density $\mathbf{j}+\delta\mathbf{j}=e\nu_F\hat{D}\int d\varepsilon\mathbf{F}_1$ and due to charge neutrality (neglecting accumulation of charge) we get the potential, $eV+e\,\delta V=\int_{\varepsilon_c}^{\infty}d\varepsilon F_0$, where ε_c is a constant energy near the Fermi level and chosen so that $F|_{\varepsilon_c}=1$, and $\nu_F=\int d\mathbf{p}_F v_F^{-1}$ is the density of states at the Fermi level. Upon integration of Eqs. (2.9) and (2.10) over the energy ε the collision integrals vanish and we arrive at the diffusion equations for the potential and density of current, respectively,

$$\nabla\cdot\hat{\sigma}\nabla V=0,\quad \mathbf{j}=-\hat{\sigma}\nabla V,\tag{2.12}$$

$$\delta\mathbf{j}+\hat{\sigma}\nabla\,\delta V=\delta\mathbf{j}^s,\quad \nabla\cdot\delta\mathbf{j}=0,\tag{2.13}$$

where the conductivity tensor $\hat{\sigma}(\mathbf{r})=e^2\nu_F\hat{D}(\mathbf{r})$ depends in general on the position $\mathbf{r}$, and $\delta\mathbf{j}^s=e\nu_F\int d\varepsilon\overline{\mathbf{l}\delta F^s}$ is the Langevin source of fluctuations of the current density. After integrating over ε in Eq. (2.6) and averaging over $\mathbf{p}$ (at the Fermi surface) we use then Eqs. (2.8) and (2.11) to obtain the correlation function of the Langevin sources

$$\langle\delta j_\alpha^s(\mathbf{r},t)\,\delta j_\beta^s(\mathbf{r}',t')\rangle=\delta(t-t')\,\delta(\mathbf{r}-\mathbf{r}')\sigma_{\alpha\beta}(\mathbf{r})\Pi(\mathbf{r}),$$

$$\Pi(\mathbf{r})=2\int d\varepsilon f_0(\varepsilon,\mathbf{r})[1-f_0(\varepsilon,\mathbf{r})],\tag{2.14}$$

where f_0 is the symmetric part of the average distribution function $f=f_0+\mathbf{l}\cdot\mathbf{f}_1$.

The physical interpretation of Eq. (2.14) is now transparent: the function Π describes the local broadening of the distribution function and can be thought of as an effective (noise) temperature. Then we see that the correlator (2.14) takes an equilibriumlike form of the fluctuation-dissipation theorem. This is a direct consequence of our diffusion approximation. In the diffusive regime all microscopic details of the transport and fluctuation mechanisms are hidden in the same conductivity matrix, which appear in the correlator of the fluctuation sources [Eq. (2.14)] as well as in the diffusion equations [Eqs. (2.12) and (2.13)]. It is this fact which leads to the universality of shot noise that is independent of microscopic mechanisms of the noise.

Next, subtracting the fluctuating part from Eqs. (2.9) and (2.10) we get the equations for the average distribution function *f*,

$$\nabla\cdot\hat{\sigma}\nabla f_0+e^2\nu_F\overline{I[f]}=0,\quad f=f_0-\mathbf{l}\cdot\nabla f_0, \tag{2.15}$$

which complete the set of coupled equations to be solved. Now we specify the boundary conditions to be imposed on Eqs. (2.12), (2.13), and (2.15). First, we assume that for a given energy there is no current through the surface S [see Fig. 1(a)]. Second, since the contacts with area L_n are perfect conductors the average potential V and its fluctuations δV are independent of position $\mathbf{r}$ on L_n. Third, the contacts are assumed to be in thermal equilibrium with outside reservoirs.[37] Then we write the boundary conditions for (2.12) and (2.13), respectively, as

$$d\mathbf{s}\cdot\mathbf{j}(\mathbf{r})|_S=0,\quad V(\mathbf{r})|_{L_n}=V_n, \tag{2.16}$$

$$d\mathbf{s}\cdot\delta\mathbf{j}(\mathbf{r},t)|_S=0,\quad \delta V(\mathbf{r},t)|_{L_n}=\delta V_n(t), \tag{2.17}$$

and for Eq. (2.15),

$$f_0(\varepsilon,\mathbf{r})|_{L_n}=f_{T_n}(\varepsilon-eV_n),\quad d\mathbf{s}\cdot\hat{\sigma}(\mathbf{r})\nabla f_0(\varepsilon,\mathbf{r})|_S=0, \tag{2.18}$$

where $f_{T_n}(\varepsilon)=[1+\exp(\varepsilon/T_n)]^{-1}$ is the equilibrium distribution function at temperature T_n, and $d\mathbf{s}$ is a vector area element perpendicular to the surface.

Equations (2.12), (2.13), and (2.15) with the boundary conditions (2.16), (2.17), and (2.18) are now a complete set of equations. In principle, these equations can be solved exactly, which would allow us to evaluate S^c_{nm} for an arbitrary multiterminal geometry of the conductor and for an arbitrary disorder distribution.

III. SOLUTION OF THE DIFFUSION EQUATIONS

A. Multiterminal conductance matrix

The multiterminal conductance matrix is defined as follows: $I_n=\Sigma_m G_{nm}V_m$ (throughout the paper the sum over the contacts m runs from $m=1$ to $m=N$, and we omit the limits for convenience). To calculate G_{nm} we need to solve Eqs. (2.12) with boundary conditions (2.16). Following Büttiker[26] we introduce characteristic potentials $\phi_n(\mathbf{r})$, $n=1,\dots,N$, associated with the corresponding contacts. These functions satisfy the diffusion equation and the boundary conditions:

$$\nabla\cdot\hat{\sigma}\nabla\phi_n=0, \tag{3.1}$$

$$d\mathbf{s}\cdot\hat{\sigma}\nabla\phi_n|_S=0,\quad \phi_n|_{L_m}=\delta_{nm}, \tag{3.2}$$

so that they are always positive $\phi_n(\mathbf{r})\geqslant 0$, $n=1,\dots,N$ and obey the sum rule (see Appendix A),

$$\sum_n \phi_n(\mathbf{r})=1. \tag{3.3}$$

The potential V can be expressed in terms of characteristic potentials

$$V(\mathbf{r})=\sum_n \phi_n(\mathbf{r})V_n \tag{3.4}$$

to satisfy the diffusion Eq. (2.12) and boundary conditions (2.16). Then the outgoing current through the mth contact is $I_m=\int_{L_m}d\mathbf{s}\cdot\mathbf{j}=-\Sigma_n\int_{L_m}d\mathbf{s}\cdot\hat{\sigma}\nabla\phi_n V_n$, and using the definition of the conductance matrix we get

$$G_{mn}=-\int_{L_m}d\mathbf{s}\cdot\hat{\sigma}\nabla\phi_n. \tag{3.5}$$

We note here that the multiplication of the integrand by ϕ_m does not change the integral in the right-hand side of this equation. Moreover, the boundary conditions (3.2) for the characteristic potentials allows us to extend the integral to the entire surface. Doing so and taking into account Eq. (3.1), we then replace the surface integral by an integral over the volume of the conductor. We are then left with another useful formula for G_{nm},

$$G_{mn}=-\int d\mathbf{r}\nabla\phi_m\cdot\hat{\sigma}\nabla\phi_n. \tag{3.6}$$

From this expression and from the sum rule for ϕ_n it immediately follows that $G_{nm}=G_{mn}$, $\Sigma_n G_{nm}=0$, and $G_{nn}<0$, as it should be. In Appendix A we use a similar procedure to prove another quite natural property of the conductance matrix: $G_{nm}>0$ for $n\neq m$.

B. Energy transport coefficients

We have already seen that the local source of noise is defined by the effective noise temperature Π [see Eq. (2.14)], which describes the broadening of the distribution function. Another important quantity is given by the energy density $\Upsilon(\mathbf{r})$ acquired by the electron gas due to the broadening of the distribution function (effective heat density). It is given explicitly by the integral

$$\Upsilon=\nu_F\int_{\varepsilon_c}^{\infty}d\varepsilon\,\varepsilon[f_0-\theta(\varepsilon-eV)]=\Lambda-\frac{1}{2}\nu_F(eV)^2, \tag{3.7}$$

where $\theta(\varepsilon-eV)$ is the local equilibrium distribution function at zero temperature, and $\Lambda(\mathbf{r})=\nu_F\int_{\varepsilon_c}^{\infty}d\varepsilon\,\varepsilon f_0(\varepsilon,\mathbf{r})-\nu_F\varepsilon_c^2/2$ is the total energy density (up to irrelevant constant).

To calculate Υ we integrate the first of Eqs. (2.15) over ε with the weight of ε and use the expression (3.7) for Λ. Then the electron-electron collision integral vanishes, and we arrive at the following equation:[38]

$$\nabla\cdot\hat{D}\nabla\Lambda=\nabla\cdot\hat{D}\nabla\Upsilon+\mathbf{j}\cdot\mathbf{E}=q, \tag{3.8}$$

where we introduced the rate of energy relaxation (or absorption) due to phonons, $q(\mathbf{r})=-\nu_F\int_{\varepsilon_c}d\varepsilon\varepsilon\overline{I_{e-ph}[f]}$. Equation (3.8) expresses energy conservation: the work done on the system by the electric field, $\mathbf{j}\cdot\mathbf{E}$, is equal to the energy flux to the lattice, q, plus the heating of the electron gas, $-\nabla\cdot\hat{D}\nabla\Upsilon$. Integration of Eqs. (2.18) gives us the boundary conditions for Λ,

$$\Lambda|_{L_n}=\Lambda_n=\nu_F\left[\frac{\pi^2}{6}T_n^2+\frac{1}{2}(eV_n)^2\right],$$

$$d\mathbf{s}\cdot\hat{D}\nabla\Lambda|_S=0. \tag{3.9}$$

We assume now that electron-phonon interaction is weak (the general case is discussed in Sec. III C). Then the energy exchange between the electron gas and the lattice occurs in the metallic reservoirs far away from the conductor, and inside the conductor we have $q=0$. Equation (3.8) for Λ with the boundary conditions (3.9) can be solved in terms of ϕ_n: $\Lambda(\mathbf{r})=\Sigma_n\phi_n(\mathbf{r})\Lambda_n$. Substituting this expression into Eq. (3.7) and using Eq. (3.4) for V, we obtain Υ,

$$\Upsilon=\nu_F\sum_{n,m}\phi_n\phi_m\left[\frac{\pi^2}{6}T_n^2+\frac{e^2}{4}(V_n-V_m)^2\right]. \tag{3.10}$$

On the other hand, in perfect metallic reservoirs (where $\sigma\to\infty$) the term $\mathbf{j}\cdot\mathbf{E}\sim\mathbf{j}^2/\sigma$ can be neglected in Eq. (3.8). Integration of this equation over the volume of the nth metallic reservoir gives the total amount of energy transferred to (or absorbed from) the lattice in this reservoir, $Q_n=\int d\mathbf{r}q(\mathbf{r})=-\int_{L_n}d\mathbf{s}\cdot\hat{D}\nabla\Upsilon$. In the particular case of thermal equilibrium between the reservoirs, i.e., $T_n=T$, $n=1,\ldots,N$, we can use Eq. (3.10) to get the Joule heat in the nth reservoir,

$$Q_n=\frac{1}{2}\sum_m G_{nm}(V_n-V_m)^2. \tag{3.11}$$

For a two-terminal conductor $(V_1-V_2)^2=V^2$, $G_{12}=G_{21}=G$, we have $Q_1=Q_2=GV^2/2$, while the total Joule heat is $Q_1+Q_2=IV$. We see in this case that the heat contributions released on each side of the two-terminal conductor are equal.[39] This general conclusion holds for an arbitrary shape of the conductor and arbitrary disorder distribution. This fact is a consequence of electron-hole symmetry.

The following simple analysis of the Eq. (3.11) exhibits its physical meaning. On one hand, the total amount of Joule heat, $\frac{1}{2}\Sigma_{nm}G_{nm}(V_n-V_m)^2=-\Sigma_nI_nV_n=\int d\mathbf{r}\mathbf{j}\cdot\mathbf{E}$, is simply equal to the total work done by the electric field on the system. On the other hand, the value $\frac{1}{2}e^2\nu_F(V_n-V_m)^2$ can be thought of as the gauge invariant difference of energy densities Λ [i.e., minus the density of the potential energy $e^2\nu_F(V_m-V_n)V_n$] applied to the contacts of the conductor. Then the energy transport coefficients, $G_{nm}/e^2\nu_F$, are determined by the conductance matrix. The last fact is a manifestation of the Wiedemann-Franz law, which holds for diffusive conductors [together with Eqs. (3.10) and (3.11)] in the cases of cold and hot electrons, as soon as the electron-phonon interaction is weak enough. To show the Wiedemann-Franz law in its usual form, we consider the thermal transport in multiterminal conductors in the absence of charge transport $V_n=0$, $n=1,\ldots,N$. In this case we can use again Eq. (3.10) to calculate the thermal current Q_n,

$$Q_n=\frac{\pi^2}{6e^2}\sum_m G_{nm}T_m^2. \tag{3.12}$$

In particular, close to thermal equilibrium $T_m=T+\Delta T_m$, we have

$$Q_n=\frac{\pi^2T}{3e^2}\sum_m G_{nm}\Delta T_m,\quad \Delta T_m\ll T, \tag{3.13}$$

where $(\pi^2T/3e^2G_{nm})$ is the thermal conductance matrix. This is now the Wiedemann-Franz law in its usual form.

C. Multiterminal spectral density of noise

In this section we derive the general formula for the multiterminal spectral density of shot noise in the case of arbitrary electron-phonon interaction. We multiply the first of Eqs. (2.13) by $\nabla\phi_n$ and integrate it over the volume of the conductor. Then we evaluate the first term in the left-hand side of the equation integrating by parts and using the second of Eqs. (2.13), $\int d\mathbf{r}\nabla\phi_n\cdot\delta\mathbf{j}=\oint d\mathbf{s}\cdot\delta\mathbf{j}\phi_n$. Taking into account the boundary conditions (2.17) for $\delta\mathbf{j}$ and (3.2) for ϕ_n we get $\int d\mathbf{r}\nabla\phi_n\cdot\delta\mathbf{j}=\delta I_n$. Integration by parts in the second term of the left-hand side of this equation gives $\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\delta V=\oint d\mathbf{s}\cdot\hat{\sigma}\nabla\phi_n\delta V=-\Sigma_kG_{nk}\delta V_k(t)$, where we used Eqs. (3.1) and (3.2) for ϕ_n, the boundary condition (2.17) for δV, and Eq. (3.5) for the conductance matrix G_{nm}. This leads us to the solution of the Langevin equation (2.13) in terms of characteristic potentials:

$$\delta\tilde{I}_n\equiv\delta I_n-\sum_m G_{nm}\delta V_m=\int d\mathbf{r}\nabla\phi_n\cdot\delta\mathbf{j}^s. \tag{3.14}$$

Now, using the correlator (2.14) for the Langevin sources $\delta\mathbf{j}^s$, we express the generalized multiterminal spectral density S_{nm} defined as

$$S_{nm}=\int_{-\infty}^{\infty}dt\langle\delta\tilde{I}_n(t)\delta\tilde{I}_m(0)\rangle \tag{3.15}$$

in terms of characteristic potentials,

$$S_{nm}=\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_m\Pi, \tag{3.16}$$

with the properties: $S_{nm}=S_{mn}$, $\Sigma_nS_{nm}=0$, and $S_{nn}>0$. In equilibrium $\Pi(\mathbf{r})=2T$, and Eq. (3.16) together with Eq. (3.6) lead to the result for the thermal noise,

$$S_{nm}=-2G_{nm}T, \tag{3.17}$$

which is again a manifestation of the fluctuation-dissipation theorem.

The formula (3.16) is one of the central results of the paper. It is valid for elastic and inelastic scatterings and for an arbitrary multiterminal diffusive conductor. The relation of S_{nm} to the measured noise is now as follows. If, say, the voltages are fixed, then $\delta I_n(t)=\delta\tilde{I}_n(t)$, and the matrix $S_{nm}=S^c_{nm}$ is directly measured. On the other hand, when currents are fixed, S_{nm} can be obtained from the measured voltage correlator $S^v_{nm}=\int_{-\infty}^{\infty}dt\langle\delta V_n(t)\delta V_m(0)\rangle$ by tracing it with conductance matrices: $S_{nm}=\Sigma_{n'm'}G_{nn'}G_{mm'}S^v_{n'm'}$. The physical interpretation of Eq. (3.16) becomes now transparent: Π describes the broadening of the distribution function (effective temperature) that is induced by the voltage applied to the conductor and $\hat{\sigma}\Pi$ can thus be thought of as a local noise *source* [see the discussion following Eq. (2.14)], while ϕ_n can be thought of as the *probe* of this local noise. In particular, this means that only S_{nm} is of physical relevance but not the current or voltage correlators themselves.

Let us consider now one important application of Eq. (3.16). In an experiment one can measure the local broadening Π of the nonequilibrium distribution function f_0 (effective noise temperature Π) at some point $\mathbf{r}=\mathbf{r}_0$ on the surface of the conductor by measuring the voltage fluctuations in a noninvasive voltage probe. This is an open contact with a small area on the surface of the conductor around the point $\mathbf{r}=\mathbf{r}_0$. The contact is not attached to the reservoir so that it does not cause the equilibration of the electron gas, and as a result $\Pi=\text{const}$ around the point $\mathbf{r}_0$. Then, Eq. (3.16) can be rewritten as follows: $S=\int d\mathbf{r}\nabla\phi\cdot\hat{\sigma}\nabla\phi\Pi=\Pi(\mathbf{r}_0)\int d\mathbf{r}\nabla\phi\cdot\hat{\sigma}\nabla\phi$, where ϕ is the characteristic potential corresponding to the noninvasive probe. Using Eqs. (3.6) we get $S=R^{-1}\Pi(\mathbf{r}_0)$, where R is the resistance of the contact that comes from the volume around $\mathbf{r}_0$. Finally, taking into account Eqs. (3.14) and (1.1) and the fact that there is no current through the voltage probe $\delta I=0$, we obtain

$$S^v=R\Pi(\mathbf{r}_0). \tag{3.18}$$

This means that Π can be directly measured, which gives an important information about nonequilibrium processes in the conductor. Equation (3.18) resembles the fluctuation-dissipation theorem. This is so because there is no transport through the noninvasive probe, and therefore one can think of the probe as being in local equilibrium with the effective temperature Π. For this reason our consideration restricted to the diffusion regime can in principle be applied to the case of the tunnel coupling between the probe and conductor. A possible experiment that could measure shot noise at local tunneling contacts is discussed in detail in Ref. 40. The above result can be easily generalized to take into account the equilibration by the contact (see Sec. VI). There will be then an additional noise suppression factor in Eq. (3.18).

We note that Eq. (3.16) together with Eqs. (2.9), (2.10), and (2.18) for the average distribution function f and Eqs. (3.1) and (3.2) for the characteristic potentials can serve as a starting point for numerical evaluations of S_{nm}. For purely elastic scattering as well as for hot electrons it is even possible to get closed analytical expressions for S_{nm} as we will show next. The physical conditions for different transport regimes are discussed in Ref. 21. In Secs. IV and V we will consider the charge transport ($T_n=T$, $n=1,\ldots,N$), and in Sec. VI we will discuss the thermal transport ($V_n=0$, $n=1,\ldots,N$).

IV. ELASTIC SCATTERING

In the case of purely elastic scattering, $I[f]=0$, the average distribution function satisfies the diffusion equation

$$\nabla\cdot\hat{\sigma}\nabla f_0=0, \tag{4.1}$$

and the boundary conditions (2.18) with $T_n=T$ (i.e., in the charge-transport regime). Using this equation one can prove (see Appendix A) that for elastic scattering cross correlations ($n\neq m$) are always negative, in agreement with the general conclusion of Ref. 27.

Equation (4.1) can be solved in terms of ϕ_n: $f_0=\Sigma_n\phi_n f_T(\varepsilon-eV_n)$. Substituting this solution into Eq. (2.14) and using the sum rule (3.3) for ϕ_n, we can express Π in the following form:[41]

$$\Pi=2\int d\varepsilon\sum_{k,l}\phi_k\phi_l f_T(\varepsilon-eV_k)[1-f_T(\varepsilon-eV_l)]. \tag{4.2}$$

Performing the integration over ε we obtain,

$$\Pi=e\sum_{k,l}\phi_k\phi_l(V_k-V_l)\coth\left[\frac{e(V_k-V_l)}{2T}\right], \tag{4.3}$$

which in combination with Eq. (3.16) gives the final expression for S_{nm}, which is valid for purely elastic scattering. Equation (4.3) describes the crossover from the shot noise in multiterminal diffusive conductors ($T\to 0$),

$$\Pi=e\sum_{k,l}\phi_k\phi_l|V_k-V_l|, \tag{4.4}$$

to the equilibrium Johnson-Nyquist noise given by Eq. (3.17).

A. Universality of noise

Now we are in the position to generalize the proof of universality of the 1/3-suppression of shot noise[9–12] to the case of an arbitrary *multiterminal* diffusive conductor. To be specific, we choose $V_n=0$, for $n\neq 1$, i.e., only contact $n=1$ has a nonvanishing voltage. Then, using the sum rule (3.3) for ϕ_n, we get

$$\Pi=2e\phi_1(1-\phi_1)V_1\coth(eV_1/2T)+2T(1-\phi_1)^2+2T\phi_1^2.$$

To get S_{1n} we substitute this equation into Eq. (3.16) and evaluate the first term as follows: $\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_1\phi_1(1-\phi_1)=\oint d\mathbf{s}\cdot\hat{\sigma}\nabla\phi_n(\phi_1^2/2-\phi_1^3/3)=-G_{1n}/6$, where we used Eqs. (3.1) and (3.2). Similarly, for the integrals in the second and third term we get: $\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_1(1-\phi_1)^2=\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_1\phi_1^2=-G_{1n}/3$. Combining these results we arrive at

$$S_{1n}=-\frac{1}{3}G_{1n}[4T+eV_1\coth(eV_1/2T)]. \tag{4.5}$$

When $V_1=0$ we get $S_{1n}=-2G_{1n}T$, and the formula for the Johnson-Nyquist noise is recovered. When $T=0$, we express S_{1n} in terms of outgoing currents, $I_n=G_{1n}V_1$:

$$S_{1n}=-\frac{1}{3}e|I_n|, \quad n\neq 1,$$

$$S_{11}=\frac{1}{3}e|I_1|. \tag{4.6}$$

We note that the above derivation is valid for arbitrary impurity distribution and shape of the conductor, and for an arbitrary electron spectrum (band structure). In this sense the suppression factor $\frac{1}{3}$ is indeed universal. This generalizes the known universality of a two-terminal conductor[12] to a multiterminal geometry.

Finally, we mention here some inequalities (derived in Appendix A), which can be used to estimate the spectral density S_{nm} in the $T=0$ limit. First, the correlations are bounded from below,

$$S_{nn}\geqslant\frac{1}{3}e|I_n|, \tag{4.7}$$

but due to the nonlocality of the noise (see the discussion in Sec. IV C) there can be no upper bound in terms of the current I_n through the same contact. In other words, the current I_n flowing through the nth contact creates the noise $\frac{1}{3}eI_n$ in this contact. However, other contacts also contribute to the noise in the nth contact, and this contribution is not universal and makes the noise arbitrarily larger compared to the value $\frac{1}{3}eI_n$. Nevertheless, we can write: $\Pi<\max\{|V_k-V_l|\}$, $k,l=1,\ldots,N$, which gives the rough estimate

$$S_{nn}<e|G_{nn}|\max\{|V_k-V_l|\}. \tag{4.8}$$

In contrast, the cross correlations possess an upper bound,

$$|S_{nm}|\leqslant\frac{1}{2}(S_{nn}+S_{mm}). \tag{4.9}$$

S_{nm} vanishes when the nth and mth contacts are completely disconnected.

B. Wide and star-shaped conductors

Next we specialize to two experimentally important cases. First we consider a multiterminal conductor of a star geometry with N long leads (but with otherwise arbitrary shape), which join each other at a small crossing region [see Fig. 1(b)]. The resistance of this region is assumed to be much smaller than the resistance of the leads. In the second case the contacts are connected through a wide region [see Fig. 1(c)], where again the resistance of the conductor comes mainly from the regions near the contacts, while the resistance of the wide region is negligible.

Both shapes are characterized by the requirement that $w/L\ll 1$, where w and L are the characteristic sizes of the contact and of the entire conductor, respectively. In both cases the conductor can be divided (more or less arbitrary) into N subsections Γ_k, $k=1,\ldots,N$, associated with a particular contact so that the potential V is approximately constant (for $w/L\ll 1$) on the dividing surfaces C_k. Each subsection then can be thought of as a two-terminal conductor with the corresponding characteristic potential $\theta_k(\mathbf{r})$,

$$\nabla\cdot\hat{\sigma}\nabla\theta_k=0, \quad d\mathbf{s}\cdot\hat{\sigma}\nabla\theta_k|_S=\theta_k|_{L_k}=0, \quad \theta_k|_{C_k}=1. \tag{4.10}$$

We will show now that both the multiterminal conductance matrices G_{nm} and the spectral densities S_{nm}, can be expressed in terms of the conductances G_k of these subsections,

$$G_k=-\int_{L_k}d\mathbf{s}\cdot\hat{\sigma}\nabla\theta_k=\int_{C_k}d\mathbf{s}\cdot\hat{\sigma}\nabla\theta_k. \tag{4.11}$$

Since each potential ϕ_n is approximately constant in the central region of the multiterminal conductor, we can write

$$\phi_n(\mathbf{r})|_{C_k}=\alpha_n=\text{const.}, \quad \sum_n \alpha_n=1, \tag{4.12}$$

for an arbitrary $k=1,\ldots,N$, where the second equation follows from the sum rule for ϕ_n. Comparing Eqs. (3.1), (3.2), and (4.12) with the definition of θ_k Eq. (4.10), we immediately obtain

$$\phi_n(\mathbf{r})|_{\mathbf{r}\in\Gamma_k}=\alpha_n\theta_k(\mathbf{r})+[1-\theta_k(\mathbf{r})]\delta_{nk}. \tag{4.13}$$

The calculation of G_{nm} and S_{nm} is now straightforward. We substitute Eq. (4.13) into Eq. (3.5) and use Eq. (4.11) to get

$$G_{nm}=(\alpha_m-\delta_{nm})G_n, \quad \alpha_m=G_m/G, \tag{4.14}$$

where $G\equiv\Sigma_n G_n$, and the equation for α_m follows from $\Sigma_n G_{nm}=0$. Substituting Eq. (4.13) into Eq. (3.16) and applying similar arguments as above in the proof of the 1/3-suppression we find the explicit expressions (for details of the derivation see Appendix B)

$$S_{nm}=\frac{1}{3}e\sum_k \alpha_n\alpha_k(J_k+J_n)(\delta_{nm}-\delta_{km})-\frac{2}{3}G_{nm}T,$$

$$J_n=\sum_l G_l(V_n-V_l)\coth\left[\frac{e(V_n-V_l)}{2T}\right]. \tag{4.15}$$

We note that this result is a consequence of the above approximation (4.12). Comparing the resistance of the subsections to the resistance of the central region of the conductor (which is neglected) we find that the corrections to Eq. (4.12) and consequently to Eq. (4.15) are of order w/L in 3D and for a star geometry in 2D, and up to corrections of order $[\ln(L/w)]^{-1}$ for wide conductors in 2D.

In principle, Eq. (4.15) and (4.14) allow us to calculate the noise for arbitrary voltages and temperature, but for illustrative purposes we consider the simple case of a cross-shaped conductor with four equivalent leads (i.e., $\alpha_n=1/4$) and $T=0$. Suppose the voltage is applied to only one contact, say $V_1>0$, $V_{n\neq 1}=0$, and $I=-I_1=3I_{n\neq 1}>0$. Then, from Eq. (4.15) we obtain: $S_{11}=\frac{1}{3}eI$, $S_{12}=S_{13}=S_{14}=-\frac{1}{9}eI$, all being in agreement with the universal 1/3-suppression proven above. Then, $S_{22}=S_{33}=S_{44}=\frac{2}{9}eI$, and $S_{23}=S_{24}=S_{34}=-\frac{1}{18}eI$. These numbers seem to be new[42] and it would be interesting to test them experimentally.

C. Nonlocality and exchange effect

We are now in the position to address the issue of *nonlocality* and *exchange* effect in shot noise ($T=0$) in multiterminal conductors. For this we consider for instance a star geometry and assume that the current enters the conductor through the nth contact, i.e., $I_n=-I$, and leaves it through the mth contact, i.e., $I_m=I$, while the other contacts are open, i.e., $I_k=0$ for $k\neq n,m$. From Eq. (4.14) we obtain for the conductance $G_nG_m/(G_n+G_m)$ (two contacts are in series), and we see that it does not depend on the other leads, which simply reflects the *local* nature of diffusive transport. However, contrary to one's first expectation, this locality does *not* carry over to the noise in general. Indeed, from Eq. (4.15) it follows that $S_{nm}=-\frac{1}{3}(\alpha_n+\alpha_m)eI$. The additional suppression factor $0<\alpha_n+\alpha_m<1$ for $N>2$ reflects the *nonlocality* of the current noise. For instance, for a cross with $N=4$ equivalent leads we have $\alpha_m=\alpha_n=1/4$, and thus $S_{nm}=-\frac{1}{6}eI$. An analogous reduction factor was obtained in Ref. 9 under a different point of view. Hence, one cannot disregard open contacts simply because no current is flowing through them; on the contrary, these open contacts, which are still connected to the reservoir induce equilibration of the electron gas and thereby reduce its current noise. We emphasize that this nonlocality is a classical effect in the sense that no quantum phase interference is involved (phase coherent effects are *not* contained in our Boltzmann approach). On the other hand, the origin of this nonlocality can be traced back to the nonlinear dependence of Π on the distribution f in Eq. (2.14), which is a consequence of the Pauli exclusion principle.

Next we discuss exchange effects[27] in a four terminal conductor. According to Blanter and Büttiker[23] they can be probed by measuring S_{13} in three ways: $V_n=V_0\delta_{n2}$ (A), $V_n=V_0\delta_{n4}$ (B), and $V_n=V_0\delta_{n2}+V_0\delta_{n4}$ (C). Then we take $\Delta S_{13}=S_{13}^C-S_{13}^A-S_{13}^B$ as a measure of the exchange effect. This experiment is analogous to the experiment of Hanbury Brown and Twiss in optics.[43] It measures the interference of electrons coming from mutually incoherent sources, which is caused by the indistinguishability of the electrons. Naively, one might expect that this interference effect averages to zero in diffusive conductors. However, it comes now as some surprise that in our semiclassical Boltzmann approach ΔS_{13} turns out to be nonzero in general and can even be of the order of the shot noise itself. Again, the reason for that is that Π is nonlinear in f_0 [see Eq. (2.14)], which is the consequence of the Pauli exclusion principle. So, the value $\Pi^C-\Pi^A-\Pi^B$, which enters ΔS_{13} is not necessarily zero. Indeed, while exchange effects vanish for cross-shaped conductors (in agreement with Ref. 23 up to corrections of order w/L, which are neglected in our approximation), it is not so for an H-shaped conductor [see Fig. 1(d)]. Calculations similar to those leading to Eq. (4.15) give for this case:

$$\Delta S_{13}=\frac{1}{24}\,\frac{eV_0G^2G_0}{(G+4G_0)^2}, \qquad (4.16)$$

where $G_n=G/4$ are the conductances (all being equal) of the outer four leads, while the conductance of the connecting wire in the middle is denoted by G_0. This exchange term ΔS_{13} vanishes for $G_0\to\infty$, because then the case of a simple cross is recovered, and also for $G_0\to 0$, because then the first and third contacts are disconnected. ΔS_{13} takes on its maximum value for $G_0=G_n$ and becomes equal to $\frac{1}{60}eI^A$, where I^A is the current through the second contact for case (A).

Although ΔS_{13} is positive in the example considered above this is not the case in general. For an arbitrary four-terminal geometry of the conductor the exchange effect can be expressed in terms of characteristic potentials:

$$\Delta S_{nm}=-4eV_0\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_m\phi_k\phi_l, \qquad (4.17)$$

where all indices are different. From this general formula it follows that $\Delta S_{nm}=\Delta S_{kl}$, and $\Delta S_{nm}+\Delta S_{nl}+\Delta S_{nk}=0$. The last equation means that the exchange effect can change sign, i.e., cross correlations can be either suppressed or enhanced.

On the other hand, the setup can be slightly modified: instead of cross correlations, the noise density in one of the contacts of a multiterminal ($N>2$) diffusive conductor is measured, say S_{11}, while the electrons are injected through the contacts 2 (A), 3 (B), and 2 and 3 (C). Again, $\Delta S_{11}=S_{11}^C-S_{11}^A-S_{11}^B$ is a measure of the exchange effect. Then, it follows from Eq. (4.17) that

$$\Delta S_{11}=-4eV_0\int d\mathbf{r}\nabla\phi_1\cdot\hat{\sigma}\nabla\phi_1\phi_2\phi_3<0, \qquad (4.18)$$

i.e., the correlations are always suppressed due to the interference effect, which is a direct manifestation of the Pauli principle. In the particular case of star-shaped conductors we have

$$\Delta S_{11}=-\frac{4}{3}eV_0\frac{G_1G_2G_3}{G^2}, \quad G=\sum_{n=1}^{N}G_n. \qquad (4.19)$$

The suppression of noise due to the interference of mutually incoherent electrons was recently observed in an experiment with a ballistic electron beam splitter.[44] We have shown here that this effect is also observable in mesoscopic diffusive conductors.

V. HOT ELECTRONS

We consider now the case of "hot" electrons where $I_{ee}\neq 0$, but still $I_{e-ph}=0$, and we assume that electron-electron scattering is sufficiently strong to cause thermal equilibration of the electron gas (i.e., $l_{ee}=\sqrt{D\tau_{ee}}\ll L$, where D is the diffusion coefficient and τ_{ee} the electron-electron relaxation time). The average distribution then assumes the Fermi-Dirac form:

$$f_0(\varepsilon,\mathbf{r})=\left\{1+\exp\left[\frac{\varepsilon-eV(\mathbf{r})}{T_e(\mathbf{r})}\right]\right\}^{-1}, \qquad (5.1)$$

with the local electron temperature $T_e(\mathbf{r})$. Substituting this f_0 into Eq. (2.14) we immediately get $\Pi(\mathbf{r})=2T_e(\mathbf{r})$. On the other hand, from Eq. (3.7) it follows that $[T_e(\mathbf{r})]^2=(6/\pi^2\nu_F)\Upsilon(\mathbf{r})$, where $\Upsilon(\mathbf{r})$ is given by Eq. (3.10) with $T_n=T$ (i.e., in the charge-transport regime). Thus, we finally obtain

$$\Pi=2T\left[1+2\sum_{n,m}\phi_n\phi_m(\beta_n-\beta_m)^2\right]^{1/2},\quad \beta_n=\frac{\sqrt{3}eV_n}{2\pi T},\tag{5.2}$$

which in combination with Eq. (3.16) gives the general solution for the case of hot electrons. We would like to note here that the cross correlations are always negative also in the case of hot electrons (the proof is given in Appendix A).

A. Universality of noise

Next we show that the shot-noise suppression factor $\sqrt{3}/4$ (Refs. 21 and 22) for hot electrons in a multiterminal conductor is also *universal*. As before we can consider the case where the voltage is applied to only one contact: $V_n=V_1\delta_{n1}$. Then

$$\Pi=2T\sqrt{1+4\beta^2(\phi_1-\phi_1^2)},\tag{5.3}$$

where $\beta\equiv\beta_1$. Using the relation

$$2\sqrt{1-\Phi^2}\nabla\Phi=\nabla\{\arcsin\Phi+\Phi\sqrt{1-\Phi^2}\},$$

$$\Phi=\beta(1+\beta^2)^{-1/2}(2\phi_1-1),\tag{5.4}$$

we transform the volume integral in Eq. (3.16) into a surface integral and obtain the spectral density of noise:

$$S_{1n}=-G_{1n}T\left[1+\left(\beta+\frac{1}{\beta}\right)\arctan\beta\right],\quad \beta=\frac{\sqrt{3}eV_1}{2\pi T}.\tag{5.5}$$

This expression describes the crossover from the thermal noise ($\beta\ll 1$) given by Eq. (3.17) to the transport noise ($\beta\gg 1$)

$$S_{1n}=-\frac{\sqrt{3}}{4}e|I_n|,\quad n\neq 1,$$

$$S_{11}=\sqrt{\frac{3}{4}}e|I_1|.\tag{5.6}$$

This general result shows that in the case of hot electrons the shot-noise suppression factor $\sqrt{3}/4$ is indeed universal, i.e., it does not depend on the shape of the multiterminal diffusive conductor nor on its disorder distribution.[45]

The origin of this universality becomes clear from the following argumentation. We have seen that the distribution of the effective noise temperature $\Pi(\mathbf{r})$ for the case of hot electrons is controlled by the transport equations for the energy [Eqs. (3.8) and (3.9)] through the heat density $\Upsilon(\mathbf{r})$. The spectral density of noise, in turn, is given by $\Pi(\mathbf{r})$ through the transport equations for charge, [Eqs. (2.13) and (2.17)]. On the other hand, according to the Wiedemann-Franz law, both the energy and charge transports are determined by the same kinetic coefficients, namely, by $\hat{\sigma}(\mathbf{r})$. Thus, the physical origin of the universality of the suppression factor $\sqrt{3}/4$ can be traced back to the Wiedemann-Franz law. Conversely, a violation of the Wiedemann-Franz law will cause deviations from universality.

We would like to note here that the universality of the noise (for cold and hot electrons) has been proven here for the case where the voltage is applied to only one contact of a multiterminal diffusive conductor, which made it possible to express the spectral densities S_{nm} in terms of conductances G_{nm}. This is no longer possible in general. Nevertheless, in the case of a 2D geometry and isotropic conductivity, $\sigma_{\alpha\beta}(\mathbf{r})=\sigma(\mathbf{r})\delta_{\alpha\beta}$, both G_{nm} and S_{nm} are of the same universality class. Indeed, one can easily see that they are invariant under conformal transformation of coordinates. G_{nm} and S_{nm} can be expressed in terms of characteristic potentials ϕ_n, which satisfy the conformal invariant diffusion equation (3.1) and boundary conditions (3.2). Moreover, the combination $d\mathbf{r}\nabla\phi_n\cdot\nabla\phi_m$ does not change with the conformal transformation of coordinates, which finally makes the integrals for G_{nm} [Eq. (3.6)] and for S_{nm} [Eq. (3.16)] conformal invariant.

We close this section by another illustrative example. Let us consider again a cross-shaped conductor with four equivalent leads,[46] $G_n=G/4$, at $T=0$ and where we choose $V_n=V_1\delta_{1n}$, $I=-I_1=3I_{n\neq 1}>0$. We then find $S_{11}=(\sqrt{3})/4eI$, and $S_{1n}=-(1/4\sqrt{3})eI$, for $n\neq 1$, while $S_{nn}=(35\sqrt{3}/108-2/3\pi)eI$, and $S_{n\neq m}=-(13\sqrt{3}/108-1/3\pi)eI$, for $n,m\neq 1$. These new numbers are consistent with the universal factor $\sqrt{3}/4$.

VI. NOISE INDUCED BY THERMAL TRANSPORT

In this section we address a new phenomenon, namely the current noise in multiterminal diffusive conductors in the presence of thermal transport. We assume no energy relaxation in the conductor due to phonons, $I_{e-ph}=0$, and no voltage is applied to the contacts, $V_n=0$, $n=1,\ldots,N$, which are kept in equilibrium at different temperatures T_n. The thermal transport is considered in Sec. III B, where the outgoing thermal currents Q_n are calculated [see Eqs. (3.12) and (3.13)]. We turn now to the calculation of the spectral density of noise.

A. Elastic scattering

To calculate Π we need to know the distribution function f_0. It obeys Eq. (4.1) with the boundary conditions (2.18) in the contacts. The solution then reads explicitly,

$$f_0=\sum_n\phi_n f_{T_n},\tag{6.1}$$

and with the help of Eq. (3.3) we get,

$$\Pi=\sum_{kl}\phi_k\phi_l Z_{kl},$$

$$Z_{kl}=T_kT_l\int_{-\infty}^{\infty}ds[1-\tanh(T_ks)\tanh(T_ls)].\tag{6.2}$$

This together with Eq. (3.16) gives the spectral density of noise S_{nm}.

In equilibrium $T_n=T$, $Z_{kl}=2T$ and Eq. (6.2) and we find for the equilibrium noise, $S_{nm}=-2G_{nm}T$. On the other hand, if for example $T_k\gg T_l$, then $Z_{kl}=(2\ln 2)T_k$. We consider then two situations, where, e.g., either $T_1=T$ and $T_{n\neq 1}=0$, or $T_1=0$ and $T_{n\neq 1}=T$. In other words, only one contact is either heated up to high enough temperature T or cooled down to zero temperature and the other contacts are kept at the same temperature. Using the sum rule for ϕ_n and carrying out the integration in Eq. (3.16) we obtain for both cases,

$$S_{1n}=-\frac{2}{3}(1+\ln 2)G_{1n}T. \tag{6.3}$$

S_{1n} for these two situations can be expressed in terms of thermal currents Q_n

$$S_{1n}=\pm 4(1+\ln 2)(e/\pi)^2T^{-1}Q_n, \tag{6.4}$$

with the sign depending on the sign of Q_n.

B. Hot electrons

We consider now the case of hot electrons, where $\Pi=2T_e$, while $T_e^2=\Sigma_k\phi_kT_k^2$ [see Eq. (3.10)]. Substituting this into Eq. (3.16) we get,

$$S_{nm}=2\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_m\sqrt{\sum_k \phi_kT_k^2}. \tag{6.5}$$

In particular, if the electron gas in the conductor is pushed out of equilibrium by heating (or cooling) one of the contacts (with $n=1$) while the other contacts are kept at the temperature $T_n=T_2$, $n\neq 1$, the integral can be calculated explicitly, and we have

$$S_{1n}=-\frac{4}{3}G_{1n}\frac{T_1^2+T_2^2+T_1T_2}{T_1+T_2}. \tag{6.6}$$

In the cases where $T_1=T\gg T_2$, and $T_2=T\gg T_1$, we obtain with the help of Eq. (3.13) that the spectral density S_{1n} can be expressed in terms of thermal currents:

$$S_{1n}=\pm 8(e/\pi)^2T^{-1}Q_n. \tag{6.7}$$

Expressions (6.4) and (6.7) are analogous to Eqs. (4.6) and (5.6) and reflect the universality of the noise in the presence of thermal transport.

VII. CONCLUSION

In conclusion, we have systematically studied the transport and noise in multiterminal diffusive conductors. Applying a diffusion approximation to the Boltzmann-Langevin kinetic equation we have derived the diffusion equations for the distribution function and its fluctuations. We then solved these equations in general terms of well defined "characteristic potentials" and we derived exact formulas for the conductance matrix, energy-transport coefficients, and the multiterminal spectral density of noise. In this way we have obtained the following results. In both regimes of cold and hot electrons the shot noise turns out to be universal in the sense that it depends neither on the geometry of a multiterminal conductor and the spectrum of carriers, nor on the disorder distribution. We have studied the noise in the presence of thermal transport and find that being expressed in terms of thermal currents it is also universal. We believe that the origin of this universality lies in the fact that in the diffusive regime the correlator of the local current densities (Langevin sources) takes an equilibriumlike form of the fluctuation-dissipation theorem involving an effective noise temperature. Thus, the transport and noise properties are determined by the same conductivity tensor. One can surmise then that the proven universality holds as long as the energy transport is governed by the Wiedemann-Franz law.

The exchange effect is proven to be nonzero even within our semiclassical Boltzmann approach. The exchange effect can change sign when measured in cross-correlations, and (in agreement with the Pauli principle) it gives always negative contribution to the autocorrelations. The exchange effect comes from a nonlinear dependence on the local distribution function. Similarly, we show that the same nonlinearities are responsible for nonlocal effects such as the suppression of shot noise by open leads even at zero electron temperature.

Finally, we have proposed a possible experiment that would allow one to locally measure the effective noise temperature, and we have given new suppression factors for shot noise in various geometries, which can be tested experimentally.

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APPENDIX A

In this appendix we derive some properties of the characteristic potentials ϕ_n, multiterminal conductance matrix G_{nm}, and spectral density of shot noise S_{nm}. First, we show that $\phi_n\geqslant 0$ in the conductor. According to the boundary condition (3.2), being negative ϕ_n would take on its minimum value at some point $\mathbf{r}=\mathbf{r}_0$ in the conductor. If this happened inside the conductor, then $\nabla\phi_n(\mathbf{r}_0)=0$, and $\nabla\cdot\hat{\sigma}(\mathbf{r}_0)\nabla\phi_n(\mathbf{r}_0)>0$, because $\hat{\sigma}$ is a positive definite matrix. This, however, would then contradict Eq. (3.1). If ϕ_n took on its minimum value on the open surface of the conductor $(\mathbf{r}_0\in S)$, then $\nabla_\parallel\phi_n(\mathbf{r}_0)=0$, and according to Eq. (3.2) $\nabla_\perp\phi_n(\mathbf{r}_0)=0$. Again, $\nabla\cdot\hat{\sigma}(\mathbf{r}_0)\nabla\phi_n(\mathbf{r}_0)>0$ in contradiction with Eq. (3.1). Thus, we see that the characteristic potentials cannot be negative $\phi_n\geqslant 0$.

The sum rule (3.3) for ϕ_n follows from the observation that the function $\phi(\mathbf{r})\equiv 1$ is a unique solution of the diffusion equation $\nabla\cdot\hat{\sigma}\nabla\phi=0$ with the boundary conditions $d\mathbf{s}\cdot\hat{\sigma}\nabla\phi|_S=0$, and $\phi|_{L_m}=1$, $\forall m$. It follows from Eqs. (3.1) and (3.2) that the function $\Sigma_n\phi_n(\mathbf{r})$ obeys the same equation and boundary conditions, and therefore $\phi=\Sigma_n\phi_n(\mathbf{r})=1$. This sum rule can also be deduced from the fact that physical observables are invariant under a global shift of the energy scale by a constant value.[26]

Now we prove that $G_{nm}>0$ for $n\neq m$. We note that the multiplication of the integrand in Eq. (3.5) by ϕ_m^2 and extension of the integral to the whole surface does not change the integral, so that $G_{nm}=-\oint d\mathbf{s}\cdot\hat{\sigma}\nabla\phi_n\phi_m^2$. Then, using Eq. (3.1) we replace the surface integral by the integral over the volume of the conductor, $G_{nm}=-2\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_m\phi_m$. Finally, we calculate this integral by parts and take into account the boundary conditions (3.2) for ϕ_n to get for the conductance matrix,

$$G_{nm}=2\int d\mathbf{r}\nabla\phi_m\cdot\hat{\sigma}\nabla\phi_m\phi_n, \quad n\neq m, \tag{A1}$$

from which it follows that $G_{nm}>0$. Interestingly, a similar procedure for $n=m$ gives,

$$G_{nn}=-2\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_n\phi_n, \tag{A2}$$

and $G_{nn}<0$.

Now we prove that in the case of elastic scattering cross correlations S_{nm}, $n\neq m$, are always negative. We calculate S_{nm} in Eq. (3.16) with Π from Eq. (2.14) integrating by parts,

$$S_{nm}=\frac{1}{2}\oint d\mathbf{s}\cdot\hat{\sigma}(\nabla\phi_n\phi_m+\nabla\phi_m\phi_n)\Pi - \frac{1}{2}\int d\mathbf{r}\nabla\Pi\cdot\hat{\sigma}(\nabla\phi_n\phi_m+\nabla\phi_m\phi_n).$$

From Eq. (2.18) it follows that only contact surfaces contribute to the first integral, and with Eq. (3.5) we get for the first term: $-G_{nm}(T_n+T_m)$. In the second term we again calculate the integral by parts and use Eq. (2.14) to get: $-\frac{1}{2}\oint d\mathbf{s}\cdot\hat{\sigma}\nabla\Pi\,\phi_n\phi_m-2\int\int d\mathbf{r}d\varepsilon\,\phi_n\phi_m\nabla f_0\cdot\hat{\sigma}\nabla f_0$. According to Eqs. (2.18) and (3.2) the surface integral disappears and we arrive at the following result:

$$S_{nm}=-G_{nm}(T_n+T_m)-2\int\int d\mathbf{r}d\varepsilon\,\phi_n\phi_m\nabla f_0\cdot\hat{\sigma}\nabla f_0, \tag{A3}$$

for $n\neq m$, from which it follows that the cross correlations are negative.

We apply similar arguments to prove that the cross correlations are always negative also in the case of hot electrons. We calculate the integral in $S_{nm}=2\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_m T_e$ and use the boundary conditions for ϕ_n and T_e to get

$$S_{nm}=-G_{nm}(T_n+T_m)+\int d\mathbf{r}\phi_n\phi_m\nabla\cdot\hat{\sigma}\nabla T_e.$$

Then we use $[T_e(\mathbf{r})]^2=(6/\pi^2\nu_F)\Upsilon(\mathbf{r})$ and Eq. (3.8) ($q=0$) to write $T_e\nabla\cdot\hat{\sigma}\nabla T_e=-\nabla T_e\cdot\hat{\sigma}\nabla T_e-3(e/\pi)^2\nabla V\cdot\hat{\sigma}\nabla V$. Substituting this into the above equation we obtain

$$S_{nm}=-G_{nm}(T_n+T_m)-\int d\mathbf{r}\phi_n\phi_m T_e^{-1}[\nabla T_e\cdot\hat{\sigma}\nabla T_e + 3(e/\pi)^2\nabla V\cdot\hat{\sigma}\nabla V], \tag{A4}$$

which shows the negativity of cross correlations.

The inequality (4.7) can be derived as follows. First, we note that according to Eq. (4.3) correlations grow with temperature. Therefore, without loss of generality we can put $T=0$ in the following. Then, one can easily see that

$$\Pi=e\sum_{k,l}\phi_k\phi_l|V_k-V_l|\geqslant 2e\phi_n\sum_l \phi_l|V_l-V_n|$$

$$\geqslant 2e\phi_n\Big|\sum_l \phi_l(V_l-V_n)\Big|=2e\phi_n|V-V_n|.$$

We substitute this into Eq. (3.16) and write another set of inequalities,

$$S_{nn}\geqslant 2e\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_n\phi_n|V-V_n| \geqslant 2e\left|\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_n\phi_n(V-V_n)\right|.$$

Integrating by parts,

$$2\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_n\phi_n(V-V_n)$$

$$=\oint d\mathbf{s}\cdot\hat{\sigma}\nabla\phi_n\phi_n^2(V-V_n)-\int d\mathbf{r}\nabla V\cdot\hat{\sigma}\nabla\phi_n\phi_n^2$$

$$=-\frac{1}{3}\oint d\mathbf{s}\cdot\hat{\sigma}\nabla V\phi_n^3=\frac{1}{3}I_n,$$

we obtain expression (4.7). The inequality Eq. (4.9) immediately follows from

$$2|\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_m|\leqslant\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_n+\nabla\phi_m\cdot\hat{\sigma}\nabla\phi_m, \tag{A5}$$

and evidently holds also for inelastic scattering.

APPENDIX B

The derivation of Eq. (4.15) proceeds as follows. We use Eq. (4.13) to replace the integral in Eq. (3.16) over the volume of the conductor by the sum of integrals over subsections Γ_k,

$$S_{nm}=\sum_k(\alpha_n-\delta_{nk})(\alpha_m-\delta_{mk})\int_{\Gamma_k}d\mathbf{r}\nabla\theta_k\cdot\hat{\sigma}\nabla\theta_k\Pi. \tag{B1}$$

Using the contraction

$$Z_{lq}=e(V_l-V_q)\coth\left[\frac{e(V_l-V_q)}{2T}\right] \tag{B2}$$

we express Π in terms of θ_k,

$$\Pi|_{\Gamma_k}=\sum_{lq}\,[\alpha_l\theta_k+(1-\theta_k)\delta_{lk}][\alpha_q\theta_k+(1-\theta_k)\delta_{qk}]Z_{lq}\,. \tag{B3}$$

Then we substitute this expression into Eq. (B1), calculate the integrals over Γ_k with the help of Eqs. (4.11),

$$\int_{\Gamma_k}d\mathbf{r}\nabla\theta_k\cdot\hat{\sigma}\nabla\theta_k\theta_k^2=2\int_{\Gamma_k}d\mathbf{r}\nabla\theta_k\cdot\hat{\sigma}\nabla\theta_k\theta_k(1-\theta_k)$$
$$=\int_{\Gamma_k}d\mathbf{r}\nabla\theta_k\cdot\hat{\sigma}\nabla\theta_k(1-\theta_k)^2=\frac{1}{3}G_k\,, \tag{B4}$$

and use the symmetry $Z_{lq}=Z_{ql}$ to get

$$S_{nm}=\frac{1}{3}\sum_{klq}\,(\alpha_n-\delta_{nk})(\alpha_m-\delta_{mk})$$
$$\times(\alpha_l\alpha_q+\alpha_l\delta_{qk}+\delta_{lk}\delta_{qk})G_kZ_{lq}\,. \tag{B5}$$

Finally, introducing the notation, $J_q=e^{-1}\Sigma_lG_lZ_{ql}$, and using $Z_{kk}=2T$ and $\alpha_k=G_k/G$, we carry out the summation over k in Eq. (B5) to arrive at Eq. (4.15) for S_{nm}.

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[1] W. Schottky, Ann. Phys. (Leipzig) **57**, 541 (1918).
[2] For a recent review, see M. J. M. de Jong, and C. W. J. Beenakker, in *Mesoscopic Electron Transport*, Vol. 345 of *NATO Advanced Studies Institute, Series E: Applied Science*, edited by L. L. Sohn, L. P. Kouwenhoven, and G. Schön (Kluwer Academic, Dordrecht, 1997).
[3] V. A. Khlus, Zh. Éksp. Teor. Fiz. **93**, 2179 (1987) [Sov. Phys. JETP **66**, 1243 (1987)].
[4] R. Landauer, Physica D **38**, 226 (1989).
[5] G. B. Lesovik, Pis'ma Zh. Éksp. Teor. Fiz. **49**, 513 (1989) [JETP Lett. **49**, 592 (1989)].
[6] B. Yurke and G. P. Kochanski, Phys. Rev. B **41**, 8184 (1990).
[7] M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990).
[8] I. O. Kulik and A. N. Omel'yanchuk, Fiz. Nizk. Temp. **10**, 305 (1984) [Sov. J. Low Temp. Phys. **10**, 158 (1984)].
[9] C. W. J. Beenakker and M. Büttiker, Phys. Rev. B **46**, 1889 (1992).
[10] K. E. Nagaev, Phys. Lett. A **169**, 103 (1992).
[11] M. J. M. de Jong and C. W. J. Beenakker, Phys. Rev. B **46**, 13 400 (1992).
[12] Yu. V. Nazarov, Phys. Rev. Lett. **73**, 134 (1994).
[13] B. L. Altshuler, L. S. Levitov, and A. Yu. Yakovets, Pis'ma Zh. Éksp. Teor. Fiz. **59**, 821 (1994) [JETP Lett. **59**, 857 (1994)].
[14] M. J. M. de Jong and C. W. J. Beenakker, Phys. Rev. B **51**, 16 867 (1995).
[15] F. Liefrink, J. I. Dijkhuis, M. J. M. de Jong, L. W. Molenkamp, and H. van Houten, Phys. Rev. B **49**, 14 066 (1994).
[16] A. H. Steinbach, J. M. Martinis, and M. H. Devoret, Phys. Rev. Lett. **76**, 3806 (1996).
[17] R. J. Schoelkopf, P. J. Burke, A. A. Kozhevnikov, D. E. Prober, and M. J. Rooks, Phys. Rev. Lett. **78**, 3370 (1997).
[18] M. Henny, H. Birk, R. Huber, C. Strunk, A. Bachtold, M. Krüger, and C. Schönenberger, Appl. Phys. Lett. **71**, 773 (1997).
[19] M. Henny, S. Oberholzer, C. Strunk, and C. Schönenberger, Phys. Rev. B **59**, 2817 (1999).
[20] A. Shimitzu and M. Ueda, Phys. Rev. Lett. **69**, 1403 (1992).
[21] K. E. Nagaev, Phys. Rev. B **52**, 4740 (1995).
[22] V. I. Kozub and A. M. Rudin, Phys. Rev. B **52**, 7853 (1995).
[23] Ya. M. Blanter and M. Büttiker, Phys. Rev. B **56**, 2127 (1997).
[24] B. B. Kadomtsev, Zh. Éksp. Teor. Fiz. **32**, 943 (1957) [Sov. Phys. JETP **5**, 771 (1957)].
[25] Sh. M. Kogan and A. Ya. Shul'man, Zh. Éksp. Teor. Fiz. **56**, 862 (1969) [Sov. Phys. JETP **29**, 467 (1969)].
[26] M. Büttiker, J. Phys.: Condens. Matter **5**, 9361 (1993).
[27] M. Büttiker, Phys. Rev. B **46**, 12 485 (1992).
[28] Th. Martin and R. Landauer, Phys. Rev. B **45**, 1742 (1992).
[29] M. J. M. de Jong and C. W. J. Beenakker, Physica A **230**, 219 (1996).
[30] For a critical discussion of the universality of noise, see R. Landauer, Physica B **227**, 156 (1996); (unpublished).
[31] E. V. Sukhorukov and D. Loss, Phys. Rev. Lett. **80**, 4959 (1998).
[32] V. L. Gurevich and A. M. Rudin, Phys. Rev. B **53**, 10 078 (1996).
[33] Y. Naveh, D. V. Averin, and K. K. Likharev, Phys. Rev. Lett. **79**, 3482 (1997).
[34] K. E. Nagaev, Phys. Rev. B **57**, 4628 (1998).
[35] This contribution is of order $(eV/\varepsilon_F)(l/L)$ in the diffusive regime, where the mean free path l is much smaller than the size of the conductor L.
[36] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981), p. 330.
[37] We mention here that this assumption is an idealization. In a real experiment the heating of the electron gas in the reservoirs caused by the electron-electron interaction is hard to avoid, which makes the experimental observation of the suppression factor of 1/3 a difficult task. On the other hand, the universality of noise (see Sec. IV A) allows us to pull out the contact surfaces L_n far into the reservoirs. Thus, a proper choice of the metallic reservoirs provides in principle the possibility to cool down the electron gas. Therefore, we believe that the above assumption is justified. The role of reservoirs is thoroughly studied in Ref. 19.
[38] A similar equation but in less general form has been used in Refs. 21, 22, and 29 for the calculation of the shot noise in two-terminal conductors.
[39] Joule heat in two-terminal diffusive conductors was studied by M. Rokni and Y. Levinson, Phys. Rev. B **52**, 1882 (1995).
[40] Th. Gramespacher and M. Büttiker, Phys. Rev. Lett. **81**, 2763 (1998).
[41] In this form Π together with Eq. (3.16) can be compared to the general result [Eq. (1.16)] of Ref. 27. We get $(e^2/h)Tr[\mathbf{A}_{kl}(n)\mathbf{A}_{lk}(m)]=2\int d\mathbf{r}\nabla\phi_n\cdot\hat{\sigma}\nabla\phi_m\phi_k\phi_l$, where $\mathbf{A}_{kl}(n)$ are scattering matrices containing all information about disorder and the shape of the conductor. The proof of this formula can be considered as a proof of the equivalence of the classical and the quantum approach to the calculation of the current noise spectrum in diffusive conductors with purely elas-

tic scattering. However, we leave this as an open problem.

[42] Our S_{24} differs from the one obtained in Ref. 23.

[43] R. Hanbury-Brown and R. Q. Twiss, Nature (London) **177**, 27 (1956).

[44] R. C. Liu, B. Odom, Y. Yamamoto, and S. Tarucha, Nature (London) **391**, 263 (1998).

[45] Our universality proof generalizes the discussion of Ref. 29, which considers a quasi-one-dimensional wire.

[46] In the case of hot electrons the expressions for S_{nm}, which are analogous to Eq. (4.15) are rather lengthy and we shall omit them here.