Low-energy effective action of lightly doped two-leg t-J ladders

Yu-Wen Lee* and Yu-Li Lee†
NCTS and Physics Department, National Tsing Hua University, Hsinchu, Taiwan
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We propose a low-energy effective theory of lightly doped two-leg t-J ladders with the help of slave fermion technique. The continuum limit of this model consists of two kinds of Dirac fermions, which are coupled to the O(3) nonlinear sigma model in terms of the gauge coupling with opposite sign of ''charges.'' In addition to the gauge interaction, there is another kind of attractive force between these Dirac fermions, which arises from the short-ranged antiferromagnetic order. We show that the latter is essential to determine the low energy properties of lightly doped two-leg t-J ladders. The effective Hamiltonian we obtain is a bosonic Gaussian model and the boson field basically describes the particle-density fluctuation. We also find two types of gapped spin excitations. Finally, we discuss the possible instabilities: charge density wave and singlet superconductivity (SC). We find that the SC instability dominates in our approximation. Our results indicate that lightly doped ladders fall into the universality class of Luther-Emery model. [S0163-1829(99)08801-3]

I. INTRODUCTION

The properties of ladder systems described by t-J and Hubbard models have been the subject of intensive studies recently.[1–13] The reason is that there are systems such as $(VO)_2P_2O_7$ (Ref. 14) and $SrCu_2O_3$ (Ref. 15) which can be the possible realization of these models. Experiments on magnetic susceptibility and neutron inelastic scattering show the existence of a finite spin gap. Moreover, a recent measurement shows the sign of superconductivity in the doped spin ladders.[16] Therefore, the study of these systems can offer new insights into the nature of magnetic, charge density, and pairing correlations in strongly correlated electron systems.

The undoped ladder systems show unusual magnetic behaviors. When the number of chains is even, there is a spin gap. When it is odd, the spin excitation is gapless. This can be easily understood by considering the limit of strong rung interactions. For two-leg ladders, since the spins on every rung forms a singlet first in this limit, it can be considered as a set of weakly coupled rung singlets. The spin excitation is formed by turning over one spin on a rung and this costs finite energy. Along the same reasoning, a three-leg ladder is effectively equivalent to a spin-1/2 chain and the latter has gapless spin excitations. Numerical studies[17] show that the spin gap for two-leg ladders persists even at the experimentally interested isotropic point. (By this we mean that the rung interaction is equal to the intrachain interaction.) This indicates that even at the isotropic point, the rough picture of a ground state dominated by rung singlets is robust. People have investigated spin ladders by the semiclassical (large spin) approach.[18] Remarkably, this approach qualitatively captures the basic feature of this system as in the case of spin chains. Besides, the spectrum predicted by the nonlinear sigma model is in good agreement with other approximations[19] both in the strong and weak rung interaction limit. These facts imply that the nonlinear sigma model correctly describes the low-energy sector of the spin-liquid state of ladder systems though it ignores fluctuations along the rung. This is one of the motivations that we would like to use the large spin approach to study the lightly doped case.

The effect of doping on the antiferromagnetism is an important and unsettled issue in strongly correlated electron systems. The main difficulty lies in hole motions, which cause frustration of the antiferromagnetic (AF) order. Theoretically, we have no adequate analytical methods to deal with the competition between charge and magnetic fluctuations because there is no obvious small parameter that facilitates an expansion about a tractable model. In the present paper, we employ the large spin expansion which permits us to map the original model to a continuum-field theory. And this allows us to tackle the problem with analytical methods due to the 1-d nature of this system. Mean-field studies[2–4] and numerical calculations[7–9] show that the spin gap in undoped two-leg ladders still persists at low-doping concentration. This implies that the underlying short-ranged AF order is not destroyed too much by hole motions as the doping concentration is low. Thus, after we resolve the constraint in the slave fermion representation, we assume that spin variables have a strong short-ranged AF order. This leads to the t'-J model proposed by Wiegman, Wen, Lee, and Shankar.[20,21] Because of this background AF order, it is natural to say that there are two kinds of holes (on A and B sublattice) with opposite sign of (fictitious) charges and they are coupled to the staggered magnetization through the Berry phase. In contrast to previous studies,[20] we keep the nearest-neighbor attractive four-fermion interactions between A and B holes. This attractive force can be understood as the following: the energy of two holes on the same spin singlet is lower than that of two holes on different singlets because there are less broken bonds in the former. And this is equivalent to an attractive force between holes on different sublattices. We discuss this model in the absence of the quartic fermion interaction first. To study the low-energy physics we can linearize the dispersion relation of fermions about the Fermi points. It turns out that we have four branches of massless Dirac fermions coupled to the nonlinear sigma model. From the study of Schwinger model, we know that massless Dirac fermions will screen the long-range Coulomb force or give a mass term to the gauge field on account of the chiral anomaly. Therefore, the gauge fields are in the Higgs phase.[22] Because of the Gauss law, all excitations must be gauge singlets (or mesons in the jargon of particle physics). In the present case, we have three gapless spin-singlet collective modes. There are also gapped spin-1/2 excitations that carry the electronic charge but no magnons. The superconductivity (SC) instability is enhanced. Then we switch on the nearest-neighbor attractive four-fermion interactions. Two of the gapless modes become massive and the long-range Coulomb force is not screened, i.e. the gauge fields are in the confining phase. Consequently, we have only one gapless charge mode, which is spin singlet. We also have gapped spin-triplet excitations and electronlike collective modes, which are spin-1/2 and carry electronic charges. We compute the exponents of pairing correlation function and $4k_F$ charge density wave (CDW) susceptibility. In our approximation, turning on the quartic fermion interactions enhances the tendency toward SC relative to the tendency toward CDW. Therefore, a weak interladder interaction will lead to SC, which has been predicted by other approaches.[2–4,7–9] Our analysis indicates that these two types of attractive forces play different roles in two-leg t-J ladders. While the formation of spin gap and spin-hole bound states are due to the gauge interaction, four-fermion interactions between A and B holes are responsible for the pairing between holes. In addition, inclusion of the latter drastically changes the low energy properties of doped ladders such that they fall into the universality class of Luther-Emery model.[23] Our results confirm the conclusions from numerical studies.[9]

The rest of the paper is organized as follows. In Sec. II we derive the low-energy effective action. In Sec. III we discuss the implications of this action. We study the effect of gauge interaction in Sec. III A. In Sec. III B we take into account the four-fermion interaction between A and B holes. Section IV is the conclusion. We give a summary of the bosonization rules we used in the Appendix.

II. DERIVATION OF THE EFFECTIVE ACTION

Since the first part of our derivation is valid for general t-J models, we will not write down the ladder index explicitly until it is necessary. We start from the following model:

$$H=-t\sum_{\langle i,j\rangle} X_{\sigma 0}(i)X_{0\sigma}(j)+\sum_i \epsilon_d X_{\sigma\sigma}(i) + \frac{J}{4}\sum_{\langle i,j\rangle}\left[X_{\sigma\sigma'}(i)X_{\sigma'\sigma}(j)-\frac{1}{2}X_{\sigma\sigma}(i)X_{\sigma'\sigma'}(j)\right], \quad (1)$$

where $\langle i,j\rangle$ means the nearest-neighbor sites, $X_{ab}\equiv|a\rangle\langle b|$ and $|a\rangle=|0\rangle,|\uparrow\rangle,|\downarrow\rangle$ corresponding to the empty site and spin-up (spin-down) sites. Since transitions between empty and occupied states include a change in the fermonic number, the operators $X_{\sigma 0}(i), X_{0\sigma}(i)$ are fermonic and the operators $X_{\sigma\sigma'}(i), X_{00}(i)$ are bosonic. It is easy to check that they satisfy the following graded Lie algebra called Spl(1,2):[24]

$$[X_{ab}(i),X_{cd}(j)]_{\pm}=\delta_{ij}[\delta_{bc}X_{ad}(i)\pm\delta_{ad}X_{cb}(i)], \quad (2)$$

where $(+)$ should be used only if both operators are fermonic. In the limit of zero doping, $\epsilon_d\to-\infty$ and the model is reduced to the spin-1/2 Heisenberg model.

Among various methods to deal with Eqs. (1) and (2), there are two popular representations of the above graded Lie algebra—slave fermion and slave boson. We adopt the former and introduce a vacuum state annihilated by operators f_i and $b_\sigma(i)$. Then the X operators can be represented as follows:

$$X_{0\sigma}(i)=f_i^+b_\sigma(i),$$
$$X_{\sigma 0}(i)=f_ib_\sigma^+(i),$$
$$X_{\sigma\sigma'}(i)=b_\sigma^+(i)b_{\sigma'}(i),$$
$$X_{00}(i)=f_i^+f_i \quad (3)$$

with the constraint $b_\sigma^+(i)b_\sigma(i)+f_i^+f_i=1$ and $\sigma=1,-1$ for spin up and down, respectively. Here f_i, f_i^+ satisfy canonical anticommutation relations and $b_\sigma(i)$, $b_\sigma^+(i)$ satisfy canonical commutation relations. To use the large spin expansion, we replace the above constraint with this one: $b_\sigma^+(i)b_\sigma(i)+f_i^+f_i=2S$, which is called the spin s representation of Spl(1,2). Now the X operators represent transitions between states $|s\rangle$ and $|s-1/2\rangle$. With the help of Eq. (3), we can write down the path integral representation of the t-J model

$$Z=\int D[f]D[f^+]D[b_\sigma]D[b_\sigma^+]\delta[b_\sigma^+(i)b_\sigma(i)+f_i^+f_i-2S]\exp(-I),$$

$$I=\int_0^\beta d\tau\sum_i [f_i^+(\partial_\tau-\mu)f_i+b_\sigma^+(i)\partial_\tau b_\sigma(i)]+\int_0^\beta d\tau H,$$

$$H=t\sum_{\langle i,j\rangle} f_j^+f_ib_\sigma^+(i)b_\sigma(j)+\frac{J}{4}\sum_{\langle i,j\rangle}\left[b_\alpha^+(i)b_\beta(i)b_\beta^+(j)b_\alpha(j)-\frac{1}{2}b_\alpha^+(i)b_\alpha(i)b_\beta^+(j)b_\beta(j)\right]. \quad (4)$$

In the above equation, ϵ_d has been absorbed into μ, the chemical potential of holes in terms of the constraint in the path integral measure. Based on the property of Grassmann variables: $(f^+f)^2=0$, we can resolve the constraint in the measure[24] as the following:

$$b_1=\sqrt{2S}\left(1-\frac{1}{4S}f^+f\right)\cos\frac{\theta}{2}\exp\frac{i}{2}(\chi-\phi),$$

$$b_2=\sqrt{2S}\left(1-\frac{1}{4S}f^+f\right)\sin\frac{\theta}{2}\exp\frac{i}{2}(\chi+\phi) \quad (5)$$

where $\mathbf{S}=(S-\frac{1}{2}f^+f)\mathbf{\Omega}$ and

$$\mathbf{\Omega}=(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta).$$

With the new variable $\mathbf{\Omega}$, we can rewrite the partition function as follows:

$$Z=\int D[\mathbf{\Omega}]D[f]D[f^{+}]\exp(-I),$$

$$I=\int_{0}^{\beta}d\tau\sum_{j}\left[f_{j}^{+}(\partial_{\tau}-\mu)f_{j}+i\left(S-\frac{1}{2}f_{j}^{+}f_{j}\right)\mathbf{A}(\mathbf{\Omega}_{j})\cdot\partial_{\tau}\mathbf{\Omega}_{j}\right]+\int_{0}^{\beta}d\tau H,$$

$$H=\overline{t}\sum_{\langle i,j\rangle}f_{j}^{+}f_{i}\langle\mathbf{\Omega}_{i}|\mathbf{\Omega}_{j}\rangle_{1/2}+\frac{\overline{J}}{8}\sum_{\langle i,j\rangle}\left(1-\frac{1}{2S}f_{i}^{+}f_{i}\right)\left(1-\frac{1}{2S}f_{j}^{+}f_{j}\right)\mathbf{\Omega}_{i}\cdot\mathbf{\Omega}_{j} \quad (6)$$

where $\overline{t}\equiv 2St$ and $\overline{J}\equiv 4JS^{2}$. (Note that when $S=1/2$, $\overline{t}=t$ and $\overline{J}=J$.) $\langle\mathbf{\Omega}_{1}|\mathbf{\Omega}_{2}\rangle_{S}$ represents the overlap of two spin-S coherent states, $|\mathbf{\Omega}_{1}\rangle$ and $|\mathbf{\Omega}_{2}\rangle$. $\mathbf{A}(\mathbf{\Omega})$ is the monopole vector potential, which satisfies $\nabla\times\mathbf{A}=\mathbf{\Omega}$. (Here we have chosen the gauge such that $\mathbf{A}\cdot\partial_{\tau}\mathbf{\Omega}=-\cos\theta\partial_{\tau}\phi$ and χ is independent of τ.)

Now we focus ourselves on two-leg t-J ladders. Using the notation of Eq. (6), the action can be written as follows:

$$I=\int_{0}^{\beta}d\tau\sum_{m}\sum_{j}\left[f_{j,m}^{+}(\partial_{\tau}-\mu)f_{j,m}+i\left(S-\frac{1}{2}f_{j,m}^{+}f_{j,m}\right)\times\mathbf{A}(\mathbf{\Omega}_{j,m})\cdot\partial_{\tau}\mathbf{\Omega}_{j,m}\right]+\int_{0}^{\beta}d\tau H,$$

$$\begin{aligned}H=&\,\overline{t}\sum_{m}\sum_{\langle i,j\rangle}f_{j,m}^{+}f_{i,m}\langle\mathbf{\Omega}_{i,m}|\mathbf{\Omega}_{j,m}\rangle_{1/2}\\&+\overline{t}_{\perp}\sum_{j}[f_{j,2}^{+}f_{j,1}\langle\mathbf{\Omega}_{j,1}|\mathbf{\Omega}_{j,2}\rangle_{1/2}+\text{H.c.}]\\&+\frac{\overline{J}}{4}\sum_{m}\sum_{j}\left(1-\frac{1}{2S}f_{j,m}^{+}f_{j,m}\right)\times\left(1-\frac{1}{2S}f_{j+1,m}^{+}f_{j+1,m}\right)\mathbf{\Omega}_{j,m}\cdot\mathbf{\Omega}_{j+1,m}\\&+\frac{\overline{J}_{\perp}}{4}\sum_{j}\left(1-\frac{1}{2S}f_{j,1}^{+}f_{j,1}\right)\left(1-\frac{1}{2S}f_{j,2}^{+}f_{j,2}\right)\mathbf{\Omega}_{j,1}\cdot\mathbf{\Omega}_{j,2},\end{aligned} \quad (7)$$

where $m=1,2$ is the chain index. To proceed, we have to make some assumptions. First of all, we assume that there is a short-ranged AF order such that we can parametrize $\mathbf{\Omega}_{j,m}$ as the following:[18]

$$\mathbf{\Omega}_{j,m}=(-1)^{j+m}\sqrt{1-a^{2}\mathbf{L}_{m}^{2}}\,\mathbf{n}(x)+a\mathbf{L}_{m}(x), \quad (8)$$

where $\mathbf{n}^{2}=1$, $\mathbf{n}\cdot\mathbf{L}_{m}=0$ and a is the lattice spacing. $\mathbf{n}$ is the order parameter and $\mathbf{L}_{m}$ are the fast modes that will be integrated out. Comparing the t term and J term in Eq. (7), it is clear that in the large spin limit J term dominates and thus Eq. (8) is valid. For small spins, we expect it is still a good assumption if there are strong short-range AF correlations and the latter is reflected by the existence of a finite spin gap in two-leg ladders. However, some conditions are required for a nonvanishing spin gap. First, according to numerical studies,[7–9] the hole concentration must be low. Second, J/t should be of order one. As has been noticed in Ref. 9, the Nogaoka theorem is applicable in ladder systems. This theorem says that the one-hole ground state is ferromagnetic at $J=0$. This phase may be stable when J/t is very small.[11] Besides, numerical studies[7–9] show that the phase separation occurs when $J/t>2$. Therefore, the qualitative feature of the following results may be valid for the spin one-half case when the ratio of J/t is order of one and the hole concentration is low.

Because of the background AF order, the original lattice is divided into two sublattices and it is natural to have two kinds of holes. We will call them A holes and B holes when $j+m$ is even and odd, respectively. An immediate consequence of Eq. (8) is that the hole cannot hop coherently between different sublattices, i.e., the intersublattice hopping is forbidden. This can be understood as the following: The amplitude for a hole hopping from site i to j is the product of the overlap of *spin* and *orbital* wave functions, i.e., $-t_{ij}\langle\mathbf{\Omega}_{j}|\mathbf{\Omega}_{i}\rangle$. In this case, it is $-t\langle\mathbf{\Omega}_{A}|\mathbf{\Omega}_{B}\rangle$. In view of Eq. (8), this is zero if we have perfect Néel order (recall that $\langle\mathbf{\Omega}|-\mathbf{\Omega}\rangle=0$) and exponentially small in a system with strong short-range AF order. Thus, t and $t_{\perp}$ terms are effectively removed from the low-energy effective Hamiltonian in the large S analysis. One of their effects is to renormalize the parameters in low-energy physics. It seems that the hole can hop coherently within the same sublattices under the hypothesis Eq. (8). However, the question of whether the hole can have coherent hopping is related to whether we have well-defined quasiparticles in low energy. As pointed out in Ref. 25, the latter depends on the density of states of spin waves in low energy. It turns out that there can be no well-defined quasiparticles in one dimension if the spin excitation is gapless. Fortunately, there is a spin gap in our system. Following the calculations in Ref. 25, there will be a sharp peak at the energy scale $-t$ in the spectral function of one-particle Green function and this implies a well-defined quasiparticle band at the bottom of the hole spectrum. Consequently, via the interactions with spins, the holes are able to acquire a kinetic energy, whether there is a ‘‘bare hopping term’’ or not. In the following, we shall introduce a t' term, i.e., hopping within the same sublattice, to represent the kinetic energy of holes. According to numerical calculations,[9] t' is of the same order as J.

Substituting Eq. (8) into Eq. (7), adding a gauge-invariant t' term to Eq. (7) and expanding the action to the quadratic terms of $\mathbf{L}_{m}$ then integrating out $\mathbf{L}_{m}$, we get to the following effective action:

$$I=I_{n}+I_{h},$$

$$I_{n}=\frac{1}{2g^{2}}\int_{0}^{\beta}d\tau\int dx\left(\frac{1}{v_{s}^{2}}|\partial_{\tau}\mathbf{n}|^{2}+|\partial_{x}\mathbf{n}|^{2}\right),$$

$$I_{h}=\int_{0}^{\beta}d\tau\sum_{m}\sum_{j}[f_{A,m}^{+}(j)(\partial_{\tau}-ia_{0}-\mu)f_{A,m}(j)+f_{B,m}^{+}(j)(\partial_{\tau}+ia_{0}-\mu)f_{B,m}(j)]+\int_{0}^{\beta}d\tau H_{h} \quad (9)$$

and

$$H_h=H_0+H_1+H_2,$$

$$H_0=\overline{t'}\sum_m\sum_j [f^+_{A,m}(j+2)f_{A,m}(j)\exp(ia_x\Delta x)+\text{H.c.}]$$
$$+(A\to B,a_x\to -a_x),$$

$$H_1=-\frac{J}{4}\sum_m\sum_j [:f^+_{A,m}(j)f_{A,m}(j)::f^+_{B,m}(j+1)$$
$$\times f_{B,m}(j+1):+(A\leftrightarrow B)],$$

$$H_2=-\frac{J_\perp}{4}\sum_j [:f^+_{A,1}(j)f_{A,1}(j)::f^+_{B,2}(j)f_{B,2}(j):+(A\leftrightarrow B)], \tag{10}$$

where $g^2=2/\overline{J}a(1-\delta/2S)^2$, $v_s=(\overline{J}a/2S)(1-\delta/2S)\sqrt{1+(\overline{J_\perp}/2\overline{J})}$, $\overline{t'}\equiv 2St'$, $\Delta x=2a$, and δ is the hole concentration. In the derivation of above equations, we have used a property of spin-coherent states: $\langle\mathbf{\Omega}_1|\mathbf{\Omega}_2\rangle_S\approx\exp -iS\mathbf{A}\cdot(\mathbf{\Omega}_1-\mathbf{\Omega}_2)$ when $\mathbf{\Omega}_1\approx\mathbf{\Omega}_2$. The gauge fields $a_\mu\equiv\frac{1}{2}\mathbf{A}\cdot\partial_\mu\mathbf{n}$. It is clear that A and B holes carry opposite sign of charges because they are on different sublattices in which the staggered magnetizations have opposite directions. We propose that the low-energy sector of lightly doped two-leg ladders can be described by Eqs. (9) and (10).

We conclude this section with a brief summary of the CP^1 representation of the nonlinear sigma model,[26] which will be used later. The order parameter $\mathbf{n}$ can be parametrized with a normalized spinor as

$$\mathbf{n}=\bar{z}\sigma z,\quad \bar{z}z=1.$$

The CP^1 version of the nonlinear sigma model in the Euclidean space is

$$Z=\int D[a_\mu]D[z]D[\bar{z}]D[\lambda]\exp(-I),$$

$$I=\frac{1}{e^2}\int d^2x[|\partial_\mu z-ia_\mu z|^2+i\lambda(|z|^2-1)], \tag{11}$$

where $x_0=v_s\tau$, $e^2=2g^2v_s$, and λ is the Lagrangian multiplier. An appropriate choice of the gauge makes a_μ in Eq. (11) equivalent to the ones in Eqs. (9) and (10) and thus we can use the same notation to denote them. In the CP^{N-1} model at large N, the z quanta become massive, i.e., λ acquires a nonvanishing mean value. At distances larger than this scale, the effective action of gauge fields will have the usual Maxwell term. Accordingly, the low-energy excitation is a massive triplet, which can be considered as the bound state of $\bar{z}$ and z.

III. EXCITATION SPECTRUM

To discuss the low-energy physics, we linearize the dispersion relation of fermions about their Fermi points, which satisfy $k_Fa=(\pi/2)(1-\delta)$. (We have assumed that the Hamiltonian is invariant when we interchange the chain index and A and B holes. Besides, remember that a is the lattice spacing of the original lattice.) After doing that, H_0 becomes

$$H_0=E_0+v_0\sum_m\int dx[:\psi^+_{A,m}\alpha(-i\partial_x-a_x)\psi_{A,m}:$$
$$+(A\to B,a_x\to -a_x)], \tag{12}$$

where $v_0=4\overline{t'a}\sin 2k_Fa=4\overline{t'a}\sin\pi\delta$ is the Fermi velocity, ψ is the two-component Dirac fermion, and $\alpha=\sigma_3$. Now we can analyze the effects of different interactions.

A. The effect of gauge coupling

We set $H_1=H_2=0$ first. Then we rescale the imaginary time: $v_0\tau\to\tau$ and do analytical continuation to the real-time formalism. The effective action is

$$I=I_0+I_n,$$

$$I_0=\sum_m\int d^2x[\overline{\psi}_{A,m}\gamma^\mu(i\partial_\mu-a_\mu-eA_\mu)\psi_{A,m}$$
$$+(A\to B,a_\mu\to -a_\mu)] \tag{13}$$

and I_n has the same form as the one in Eq. (9) and γ_μ is the Dirac γ matrices. Here A_μ are the external electromagnetic fields. In terms of the standard bosonization rules (see the Appendix), I_0 can be bosonized as the following:

$$I_0=\frac{1}{2}\sum_m\int d^2x[(\partial_\mu\phi_{A,m})^2+(\partial_\mu\phi_{B,m})^2]$$
$$-\frac{e}{\sqrt{\pi}}\sum_m\int d^2x\,A_\mu\epsilon^{\mu\nu}\partial_\nu(\phi_{A,m}+\phi_{B,m})$$
$$+\frac{1}{\sqrt{\pi}}\sum_m\int d^2x\,a_\mu\epsilon^{\mu\nu}\partial_\nu(\phi_{A,m}-\phi_{B,m}). \tag{14}$$

We define $\phi_{\pm,m}\equiv(1/\sqrt{2})(\phi_{A,m}\pm\phi_{B,m})$. Then I_0 can be written as

$$I_0=\frac{1}{2}\sum_m\int d^2x[(\partial_\mu\phi_{+,m})^2+(\partial_\mu\phi_{-,m})^2]$$
$$-\sqrt{\frac{2}{\pi}}e\sum_m\int d^2x\,A_\mu\epsilon^{\mu\nu}\partial_\nu\phi_{+,m}$$
$$+\sqrt{\frac{2}{\pi}}\sum_m\int d^2x\,a_\mu\epsilon^{\mu\nu}\partial_\nu\phi_{-,m}. \tag{15}$$

The above action can be further simplified if we define the following fields:

$$\Phi_1 \equiv \frac{1}{\sqrt{2}}(\phi_{+,1}+\phi_{+,2}),$$

$$\Phi_2 \equiv \frac{1}{\sqrt{2}}(\phi_{+,1}-\phi_{+,2}),$$

$$\Phi_3 \equiv \frac{1}{\sqrt{2}}(\phi_{-,1}+\phi_{-,2}),$$

$$\Phi_4 \equiv \frac{1}{\sqrt{2}}(\phi_{-,1}-\phi_{-,2}). \tag{16}$$

With the help of the above canonical transformation, we obtain the following low-energy effective action

$$I_0 = \frac{1}{2}\sum_{\alpha=1}^{4}\int d^2x(\partial_\mu \Phi_\alpha)^2 - \frac{2}{\sqrt{\pi}} e\int d^2x\, A_\mu \epsilon^{\mu\nu}\partial_\nu \Phi_1 + \frac{2}{\sqrt{\pi}}\int d^2x\, \Phi_3 \epsilon^{\mu\nu}\partial_\mu a_\nu . \tag{17}$$

What can we learn from Eqs. (11) and (17)? First, there are three gapless spin-singlet excitations: Φ_1, Φ_2, and Φ_4. Second, to understand the coupled system of Φ_3 and z, we have to investigate the gauge-field dynamics. This can be done by integrating out the Φ_3 field since its action is quadratic and we obtain

$$\frac{1}{\pi^2}\int d^2x d^2y \tilde{f}(x)\ln|x-y|\tilde{f}(y),$$

where $\tilde{f} \equiv \epsilon^{\mu\nu}\partial_\mu a_\nu$ is the dual-field strength. If we choose the Lorentz gauge as $\partial_\mu a^\mu = 0$, then the above equation becomes the mass term of a_μ. That is to say, the fluctuations of the Φ_3 field will screen the long-range Coulomb force. It follows that there are massive spin-1/2 excitations and they are neutral with respect to the a_μ fields in view of the gauge-field mass. (This is easy to be seen by integrating Gauss's law from $-\infty$ to $+\infty$.) Since the z quantum has charge 1, where is the compensating charge from? It must come from the hole sector. We can understand this as the following.[21]

Let us integrate out all fields except Φ_3. The action we obtain must be of the form $\cos 8\sqrt{\pi} n \Phi_3$ where n is an integer. (Of course, the renormalization of the kinetic term is possible.) This is because the coefficient of Φ_3 in Eq. (17) is proportional to the instanton density: $(1/4\pi)\,\epsilon^{\mu\nu}\partial_\mu a_\nu$. The action is invariant under the transformation: $\Phi_3 \rightarrow \Phi_3 + \sqrt{\pi}/4$ and the cosine just has the form that satisfies the requirement. If the cosine is relevant, this translation symmetry is spontaneously broken and the value of Φ_3 is pinned at some minimum of the potential. In addition, its solitons carry exactly the charge needed to match the charge of the z quantum. The Gauss law demands that at least one of the cosines in question must be relevant and thus the excitations corresponding to the Φ_3 sector are massive. These solitons are nothing but the A (or B) holes. They carry the electronic charges as well as the U(1) charges. As a consequence, we expect these massive doublets also carry the electronic charges, i.e., they carry the same quantum numbers as electrons. We should emphasize that there are no $z\bar{z}$ bound states, i.e., massive triplets, in this model. As we shall see later, the absence or presence of this type of excitations is one of the distinctions between the Higgs phase and confining phase in our model.

After identifying the spectrum, we would like to examine the pairing correlation function. Because of the spin gap, the pair field of singlet SC can be defined as the following:

$$\Delta(j) \equiv f_{A,1}(j) f_{B,2}(j). \tag{18}$$

Since the correlation functions of Φ_3 decay exponentially, the long-distance behavior of the pairing correlation function can be calculated by the following operators:

$$\Delta(j) \sim \exp(-i\sqrt{\pi}\Theta_1)\exp(-i\sqrt{\pi}\Phi_2)\exp(-i\sqrt{\pi}\Theta_4) + \exp(-i\sqrt{\pi}\Theta_1)\exp(i\sqrt{\pi}\Phi_2)\exp(-i\sqrt{\pi}\Theta_4), \tag{19}$$

where Θ_α is the dual field of Φ_α. The pairing correlation function behaves as $\langle \Delta(j)\Delta^+(0)\rangle \xrightarrow{|j|\to\infty} 1/|j|^{3/2}$. Compared with the one of free fermions, which behaves like $1/|j|^2$, we see that the pairing susceptibility is enhanced.

To sum up, if we consider the gauge coupling only, then the low-energy effective Hamiltonian consists of three gapless spin-singlet modes and one massive spin-1/2 modes, which carry the electronic charges. (It is possible that there are massive spin-singlet modes. However, the gapless modes and the massive doublet predominant the long-distance behavior of correlation functions.) Compared with the numerical results,[9] there are too many gapless modes but no magnons. It is not enough to consider the gauge interactions only. We will see in the next section the importance of taking into account H_1 and H_2 to get the correct low-energy properties.

B. The role of H_1 and H_2

Here we take H_1 and H_2 into account. The continum limit of them are as follows:

$$H_1 = -g_1\sum_m \int dx{:}\psi^+_{A,m}\psi_{A,m}{::}\psi^+_{B,m}\psi_{B,m}{:} + g_1\cos\pi\delta\sum_m \int dx[\psi^+_{A,L,m}\psi_{A,R,m}\psi^+_{B,R,m}\psi_{B,L,m} + (R\leftrightarrow L)],$$

$$H_2 = -g_2\int dx[{:}\psi^+_{A,1}\psi_{A,1}{::}\psi^+_{B,2}\psi_{B,2}{:} + (A\leftrightarrow B)] - g_2\int dx[\psi^+_{A,R,1}\psi_{A,L,1}\psi^+_{B,L,2}\psi_{B,R,2} + \psi^+_{A,L,1}\psi_{A,R,1}\psi^+_{B,R,2}\psi_{B,L,2} + (A\leftrightarrow B)], \tag{20}$$

where $g_1 = 2Ja/v_0$, $g_2 = J_\perp a/v_0$, and $\psi_{L,R}$ are left-handed and right-handed fermions, respectively. With the same convention used in the previous section, the bosonized forms of the above equations are as follows:

$$H_1=-\frac{g_1}{2\pi}\sum_m\int dx[(\partial_x\phi_{+,m})^2-(\partial_x\phi_{-,m})^2]$$
$$+g_1'\sum_m\int dx\cos\sqrt{8\pi}\phi_-,m,$$

$$H_2=-\frac{g_2}{\pi}\int dx(\partial_x\phi_{+,1}\partial_x\phi_{+,2}-\partial_x\phi_{-,1}\partial_x\phi_{-,2})$$
$$-2g_2'\int dx\cos\sqrt{2\pi}(\phi_{+,1}-\phi_{+,2})\cos\sqrt{2\pi}$$
$$\times(\phi_{-,1}+\phi_{-,2}). \tag{21}$$

We can do the following canonical transformation to diagonalize the quadratic part of the Hamiltonian:

$$\Phi_1=\sqrt{\frac{K_1}{2}}(\phi_{+,1}+\phi_{+,2}),$$
$$\Phi_2=\sqrt{\frac{K_2}{2}}(\phi_{+,1}-\phi_{+,2}),$$
$$\Phi_3=\sqrt{\frac{K_3}{2}}(\phi_{-,1}+\phi_{-,2}), \tag{22}$$
$$\Phi_4=\sqrt{\frac{K_4}{2}}(\phi_{-,1}-\phi_{-,2}).$$

The parameters in the above equations are as follows:

$$K_1=\sqrt{1-\frac{g_1+g_2}{\pi}},$$
$$K_2=\sqrt{1-\frac{g_1-g_2}{\pi}},$$
$$K_3=\sqrt{1+\frac{g_1+g_2}{\pi}}, \tag{23}$$
$$K_4=\sqrt{1+\frac{g_1-g_2}{\pi}}.$$

These relations are valid only in the weak-coupling limit. In our case, this corresponds to the large-spin limit. [Remember that $\overline{t'}(=2St')$ appears in the denominator of the definition of g_i.] We can see that $K_1<1$ and $K_3>1$ because $g_i>0$. (This corresponds to the attractive force between holes.) The effective Hamiltonian of the hole sector is

$$H_h=\frac{1}{2}\sum_{\alpha=1}^{4}K_\alpha\int dx[(\partial_x\Theta_\alpha)^2+(\partial_x\Phi_\alpha)^2]$$
$$+2g_1'\int dx\cos\sqrt{\frac{4\pi}{K_3}}\Phi_3\cos\sqrt{\frac{4\pi}{K_4}}\Phi_4$$
$$-2g_2'\int dx\cos\sqrt{\frac{4\pi}{K_2}}\Phi_2\cos\sqrt{\frac{4\pi}{K_3}}\Phi_3. \tag{24}$$

Here, we neglect the gauge coupling temporarily and will discuss it later. The relevancy of these interactions is determined by their scaling dimensions. They are $\Delta_1=1/K_3+1/K_4$ and $\Delta_2=1/K_2+1/K_3$, which correspond to g_1' and g_2' terms, respectively. From Eq. (23), $K_2<1$ and $K_4>1$ at the isotropic point. This implies $\Delta_1<2$. Therefore, g_1' term is a relevant operator in the sense of renormalization group. The Φ_3 and Φ_4 field are pinned at some values and both acquire gaps. They are decoupled from the low-energy theory. Taking into account these facts, the effective Hamiltonian becomes

$$H_h=\frac{1}{2}\sum_{\alpha=1,2}K_\alpha\int dx[(\partial_x\Theta_\alpha)^2+(\partial_x\Phi_\alpha)^2]$$
$$-g\int dx\cos\sqrt{\frac{4\pi}{K_2}}\Phi_2, \tag{25}$$

where $g=2g_2'\langle\cos\sqrt{4\pi/K_3}\Phi_3\rangle$. In the large spin limit, $K_2>1/2$. Thus the scaling dimension of g term is less than two. It is a relevant operator. The Φ_2 field is also massive. In the low-energy limit, we obtain our effective Hamiltonian as the following:

$$H_{\rm eff}=\frac{K_1}{2}\int dx[(\partial_x\Theta_1)^2+(\partial_x\Phi_1)^2]. \tag{26}$$

Now we would like to discuss the implications of our results. First of all, the low-energy effective Hamiltonian consists of a gapless spin-singlet mode and from Eq. (22), this mode describes the total charge-density fluctuation. Furthermore, there is only one free parameter K_1, the compactification radius of the Φ_1 field, which has to be determined from experiments. This supports the suggestion proposed in Ref. 9. Second, since the Φ_3 field is pinned at some value, it cannot affect the nonlinear sigma model too much. Especially, it is unable to screen the long-range Coulomb force. As a result, the gauge field is in the confining phase. This leads to two types of spin excitations. One is the massive triplet (in the CP^1 language, it is the $z\bar{z}$ bound state), which is the same as the magnon in the undoped case. The other is the bound state of holes and z quanta, which carries the same quantum numbers as the electron. In fact, this excitation can be considered as the breaking of a hole pair and we have two quasiparticles. Each carries electric charge one and spin one-half. The latter is also observed in Ref. 9. In that paper, Troyer *et al.* find that the spin gap is determined by the bound state of spinons and holons. We cannot compare the gaps of these excitations in our approach. However, we give a picture about the formation of the unexpected spin excitation. The Berry phase term gives rise to the necessary attractive force between spinons and holons. Since this force is a gauge interaction and the latter in 1d is a linear confining potential, this results in bound states of spinons and holons.

After we understand the spectrum, we can calculate the asymptotic behavior of various correlation functions. The most important ones are pairing and CDW correlation functions. The definition of the pair field is the same as Eq. (18). The long-distance behavior of the pairing correlation function can be calculated by the operator

$$\Delta(j)\sim\exp(-i\sqrt{\pi K_1}\Theta_1) \tag{27}$$

because other fields are massive. The result is

$$\langle \Delta(j)\Delta^{+}(0)\rangle \overset{|j|\to\infty}{\to} \frac{1}{|j|^{K_1/2}}. \tag{28}$$

Next we shall examine the $2k_F$ CDW susceptibility. The corresponding order parameter O_{CDW} can be expressed as the product of hole operators and spin operators. As emphasized in the previous paragraph, matter fields do nothing much on the spin sector of the effective action. We can find the contributions of spin operators from results of the undoped case. The work of Shelton *et al.*[27] showed that the low-energy theory of two-leg ladders is described by four decoupled noncritical Ising models with three of them in the ordered phase and one in the disordered phase (or vice versa). Moreover, the spin part in O_{CDW} can be expressed in terms of the order (σ) and disorder (μ) parameter fields of the Ising model as $\mu_1\mu_2\mu_3\mu_0$ or $\sigma_1\sigma_2\sigma_3\sigma_0$. It is clear that the correlation functions of the above operators decay exponentially to zero. This implies that $\langle O_{CDW}(x)O_{CDW}(0)\rangle$ shows the same behavior. Then we have to consider the $4k_F$ CDW susceptibility or $\langle O^2_{CDW}(x)O^2_{CDW}(0)\rangle$. Using the OPE $\mu(z)\mu(\omega)\sim 1/(z-\omega)^{1/4}+\cdots$, it is straightforward to see that the spin part of this correlation function contributes a nonvanishing constant to it. Thus, the long-distance behavior of $\langle O^2_{CDW}(x)O^2_{CDW}(0)\rangle$ can be determined solely through its hole part. If we define the operator

$$O_h(x)\equiv \psi^{+}_{R,A,1}\psi^{+}_{R,B,2}\psi_{L,A,1}\psi_{L,B,2}\sim \exp\left(-i\sqrt{\frac{4\pi}{K_1}}\Phi_1\right), \tag{29}$$

then

$$\langle O^2_{CDW}(x)O^2_{CDW}(0)\rangle\sim\langle O_h(x)O_h(0)\rangle\sim\frac{1}{|x|^{2/K_1}}. \tag{30}$$

In the second line of Eq. (29), we keep the gapless mode only. From Eqs. (28) and (30), it is clear that SC dominates when $K_1<2$ and CDW dominates when $K_1>2$. In the large spin limit, $K_1\approx 1$. Therefore, we conclude that SC susceptibility dominates in two-leg t-J ladders and a weak interladder interaction will lead to superconductivity at low temperature. Also the exponents of pairing and CDW susceptibility satisfy the relation: $K_{SC}K_{CDW}=1$. We arrive at the same conclusions as previous numerical investigations. Although we heavily rely on the large spin approximation, our results should capture the basic feature of the system with spin one-half.

In summary, after we take H_1 and H_2 into account, the low-energy effective Hamiltonian only consists of one gapless charge mode, which describes the charge-density fluctuation. The spin excitations are electronlike quasiparticles and magnons and both have energy gaps. The $2k_F$ CDW susceptibility shows exponentially decaying behavior while those of $4k_F$ CDW and singlet SC both show power-law behavior. With the above results, we conclude that lightly doped two-leg ladders fall into the universality class of Luther-Emery model. In addition, this phase is dominated by singlet SC susceptibility according to our analysis.

There are two related works that should be mentioned. The first is the paper by Ichinose and Matsui.[12] They also adopted the slave-fermion and CP^1 boson technique to treat the two-leg t-J ladder. The other is the work of Ivanov and Lee.[13] They use the slave-boson scheme and arrive at the same conclusion that the low-energy sector of the two-leg t-J ladder is a Luther-Emery liquid. For quantitative description of this system, they introduce a new order parameter $n_{\text{pair}}\equiv(n_1-\delta)(n_2-\delta)$ where n_i is the hole density in the ith chain in addition to the ones we considered. They suggested that the long-distance behavior of the $2k_F$ part of $\langle n_{\text{pair}}(x)n_{\text{pair}}(0)\rangle$ (notice that their $2k_F$ is equivalent to our $4k_F$.) is as the following

$$\frac{A}{x^{\alpha_1}}+\frac{B}{x^{\alpha_1+2}},$$

where A and B depend on the average pair overlap: $A\gg B$ at $\xi_{\text{pair}}\delta\ll 1$ and $A\ll B$ at $\xi_{\text{pair}}\delta\gg 1$. Here ξ_{pair} is the size of the hole pair and is of the order of the lattice spacing. α_1 is the exponent of $\langle O^2_{CDW}(x)O^2_{CDW}(0)\rangle$. We calculate this correlation function and we obtain a power-law behavior with the exponent equal to $2/K_1$, which is the same as the one of $\langle O^2_{CDW}(x)O^2_{CDW}(0)\rangle$. We do not find an exponent equal to $2/K_1+2$ even including higher harmonics. Since our approach is valid when $\delta\ll 1$, we cannot tell whether the second term exists or not at larger doping concentration.

IV. CONCLUSION

Anderson has proposed that the spin-liquid state may evolve into a superconductor upon doping. However, it has been proved notoriously difficult to have any concrete analytic result to confirm this idea in two dimensions. A doped spin ladder may provide a good place to study this mechanism though there is no true long range order here. This is so because the undoped two-leg ladder is a kind of spin-liquid state and it is simpler to deal with this problem both analytically and numerically. Soon after the discovery of high T_c superconductivity a model was proposed to describe the doped spin-liquid state by coupling holes to the nonlinear sigma model and the gauge interaction between A and B holes provides the necessary attraction for pairing. Our approach is basically to apply the same idea to ladder systems. In contrast to previous studies, the gauge interaction plays a minor role on the formation of hole pairing in two-leg t-J ladders. Instead, it is mainly due to the effective attraction between the nearest-neighbor holes. Therefore, upon doping, these holes are inclined to stay on the same preexisting singlets. Although this may be a unique characteristic of two-leg ladders exclusively, this point deserves further examination in higher dimensions. We also point out that the gauge interaction is responsible for the electronlike collective mode found in Ref. 9. The existence of this type of excitations is independent of the quartic fermion interactions. The absence or presence of the latter determines whether the long-range Coulomb force is screened or not and thus affects the magnitude of the gap of excitations with the nontrivial spin-quantum number. More importantly, it does affect the existence of the magnon: there are no magnons without the quartic fermion interactions. We have to emphasize that both the effective attraction between nearest-neighbor holes and gauge interactions are due to the strong short-range AF background, which has its origin in the strong repulsive interactions between electrons.

The main assumption we made is to replace Eq. (7) with Eqs. (9) and (10). This is justified for undoped case and for lightly doped two-leg ladders. The spin excitations in the undoped three-leg ladder are gapless and thus the distortion of the short-range AF background arising from hole motions will be so serious such that Eq. (8) may be far from the real situation. Moreover, there may be no coherent peak in the spectral function of one-particle Green function and we are unable to simply add a t' term to the effective Hamiltonian. In other words, the t term should play a more important role in this case. As has been shown in Ref. 10, upon doping, the three-leg t-J ladder has two components—a conducting Luttinger liquid coexisting with an insulating spin-liquid phase. In order to discuss its low-energy physics in the same spirit as the present paper, we need more understanding about the ground state of the undoped case and the behavior of holes in this ground state. Work along this line is under progress.

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APPENDIX: BOSONIZATION RULES

We list all bosonization rules we used in the following:

$$\psi_L(z)=\frac{1}{\sqrt{2\pi a}}\exp\{-i\sqrt{4\pi}\phi_L(z)\},$$

$$\psi_R(\bar{z})=\frac{1}{\sqrt{2\pi a}}\exp\{i\sqrt{4\pi}\phi_R(\bar{z})\},$$

$$i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi=\frac{1}{2}(\partial_{\mu}\phi)^2,$$

$$:\bar{\psi}\gamma_{\mu}\psi:=\frac{1}{\sqrt{\pi}}\epsilon_{\mu\nu}\partial^{\nu}\phi,$$

where $z=\tau+ix$, $\bar{z}$ is its complex conjugate and a is the short-distance cutoff. $\psi_L(z)$ and $\psi_R(\bar{z})$ are left and right fermions, respectively. $\phi_L(z)$ and $\phi_R(\bar{z})$ are bosonic left and right movers, respectively. In terms of them, we define $\phi(\tau,x)=\phi_L+\phi_R$ and $\Theta(\tau,x)=\phi_L-\phi_R$.

*Electronic address: d857312@phys.nthu.edu.tw
†Electronic address: yllee@phys.nthu.edu.tw

[1] For a review, see E. Dagotto and T. M. Rice, Science **271**, 618 (1996).
[2] E. Dagotto, J. Riera, and D. Scalapino, Phys. Rev. B **45**, 5744 (1992).
[3] T. M. Rice, S. Gopalan, and M. Sigrist, Europhys. Lett. **23**, 445 (1993).
[4] M. Sigrist, T. M. Rice, and F. C. Zhang, Phys. Rev. B **49**, 12 058 (1994).
[5] R. M. Noack, S. R. White, and D. J. Scalapino, Phys. Rev. Lett. **73**, 882 (1994).
[6] D. V. Khveshchenko and T. M. Rice, Phys. Rev. B **50**, 252 (1994); D. V. Khveshchenko, *ibid.* **50**, 380 (1994); L. Balents and M. P. A. Fisher, *ibid.* **53**, 12 133 (1996).
[7] C. A. Hayward, D. Poilblanc, R. M. Noack, D. J. Scalapino, and W. Hanke, Phys. Rev. Lett. **75**, 926 (1995); C. A. Hayward and D. Poilblanc, Phys. Rev. B **53**, 11 721 (1996).
[8] D. Poilblanc, D. J. Scalapino, and W. Hanke, Phys. Rev. B **52**, 6796 (1995).
[9] M. Troyer, H. Tsunetsugu, and T. M. Rice, Phys. Rev. B **53**, 251 (1996).
[10] T. M. Rice, S. Haas, M. Sigrist, and F. C. Zhang, Phys. Rev. B **56**, 14 655 (1997).
[11] T. F. A. Müller and T. M. Rice, Phys. Rev. B **58**, 3425 (1998).
[12] I. Ichinose and T. Matsui, Phys. Rev. B **57**, 13 790 (1998).
[13] D. A. Ivanov and P. A. Lee, Phys. Rev. B **57**, 2118 (1998).
[14] D. C. Johnston, J. W. Johnson, D. P. Goshorn, and A. J. Jacobson, Phys. Rev. B **35**, 219 (1987).
[15] Z. Hiroi, M. Takano, M. Azuma, Y. Takeda, and Y. Bando, Physica C **185–189**, 523 (1991); M. Takano, Z. Hiroi, M. Azuma, and Y. Takeda, Jpn. J. Appl. Phys. Ser., **7**, 3 (1992); M. Azuma, Z. Hiroi, M. Takano, K. Ishida, and Y. Kitaoka, Phys. Rev. Lett. **73**, 3463 (1994).
[16] M. Uehara, T. Nagata, J. Akimitsu, H. Takahashi, N. Môri, and K. Kinoshita, J. Phys. Soc. Jpn. **65**, 2764 (1996).
[17] T. Barnes, E. Dagotto, J. Riera, and E. S. Swanson, Phys. Rev. B **47**, 3196 (1993); S. R. White, R. M. Noack, and D. J. Scalapino, Phys. Rev. Lett. **73**, 886 (1994).
[18] G. Sierra, J. Phys. A **29**, 3299 (1996); S. Dell'Aringa, E. Ercolessi, G. Morandi, P. Pieri, and M. Roncaglia, Phys. Rev. Lett. **78**, 2457 (1997).
[19] M. Reigrotzki, H. Tsunetsugu, and T. M. Rice, J. Phys.: Condens. Matter **6**, 9235 (1994).
[20] P. B. Wiegmann, Phys. Rev. Lett. **60**, 821 (1988); X. G. Wen, Phys. Rev. B **39**, 7223 (1989); R. Shankar, Phys. Rev. Lett. **63**, 203 (1989); P. A. Lee, *ibid.* **63**, 680 (1989).
[21] R. Shankar, Nucl. Phys. B **330**, 433 (1990). In this paper, the author has considered the possibility of including an attractive quartic fermion interaction between A and B holes in the single chain t-J model.
[22] In the use of this terminology, we follow the definition of the paper by D. J. Gross, I. R. Klebanov, A. V. Matytsin, and A. V. Smilga, Nucl. Phys. B **461**, 109 (1996). For the $1+1$ dimensional gauge theories, the Higgs phase means that the potential $V(R)$ between two fractional probe charges separated by a large distance R is of the form $V(R)\sim e^{-\mu R}/R$, μ being the screening mass. On the other hand, we say that the gauge fields are in the confining phase when $V(R)\sim\sigma R$ with a nonvanishing σ.
[23] A. Luther and V. J. Emery, Phys. Rev. Lett. **33**, 589 (1974).
[24] See A. M. Tsvelik, *Quantum Field Theory in Condensed Matter Physics* (Cambridge University Press, Cambridge, 1995).
[25] C. L. Kane, P. A. Lee, and N. Read, Phys. Rev. B **39**, 6880 (1989); S. Schmitt-Rink, C. M. Varma, and A. E. Ruckenstein, Phys. Rev. Lett. **60**, 2793 (1988).
[26] See A. M. Polyakov, *Gauge Fields and Strings* (Harwood Academic, Chur, Switzerland, 1987).
[27] D. G. Shelton, A. A. Nersesyan, and A. M. Tsvelik, Phys. Rev. B **53**, 8521 (1996); D. G. Shelton and A. M. Tsvelik *ibid.* **53**, 14 036 (1996).