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Fano interference of collective excitations in semiconductor quantum wells and lasing without inversion

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Absorption cancellation via tunneling induced Fano interference in semiconductor quantum wells is studied in the presence of the Coulomb interaction between electrons. For a small subband dispersion, gain or loss is determined by single-electron Fano interference. For a large subband dispersion, collective excitations dominate the absorption spectrum and are crucial for the observability of tunneling induced transparency, which exists in spite of subband dispersion. Pumping destroys collective excitations; therefore gain without inversion is possible only for small subband dispersion. [S0163-1829(99)04820-1]

Many interesting phenomena in optics such as electromagnetically induced transparency[1] (EIT) and lasers without population inversion[2,3] (LWI) originate from atomic coherence and interference of radiative processes. One common feature is Fano interference,[4] where two decay processes from two different states interfere due to coupling of these states to the same reservoir. This interference occurs even though the processes are irreversible.

After the successful observation of these phenomena in atomic media, it is natural to extend them to semiconductor materials.[5] Motivated by the progress in quantum cascade lasers,[6] some theoretical aspects of EIT and LWI in semiconductors,[7] and specifically in intersubband transitions,[5,8] have been discussed. The band dispersion, unfavorable for LWI, is smaller in intersubband transitions than in interband transitions. Tunneling induced transparency (TIT) in intersubband transitions has recently been observed,[9] providing further motivation for a detailed study of LWI in these systems.

A rigorous theory of semiconductor LWI requires the inclusion of many-body effects in connection with relaxation and subband dispersion (for a review see Ref. 10). The Coulomb interaction of electrons in semiconductors has been studied in the local-density approximation[8] without taking relaxation into account. In this paper we study TIT and LWI of collective excitations in intersubband transitions[11] and excitons in interband transitions, using the formalism of semiconductor Bloch equations generalized for a multiband semiconductor.

We show that for materials having a large subband dispersion, e.g., InAs, absorption cancellation is possible only by virtue of the Coulomb interaction. TIT is made possible in this case due to the interference of collective excitations[12] and exists even at very large subband dispersion. Injection of electrons in the upper lasing level destroys the collective excitations and limits LWI to small subband dispersions. In this case (e.g., GaAs), TIT and LWI originate from interference of single particles and are not diminished by pumping.

We label the states of electrons in multiple quantum wells by the subband index μ and the quasimomentum in the plane of the well $\mathbf{k}$ (see Fig. 1). The Hamiltonian in the rotating frame (see Ref. 13) for the interaction of electrons with a laser field $\mathcal{E}$ and their Coulomb interaction in terms of the electron creation and annihilation operators $a_{\mu\mathbf{k}}$ is[14]

$$H=\sum_{\mu,\mathbf{k}} \hbar\Delta_{\mu\mathbf{k}} a^{\dagger}_{\mu\mathbf{k}} a_{\mu\mathbf{k}} - \sum_{\{\mu,\nu\}\mathbf{k}} (\hbar\Omega_{\mu\nu} a^{\dagger}_{\mu\mathbf{k}} a_{\nu\mathbf{k}} + \hbar\Omega^{*}_{\mu\nu} a^{\dagger}_{\nu\mathbf{k}} a_{\mu\mathbf{k}}) + \frac{1}{2}\sum_{\mu,\nu,\nu',\mu',\mathbf{k},\mathbf{k}',q} \hbar V_{\mathbf{q}}^{\mu\nu\nu'\mu'} a^{\dagger}_{\mu,\mathbf{k}+\mathbf{q}} a^{\dagger}_{\nu,\mathbf{k}'-\mathbf{q}} a_{\nu',\mathbf{k}'} a_{\mu',\mathbf{k}}, \tag{1}$$

where $\Delta_{\mu\mathbf{k}}$ is the detuning of the state $\{\mu,\mathbf{k}\}$ (see Fig. 1) $\Omega_{\mu\nu}=\wp_{\mu\nu}\mathcal{E}/\hbar$ is the Rabi frequency of the electromagnetic field, with $\wp_{\mu\nu}=\int f^{*}_{\mu} z f_{\nu} dz$ denoting the dipole matrix element. The two-dimensional Coulomb potential is $V_{\mathbf{q}}=e^2/[2\hbar\varepsilon_0\varepsilon_b\varepsilon(\mathbf{q})|\mathbf{q}|]$, where ε_b is the background dielectric constant and the resonant screening $\varepsilon(\mathbf{q})$ is given by the static Lindhard formula.[10] The Coulomb interactions be-

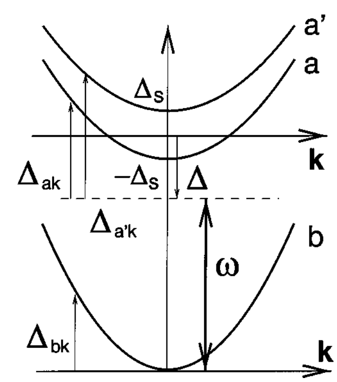

FIG. 1. Scheme of the momentum states of subbands a and a' coupled by a laser field with the frequency ω to subband b. Here $\Delta_{a\mathbf{k}}$, $\Delta_{a'\mathbf{k}}$, and $\Delta_{b\mathbf{k}}$ are the detunings of the states from the field and $\Delta=-(\Delta_{a0}+\Delta_{a'0})/2$.

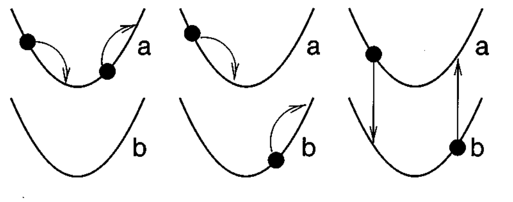

FIG. 2. Change of electron states in processes corresponding to parts of the Coulomb interaction (from left to right): band gap renormalization $V_{\mathbf{q}}^{aaaa}$, excitonic enhancement $V_{\mathbf{q}}^{abba}$, and depolarization $V_{\mathbf{q}}^{abab}$.

tween different subbands $V_{\mathbf{q}}^{\mu\nu\nu'\mu'}$ are $V_{\mathbf{q}}$ multiplied by the corresponding form factor Form(μ,ν,ν',μ') which is determined by the envelope wave functions $f_\mu(z)$ of states in the well (see Refs. 11 and 14 for details). We call Fock terms those with $\mu=\mu'$, $\nu=\nu'$, and $\mathbf{q}\neq\mathbf{0}$ and ''depolarization'' terms those with $\mu=\nu'\neq\nu=\mu'$; we retain only $\mathbf{q}=\mathbf{0}$ depolarization terms. Depolarization terms have a finite limit at $\mathbf{q}\to 0$; we define the effective well width in this limit as Form$(\mu,\nu,\mu,\nu)/|\mathbf{q}|\to a_{W\mu\nu}$ as $\mathbf{q}\to 0$. Processes corresponding to these contributions are depicted in Fig. 2.

We can write the semiconductor Bloch equations[10] for the reduced density matrix $\sigma_{\mu\nu\mathbf{k}}=\langle a_{\nu\mathbf{k}}^\dagger a_{\mu\mathbf{k}}\rangle$, where angular brackets denote a quantum statistical average with the full many-body density matrix. The Hamiltonian part of the evolution of the density matrix for an arbitrary number of subbands is

$$
\begin{aligned}
\dot{\sigma}_{\mu\nu\mathbf{k}}\big|_{\mathrm{coh}} = & -i(\Delta_{\mu\mathbf{k}}-\Delta_{\nu\mathbf{k}})\sigma_{\mu\nu\mathbf{k}} + i\sum_{\{\kappa,\lambda\}}[\Omega_{\kappa\lambda}(\delta_{\mu\kappa}\sigma_{\lambda\nu\mathbf{k}} \\
& -\sigma_{\mu\kappa\mathbf{k}}\delta_{\lambda\nu}) + \Omega_{\kappa\lambda}^*(\delta_{\mu\lambda}\sigma_{\kappa\nu\mathbf{k}}-\sigma_{\mu\lambda\mathbf{k}}\delta_{\kappa\nu})] \\
& + i\sum_{\kappa\mathbf{q}\neq 0}(V_{\mathbf{q}}^{\mu\kappa\kappa\mu}\sigma_{\mu\kappa\mathbf{k}+\mathbf{q}}\sigma_{\kappa\nu\mathbf{k}} - V_{\mathbf{q}}^{\nu\kappa\kappa\nu}\sigma_{\mu\kappa\mathbf{k}}\sigma_{\kappa\nu\mathbf{k}+\mathbf{q}}) \\
& + i\sum_{\kappa,\mathbf{k}'}(V_0^{\kappa\nu\kappa\nu}\sigma_{\mu\kappa\mathbf{k}}\sigma_{\kappa\nu\mathbf{k}'} - V_0^{\mu\kappa\mu\kappa}\sigma_{\mu\kappa\mathbf{k}'}\sigma_{\kappa\nu\mathbf{k}}).
\end{aligned}
\tag{2}
$$

We treat the relaxation part of the evolution as the usual interaction with the reservoir in a rate approximation.[13] In our case, transition rates between subbands (mostly with emission of a phonon) are $w_{\mu\nu}$, a collision rate of electrons in subband μ with all others is $\gamma_{\mu\mathrm{coll}}$, and a dephasing rate of polarization between subbands, e.g., due to interface roughness, is $\gamma_{\mu\nu\mathrm{deph}}$, etc. A more rigorous treatment of electron collisions can be found, e.g., in Ref. 15. However, our approximation was found to describe adequately the linewidth of intersubband transitions[9] that depends weakly on the density of electrons. We describe in more detail the tunneling processes that experience Fano interference. Discrete subbands interact with the same quasicontinuum modes of the reservoir. The rate of tunneling from and to state μ is $r_\mu\propto|g_\mu|^2$, where the coupling constant g_μ has the phase ϕ_μ. The frequency associated with the band in the rotating frame is ω_μ and the reservoir occupation number in the modes having the energy approximately equal to $\hbar\omega_\mu$ is $\overline{n}$. The contribution of the Fano interference is then[16]

$$
\begin{aligned}
\dot{\sigma}_{\mu\nu\mathbf{k}}\big|_{\mathrm{Fano}} = & \overline{n}\sqrt{r_\nu r_\mu}\exp(i\phi_\mu - i\phi_\nu + i\omega_\mu t - i\omega_\nu t) \\
& - \sum_\kappa \frac{\sqrt{r_\mu r_\kappa}}{2}\exp(i\phi_\mu - i\phi_\kappa + i\omega_\mu t - i\omega_\kappa t)\sigma_{\kappa\nu\mathbf{k}} \\
& - \sum_\kappa \frac{\sqrt{r_\nu r_\kappa}}{2}\exp(i\phi_\kappa - i\phi_i + i\omega_\kappa t - i\omega_i t)\sigma_{\mu\kappa\mathbf{k}}.
\end{aligned}
\tag{3}
$$

Usually we envision each level interacting with two reservoirs: The electrons are injected with rates r_i into the wells from the first one (emitter) and the decay of the electron occupancy with rates γ_i is due to coupling to the second reservoir (collector). It is essential for the creation of coherence that the relative phases of coupling to the emitter and collector are different so that the injection and the decay happen to different linear combination of the two quantum states. In all calculations we use phases 0 and 0 for coupling to the collector and 0 and π for coupling to the emitter for states a and a', respectively. This is the case of absorption cancellation.[5]

The subband dispersion is $\delta_{\mu\nu\mathbf{k}}=\Delta_{\mu\mathbf{k}}-\Delta_{\mu 0}-\Delta_{\nu\mathbf{k}}+\Delta_{\nu 0}$. The inhomogeneous broadening associated with the dispersion of two subbands is estimated for parabolic subbands as $\delta_{F\mu\nu}=\hbar k_F^2/2m_{r\mu\nu}$, where k_F is the largest of the two Fermi momenta and the reduced mass of the pair of subbands is given by $m_{r\mu\nu}^{-1}=m_\mu^{-1}-m_\nu^{-1}$. The intensity gain coefficient $G_{\mu\nu}=-2\omega|\wp_{\mu\nu}|^2/(\hbar c\varepsilon_0 n_b)\mathrm{Im}(P_{\mu\nu}/\Omega_{\mu\nu})$ is proportional to the polarization between subbands. Here $P_{\mu\nu}=\Sigma_{\mathbf{k}}\sigma_{\mu\nu\mathbf{k}}$, the sum runs over momenta of all transitions in a unit volume, ω is the cyclic frequency of the field, and $n_b=\sqrt{\varepsilon_b}$ is the index of refraction.

In the following derivation we approximate all form factors by unity in the Fock terms. We may do this for only low to moderate electron densities (see Ref. 11). Additionally we assume that the rates of relaxation processes are momentum independent; then it is possible to express the incoherent terms only via total polarizations $P_{\mu\nu}$. We sum the equations (2) and obtain

$$
\begin{aligned}
i\dot{P}_{\mu\nu}\big|_{\mathrm{coh}} = & \Delta_{\mu\nu}P_{\mu\nu} + S_{\mu\nu} - \sum_\kappa (V_0^{\nu\kappa\nu\kappa} - V_0^{\mu\kappa\mu\kappa})P_{\mu\kappa}P_{\kappa\nu} \\
& - \sum_{\{\kappa,\lambda\}}[\Omega_{\kappa\lambda}(\delta_{\mu\kappa}P_{\lambda\nu} - P_{\mu\kappa}\delta_{\lambda\nu}) \\
& + \Omega_{\kappa\lambda}^*(\delta_{\mu\lambda}P_{\kappa\nu} - P_{\mu\lambda}\delta_{\kappa\nu})],
\end{aligned}
\tag{4}
$$

where $S_{\mu\nu}=\Sigma_{\mathbf{k}}\delta_{\mu\nu\mathbf{k}}\sigma_{\mu\nu\mathbf{k}}$ is the inhomogeneous part of the polarization. This proves, under the above assumptions for an arbitrary number of subbands, that if the subband dispersion is negligible, then the exchange contribution cancels out and the only effect of the Coulomb interaction on the total polarization is depolarization V_0 that becomes a rigid frequency shift of the spectrum if the upper subband coherence is small.

We numerically solve the semiconductor Bloch equations (2) for a multiple quantum well in which two upper subbands a and a' are closely spaced by $2\Delta_s\ll\omega$ and one lower subband b is coupled to both a and a' by a laser field.[5] As a result we find the total polarizations in the laser transitions

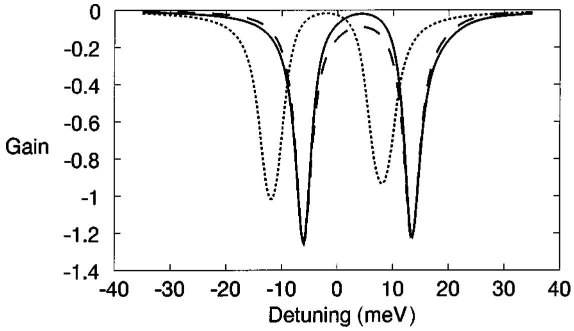

FIG. 3. Gain spectrum for a small subband dispersion (corresponding to a GaAs QW) as a function of detuning Δ with no injection into subbands a and a' with Coulomb terms and Fano interference (solid curve), with Coulomb terms without Fano interference (dashed curve), and without Coulomb terms with Fano interference (dotted curve).

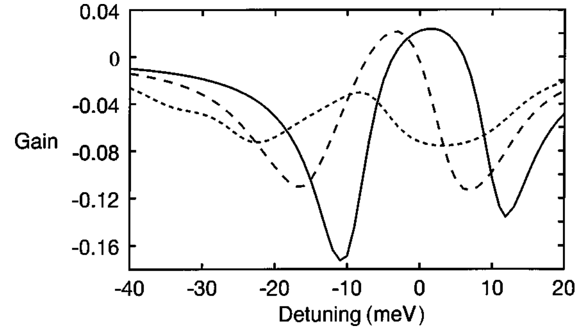

FIG. 4. Gain spectrum as a function of detuning Δ with the injection into subbands a and a' for $m_a=0.042$ (corresponding to a GaAs QW), solid curve; for $m_a=0.033$, dashed curve; and $m_a=0.028$ (corresponding to an InAs QW), dotted curve.

$P_{ab}+P_{a'b}$ for a very small Rabi frequency Ω_{ab} (linear gain or loss). The detuning of the laser field from resonance with the center of gravity of the two upper subbands is $\Delta=-(\Delta_a+\Delta_{a'})/2$; see Fig. 1. In all of the above calculations we will assume an ambient temperature of 12 K.

The case of small subband dispersion is illustrated by the parameters corresponding to a GaAs quantum well with the intersubband transition frequency of 100 meV, $\Delta_s=10$ meV, and $\epsilon_b=12.9$. The masses of the subbands according to Kane's model[10] are $m_b=0.069m_0$, $m_a=0.077m_0$, and $m_a'=0.079m_0$. For TIT, the lower subband b is populated up to a Fermi momentum k_F by injection with the rate $r_b=0.1$ meV, chemical potential $\mu_b=37$ meV, and decay rate $\gamma_b=0.01$ meV. No carriers are injected into upper subbands a and a', but they can tunnel out with rate $\gamma_a=3$ meV. We take the depolarization values to be $k_F a_{Wab}=0.2$ and $k_F a_{Waa'}=1$, the dephasing rate in each transition $\gamma_{\mathrm{deph}}=0.2$ meV, and the effective collision rate $\gamma_{\mathrm{coll}}=0.1$ meV. The numerical result confirms the above prediction: The absorption line shapes calculated with and without the Coulomb terms are not very much different; the line is blueshifted due to depolarization (Fig. 3). If there were no Fano interference included, the positions of the absorption peaks would not change; however, the absorption in its local minimum between the peaks would be several times higher. This demonstrates that TIT is still preserved in the presence of many momentum states and the Coulomb interaction; this prediction is already confirmed by experiments.[9]

LWI can be achieved by injection to the upper subbands. Now we take for the upper subbands $r_a=2$ meV and $\mu_a=34$ meV and for the lower subbands $r_b=1.5$ meV and $\gamma_b=2$ meV. Still there is no inversion between any of the momentum states of any of the upper and the lower subbands. In this case (the right curve in Fig. 4) gain without population inversion is obtained between the resonance peaks. This is purely due to Fano interference: We would have obtained loss at any detuning without it.

The case of large subband dispersion is illustrated by an InAs quantum well with the same energies of subbands at zero momentum. There $\epsilon_b=15.7$ and the masses of the subbands are $m_b=0.027m_0$, $m_a=0.037m_0$, and $m_a'=0.041m_0$. For TIT the rates for the lower subband are $r_b=0.1$ meV and $\gamma_b=0.01$ meV, the chemical potential is $\mu_b=80$ meV, and the tunneling rates from the upper subbands are $\gamma_a=5$ meV. Without the inclusion of the Coulomb terms we obtain a wide spectrum of absorption that is an overlap of the two shifted flat line shapes (see Fig. 5, dashed line). Each flat distribution corresponds to coupling to one of the upper subbands and their widths are given by the inhomogeneous broadening $|\delta_F|$ due to subband dispersion. Large inhomogeneous broadening is known to destroy absorption cancellation. The calculation shows that absorption is maximal in the center of the line confirming that no absorption cancellation would have existed for single particle excitations.

The situation changes dramatically with the inclusion of Coulomb terms (Fig. 5, solid line). We see two narrow lines corresponding to *collective excitations* corresponding to each of the upper subbands[11]). The width of the peaks is not determined by the subband dispersion, but rather by the homogeneous width given by tunneling, collisions, and dephasing. Assuming a stepwise distribution of electron occupation numbers and a constant characteristic value of the Coulomb potential V_{ch}, we approximately solve Eq. (2) to obtain a criterion that the spectrum is close to that of a sole collective excitation

$$V_{ch}(P_{bb}-P_{aa}) \gg \frac{|\delta_F|}{\ln\left(\frac{\delta_F}{\gamma}\right)}, \tag{5}$$

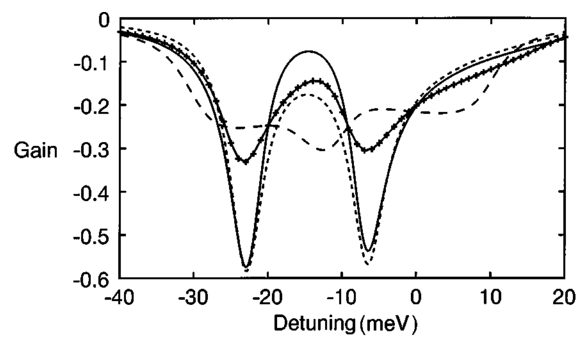

FIG. 5. Gain spectrum for a large subband dispersion (corresponding to an InAs QW) as a function of detuning Δ with no injection into subbands a and a' with Coulomb terms and Fano interference (solid curve), with Coulomb terms without Fano interference (dotted curve), and without Coulomb terms with Fano interference (dashed curve). The curve with crosses corresponds to injection into subbands a and a' with the account of all terms.

where γ is the homogeneous width. If one neglects Fano interference, the absorption between the peaks turns out to be significantly higher (Fig. 5, dotted line). This shows that the absorption cancellation exists in our case and is only possible due to Fano interference of collective rather than single-particle modes.

We try to obtain LWI by introducing the electron injection to the upper subbands with $r_a = 0.5$ meV and $\mu_a = 55$ meV. Surprisingly, we obtain an increase in absorption (Fig. 5, crossed line). One can see that now the left-hand side of the collectivization criterion (5) is smaller and less oscillator strength is concentrated in the collective mode. Therefore, the inhomogeneous broadening is eliminated to a smaller degree and absorption cancellation is less perfect. At large subband dispersion, this increase of absorption outweighs the increase of emission due to the presence of carriers in the upper subbands. It is still possible to achieve LWI when the subband dispersion is appreciable but not too large, i.e., comparable to either the splitting between the subbands or any relaxation rate; see Fig. 4, middle curve.

Fano interference can be observed as well in interband transitions, i.e., when the lower subband belongs to the valence band. This case is analogous to the intersubband problem, but has a much larger subband dispersion. We illustrate it by the example of GaAs with masses $m_a = 0.069 m_0 = m_{a'}$ and $m_b = -0.377 m_0$. We set $\Delta_s = 6$ meV, $\gamma_a = 2$ meV, and $\gamma_b = 0.1$ meV, with the collision and dephasing rates the same as above. The Coulomb interaction causes the appearance of two exciton lines separated from the free-electron absorption at higher frequencies; see Fig. 6. The absorption between them is significantly smaller than one would obtain without taking into account Fano interference.

It is known that the usual lasing in exciton lines (with direct photon emission) is not possible: Injection of electrons and holes destroys excitons before inversion is created because of the decrease of the population difference between the bands and because of screening of the Coulomb potential; then, only the single-particle transitions are observed. One might hope that lasing without inversion in exciton lines would be possible. However, our calculation shows that in this case of extremely large subband dispersion, the disappearance of excitons overshadows the increase in emission as per the discussion above and lasing without inversion does not occur in exciton lines.

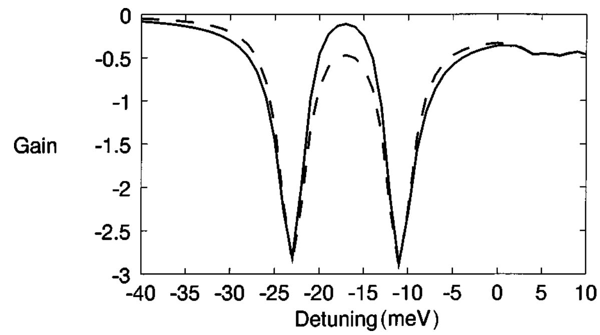

FIG. 6. Gain spectrum for interband transitions (corresponding to a GaAs QW) as a function of detuning Δ with no injection into subbands a and a' with Coulomb terms and Fano interference (solid curve) and with Coulomb terms without Fano interference (dotted curve).

In conclusion, we predict the existence of tunneling induced transparency and gain without population inversion via Fano interference. In the limit of a small subband dispersion, the Coulomb interaction does not modify the gain line shape. In the limit of a large subband dispersion, tunneling induced transparency exists only due to Fano interference of collective excitations induced by Coulomb interaction. Electron injection to the upper subbands in this case gives gain without inversion only for a small subband dispersion. Gain without inversion, however, was not possible for a large subband dispersion, including the case of interband transitions.

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