Transmission of *L*-mode phonons from a superlattice into a liquid by effective acoustic impedance matching

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A superlattice placed between solids and liquids resonantly enhances the transmission of longitudinal phonons incident normally to the superlattice layers in spite of a large mismatch of their acoustic impedances. This resonant transmission originates from the surface vibrational mode at the interface between the liquids and the superlattice. To characterize this resonance, an effective acoustic impedance is introduced for the superlattice structure. If the acoustic impedances of the liquids coincide with the effective impedance of the superlattice, the transmission rate has a maxium value in a resonance condition. [S0163-1829(99)01517-9]

Recently, a wave-front imaging experiment has been developed[1] for observing the shape of a vibrational wave front on crystalline solid surfaces. A focused acoustic beam from an ultrasonic transducer excites a small spot on one face of a solid that is immersed in a liquid bath. An identical transducer is focused on the opposite face and receives the acoustic wave transmitted through the crystal. The receiving transducer is raster-scanned, and gets a spatial distribution of the transmitted flux that appears on the solid surface and gives data of the expanding wave front striking the crystal surface.

According to previous studies on acoustic vibrational field in multilayered structures, displacements of the acoustic field have spatial quasiperiodic properties at passing frequencies in infinite-size structures. However, at stopping band frequencies, the displacements attenuate exponentially.[2] Semi-infinite multilayers have a resonant surface mode with a large vibrational amplitude at the stress-free surface.[3] These features are investigated for transverse vibrations, and they are the same also for longitudinal vibrations. Experimentally, the surface vibrational mode at the free surface of the multilayers has been detected with the pump and probe method.[4] Meanwhile, resonant transmissions of phonons are reported for triple-superlattice structures and a bulk material sandwiched by two superlattices.[5,6] These analyses show that the resonant frequencies are at acute frequency regions in stop bands of the superlattices.

If the wave front imaging experiment is applied on multilayered materials or a superlattice (SL), observation of the resonant surface modes on the superlattice surface is expectable with precise time and space resolutions. In the present paper, we study a finite-size SL as in Fig. 1 with configuration that the free surface of the SL is immersed in liquid. In the SL, two kinds of solid layers (A and B) are alternately stacked. The substrate is the same solid as in the layer B for simplicity.

From this substrate, longitudinal (L)-mode phonons are incident normally to interfaces of the layers. Therefore, the L mode does not couple with the other transverse modes which do not penetrate into liquids. To observe the L-mode phonons which penetrate into liquid, we have a way to detect pressures in the liquid. The advantage of this system is its freedom of layer structures and controllability of the surface vibrational mode in the phonon transmission. In this system, we show a resonant transmission into liquids from the substrate for L-mode phonons, which has broader peaks of the transmission rate against frequencies than the above-mentioned resonant peaks caused in solid SL's without using the resonant surface mode.

Making use of the transfer-matrix method, we express the relation of phonon amplitudes between the substrate and the immersed surface of the SL:

$$\begin{bmatrix} a_N \\ \kappa \ \ a_N \end{bmatrix} = G^N \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}, \tag{1}$$

where N is the number of the bilayers (AB's); a_0 and b_0 are incident amplitude and reflected amplitude in the substrate, respectively. The quantity a_N is the amplitude of the L-mode phonons reached at the immersed surface of the SL from the substrate. The value κ expresses the amplitude reflection co-

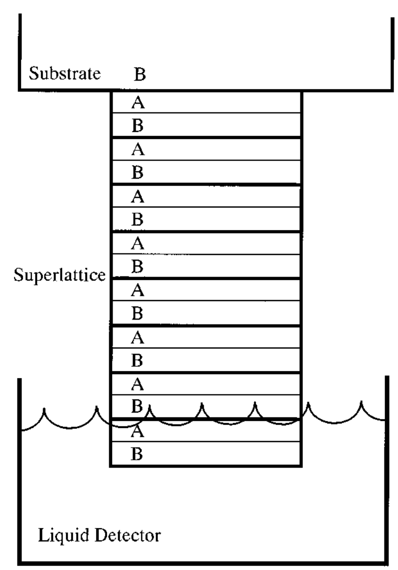

FIG. 1. Configuration of a superlattice (SL) and liquid detector: The free surface of the SL is immersed in liquid. Two kinds of solid layers (A and B) are alternately stacked in the SL. The substrate is the same solid as in the layer B. From this substrate, L-mode phonons are incident perpendicularly to the interfaces of the layers.

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efficient from the immersed surface. The matrix G is the transfer matrix for one bilayer, and each element of the matrix G^N becomes

$$(G^N)_{11}=\cos N\gamma+i\ s(\gamma)g(\alpha,\beta), \tag{2}$$

$$(G^N)_{22}=\cos N\gamma-i\ s(\gamma)g(\alpha,\beta), \tag{3}$$

$$(G^N)_{12}=i\ e^{i\beta}s(\gamma)\frac{1}{2}(Z_A/Z_B-Z_B/Z_A)\sin\alpha, \tag{4}$$

$$(G^N)_{21}=-i\ e^{-i\beta}s(\gamma)\frac{1}{2}(Z_A/Z_B-Z_B/Z_A)\sin\alpha, \tag{5}$$

where

$$g(\alpha,\beta)=\cos\alpha\sin\beta+\frac{1}{2}(Z_A/Z_B+Z_B/Z_A)\sin\alpha\cos\beta. \tag{6}$$

The values α and β are the products of wave numbers of L-mode phonons and thicknesses for the layer A and for the layer B, respectively. Acoustic impedances are expressed by the values Z_A for the layer A and by the value Z_B for the layer B. Function $s(\gamma)$ is defined as $s(\gamma)=\sin N\gamma/\sin\gamma$, and γ is given as follows:[7]

$$\cos\gamma=\cos\alpha\cos\beta-\frac{1}{2}(Z_A/Z_B+Z_B/Z_A)\sin\alpha\sin\beta. \tag{7}$$

From the above expressions, we get an amplitude reflection coefficient, which is a ratio of the reflected amplitude to the incident amplitude in the substrate as follows:

$$r=\frac{b_0}{a_0}=\frac{\kappa(G^N)_{11}-(G^N)_{21}}{(G^N)_{22}-\kappa(G^N)_{12}}. \tag{8}$$

Therefore, the reflection rate of the SL becomes $R=|r|^2$. The amplitude transmission coefficient that expresses the fraction of the amplitude getting through the SL to the immersed surface is

$$t=\frac{a_N}{a_0}=(G^N)_{11}+r(G^N)_{12}=\frac{1}{(G^N)_{22}-\kappa(G^N)_{12}}, \tag{9}$$

where we use $\det[G^N]=1$. An acoustic impedance Z_D of the liquid detector has a relation with the value κ as follows:

$$\frac{Z_D}{Z_B}=\frac{1-\kappa}{1+\kappa}. \tag{10}$$

Knowing these expressions, we get the transmission rate of L-mode phonons into the liquid from the substrate

$$T=\frac{Z_D}{Z_B}\left|\frac{a_D}{a_0}\right|^2=(1-\kappa^2)|t|^2, \tag{11}$$

where a_D is the amplitude of the L-mode phonons that penetrate into the liquid, and has a relation $a_D/a_0=(1+\kappa)t$, if

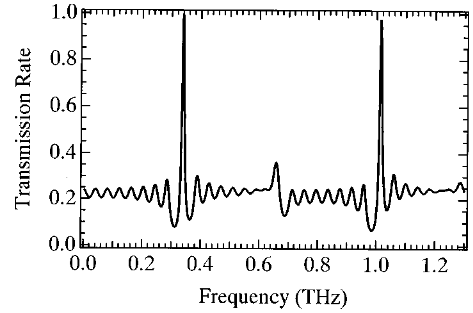

FIG. 2. The transmission rate of the L-mode phonons into distilled water has resonant peaks in certain frequencies. The layer A of the SL consists of GaAs, the layer B is of AlAs, and the liquid is assumed as distilled water. The number of the bilayers (AB's) in the SL is eight, and each layer (A or B) has 15 monolayers with (100) interfaces.

we neglect reflection amplitude of the phonons in the liquid back to the immersed surface.

In Fig. 2, we show the results of numerical calculations for the SL that has the layer A of GaAs and the layer B of AlAs; and a liquid is assumed as distilled water. The number of the bilayers in the SL is eight, and each layer (A or B) has 15 monolayers with (100) interfaces. Acoustic impedances of each composition in the SL are 25.2 in GaAs, 21.2 in AlAs, and 1.48 in the distilled water for the L-mode phonons in unit of $10^5\,\mathrm{g\,cm^{-2}\,s^{-1}}$. The transmission rate T for the L mode phonons has some peaks close to the value of unity in certain frequencies (e.g., $\nu_G=338$ GHz), in spite of the large mismatch of the acoustic impedance of the distilled water that is less than one tenth of the layers' acoustic impedances.

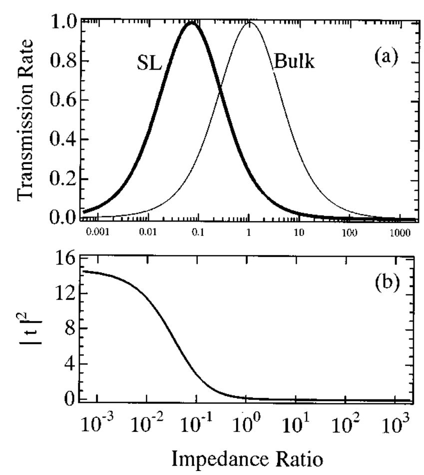

FIG. 3. (a) The transmission rate of both the SL and the bulk materials against ratios of the liquid detector's impedance to that of the immersed surface layer. Frequency is ν_G and the other conditions are the same as in Fig. 2. The peak of the transmission rate of the SL shifts to lower impedance ratios than the peak of the bulk materials. The kind of detector is expressed by the acoustic impedance, and the transverse axis is for the impedance ratios Z_D/Z_B on a log scale. In the case of distilled water, the ratio is 6.98×10^{-2}. (b) The effect on the peak for the bulk materials by the SL interfaces is expressed by $|t|^2$. This factor shifts the peak of the transmission rate to lower impedance regions which are comparative with those of liquids. The transverse axis is also for the impedance ratios on a log scale.

These peaks originate from the surface vibrational mode[2–4,8] when $Z_B < Z_A$. In the case of Fig. 2, the layer A must be of GaAs and the layer B must be of AlAs. If the two materials of the layers A and B interchanged, the surface vibrational mode does not exist at the immersed surface in the liquids. The frequency ν_G is in the stop bands which appear in infinite-size SL's. Except these resonant frequencies, the reflection rate R is near the value $\kappa^2 = 0.757$. This is a value for the bulk material which is the same solid as in the layer B.

In Fig. 3(a), the transmission rate T is plotted with thick line labeled ''SL'' against ratios (Z_D/Z_B) of the liquid detector's impedance to that of the immersed surface layer B (AlAs). Here, the frequency is $\nu_G = 338$ GHz and the other conditions are the same as in Fig. 2. The kind of liquid is expressed by the acoustic impedances Z_D. In the case of distilled water, the ratio is 6.98×10^{-2}, and it has a value that is optimal for the resonance transmission. The other liquid that is appropriate to the resonance includes toluene and benzene, and their impedance ratios are 5.40×10^{-2} and 5.48×10^{-2}, respectively.

In Eq. (11), the factor $(1-\kappa^2)$ is the transmission rate for bulk materials and has one peak only at $\kappa = 0$ $(Z_D = Z_B)$. This factor is plotted in Fig. 3(a) with thin line labeled ''bulk.'' The other factor $|t|^2$ expresses an effect of the SL to shift the peak to lower impedance ratios which are comparative with those of liquids. The latter factor is plotted in Fig. 3(b). If the immersed surface is nearly exposed to vacuum $(\kappa \simeq 1$ or $Z_D \simeq 0)$, then $|t|^2$ has the maximum magnitude of the surface vibrations. On the other hand, this factor approaches zero as $\kappa \to -1 (Z_D \to \infty)$, when a rigid detector suppresses the surface vibrations. The factor $|t|^2$ also has one peak only at $\kappa = \mathrm{Re}\{c\}$, where $c = (G^N)_{22}/(G^N)_{12}$. If $\mathrm{Re}\{c\}$ is positive in the resonant condition, the peak of the transmission rate shifts to lower impedance ratios (As is the case in Fig. 3, when $\mathrm{Re}\{c\} = 1.15$). Exact calculation to obtain the value of κ for this peak of T gives a cubic equation. This equation is modified with expressing the value by κ_e as follows:

$$\kappa_e = \frac{(3\kappa_e^2 - 1)\mathrm{Re}\{c\}}{2\kappa_e^2 - 1 + |c|^2}. \tag{12}$$

Because of the physical nature of the amplitude reflection coefficient, we require that κ_e is real and satisfies an inequality $1 \geqslant \kappa_e > -1$. An effective impedance Z_e is defined by the relation $Z_e = Z_B(1-\kappa_e)/(1+\kappa_e)$, which is a similar expression to Eq. (10), in the condition that the resonant transmission appears. In this definition, the SL structure is expressed by only one parameter c. In the case of Fig. 3, the effective impedance Z_e is less than one tenth of the acoustic impedance Z_B that is of AlAs.

Meanwhile, the above discussion implies that the effective acoustic impedance Z_e can be made greater than that of layer B by interchanging the two materials (GaAs and AlAs) of the layers A and B. In this case, the layers' impedances satisfy $Z_B > Z_A$ and $\mathrm{Re}\{c\}$ is negative. As a result, Z_e becomes nearly ten times greater than Z_B that is of GaAs. Therefore, the impedance Z_e is no longer comparative with those of liquids, but with those of rigid solids, i.e., very high acoustic impedances.

As a conclusion, the transmission of the L-mode phonons is resonantly enhanced if the acoustic impedances Z_D of the liquids coincide with the effective impedance Z_e of the SL. The impedance Z_e is controllable by a modification of the parameter c which is defined only by the SL structures without considering the liquid natures. The resonant frequencies can be decreased by increasing the thicknesses of bilayers. Therefore, it is possible to detect this resonant transmission experimentally in lower frequencies than those of the above discussions. Further, because it is easy to make many resonant peaks against frequencies by controlling the thicknesses and the impedances of the layers in the SL's, the above-mentioned resonant transmission may be useful in decreasing the thermal resistance[9] between bulk metals and liquids like helium at low temperatures — provided we use multilayered metals instead of bulk metals.

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