Transmission fingerprints in quasiperiodic dielectric multilayers

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We investigate the optical transmission fingerprints in structures that exhibit deterministic disorders. A class of models that has attracted particular attention in this context are the quasiperiodic dielectric multilayers that obey a substitutional sequence. These substitutional sequence are characterized by the nature of their Fourier spectrum, which can be dense pure point (Fibonacci sequences), singular continuous (Thue-Morse and double-period sequences), and absolutely continuous (Rudin-Shapiro sequence). We use a transfer-matrix approach to derive the optical transmission coefficients. Numerical results are presented to illustrate the self-similar aspect of the spectra, as well as to show the optical fingerprint through a return map of the transmission coefficients. [S0163-1829(99)00514-7]

In the past 15 years, a great number of works in quasiperiodic systems have been reported. These quasiperiodic structures are formed by the superposition of two (or more) incommensurate periods, so that they can be defined as intermediate systems between a periodic crystal and the random amorphous solid.[1]

Theoretical and experimental works have been carried out to study the propagation of various excitations in these quasiperiodic systems (for a good account of this subject see Ref. 2, and the references therein). It has been observed, among others things, that they exhibit collective properties not shared by their constituents. Therefore, the long-range correlation induced by the construction of these systems is reflected someway in their various spectra, defining a novel description of disorder. Indeed, theoretical transfer matrix treatments show that these spectra are fractals.[3]

Localization due to the electronic properties of one-dimensional Schrödinger equation with discontinuous quasiperiodic potential was also studied by several authors.[4–6] In particular, Kohmoto and collaborators[7] have used the renormalization group technique to obtain the Cantor-set spectrum and to describe scaling properties of the wave function, through a dynamic trace map. Recently, we developed a method to distinguish spectral properties among quasiperiodic array.[8] We have solved the one-dimensional (1D) Schrödinger equation for electronic transport through a finite sequence of barriers obeying Fibonacci, Thue-Morse, and Cantor sequences, and we set up their transmission spectra as a function of the energy, showing an evident self-similar behavior. We plotted their return map T_{N+1} versus T_N, where T_N is the probability corresponding to the Nth sequence of the quasiperiodic structure, finding a pattern (or attractor) depending only upon the particular substitution sequence, and not upon the energy or other details. In this way, it was possible to identify these patterns as *fingerprints* for each kind of sequence.

The aim of this work is to use the same method in quasiperiodic dielectric multilayers which obey a Fibonacci, Thue-Morse, double-period, and Rudin-Shapiro sequences, looking for their *optical fingerprints* through their return map. Our main purpose is to search for *global* or *universal* properties for each of these quasiperiodic systems.

Consider a dielectric multilayer system where the Cartesian axes are chosen in such a way that the z axis is parallel to the direction normal to the planes of the layers. The multilayer system is at the region $0<z<L$ (L being its size). The regions $z<0$ and $z>L$ are considered to be filled by a transparent medium V (vacuum in general). This multilayer system is formed by a quasiperiodic array of Fibonacci, Thue-Morse, double-period, and Rudin-Shapiro types. The procedure to grow these kind of structures can be found in Ref. 9. They can be generated by their inflation rules (in what follows A, B, C, and D are building blocks): $A\rightarrow AB$, $B\rightarrow A$ (Fibonacci sequence), $A\rightarrow AB$, $B\rightarrow BA$ (Thue-Morse sequence), $A\rightarrow AB$, $B\rightarrow AA$ (double-period sequence); $A\rightarrow AC$, $B\rightarrow DC$, $C\rightarrow AB$, and $D\rightarrow DA$ (Rudin-Shapiro sequence). Each building block is characterized by a thickness and refractive index d_J and n_J, respectively ($J=A$, B, C, and D). The transparent medium V, which surrounds the multilayer system, is characterized by a refractive index n_V.

To calculate the light transmission rate (or transmittance) through the multilayer system, we use a transfer matrix approach for the electromagnetic fields. In this way, we consider that a s-polarized (TE waves) light of frequency ω is normally incident from a transparent medium V with respect to the layered system. We have chosen the s-polarization mode for simplicity, since at normal incidence both s and p polarization give the same results. The reflectance and the transmittance coefficients are simply given by $R=|M_{21}/M_{11}|^2$, and $T=|1/M_{11}|^2$, where M_{ij} ($i,j=1,2$) are the elements of the optical transfer matrix M, which links the amplitudes of the electromagnetic fields in the region $z<0$ to the amplitudes of the electromagnetic fields in the region $z>L$.

The transmission of a normal incident light wave across the interfaces $\alpha\rightarrow\beta$ (α, β being V, A, B, C, and D) is represented by the matrix

$$M_{\alpha\beta}=\frac{1}{2}\begin{pmatrix} 1+k_{z\beta}/k_{z\alpha} & 1-k_{z\beta}/k_{z\alpha} \\ 1-k_{z\beta}/k_{z\alpha} & 1+k_{z\beta}/k_{z\alpha} \end{pmatrix}, \tag{1}$$

with $k_{z\alpha}=n_\alpha\omega/c$. The propagation of the light wave within one of the layer γ ($\gamma=A$, B, C, or D) is characterized by the matrix

$$M_\gamma=\begin{pmatrix} \exp(-ik_{z\gamma}d_\gamma) & 0 \\ 0 & \exp(ik_{z\gamma}d_\gamma)\end{pmatrix}. \tag{2}$$

We assume that in each layer, the electrical field is given by

$$\vec{E}_j^{(n)}=\{0,[A_{1j}^{(n)}\exp(-ik_{zj}z)+A_{2j}^{(n)}\exp(ik_{zj}z)]\exp(-i\omega t),0\}, \tag{3}$$

where $A_{1j}^{(n)}$ and $A_{2j}^{(n)}$ ($j=A$, B, C, or D; $n=0,1,2,\ldots,N$) are their amplitudes.

Successive applications of Maxwell's electromagnetic boundary conditions at each interfaces of the finite multilayer system, yield

$$\begin{pmatrix} A_{1V}^{(0)} \\ A_{2V}^{(0)}\end{pmatrix}=M_N\begin{pmatrix} A_{1V}^{(N)} \\ 0\end{pmatrix}, \tag{4}$$

where $A_{1V}^{(0)}$ and $A_{2V}^{(0)}$ are the amplitudes of the electromagnetic field in the transparent medium V at $z<0$, while $A_{1V}^{(N)}$ is the amplitude of the electromagnetic field in medium V at $z>L$; M_N is the optical transfer matrix of the Nth generation quasiperiodic multilayer system. This transfer matrix is formed by a product of matrices $M_{\alpha\beta}$ and M_γ. The ordering of these matrices in the product depends upon the type of quasiperiodic array and the generation number N of the quasiperiodic sequence (which is the same index used in the amplitudes of the electromagnetic field). The transfer matrices of all quasiperiodic systems considered here can be straightforwardly determined (for details see Ref. 10).

We now present some numerical calculations for the transmission probability T, due to a normal propagation of light waves of frequency ω in a quasiperiodic multilayer system of the types discussed here. In these examples we adopt the optical thickness of the individual layers as quarter wavelength of the incident light wave, which is considered to have a wavelength equal to $\lambda_0=700$ nm, i.e., $n_Ad_A=n_Bd_B=n_Cd_C=n_Dd_D=\lambda_0/4$. The physical parameters used here are the same as those in Ref. 10, except for the Rudin-Shapiro case where we have adopted $n_A=2$, $n_B=3$, $n_C=4$, and $n_D=5$.

A common feature of this quasiperiodic structure is that their properties are more evident at higher order (for example, the self-similarity).[11] However, the traditional theoretical methods, used to study the optical transmission spectra in these system, do not allow us to investigate these quasiperiodic structures at higher order because their size grows exponentially when the number of generation increases. As a consequence their transmittances go to zero quickly. To overcome this problem we use the method developed in Ref. 8, i.e., given an incident frequency ω, we calculate the maximum generation number N where the transmittance T_N is less than 10^{-12}.

Figure 1(a) shows the plot of the maximum generation number N versus the reduced frequency ω/ω_0 for the Fibonacci multilayer system in the range of frequency $0.75<\omega/\omega_0<1.25$. In this region the largest generation number reached was $N=78$, which corresponds to a size of the multilayer system $L=F_{78}\sim1.44\times10^{16}$ monolayers ($F_l=F_{l-1}+F_{l-2}$ are the Fibonacci numbers, with $F_1=1$ and $F_2=2$). From there we can observe several dips and peaks forming band gaps in the frequency. These band gap regions

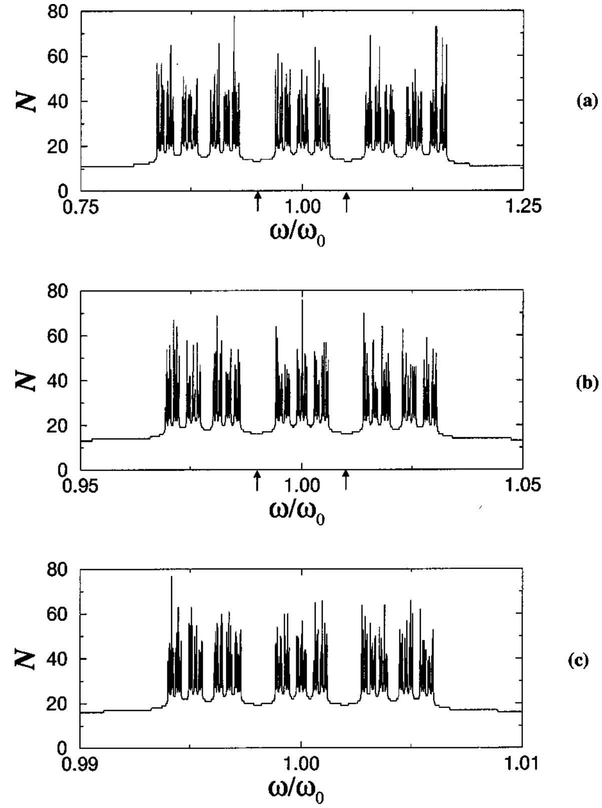

FIG. 1. Maximum generation number N versus ω/ω_0 for the Fibonacci system for three different scales. The arrows here indicate the scaling points where the self-similarity occurs.

correspond to more localized states in higher orders of Fibonacci multilayer system. Moreover, these states have self-similar properties. To illustrate this fact, we plot in Figs. 1(b) and 1(c) the same spectra as in Fig. 1(a), but with other scales of frequency. The arrows in the figures indicate the scaling points where the self-similarity occurs.

In Fig. 2(a) we plot the maximum generation number N versus the reduced frequency ω/ω_0 for the Thue-Morse multilayer system, at the frequency range $0.85<\omega/\omega_0<1.15$. As in the Fibonacci case we observe several band gaps in this frequency range. They have also self-similar properties, as shown in Figs. 2(b) and 2(c). The arrows in these figures indicate the scaling points were the self-similarity is more evident. Another characteristic of this quasiperiodic system is that its size grows as 2^n, and the maximum generation number found was $N=72$ ($L=2^{72}\sim4.72^{21}$ monolayers).

The double-period multilayer case is shown in Fig. 3(a), where we plot N versus the reduced frequency ω/ω_0 in the range $0.75<\omega/\omega_0<0.84$. As in other multilayer systems, we can find frequency band gaps distributed in a mode characteristic of this system. The largest generation number reached was $N=51$. As in the Thue-Morse case, the size of this system grows as 2^n. It corresponds to $L=2^{51}\sim2.5\times10^{15}$. The self-similarity can be observed in Figs. 3(b) and 3(c). The arrows in these figures indicate the scaling points.

For the Rudin-Shapiro multilayer case, we plot in Fig. 4(a) a complete different spectra N versus the reduced frequency ω/ω_0. We have chosen the range of frequency $1.96<\omega/\omega_0<2.04$ to illustrate our spectra, because it is the re-

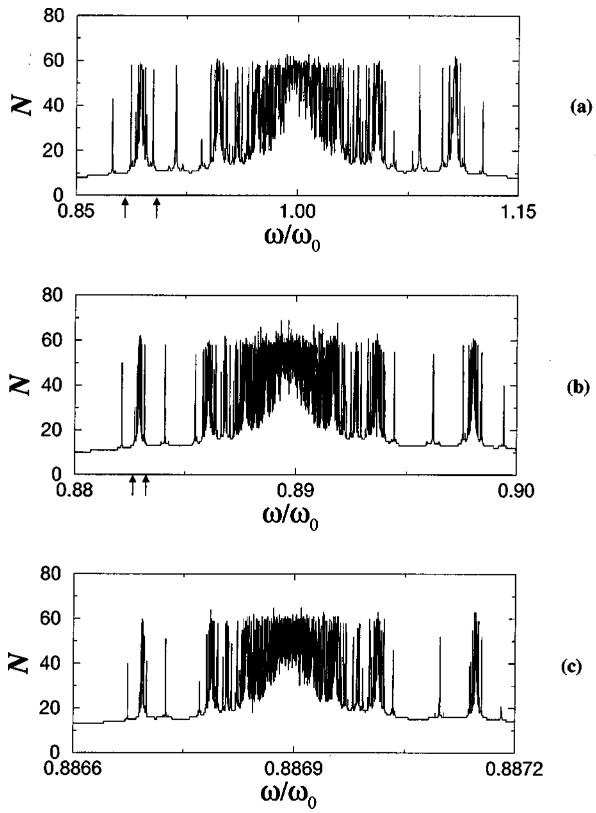

FIG. 2. Same as in Fig. 1, but for the Thue-Morse case.

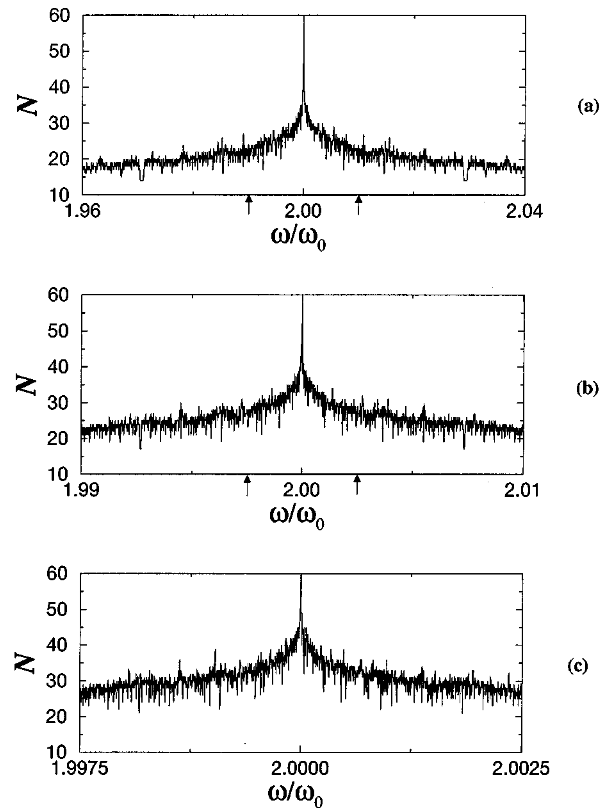

FIG. 4. Same as in Fig. 1, but for the Rudin-Shapiro case.

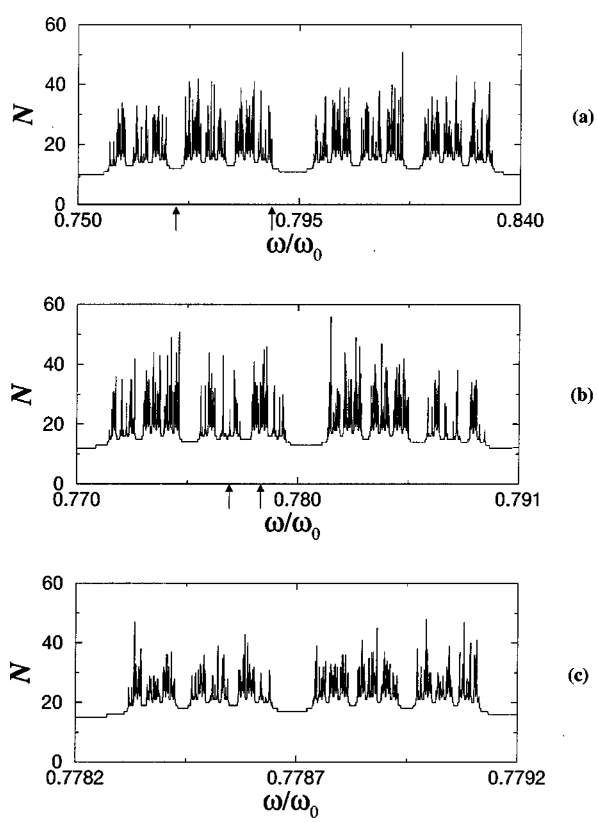

FIG. 3. Same as in Fig. 1, but for the double-period case.

gion where we can find more localized states in high order generations. The spectrum has no band gaps, as in other spectra of the quasiperiodic structures. There is also a central peak at $\omega/\omega_0 = 2.0$, where, around it, the maximum generation number decreases to small values in an exponential form. This fact was also not observed in the other quasiperiodic multilayers systems. The value of the maximum genera-

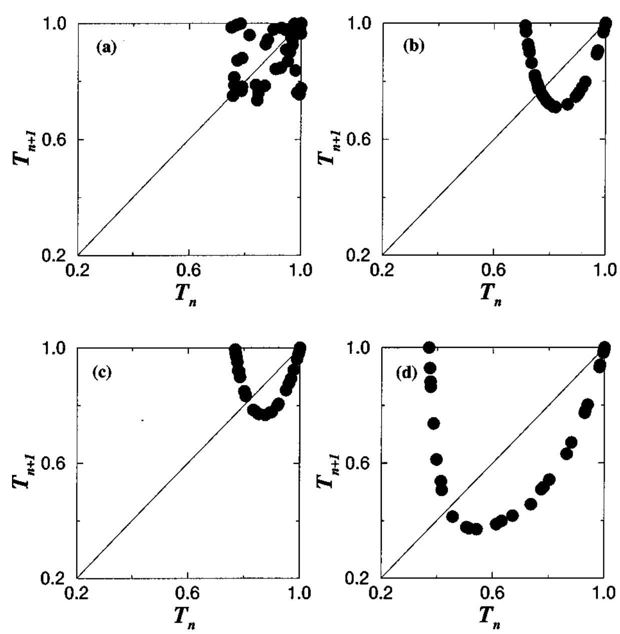

FIG. 5. Return map T_{N+1} versus T_N to characterize the optical fingerprint of the quasiperiodic structures: (a) Fibonacci, (b) Thue-Morse, (c) double-period, and (d) Rudin-Shapiro.

tion at this peak is $N=58$, and it corresponds to $L=2^{59} \sim 5.76\times 10^{17}$ monolayers (this system grows as 2^{n+1}). The self-similar properties can be observed in Figs. 4(b) and 4(c), where the arrows indicate the scaling points.

Other important point to explore here is the return map, which can be built in the following form: For a given value of the frequency, we generate a set of transmissions T_N ($T_1, T_2, \ldots, T_N$) for each generation of the sequences in the quasiperiodic multilayer system. The return map is the plot T_{N+1} versus T_N. In Fig. 5 we illustrate this return map for the systems studied here and for $\omega/\omega_0=2.0$. We have chosen this value for ω/ω_0, because it is at this value that the light waves *see* the quasiperiodic array of the layers (as it is shown in Fig. 7 of Ref. 10). The return map for the Fibonacci structure is shown in Fig. 5(a). It gives rise to a *box* whose vertices are defined by $0.725<T_N<1.0$ and $0.725<T_{N+1}<1.0$. Different than the electronic case,[8] the spectrum *depends* on the frequency of the incident light. For instance, if we take another value for the frequency, different of $\omega/\omega_0=2.0$, we should have a different pattern. In Fig. 5(b) we show the return map for the Thue-Morse quasiperiodic system. The pattern in this figure is similar to a *parabola* and, as in the electronic case, it does not depend upon the particular value of the frequency. The double-period quasiperiodic case is shown in Fig. 5(c). The pattern found is again a *parabola*, as in the Thue-Morse case, but it depends upon the choice of the frequency. Finally, in Fig. 5(d) we show the return-map for the Rudin-Shapiro quasiperiodic multilayer system. The pattern found is a kind of *parabola* with no symmetric axis, as those found for the Thue-Morse and double-period multilayer systems. It is also dependent upon the incident frequency of the light.

In conclusion, we have studied the propagation of light waves in quasiperiodic dielectric multilayer systems that obey the Fibonacci, Thue-Morse, double-period, and Rudin-Shapiro sequences. We believe that it could be an excellent way to probe experimentally localized states, since localization phenomenon is essentially due to the wave nature of the electronic states, and thus can be found in any wave phenomena. In addition, the global aspects of these sequences were found in their transmission spectra, which present a quite interesting self-similar pattern, with well defined scaling points in frequency. We have calculated also the optical transmission probabilities through the deterministic quasiperiodic sequences. As the defining rules of these sequences impose long range correlations on them, it is plausible to search for global (universal) consequences of these correlations, as exemplified in Fig. 5. The return maps $T_{n+1}\times T_n$, represent the attractors of the *dynamic* evolution $T_1, T_2, T_3, \ldots, T_n, \ldots$, and we found that the patterns of these attractors depend only upon the particular sequence being tested, for a particular value of the frequency of the incident light. In this way, we can say that they work as *fingerprints* for each kind of sequence.

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