Transient heat transport by carriers and phonons in semiconductors

A. F. Carballo Sánchez, G. González de la Cruz, Yu G. Gurevich, and G. N. Logvinov*
Departamento de Física, Centro de Investigación y de Estudios Avanzados del I.P.N., Apartado postal 14-740, 07000 México, D.F., Mexico
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The electron and phonon temperature distribution in semiconductors initially heated on the surface with a short laser pulse is calculated as a function of position and time. We solved the coupled one-dimensional heat diffusion equations for electron and phonon systems in the linear approximation in which the physical parameters of the sample are temperature independent. We also consider the heat pulse at the surface of the semiconductor as a boundary condition for each electron and phonon system. We believe that the transient heat transport experiments are a very suitable way of measuring the electron and phonon temperatures $T_{e,p}(x,t)$ in the sample, and they also yield the relaxation time associated with the different relaxation processes, e.g., electron-electron, phonon-phonon, and electron-phonon relaxation times, respectively. We provide a detailed quantitative theory for heat transient transport and find that the mechanism for electron energy relaxation time is strongly dependent on the size of the sample. For thin-film semiconductors the main relaxation process is due to heat diffusion by carriers, however the energy relaxation in larger samples is due to electron-phonon energy interaction. On the other hand, for nondegenerate semiconductors the typical ratio of the heat conductivities of electrons and phonons satisfies $\kappa_e/\kappa_p \sim 10^{-3}$; under these circumstances the phonon energy relaxation time is due to heat diffusion by phonons and it is sample size independent. It is exciting that the electron temperature distribution function can be measured experimentally by means of the thermoelectric effect in semiconductors as a function of time. [S0163-1829(99)00916-9]

I. INTRODUCTION

Up to now, thermal parameters in semiconductors have been extensively investigated by solving the heat diffusion equation in steady-state conditions.[1] There is also experimental evidence in favor of this model.[2,3] The experimental photoacoustic effect, which directly measures the pressure fluctuations in the chamber gas, yields the thermal diffusivity of the sample. However, in steady-state transport experiments the situation is somewhat more complicated because the electrons (phonons) continuously gain energy from the absorption of the incident radiation and lose it by the interaction between them.[4,5] For typical values of the heat conductivity of electrons and phonons in semiconductors, it is possible to obtain information about the physical parameters describing the diffusivity and thermal conductivity of electrons and phonons as well as the electron-phonon energy interaction. But here these photothermal experiments cannot provide insight into processes related to energy relaxation times associated with quasiparticle systems. Pulsed optical spectroscopy allows an investigation of the dynamics of nonequilibrium processes and the various relaxation mechanism in photoexcited semiconductor[6] and metallic samples.[7–11] The decay curves of transient thermoelectric effect[12] consist of all the stages (carrier generation and recombination,[13,14] heat diffusion by carrier and phonons,[5] etc.), each with its characteristic relaxation or decay time. The transient thermoelectric effect experiments provide useful information regarding relaxation mechanism, specific heat, thermal diffusivity, and more.[15,16]

Over the past decade there have been many experiments using excite-and-probe techniques to investigate nonsteady thermal processes in semiconductors. Most of these experiments have been reported where one surface of the sample is heated by means of a flashlamp or a laser pulse and the transient temperature of the other surface is measured using some photothermal technique.[6]

Above band-gap excitation of a semiconductor with an intense laser pulse produces a large number of nonequilibrium electron and hole carriers.[13,14] As the systems diffuse into the sample, the electron-hole pairs eventually recombine producing a second source of heat which also diffuses into the semiconductor. If the following inequality $\nu(\epsilon) \gg \nu_{ee}(\epsilon) \gg \nu_{\epsilon}(\epsilon), \omega$ is valid, where $\nu(\epsilon)$ is the frequency of the momentum relaxation, $\nu_{ee}(\epsilon)$ represents the electron-electron collision, $\nu_{\epsilon}(\epsilon)$ the electron energy relaxation, ω is the external perturbation frequency, and ϵ is the kinetic electron energy, the nonequilibrium electron system can be described by a Fermi-Dirac distribution function with temperature T_e.[17] Similarly, we can introduce the nonequilibrium phonon temperature T_p.

One of the simplest and at the same time quite effective models describing the process of heat transport in the local equilibrium conditions, i.e., a purely dissipative energy transport, is the so-called two-temperature model.[17] In this case an individual temperature can be assigned to each subsystem. Then the thermal problem in the system can be reduced to the determination of the space-time evolution of these two temperatures taking into consideration the energy exchange between subsystems. The two-temperature approach has been used to analyze the thermal wave propagation in semiconductors[5] and the measurements of transient transport due to the generation of nonequilibrium electron temperature which cools to the lattice.[11]

In general, the photothermal signal is not only dependent upon how heat is carried out by each quasiparticle system in

the semiconductor and its thermal parameters (i.e., electron and phonon thermal diffusivity and thermal conductivity), but also on how the energy and momentum are distributed between them, i.e.,[5] the detected signal depends on the different mechanisms of carrier and phonon interactions. For example, in insulators heat is only carried out by phonons, and the energy relaxation time is given as $\tau = l^2/\alpha_p$, where l is the thickness of the sample and α_p is the effective phonon thermal diffusivity; for crystalline Si films with thickness 100 μm, typical values of $\alpha_p = 0.88\,\mathrm{cm}^2/\mathrm{s}$ at room temperature are reported when $T_e = T_p$ and the response of the sample occurs on a $\tau \sim 10^{-4}$ s time scale.

In order to observe these thermal fluctuations such as the various quasiparticles dynamics and heat diffusion in semiconductors, several photothermal techniques have been developed. Some of these are conventional gas-microphone photoacoustic detection, photopyroelectric deflection, or remote sensing techniques such as photothermal reflection, etc., have been reviewed in Refs. 2 and 3 for a periodic thermal excitation on the surface of the semiconductor due to the absorption of a modulated electromagnetic radiation.

Recently, a novel variant of that technique in which a transient thermoelectric effect is generated by heating a sample through absorption of optical energy has been successfully developed.[12,15] Photothermal pulse heating of a sample is frequently produced by using intense light sources. The absorption of optical energy generates electron-hole pairs, increases the energy of the majority carriers, and produces a thermal flux of phonons. The decay curves of the transient thermoelectric effect thus consist of all the ultrafast processes, each one with its characteristic relaxation or decay time. From the theoretical point of view, it is well known that heat transport in solids is carried out by these quasiparticles. Frequently the interactions between them are such that each of these systems can have their own temperature and physical conditions at the boundary of the sample.[18]

At this point, we would like to draw attention to two aspects which, in the final analysis, are important in applications of the photoacoustic or thermoelectric effects. Because the source of the photoacoustic signal arises from the periodic heat flow from the semiconductor, the periodic diffusion process produces a periodic temperature variation in the semiconductor; in this case only information about the electron and phonon thermal parameters are obtained from those experiments (steady-state conditions, see Ref. 5). However, in thermoelectric experiments the important contribution to the signal comes from the different relaxation mechanisms between the quasiparticles which in general occur at different times (with transient heat transport, which is in the limit of very long laser pulse duration, the static approximation is recovered; see Sec. IV).

It is clear from the above introduction that a detailed knowledge of how, when, and where the laser radiation is initially deposited and how it is redistributed in time and space is a very complicated problem. Nevertheless, this is a problem that has to be addressed in order to obtain a better understanding of ultrafast semiconductor dynamics.

It is worth mentioning that the local equilibrium model of heat transport process has been investigated by Sobolev.[19,20] Physically, the approximation of local equilibrium is correct if the relaxation time τ (the time for the establishment of equilibrium in a small domain, although containing a large number of particles) is considerably smaller than the characteristic time of the external perturbation τ_c (the inverse of the chopper frequency in photoacoustic experiments or the laser pulse duration in ultrafast experiments). Nevertheless, many authors[19–21] have proposed an additional term $\tau \partial^2 T/\partial t^2$ in the parabolic heat diffusion equation. This new equation (hyperbolic equation) really represents the propagation of undamped heat waves instead of an energy dissipation process and it is valid if $\tau > \tau_c$. However, it is well known that the temperature distribution $T(x,t)$ is a thermodynamic parameter which describes the local average energy of the system; thus if τ satisfies $\tau > \tau_c$, the local thermodynamic equilibrium cannot be established and $T(x,t)$ loses its physical meaning.[22]

In this paper we present a theoretical investigation of transient heat transport in semiconductors in the local equilibrium approximation within the two-temperature model (electrons and phonons). With this approximation the quasiparticle temperatures are well defined and the heat parabolic equation can be used to describe the heat diffusion process. To make the calculations of carrier and phonon tractable, simplified assumptions about the transient relaxation processes of carriers are made. We shall assume that the semiconductor is optically opaque to the incident laser pulse (i.e., all the incident light is absorbed at the surface and converted into heat).

In particular, the heat source produced by recombination of carriers is neglected in the heat diffusion equations for electrons and phonons. The last approximation is only valid in the limit of strong electron-hole recombination near the surface of the semiconductor after the photoexcitation of an electron-hole pair. The carrier yields energy which is converted into heat in the sample.

Under these conditions, three energy relaxation mechanisms are important to mention: heat diffusion by the electron system and relaxation of the extra energy to the heat reservoir environment during a time τ_e, τ_p will be the characteristic relaxation time of the phonon system to the thermal reservoir, and the electron and phonon relaxation energy with a characteristic time scale τ_{ep}.

II. FUNDAMENTAL EQUATIONS

For simplification, we shall consider a semiconductor with the form of a parallelipiped. On one surface $(x=0)$ there is an incident pulsed laser excitation, the surface $x=l$ is maintained at constant temperature T_0, and the lateral faces are adiabatically isolated. In this geometry we have to solve a one-dimensional heat-diffusion equations. To evaluate the electron temperature distribution we make the following assumptions: the sample is optically opaque (i.e., the energy radiation is fully absorbed at the surface of the sample and converted into heat), the surface recombination velocity is infinite so the electron-hole pairs generated at the surface by the incident radiation rapidly recombine, and the profile of the thermal pulse is constant during the laser pulse and zero otherwise.

It is common knowledge that heat transfer in semiconductors is realized by various quasiparticles systems. In the local equilibrium model the interaction between these quasiparti-

cles in a large number of cases can be represented as if these systems have their own temperature, and boundary conditions can be formulated for each of the subsystems individually. We shall restrict our analysis to the case of monopolar semiconductors under the condition of strong phonon-phonon interaction as discussed by Gurevich and Mashkevich.[17] Therefore, the space and time evolution of the electron and phonon temperatures (T_e and T_p, respectively) are governed by one-dimensional coupled heat-diffusion equations, which were discussed in Ref. 5 [see Eqs. (1) and (3) therein].

Recently, time-dependent heat diffusion in semiconductors by electrons and phonons has been investigated in steady-state conditions, where a periodic time-dependent heat flux at the surface of the sample is used as a boundary condition for the electron and phonon systems and a fixed temperature at the opposite surface. On the other hand, in the present work we shall study transient heat transport by carriers in semiconductors (non-steady-state conditions) with boundary conditions according to the experimental situation and, of course, the solutions for the electron and phonon temperatures are different as compared with those of Ref. 5. Nevertheless, we also use the linear approximation for the thermal parameters.

In the coupled limit (when the energy relaxation time between electrons and phonons vanishes, i.e., $\tau_\epsilon = 0$) for small effective cooling length of the electron-phonon energy interaction, as compared to the sample dimensions and strong electron-phonon energy interaction, the system of quasiparticles can be described by the single temperature approximation and the coupled heat-diffusion equations reduce to the usual diffusion equation,

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t},$$

where

$$\alpha = \frac{\kappa_e + \kappa_p}{(\rho c)_e + (\rho c)_p}.$$

The notation is the same as in Ref. 5. In this case, the distance traveled by the heat varies with the square root of time, $L = (\alpha \Delta t)^{1/2}$. Choosing reasonable values for the thermal diffusivity ($\alpha \sim 1$ cm^2/s for Si at room temperature), we find that heat propagates only 1000 Å after 100 ps.

However, when the electron and phonon systems interact ($\tau_\epsilon \neq 0$), the heat diffusion by electrons and phonons is more complex. The cooling of the electron temperature to the lattice can make the heat transient time deviate from the expected $\Delta t \sim L^2/\alpha$ behavior and assume a more complicate dependence.

The temperature fluctuations $T_{e,p}(x,t)$ should be supplemented by boundary conditions at the surfaces of the sample and some initial conditions. In transient heat transport experiments, the most common mechanism to produce a heat pulse is the absorption by the sample of an intense pulsed laser beam. It is clear that when the intensity of the radiation is fixed, the light-into-heat conversion at the surface of the sample can be written in general as[23]

$$Q_{e,p}(x,t)|_{x=0} = -\kappa_{e,p}\left.\frac{\partial T_{e,p}(x,t)}{\partial x}\right|_{x=0} = Q_{e,p}, \quad 0 \leqslant t \leqslant \tau_c, \tag{1a}$$

$$\left.\frac{\partial T_{e,p}(x,t)}{\partial x}\right|_{x=0} = 0, \quad \tau_c \leqslant t \leqslant \infty, \tag{1b}$$

and

$$T_{e,p}(x,t)|_{t\leqslant 0} = T_0, \tag{1c}$$

$$T_{e,p}|_{x=l} = T_0. \tag{1d}$$

Here $Q_{ep}(x,t)$ represents the electron and phonon heat flux, $Q_{e,p} = \text{const}$, and at $x = 0$ describes the temporal form of the electron and phonon heat pulse during the time τ_c (laser pulse duration); the surface at $x = l$ remains at the ambient temperature T_0. In general, $Q_e \neq Q_p$. It is important to note that when chopped incident light is used in photothermal experiments, the boundary conditions are given by Eq. (4) in Ref. 5 and, of course, the solutions of the heat-diffusion equation obtained in this work and in Ref. 5 [see Eqs. (3)] are quite different.

The general solution of the coupled heat-diffusion equations for the electron and phonon systems can be written as follows. For $0 \leqslant t \leqslant \tau_c$,

$$\begin{aligned} T_e(x,t) = T_0 &+ \frac{1}{k^2}\left(\frac{Q_e}{\kappa_e}k_p^2 + \frac{Q_p}{\kappa_p}k_e^2\right)(l-x) + \frac{k_e^2}{k^2}\left(\frac{Q_e}{\kappa_e} - \frac{Q_p}{\kappa_p}\right)\frac{\sinh[k(l-x)]}{k\cosh(kl)} \\ &+ \frac{2k_e^2}{lk^2}\left(\frac{Q_e}{\kappa_e} - \frac{Q_p}{\kappa_p}\right)\sum_{n=0}^{\infty}\cos(\beta_n x)\left[\frac{(k^2\alpha_e + \beta_n^2\alpha_e + \lambda_{2n})e^{\lambda_{1n}t} - (k^2\alpha_e + \beta_n^2\alpha_e + \lambda_{1n})e^{\lambda_{2n}t}}{(\lambda_{1n} - \lambda_{2n})(k^2 + \beta_n^2)}\right] \\ &+ \frac{2}{lk^2}\left(\frac{Q_e}{\kappa_e}k_p^2 + \frac{Q_p}{\kappa_p}k_e^2\right)\sum_{n=0}^{\infty}\frac{\cos(\beta_n x)}{\beta_n^2}\left(\frac{(\beta_n^2\alpha_e + \lambda_{2n})e^{\lambda_{1n}t} - (\beta_n^2\alpha_e + \lambda_{1n})e^{\lambda_{2n}t}}{\lambda_{1n} - \lambda_{2n}}\right) \end{aligned} \tag{2a}$$

and

$$T_p(x,t)=T_0+\frac{1}{k^2}\left(\frac{Q_e}{\kappa_e}k_p^2+\frac{Q_p}{\kappa_p}k_e^2\right)(l-x)-\frac{k_p^2}{k^2}\left(\frac{Q_e}{\kappa_e}-\frac{Q_p}{\kappa_p}\right)\frac{\sinh[k(l-x)]}{k\cosh(kl)}$$
$$-\frac{2k_p^2}{lk^2}\left(\frac{Q_e}{\kappa_e}-\frac{Q_p}{\kappa_p}\right)\sum_{n=0}^{\infty}\cos(\beta_n x)\left(\frac{(k^2\alpha_p+\beta_n^2\alpha_p+\lambda_{2n})e^{\lambda_{1n}t}-(k^2\alpha_p+\beta_n^2\alpha_p+\lambda_{1n})e^{\lambda_{2n}t}}{(\lambda_{1n}-\lambda_{2n})(k^2+\beta_n^2)}\right)$$
$$+\frac{2}{lk^2}\left(\frac{Q_e}{\kappa_e}k_p^2+\frac{Q_p}{\kappa_p}k_e^2\right)\sum_{n=0}^{\infty}\frac{\cos(\beta_n x)}{\beta_n^2}\left(\frac{(\beta_n^2\alpha_p+\lambda_{2n})e^{\lambda_{1n}t}-(\beta_n^2\alpha_p+\lambda_{1n})e^{\lambda_{2n}t}}{\lambda_{1n}-\lambda_{2n}}\right), \tag{2b}$$

where $k^2=k_e^2+k_p^2$ represents the inverse of the cooling length,[18] $\alpha=\alpha_e+\alpha_p$, and $\beta_n=(2n+1)\pi/2l$ with $n=0,1,2,\ldots,$

$$\lambda_{1n}=\tfrac{1}{2}(-k_e^2\alpha_e-k_p^2\alpha_p-\beta_n^2\alpha+\{[k_e^2\alpha_e-k_p^2\alpha_p+\beta_n^2(\alpha_e-\alpha_p)]^2+4k_e^2k_p^2\alpha_e\alpha_p\}^{1/2}), \tag{3a}$$

$$\lambda_{2n}=\tfrac{1}{2}(-k_e^2\alpha_e-k_p^2\alpha_p-\beta_n^2\alpha-\{[k_e^2\alpha_e-k_p^2\alpha_p+\beta_n^2(\alpha_e-\alpha_p)]^2+4k_e^2k_p^2\alpha_e\alpha_p\}^{1/2}). \tag{3b}$$

The electron and phonon temperature distribution for $\tau_c\leqslant t<\infty$ together with the continuity of temperature at $t=\tau_c$, i.e., $T_{e,p}(x,t)|_{t=\tau_c}=T_{e,p}(x,\tau_c)$, where $T_{e,p}(x,\tau_c)$ are given by Eqs. (2a) and (2b), are

$$T_e(x,t)=T_0+\frac{2k_e^2}{lk^2}\left(\frac{Q_e}{\kappa_e}-\frac{Q_p}{\kappa_p}\right)$$
$$\times\sum_{n=0}^{\infty}\cos(\beta_n x)\left(\frac{(k^2\alpha_e+\beta_n^2\alpha_e+\lambda_{2n})(1-e^{-\lambda_{1n}\tau_c})e^{\lambda_{1n}t}-(k^2\alpha_e+\beta_n^2\alpha_e+\lambda_{1n})(1-e^{-\lambda_{2n}\tau_c})e^{\lambda_{2n}t}}{(\lambda_{1n}-\lambda_{2n})(k^2+\beta_n^2)}\right)$$
$$+\frac{2}{lk^2}\left(\frac{Q_e}{\kappa_e}k_p^2+\frac{Q_p}{\kappa_p}k_e^2\right)\sum_{n=0}^{\infty}\frac{\cos(\beta_n x)}{\beta_n^2}\left(\frac{(\beta_n^2\alpha_e+\lambda_{2n})(1-e^{-\lambda_{1n}\tau_c})e^{\lambda_{1n}t}-(\beta_n^2\alpha_e+\lambda_{1n})(1-e^{\lambda_{2n}\tau_c})e^{\lambda_{2n}t}}{\lambda_{1n}-\lambda_{2n}}\right), \tag{4a}$$

$$T_p(x,t)=T_0-\frac{2k_p^2}{lk^2}\left(\frac{Q_e}{\kappa_e}-\frac{Q_p}{\kappa_p}\right)$$
$$\times\sum_{n=0}^{\infty}\cos(\beta_n x)\left(\frac{(k^2\alpha_p+\beta_n^2\alpha_p+\lambda_{2n})(1-e^{-\lambda_{1n}\tau_c})e^{\lambda_{1n}t}-(k^2\alpha_p+\beta_n^2\alpha_p+\lambda_{1n})(1-e^{-\lambda_{2n}\tau_c})e^{\lambda_{2n}t}}{(\lambda_{1n}-\lambda_{2n})(k^2+\beta_n^2)}\right)$$
$$+\frac{2}{lk^2}\left(\frac{Q_e}{\kappa_e}k_p^2+\frac{Q_p}{\kappa_p}k_e^2\right)\sum_{n=0}^{\infty}\frac{\cos(\beta_n x)}{\beta_n^2}\left(\frac{(\beta_n^2\alpha_p+\lambda_{2n})(1-e^{-\lambda_{1n}\tau_c})e^{\lambda_{1n}t}-(\beta_n^2\alpha_p+\lambda_{1n})(1-e^{-\lambda_{2n}\tau_c})e^{\lambda_{2n}t}}{\lambda_{1n}-\lambda_{2n}}\right). \tag{4b}$$

Once we know the electron and phonon temperature distributions in the sample, we can calculate the response of the surrounding medium due to the laser pulse heating of the sample using one of the several alternative detection techniques mentioned in Ref. 6.

III. ELECTRON AND PHONON TEMPERATURE DISTRIBUTIONS IN NONDEGENERATE SEMICONDUCTORS

One of the most important properties of any semiconductor at temperature T is the number of electrons per unit volume in the conduction band. The determination of these as a function of temperature is straightforward through the Fermi-Dirac statistics, and one can extract some useful information about the electron and phonon thermal parameters provided only that the chemical potential

$$\mu<0, \quad |\mu|\gg T.$$

If it does, the semiconductor is described as nondegenerate and in this case the ratio between the electron and phonon thermal conductivity is satisfied; $\kappa_e/\kappa_p\sim 10^{-3}$. Then by the definition of the electron k_e^{-1} and phonon k_p^{-1} cooling length $k_e\gg k_p$, $k^{-1}=k_e^{-1}$, Gurevich *et al.*[24] show, using kinetic theory of electron gas, that electron heat capacity is proportional to the electron density in nondegenerate semiconductors, i.e., $(\rho c)_e\sim n\sim 10^{14}-10^{16}\,\mathrm{cm}^{-3}$, while for phonon gas $(\rho c)_p\sim 10^{23}\,\mathrm{cm}^{-3}$.[25] Therefore, under these circumstances, a remarkable result is obtained for the electron and phonon thermal diffusivity,

$$\alpha_e=\kappa_e/n, \quad \alpha_p=10^3\kappa_e/(\rho c)_p,$$

$$\frac{\alpha_e}{\alpha_p}=\frac{\kappa_e}{(\rho c)_e}\frac{(\rho c)_p}{(\kappa_p)}\sim 10^6-10^4,$$

thus

$$\alpha_e\gg\alpha_p.$$

With these approximations, after simplifying [$\lambda_{1n}=0$, $\lambda_{2n}=-\alpha_e(k_e^2+\beta_n^2)$] the expressions for T_e and T_p, they can be written as

$$T_e(x,t)=T_0+\frac{Q_e}{\kappa_e}\frac{\sinh[k_e(l-x)]}{k_e\cosh(k_el)}-\frac{2Q_e}{l\kappa_e}\sum_{n=0}^{\infty}\frac{\cos(\beta_nx)}{k_e^2+\beta_n^2}e^{-\alpha_e(k_e^2+\beta_n^2)t}, \quad (5a)$$

$$T_p(x,t)=T_0 \quad \text{if } 0\leqslant t\leqslant\tau_c \quad (5b)$$

and

$$T_e(x,t)=T_0-\frac{2Q_e}{l\kappa_e}\sum_{n=0}^{\infty}\frac{\cos(\beta_nx)}{k_e^2+\beta_n^2}(1-e^{\alpha_e(k_e^2+\beta_n^2)\tau_c})e^{-\alpha_e(k_e^2+\beta_n^2)t}, \quad (5c)$$

$$T_p(x,t)=T_0 \quad \text{if } \tau_c\leqslant t<\infty. \quad (5d)$$

It is clear that, in the case of nondegenerate semiconductors the phonon temperature distribution is constant and is equal to the ambient temperature.

Since this is a zeroth-order approximation for $T_p(x,t)$, the next-order approximation can be obtained from Eqs. (4). It is straightforward to show that the amplitude of the dynamical contribution to the phonon temperature fluctuation is proportional to $(\kappa_e/\kappa_p)\exp(-\lambda_{1n}t)$, and because $\kappa_e/\kappa_p\ll 1$, this amplitude is very small as compared with T_0. On the other hand, the characteristic phonon energy relaxation time τ_{T_p} is proportional to λ_{1n}^{-1}, i.e., $\tau_{T_p}\sim\lambda_{1n}^{-1}\gg\tau_{T_e}=1/\alpha_e(k_e^2+\beta_n^2)$ (electron energy relaxation time), which means that the system of phonons has enough time to redistribute the energy received from the laser pulse in this approximation.

The above expressions for the heat transient transport of electrons are quite complicated. Nevertheless, there are some important limiting cases which are simple to analyze. These cases may be grouped according to the cooling length k_e^{-1} and the thickness of the sample.

Case *a*: $k_el\ll 1$ (thin films). In this case one gets from the electron temperature

$$T_e(x,t)=T_0+\frac{Q_e}{\kappa_e}\frac{\sinh[k_e(l-x)]}{k_e\cosh(k_el)}-\frac{8Q_el}{\pi^2\kappa_e}\sum_{n=0}^{\infty}\frac{\cos[(2n+1)\pi x/2l]}{(2n+1)^2}\times e^{-(2n+1)^2(\pi^2\alpha_e/4l^2)t}, \quad 0\leqslant t\leqslant\tau_c, \quad (6a)$$

$$T_e(x,t)=T_0-\frac{8Q_el}{\pi^2\kappa_e}\sum_{n=0}^{\infty}\frac{\cos[(2n+1)\pi x/2l]}{(2n+1)^2}\times(1-e^{(2n+1)^2(\pi^2\alpha_e/4l^2)\tau_c})\times e^{-(2n+1)^2(\pi^2\alpha_e/4l^2)t}, \quad \tau_c\leqslant t<\infty. \quad (6b)$$

Case *b*: $k_el\gg 1$ (bulk),

$$T_e(x,t)=T_0+\frac{Q_e}{\kappa_e}\frac{\sinh[k_e(l-x)]}{k_e\cosh(k_el)}-\frac{2Q_el}{l\kappa_e}\sum_{n=0}^{\infty}\frac{\cos[(2n+1)\pi x/2l]}{k_e^2+(2n+1)^2(\pi^2/4l^2)}\times e^{-\alpha_e[k_e^2+(2n+1)^2(\pi^2/4l^2)]t}, \quad 0\leqslant t\leqslant\tau_c, \quad (7a)$$

$$T_e(x,t)=T_0-\frac{2Q_el}{l\kappa_e}\sum_{n=0}^{\infty}\frac{\cos[(2n+1)\pi x/2l]}{k_e^2+(2n+1)^2(\pi^2/4l^2)}\times(1-e^{\alpha_e[k_e^2+(2n+1)^2(\pi^2/4l^2)]\tau_c})\times e^{-\alpha_e[k_e^2+(2n+1)^2(\pi^2/4l^2)]t}, \quad \tau_c\leqslant t<\infty. \quad (7b)$$

IV. DISCUSSION

We now turn to the discussion of the results obtained so far for nondegenerate semiconductors. It is clear that, because of the electron-phonon interaction, there is a characteristic length $k^{-1}\simeq k_e^{-1}$ (cooling length) over which the energy acquired by electrons from the laser pulse is transferred to phonons with temperature T_0 of the ambient medium. Nonequilibrium carriers interact with phonons acting as a bulk thermostat.

From the expressions (5), (6), and (7) it follows that the electron temperature distribution depends substantially on the relationship between laser pulse excitation time τ_c and the characteristic time of the electron energy relaxation τ_{T_e}. For example, if the thickness of the semiconductor (l) is greater than the electron cooling length (k_e^{-1}), then τ_{T_e} represents the electron-phonon relaxation time $\tau_{T_e}=(\alpha_ek_e^2)^{-1}=\tau_\epsilon$, and under this situation the electron energy is mainly redistributed between the system of electrons and phonons in the range $0<x\leqslant k_e^{-1}\ll l$, and therefore heat flux across the surface of the sample at $x=l$ may be neglected.

However, if $l\ll k_e^{-1}$, the electron-phonon energy interaction is not efficient in terms of energy relaxation and in this case $\tau_{T_e}=l^2/\alpha_e$ and the electron heat flux satisfies $\operatorname{div}\mathbf{Q}_e=(\rho c)_e(\partial T_e/\partial t)$.

Therefore, it is clear from the above discussion that there are some important limiting cases which should be pointed out. (i) The electron system cannot transfer its energy to the lattice. This situation corresponds to thin-film semiconductors when $k_e^{-1}\gg l$ and the electron energy relaxation time is considerably dependent on the sample thickness and the electron thermal diffusivity, $\tau_{T_e}=l^2/\alpha_e$. (ii) For massive samples ($k_e^{-1}\ll l$), in this approximation the electron energy is completely given to the lattice and τ_{T_e} depends upon both the electron cooling length and thermal diffusivity [$\tau_{T_e}=(\alpha_ek_e^2)^{-1}$].

One can now discuss various approximations to the electron temperature fluctuation $T_e(x,t)$ depending on the relationship between the electron energy relaxation time τ_{T_e} and the laser pulse duration time τ_c. These approximations are discussed in the following.

The first approximation is to neglect the dynamical contribution in the electron temperature distribution so that the

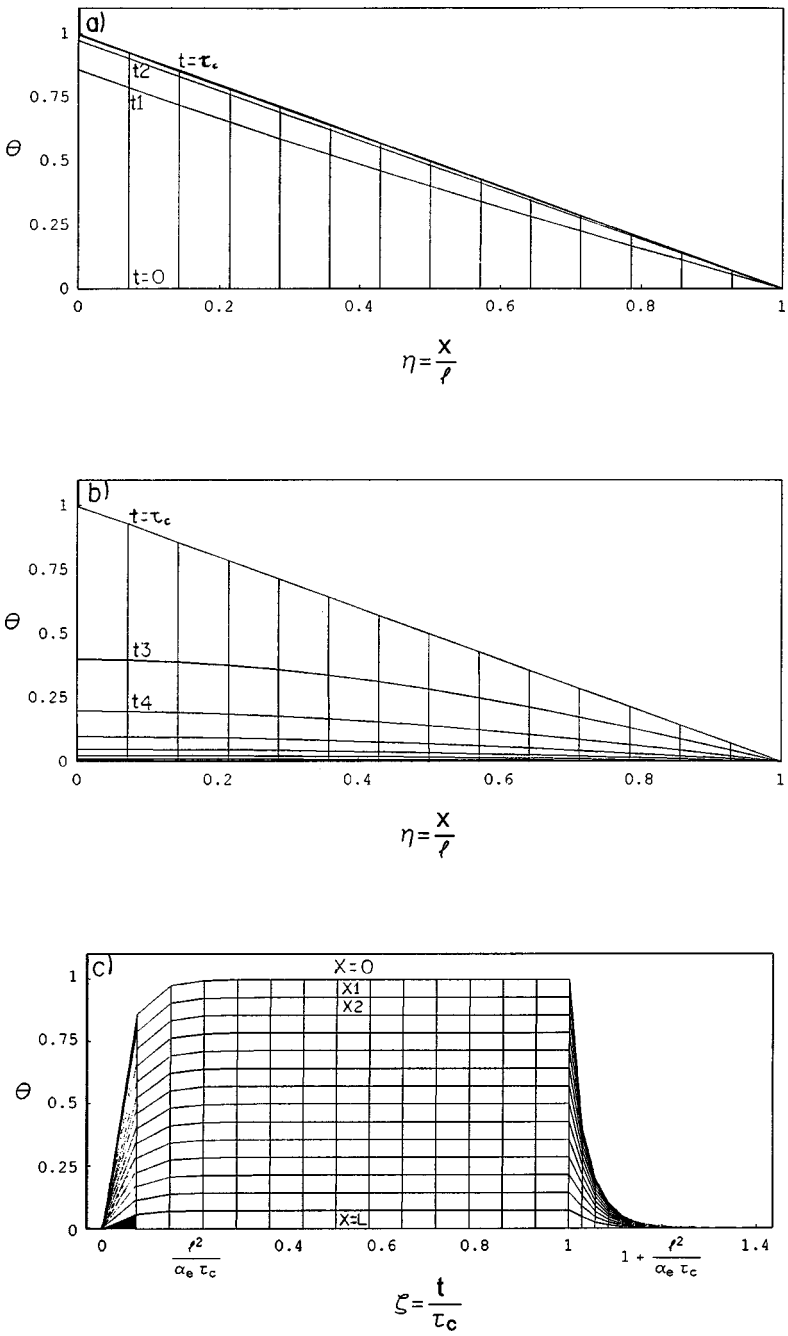

FIG. 1. Time dependence of the normalized electron temperature $\theta=(k_e/Q_e l)(T_e-T_0)$ as a function of position in the quasistatic approximation; $k_e l \ll 1$ and $\tau_c \gg \tau_{T_e}=l^2/\alpha_e$ for (a) $0<t_1<t_2<\cdots<\tau_c$, (b) $\tau_c<t_3<t_4<\cdots$, (c) position dependence of θ as a function of time at $0<x_1<x_2<\cdots<l$.

electron heat flux is almost constant. This situation is valid if the surface of the sample is illuminated during a time τ_c such that $\tau_c \gg \tau_{T_e}$. This approximation resembles the quasistatic heat diffusion in which the temperature is a linear function of the position when $\tau_{T_e}=l^2/\alpha_e$ and $k_e l \ll 1$ (see Fig. 1) or $T_e(x,t)$ is an exponential function for $\tau_{T_e}=\tau_\epsilon$ and $k_e l \gg 1$ (see Fig. 2). It increases exponentially with time in the interval $0<t<\tau_{T_e}$, and it is time independent in the regime $\tau_{T_e}<t<\tau_c$; after the laser pulse has been extinguished, the electron temperature attenuates rapidly to zero with increasing time such that at time $t>\tau+\tau_{T_e}$ the temperature fluctuation is effectively fully damped out.

On the other hand, for short laser pulse excitation ($\tau_c \ll \tau_{T_e}$), the evolution of the electron temperature distribution is the following (see Fig. 3); at fixed time in the range $0<t\leqslant\tau_c$, the electron temperature decreases exponentially with increasing distance from the surface such that, at distance $x=\sqrt{\alpha_e t}$, the temperature fluctuation has been damped out. Therefore, the initial rise in the carrier temperature due to the excess kinetic energy received during laser excitation results in a diffusion of carriers out of the interactive region ($0\leqslant x\leqslant x_c=\sqrt{\alpha_e \tau_c}$). However, the diffusion of carriers persists over a period of $t>\tau_c$ in the semiconductor until the spatial inhomogeneity returns to equilibrium.

Note that there are two regions in the sample. One of them in the interval $0<x<x_c$, where $\partial^2 Te/\partial x^2<0$, corresponds to the hot region in the semiconductor, while the cold region $x>x_c$ is described by the electron temperature such that $\partial^2 Te/\partial x^2>0$. In the hot region of the sample the electron temperature decreases in time, therefore this implies that the heat flux incoming into a small volume element in the

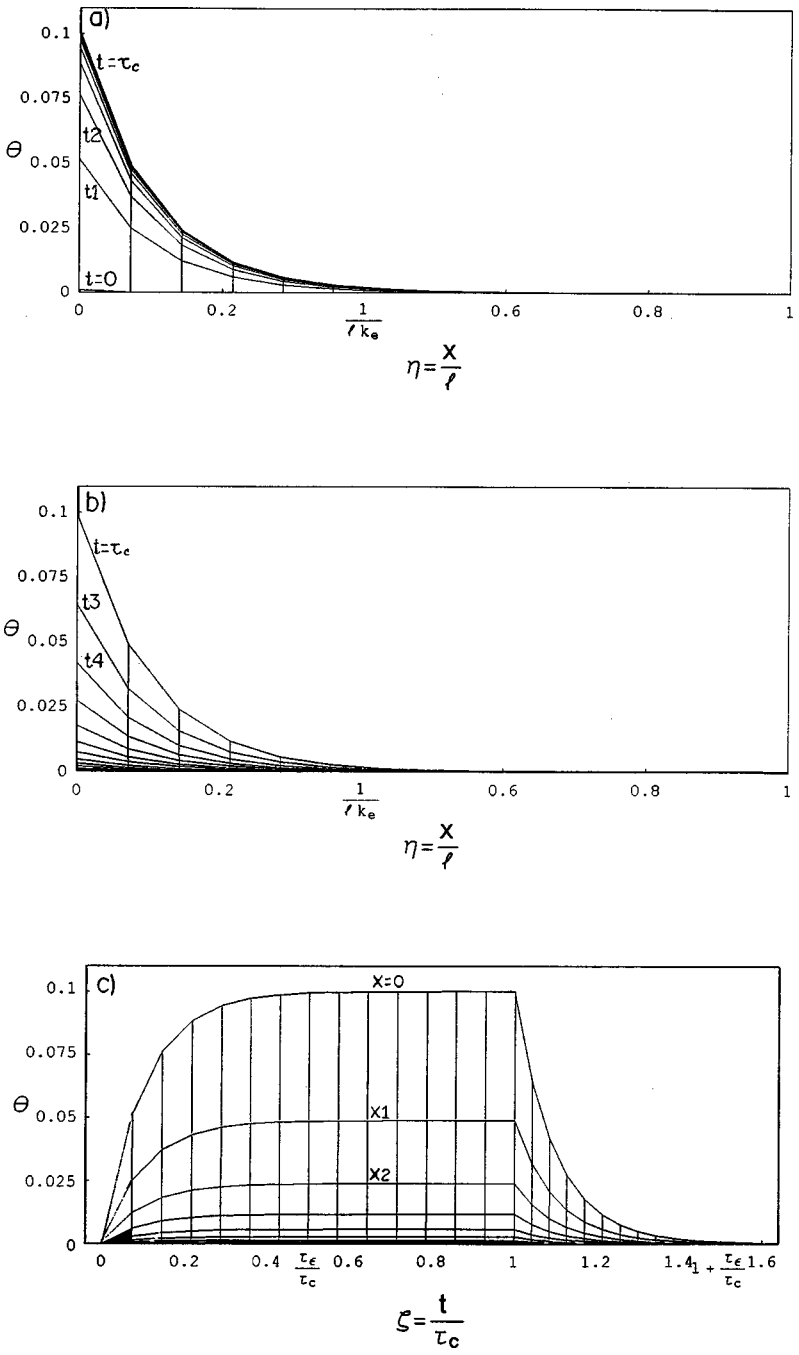

FIG. 2. The same as Fig. 1 but for $k_e l \gg 1$, $\tau_{T_e} = \tau_\epsilon$.

sample is smaller than the outgoing heat energy. However, in the cold region the heat flux decreases with respect to position. The hot carriers close to the surface of the semiconductor ($0 < x < x_c$) correspond to the photoexcited electrons with excess of kinetic energy from the incident radiation pulse. Therefore, this extra energy is transferred by diffusion to the cold region of the semiconductor ($x > x_c$) until the electron energy is fully relaxed at the surface of the sample or to the phonon system. In Fig. 3 we show the calculated results for the electron temperature distribution thin-film approximation using Eqs. (6) ($k_e l \ll 1$) during and after the short laser pulse excitation. In Figs. 3(a) and 3(b) the normalized electron temperature is shown as a function of the position for various values of t before and after the laser pulse has been switched off. In Fig. 3(c) we compare the temporal electron temperature evolution for various fixed values of the position in the semiconductor. As can be seen, all the curves increase during the laser pulse excitation heating the electron system, and subsequently the carriers lose their kinetic energy and the system returns to thermal equilibrium. Note that the cooling of the carrier system occurs after the electron temperature reaches its maximum value at $t = \tau_c$ in a region close to the surface of the sample ($0 \leqslant x \leqslant x_c$). However, the electron temperature peak shifts notably towards greater times ($t > \tau_c$) with increasing distance from the surfaces of the sample ($x_c \leqslant x \leqslant l$). This arises from the fact that after the laser pulse excitation the heat at the surface is transferred to the subsequent volume element in the sample by a diffusion process.

We now return to analyzing a massive semiconductor sample described by the inequality $k_e l \gg 1$. In this case the electron-phonon interaction is more efficient in terms of the

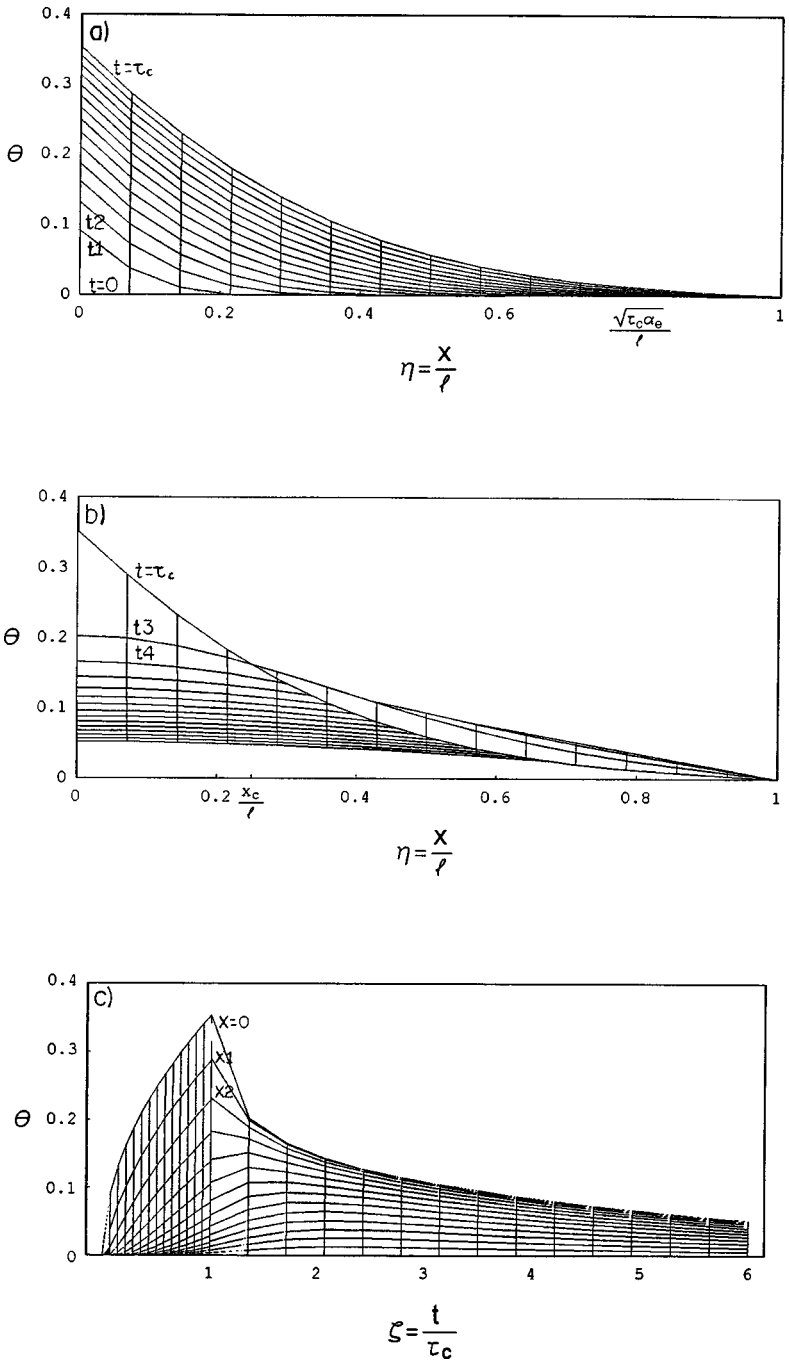

FIG. 3. Time dependence of the normalized electron temperature ($\tau_c \ll \tau_{T_e}$) in the approximation; $k_e l \ll 1$ and $\tau_{T_e} = l^2/\alpha_e$, for (a) $0 < t_1 < t_2 < \cdots \tau_c$, (b) $\tau_c < t_3 < t_4 < \cdots$, (c) position dependence of θ as a function of position of time $0 < x_1 < x_2 < \cdots < l$.

electron energy relaxation. In this approximation the characteristic relaxation time corresponds to the electron-phonon time relaxation $\tau_{T_e} = \tau_\epsilon = (\alpha_e k_e^2)^{-1}$. It is clear from the above discussion that the carriers lose their energy within a small region in the semiconductor of thickness k_e^{-1}, and the rest of the sample, i.e., $x > k_e^{-1}$, remains in equilibrium with the lattice.

Similar behavior of the diffusion heat in the semiconductor for $\tau_c \ll \tau_\epsilon$ can be observed as compared for the thin-film semiconductor approximation, as shown in Fig. 3 (for $\tau_c \ll \tau_\epsilon$), but the amplitude of the electron temperature is smaller than $T_e(x,t)$ in the limit of thin films (see Figs. 1 and 2) and the electron temperature has an exponential dependence on position.

V. CONCLUSIONS

A theoretical analysis of heat diffusion carried out by electrons in semiconductors has been performed. Solving the heat-diffusion equations with appropriate boundary conditions, we obtain the electron and phonon temperature distribution in the sample, which can be experimentally measured using the thermoelectrical effect and the photoacoustic technique, respectively. For typical parameters of the electrons and phonons in semiconductors ($k_e \gg k_p$ and $\alpha_e \gg \alpha_p$) it is possible to obtain the spatial and temporal behavior of the electron temperature. It is shown that the electron temperature is ultimately governed by the thickness of the semiconductor and the electron cooling length. In our model, the effect of the laser pulse excitation time τ_c on the electron

heat diffusion in semiconductors has been pointed out according to whether τ_c is greater or shorter than the electron energy relaxation time τ_{T_e}. If $\tau_c \gg \tau_{T_e}$, the electron heat diffusion in the sample is quasistatic while for $\tau_c \ll \tau_{T_e}$ the electron temperature represents a typical transient heat-diffusion process.

For a nondegenerate semiconductor, the electron energy relaxation time depends strongly upon the relationship between the electron cooling length and the size of the sample. In the case of thin-film semiconductors, when the electron cooling length k_e^{-1} is greater than the thickness of the sample, i.e., $k_e l \ll 1$, the electron energy relaxation time is given as $\tau_{T_e} = l^2/\alpha_e$, which means that the electron energy received from the laser pulse is transferred to the reservoir across the surface of the semiconductor at $x = l$. On the other hand, for massive semiconductors $k_e l \gg 1$ the energy relaxation time satisfies $\tau_{T_e} = (\alpha_e k_e^2)^{-1}$, and in this case the excess of the electron energy is redistributed to the phonon system.

The heat electron transient transport has also been studied for various values of laser pulse duration τ_c. Specifically, for a short laser pulse as compared with the characteristic electron energy relaxation time ($\tau_c < \tau_{T_e}$), the transient electron temperature, after the laser pulse has been disconnected, shows a different decaying curve if $0 < x < x_c$, where $\partial^2 T_e/\partial x^2 < 0$, or is greater than x_c, where $\partial^2 T_e/\partial x^2 > 0$ and $x_c \simeq \sqrt{\alpha_e \tau_c}$. The origin of the transient behavior observed in Fig. 3 for the electron temperature comes from the hot electrons, photoexcited by the laser pulse close to the surface of the semiconductor in the range $0 < x < x_c$; as a consequence, this extra energy diffuses to the cold region of the semiconductor ($l > x > x_c$) after the laser pulse has been extinguished.

We shall conclude this section by mentioning the following: in contrast to the steady-state photothermal experiments, the transient heat transport in semiconductors provides additional information about the electron thermal parameters and the electron-phonon energy interaction, which can be determined by means of the transient thermoelectric experiments.

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*Permanent address: Institute of Thermoelectricity National Academy of Sciences of the Ukraine, Chernivtsi 274000, Ukraine.

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