Strong tunneling and Coulomb blockade in a single-electron transistor

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We have developed a detailed experimental study of a single-electron transistor in a strong tunneling regime. Although weakened by strong charge fluctuations, Coulomb effects were found to persist in all samples including one with the effective conductance eight times higher than the quantum value $(6.45\ \mathrm{k}\Omega)^{-1}$. A good agreement between our experimental data and theoretical results for the strong tunneling limit is found. A reliable operation of transistors with conductances 3–4 times larger than the quantum value is demonstrated. [S0163-1829(99)04207-1]

Mesoscopic tunnel junctions between metals represent a nontrivial example of a macroscopic quantum system with discrete charge states and dissipation.[1,2] Charging effects in such systems can be conveniently studied in the so-called SET (single electron tunneling) transistors. A typical SET transistor consists of a central metallic island connected to the external leads via two tunnel junctions with resistances $R_{L,R}$ and capacitances $C_{L,R}$ (see Fig. 1). In addition to the transport voltage V, the gate voltage V_g can also be applied to the metallic island via the gate capacitance C_g.

Provided the junction resistances are large $R_{L,R} \gg R_q = \pi\hbar/2e^2 \simeq 6.45\ \mathrm{k}\Omega$ tunneling effects are weak and can be treated perturbatively.[3] For a quantitative measure of the tunneling strength we define the parameter $\alpha_t = R_q/R_0$, where $1/R_0 = 1/R_L + 1/R_R$. After each electron tunneling event the charge of a central island changes exactly by e and the energy difference between initial and final states of a SET device is typically of order $E_C = e^2/2C$, where $C = C_L + C_R + C_g$. As long as tunneling is weak $\alpha_t \ll 1$ dissipative broadening of different charge states Γ is small (at low T it is roughly $\Gamma \sim \alpha_t E_C$) and these states are well resolved in energy. This ensures nearly perfect quantization of the charge on a central island in units of e. As a result at sufficiently low $T \lesssim E_C$ Coulomb effects dominate the behavior of a SET transistor leading to a number of observable effects, such as Coulomb blockade of tunneling, modulation of the current through a SET transistor by a gate voltage V_g, Coulomb staircase on the I-V curve, etc.[1]

The situation changes if the effective resistance R_0 becomes of order of R_q or smaller, i.e., $\alpha_t \gtrsim 1$. In this case dissipation is large and the excited charge states of the system become broadened and overlap. Do strong charge fluctuations lead to a complete smearing of Coulomb effects in highly conducting mesoscopic tunnel junctions, like, e.g., in the case of Ohmic shunts?

This problem was analyzed both analytically with the aid of various nonperturbative approaches[4–8] and numerically by means of Monte Carlo simulations.[5,7,9–11] The results of these studies, although remain somewhat controversial in details, clearly demonstrate the existence of a nonvanishing Coulomb gap $E_C^* \propto E_C \exp(-2\alpha_t)$ in the spectrum of the system even for large $\alpha_t \gg 1$. Thus, strong tunneling *does not* destroy Coulomb effects, it only leads to effective renormalization of the junction capacitance. The temperature interval relevant for charging effects shrinks, but they still remain observable even at $T \gg E_C^*$.

This behavior is *qualitatively* different from that of an Ohmic resistor. The physical reason for this difference is also quite clear. It is due to different symmetries of the allowed charge states: the symmetry is continuous in the case of an Ohmic shunt, whereas only discrete e-periodic charge states are allowed in the case of a normal tunnel junction (see, e.g., Ref. 2). The latter symmetry remains the same for any SET strength, and therefore at low-T Coulomb effects survive and can be well observed even in highly conducting junctions.

In spite of all these theoretical developments, an experimental investigation of this problem was lacking. Recently Joyez *et al.*[13] carried out an experiment aimed to study

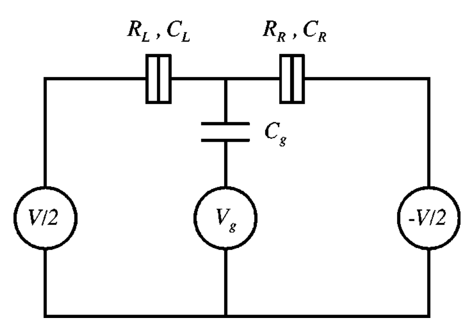

FIG. 1. Schematics of a SET transistor.

strong tunneling effects in SET transistors with higher α_t. Deviations from the standard ''orthodox'' theory[1] were detected in four different samples.[13] The data[13] for three of these samples with $\alpha_t \lesssim 0.6$ can be well explained within the perturbation theory in α_t if one retains the ''cotunneling'' terms $\propto \alpha_t^2$.[12,14] The fourth sample[13] with $\alpha_t \simeq 1.8$ is in the intermediate regime, which is quite difficult to describe theoretically, except in the limit of sufficiently high temperatures.

The main goal of the present paper is to develop a detailed experimental study of Coulomb effects in mesoscopic tunnel junctions in a *nonperturbative* strong tunneling regime, in which case discrete charge states are essentially smeared due to dissipation. For this purpose we have carried out measurements of the current-voltage characteristics of five SET transistors with low junction resistances $R_{L,R} \lesssim R_q$. The results are compared with the existing theory.[12,15] Beside its fundamental importance the problem might be also of interest in view of possible applications of SET transistors as electrometers.

EXPERIMENT

We have fabricated several SET transistors with different values of the junction resistance. The transistors were made using a standard electron-beam lithography with two-layer resist and two-angle shadow evaporation of aluminum. Five transistors with junction resistances in the range from 2 to 20 kΩ were studied. The corresponding values of α_t varied between 1.5 and 8.3. The crossection area of the tunnel junctions is estimated to be $\sim 0.01\,\mu\text{m}^2$.

Measurements were done in a dilution refrigerator capable to hold temperature from 20 mK to 1.2 K. Magnetic field of 2 T was applied to keep aluminum in the normal state. Thermocoax cables and a copper-powder filter next to the mixing chamber of the refrigerator provided necessary filtering against high-frequency noise penetration.

We have measured the I-V characteristics of the transistors at different temperatures and gate voltages. At low voltages our measurements were performed in the current-bias mode using a lock-in amplifier with 6 Hz reference signal frequency. Such a low frequency was chosen to avoid the influence of the low-pass filter with the large RC time. A computer program feedback was used to control the amplitude of the current excitation and to keep the voltage response at the same level. Because of the low frequency of modulation we could not completely suppress the slow fluctuations in the system, which can be seen on some curves.

THEORY

At not very low temperatures (or voltages) the quantum dynamics of a tunnel junction is well described by the quasiclassical Langevin equation[16,17]

$$C_j \frac{\hbar \ddot{\varphi}_j}{e} + \frac{1}{R_j}\frac{\hbar \dot{\varphi}_j}{e} = J_j + \xi_{j1}\cos\varphi_j + \xi_{j2}\sin\varphi_j, \qquad (1)$$

$j=L,R$; J_j is the current flowing through the junction; the phase variable φ_j is related to the voltage across the junction as $\hbar\dot{\varphi}_j/e = V_j$. Discrete electron tunneling is responsible for the noise terms in Eq. (1) described by the stochastic Gaussian variables $\xi_{jk}(t)$ with correlators

$$\langle \xi_{jk}(0)\xi_{j'k'}(t)\rangle = \delta_{jj'}\delta_{kk'}\frac{\hbar}{R_j}\int \frac{d\omega}{2\pi} e^{i\omega t}\omega \coth\left(\frac{\hbar\omega}{2T}\right). \qquad (2)$$

Equation (1) is supplemented by the appropriate current balance and Kirghoff equations and can be solved perturbatively in the noise terms (see Refs. 12 and 15 for details). One arrives at the I-V curve for a SET transistor:[12]

$$I(V) = \frac{V}{R_\Sigma} - I_0(V) - \frac{V}{R_\Sigma} A e^{-F}\cos\left[\frac{2\pi Q_{\text{av}}(V)}{e}\right], \qquad (3)$$

where $Q_{\text{av}}(V) = C_g V_g + [(R_L C_L - R_R C_R)/(R_L + R_R)]V$ is the average charge of the island and $R_\Sigma = R_L + R_R$. The last two terms in Eq. (3) describe deviations from the Ohmic behavior due to charging effects. In the case $R_L = R_R$ for the V_g-independent term $I_0(V)$ we find[12]

$$I_0(V) = \frac{V}{8R_q}[\text{Re}\Psi(b) - \text{Re}\Psi(a)] - \frac{E_C}{\pi e R_0}\text{Im}\,\Psi(b), \qquad (4)$$

$\Psi(x)$ is the digamma function, $a = 1 - ieV/4\pi T$ and $b = a + 2\alpha_t E_C/\pi^2 T$. The expression (4) holds for[12]

$$\max\{eV,T\} \gg w_0 = \begin{cases} \dfrac{2\alpha_t}{\pi^2} e^{-2\alpha_t + \gamma} E_C, & \alpha_t \gtrsim 1 \\ E_C, & \alpha_t \lesssim 1 \end{cases}. \qquad (5)$$

$\gamma = 0.577...$ is the Euler constant. The last term in Eq. (3) describes the modulation of the I-V curve by the gate voltage. Provided the island charge fluctuations are large the amplitude of the modulation is exponentially suppressed:[12,15] $F(T,V) \gg 1$. The general expression for the function $F(T,V)$[12] is quite complicated and is not presented here. In the limit of small voltages and for $T \ll \alpha_t E_C$ this expression becomes simpler, and we get

$$F(T,0) \simeq (T/T_0)^2, T_0 = \sqrt{12\alpha_t} E_C/\pi^2. \qquad (6)$$

As the condition (5) should be simultaneously satisfied Eq. (6) makes sense only for $\alpha_t \gtrsim 1$. The constant A in Eq. (3) has the form[12,15] $A = f(\alpha_t)e^{-2\alpha_t}$. The prefactor $f(\alpha_t)$ was also estimated in[12,15] with a limited accuracy. More accurate results for $f(\alpha_t)$ at low T can be derived by means of other techniques.[4]

RESULTS AND DISCUSSION

In what follows we will disregard a small asymmetry in the parameters of L and R junctions and put $R_0 = R_\Sigma/4$. The value of R_Σ was measured from the slope of the I-V curve at high voltages. The accuracy of these measurements was limited by nonlinearities on the I-V curves due to suppression of tunnel barriers and heating and is estimated as $\sim 3\%$. The charging energy E_C is usually determined from the offset on the I-V curve at large V. In the strong tunneling regime $\alpha_t \gtrsim 1$ a clear offset can be reached only at very high voltages where precise measurements are difficult due to other reasons. Therefore, the above method gives only a rough estimate of E_C with the accuracy within a factor 2.

TABLE I. Parameters of the samples.

Sample	R_Σ (kΩ)	α_t	E_c (K)	$10w_0$ (μeV)	$10w_0$ (mK)
I	17.4	1.48	2.25	50	600
II	12	2.15	1.1	10	100
III	10.4	2.48	1.04	5	60
IV	6.5	3.97	1.16	0.5	6
V	3.1	8.32	~0.3	5×10^{-4}	5×10^{-3}

Alternatively, one can try to determine E_C from the high temperature expansion ($T\gg E_C$) of a zero-bias conductance: $G(T)=(1/R_\Sigma)(1-E_C/3T+\cdots)$. For our samples with high α_t this asymptotics works well only at $T\gtrsim 10$ K. It is easy to observe that at this temperature the value of the term $E_C/3T$ is still very small (~0.03) and could not provide a good accuracy in determination of E_C. On top of that, we did not have means to maintain a sufficient accuracy of temperature measurements above 4.2 K in the same cooling cycle of a dilution refrigerator. Therefore, determine the parameter E_C from the best fit of the low-temperature ($\leqslant 1$ K) and low-voltage ($\leqslant 700\mu$ eV) parts of the I-V curves averaged over the gate charge. Fitting of dI/dV for different temperatures allows to determine E_C with a sufficiently high accuracy and avoid problems discussed above. Another advantage of this method is that we could fit all the curves for different temperatures with the same parameters E_C and α_t (otherwise one needs two fitting parameters for *each* such curve). Also the precise values of α_t were verified by means of this method (see Table I). The last two columns in Table I show the lowest voltage and temperature $\sim 10w_0$ where the theory is still applicable.

The data for a temperature dependent zero bias conductance averaged over the gate charge are given in Fig. 2 for four samples (filled symbols) together with the theoretical dependencies $G_{\rm av}^{\rm theor}=1/R_\Sigma-I_0/V|_{V=0}$ (solid curves). The

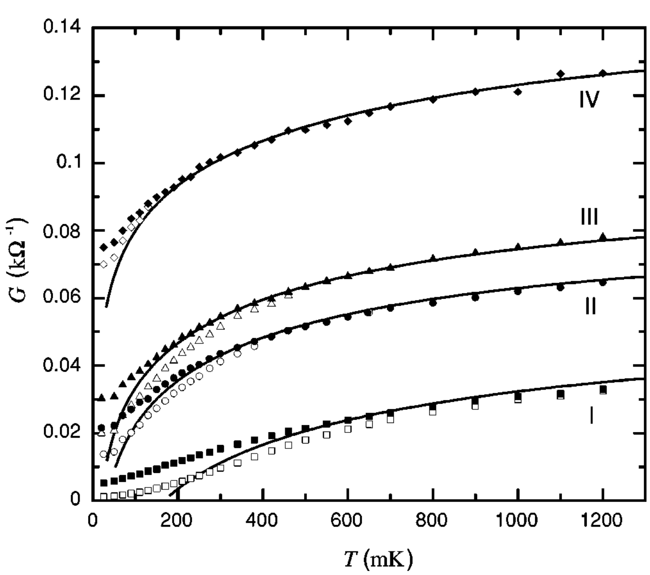

FIG. 2. The averaged over V_g (filled symbols) and the minimum (open symbols) values for the linear conductance of the samples I–IV together with a theoretical prediction for the averaged conductance (solid curves).

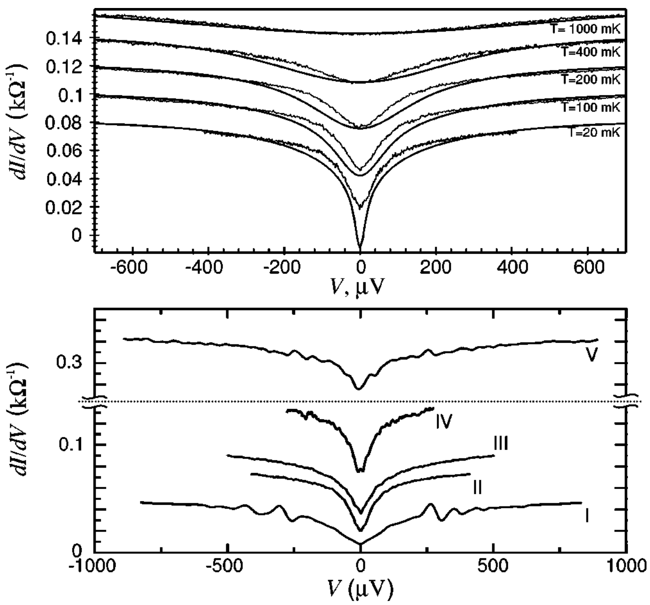

FIG. 3. The V_g-averaged differential conductance. Top panel: the data for the sample II (thinner curves) together with theoretical results (thicker curves). For the sake of clarity the curves at higher T are shifted vertically with a step 0.02 kΩ^{-1}. Bottom panel: the data for all five samples at $T\simeq 20$ mK.

Coulomb blockade-induced suppression of the conductance at low T is clearly seen even for a highly conducting sample IV. We observe a good agreement between theory and experiment except at the lowest temperatures where the theoretical results become unreliable. In this temperature interval the theoretical curves turn out to be closer to the minimum conductance, which is also shown by open symbols. At low T the system conductance shows a tendency to saturation. This is compatible with the corresponding conjecture made in Ref. 12. On the other hand, heating effects as an additional reason for this saturation cannot be excluded either.

The averaged differential conductance dI/dV is shown in the bottom panel of Fig. 3 for all five samples at the lowest temperature $T\approx 20$ mK. It is remarkable that even for the sample V with $\alpha_t>8$ a decrease of the differential conductance at small V due to charging effects is well pronounced. The values dI/dV measured for the sample II for different temperatures are presented in the top panel together with a theoretical prediction from Eq. (4). A similarly good agreement was also found for the samples III, IV, and V.

The gate modulation of the current was always found to be of a cosine form (3) except for the samples with $\alpha_t\lesssim 2$ at the lowest $T\lesssim 50$ mK. The results for the gate modulated linear conductance of the sample II are shown in Fig. 4(a). The amplitude of modulation increases with decreasing temperature in a qualitative agreement with theory. At low T, the modulation effect is considerable even for the sample IV with $\alpha_t\simeq 4$. At the lowest temperatures this effect is visible also for the sample V, but the modulation was only slightly above the noise level.

The data for the temperature dependent amplitude of conductance modulation for the samples I–IV are presented in Fig. 4(b). Solid curves correspond to the best fit with a theoretical dependence $\propto\exp(-T^2/T_0^2)$ [cf. Eqs. (3) and (6)]. Note, that for all samples the best fit value T_0 was found to

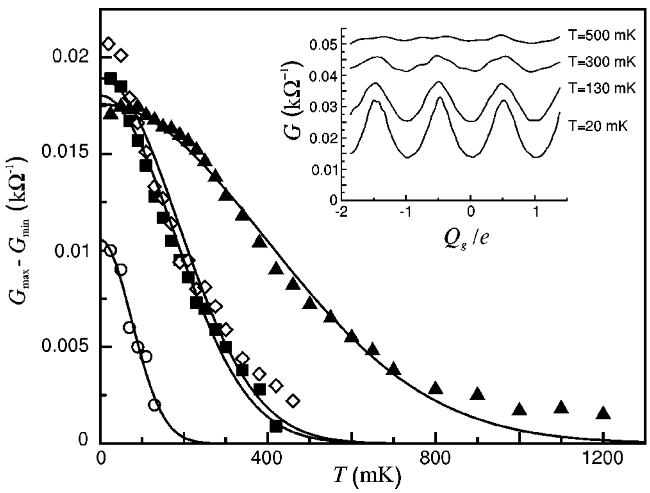

FIG. 4. The gate modulation amplitude $G_{max}-G_{min}$ of a linear conductance as a function of T for the samples I–IV (triangles, squares, diamonds and circles, respectively). Solid lines provide the best fit with a formula $A\exp[-(T/T_0)^2]$. The inset: the linear conductance of the sample II as a function of $Q_g=C_gV_g$ measured at different T.

be by a numerical factor $\sim 2-3$ smaller than the value (6). In other words, the temperature suppression of the gate modulation is *bigger* than it is predicted theoretically. We speculate that this may be due to an additional effect of noise. Another possible reason for such a discrepancy is an insufficient accuracy of the theoretical calculations of the gate modulated conductance.

Our experimental results clearly show that—in accordance with various theoretical predictions—Coulomb blockade is *not* destroyed even in the strong tunneling regime: clear signs of Coulomb suppression of the transistor conductance were observed for α_t as large as 8.3. For all α_t the characteristic energy scale for charging effects is set by the renormalized Coulomb gap E_C^*, however such effects remain well pronounced even at $T \gg E_C^*$. The I-V curves measured for all five SET transistors are in a quantitative agreement with the strong tunneling theory,[12,15] except at very low temperatures where this theory is not applicable. Along with the overall suppression of the conductance its modulation by the gate voltage was also observed at sufficiently low T. The modulation effect increases with decreasing temperature in a qualitative agreement with theoretical predictions.

Our paper was aimed to study low resistance SET transistors in the nonperturbative strong tunneling regime. Theoretically, a perturbative regime characterized by an expansion parameter $g=\alpha_t/\pi^2 \ll 1$ (see, e.g., Refs. 12 and 14) can be easily distinguished from a strong tunneling one with $\exp(-2\alpha_t)\ll 1$ (cf., e.g., Refs. 4 and 12) except within the interval $0.5 \lesssim \alpha_t \lesssim 3$ where both inequalities are (roughly) satisfied. Combining our results with those of Ref. 13 as well as with the corresponding theoretical predictions[12,14,15] we can draw a somewhat more definitive conclusion about the validity range for both regimes: tunneling can be treated perturbatively for $\alpha_t \lesssim 1-2$ while the nonperturbative tunneling regime sets in for $\alpha_t \gtrsim 2-3$.

In summary, we have operated SET transistors with effective resistances R_0 *several times smaller* than 6.5 kΩ. The experimental I-V curves averaged over the gate modulation can be well fitted by the strong tunneling theory, while the amplitude of the gate modulation is only in qualitative agreement with it.

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