Effective hypernetted-chain study of even-denominator-filling state of the fractional quantum Hall effect

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The microscopic approach for studying the half-filled state of the fractional quantum Hall effect is based on the idea of proposing a trial Fermi wave function of the Jastrow-Slater form, which is then fully projected onto the lowest Landau level. A simplified starting point is to drop the projection operator and to consider an unprojected wave function. A recent study claims that such a wave function approximated in a Jastrow form may still constitute a good starting point on the study of the half-filled state. In this paper we formalize the effective hypernetted-chain approximation and apply it to the unprojected Fermi wave function, which describes the even-denominator-filling states. We test the above approximation by using the Fermi hypernetted-chain theory, which constitutes the natural choice for the present case. Our results suggest that the approximation of the Slater determinant of plane waves as a Jastrow wave function may not be a very accurate approximation. We conclude that the lowest Landau-level projection operator cannot be neglected if one wants a better quantitative understanding of the phenomena. [S0163-1829(99)01416-2]

I. INTRODUCTION

The fractional quantum Hall effect[1] (FQHE) results from a strongly correlated incompressible liquid state[2] formed at special uniform densities $\rho_e(\nu)$ of a two-dimensional (2D) electronic system, subject to a strong-transverse magnetic field **B**. For a fully spin-polarized (spinless) system of electrons with the spin degeneracy of each quantum state $g_s=1$, the dominant sequence of fractional Hall states occurs for fillings of the lowest Landau level (LLL) $\nu=p/(q_e p+1)$, where $q_e=2,4\ldots$ is an even integer and $p=1,2,\ldots$ is an integer.

Much of the theoretical work on the FQHE is based on the study of the properties of a 2D fully spin-polarized (spinless) system of N interacting electrons embedded in a uniform positive background, with the magnetic field **B** high and temperature T low, such that only the LLL is partially filled. At $T=0$, the interaction energies $\sim \nu^{1/2}(1/4\pi\epsilon_0)(e^2/\epsilon l_0)$, where $l_0=\sqrt{\hbar/eB}$ is the magnetic length and ϵ is the dielectric constant of the background, are assumed to be weak compared with the Landau-level splitting $\hbar\omega_c$ and so all electrons are considered to remain in the LLL.

Electrons with charge $-e$ $(e>0)$ are considered to be confined in a 2D plane, subjected to a perpendicular magnetic field $\mathbf{B}=\nabla\times\mathbf{A}(\mathbf{r})$, where $\mathbf{A}(\mathbf{r})$ is the symmetric gauge vector potential $\mathbf{A}(\mathbf{r})=(-By/2,Bx/2,0)$. The many-electron system is described by the Hamiltonian $\hat{H}=\hat{K}+\hat{V}$ where $\hat{K}$ is the kinetic energy operator

$$\hat{K}=\frac{1}{2m_e}\sum_{j=1}^{N}[-i\hbar\nabla_j+e\mathbf{A}(\mathbf{r}_j)]^2, \tag{1}$$

and

$$\hat{V}=\sum_{j<k}^{N} v(|\mathbf{r}_j-\mathbf{r}_k|)-\rho_e(\nu)\sum_{j=1}^{N}\int d^2r\ \ v(|\mathbf{r}_j-\mathbf{r}|)+\frac{\rho_e(\nu)^2}{2}\int d^2r_1\int d^2r_2\ \ v(|\mathbf{r}_1-\mathbf{r}_2|). \tag{2}$$

Here m_e is the bare mass of the electrons, $z_j=x_j+iy_j$ is the location of the jth electron in complex coordinates, $v(|\mathbf{r}_j-\mathbf{r}_k|)=(1/4\pi\epsilon_0)e^2/\epsilon|z_j-z_k|$ is the Coulomb-interaction potential among charges, and $\hat{V}$ contains the electron-electron, electron-background, and background-background interaction potential.

From a theoretical point of view, the occurrence of Hall plateaus at filling factors with odd denominators can be understood, to a great extent, through the original ideas of Laughlin[3] and the composite fermion (CF) theory of Jain.[4]

In contrast to the odd denominator fillings, the nature of the groundstate at even denominators is still an intriguing problem and only recently has a theory of a compressible Fermi-liquid-like behavior at $\nu=1/q_e$ been proposed by Halperin, Lee, and Read (HLR).[5]

Based on their theory, a 2D system of electrons subjected to an external perpendicular magnetic field **B**, with the LLL filling factor $\nu=1/q_e$, is transformed to a mathematically equivalent system of fermions interacting with a Chern-Simons gauge field such that the average effective magnetic field acting on the fermions is zero. The transformed Hamiltonian of the system is, therefore, $\hat{H}'=\hat{K}'+\hat{V}$ where

$$\hat{K}'=\frac{1}{2m_e}\sum_{j=1}^{N}\{-i\hbar\nabla_j+e[\mathbf{A}(\mathbf{r}_j)-\mathbf{a}(\mathbf{r}_j)]\}^2. \tag{3}$$

The ''Chern-Simons'' vector potential $\mathbf{a}(\mathbf{r})$ generates a ''Chern-Simons'' magnetic field

$$\mathbf{b}(\mathbf{r})=\nabla\times\mathbf{a}(\mathbf{r})=q_e\phi_0\sum_{j=1}^{N}\delta(\mathbf{r}-\mathbf{r}_j)=\rho(\mathbf{r})q_e\phi_0, \quad (4)$$

where $\rho(\mathbf{r})$ is the local particle density and ϕ_0 is the magnetic-field flux quantum.

Assuming a uniform density $\rho(\mathbf{r})=\rho_e(\nu)$, the electrons at special filling factors $\nu=(\phi_0/B)\rho_e(\nu)=1/q_e$ where $q_e=2,4\ldots$ feel no net magnetic field and in principle they can form a 2D gapless Fermi liquid, which is a compressible state.[6] Subsequent experimental observation[7] of the geometric resonance of the charge carriers near $\nu=1/2$ with ultrasound waves moving perpendicular to the applied magnetic field indicated the existence of a Fermi surface at half-filling.

Parallel to the HLR approach, there is an ongoing effort to develop a microscopic approach to $\nu=1/q_e$ by explicitly writing a trial Fermi wave function as a starting point. Such a trial wave function is written[8,9] as the product of the Laughlin state for bosons at $\nu=1/q_e$ with the Slater determinant of free fermions at the corresponding density

$$\Psi^{Fermi}_{\nu=1/q_e}=\hat{P}_{LLL}[\Psi^{Bose}_{\nu=1/q_e}\mathrm{Det}\{\varphi_{\mathbf{k}}(\mathbf{r})\}], \quad (5)$$

where the Laughlin wave function for bosons at $\nu=1/q_e$ is written as

$$\Psi^{Bose}_{\nu=1/q_e}=\prod_{j<k}^{N}(z_j-z_k)^{q_e}\prod_{j=1}^{N}\exp(-|z_j|^2/4l_0^2). \quad (6)$$

The operator $\hat{P}_{LLL}$ is the LLL projection operator and $\varphi_{\mathbf{k}}(\mathbf{r})$ are normalized 2D plane waves, which fill the Fermi disk up to $k_F(\nu)$. The resulting theory is essentially equivalent[8] to the HLR theory, though in the HLR approach a transformation that involved attaching delta-function fluxes to the electrons was employed. The wave function of Eq. (5) has been tested numerically[9] and it has been found that the theoretical picture gives a good description of small size systems at $\nu=1/2$.

Up to now, we are not aware of many-body schemes that include the projection operator that can be applied to the study of states described by $\Psi^{Fermi}_{\nu=1/q_e}$ at the thermodynamic limit. A simplified starting point is to drop the projection operator from Eq. (5) and to consider a simpler unprojected Fermi wave function. In a recent paper, Chakraborty[10] presented a many-body approach to calculate the ground-state properties of the half-filled state by using the above unprojected Fermi wave function. In his approach the squared Slater determinant of the unprojected wave function was approximated by a Jastrow wave function with only two-body correlations on it. If we do not consider the projection operator, the wave function of Eq. (5) is of the Jastrow-Slater form, which can be directly treated by the Fermi hypernetted-chain (FHNC) theory.[11] In the FHNC theory there are no approximations of the wave function, but since this theory is more complicated than its Bose counterpart, the idea to write the Slater determinant as a Jastrow wave function looks quite attractive because it simplifies the calculation of many quantities.

In this paper we studied the even-denominator-filling states ($\nu=1/2$ and 1/4) of the FQHE by adopting the effective hypernetted-chain (EFHNC) technique and the FHNC theory. These methods allow us to compute physical quantities on the thermodynamic limit, without having the limitations of the exact calculations with few electrons where the extrapolation to the thermodynamic limit is not totally unambiguous. Both techniques are intrinsically approximated because there is a class of cluster diagrams that cannot be fully included in any closed form in the calculation of the radial distribution function. But differently from the EFHNC method, the FHNC theory does not involve approximations of the many-body wave function, and in this sense it constitutes a strong validity test for the EFHNC technique. The unprojected radial distribution function obtained from the EFHNC approach has a stronger oscillatory behavior than that observed by Chakraborty.[10] The FHNC results support this conclusion and are in better qualitative agreement with other numerical results.[9,12] Within the framework of the EFHNC approach we can prove analytically that the radial distribution function obtained from the unprojected Fermi wave function always has an incorrect short-range behavior due to the LLL missing projection.

Since the application of the EFHNC approach for the unprojected Fermi wave function of Eq. (5) is not straightforward, in Sec. II we present a detailed description of the EFHNC formalism we used. Numerical results are presented and discussed in Sec. III, while Sec. IV is devoted to the conclusions.

II. THE EFFECTIVE HYPERNETTED-CHAIN FORMALISM FOR THE EVEN-DENOMINATOR-FILLING STATE

Integral equation techniques such as the hypernetted-chain (HNC) theory for bosons[13] or the FHNC formalism for fermions[14] permit an accurate evaluation of the radial distribution function and related quantities associated with a Jastrow or a Jastrow-Slater wave function.

In particular they are extremely useful for calculations that are performed in the thermodynamic limit. They have been extensively and successfully applied in studies of quantum fluids such as liquid ^{4}He, ^{3}He, and nuclear matter. Recently, these methods have also been applied to problems in the newly developing areas of condensed matter theory such as the physics related to the FQHE. Because the FHNC theory was developed to treat Jastrow-Slater wave functions of the form of Eq. (5) where $\hat{P}_{LLL}$ is dropped out, applying it to this case is a natural step.

For odd denominator fillings where the Slater determinant consists of single-particle states describing the Landau levels, the FHNC has been applied[11] and proved to be rather involved. A simpler and straightforward approximation of such theory is the EFHNC approach, known also as the Lado approximation.[15]

In this approach one views the square of the Slater determinant as some positive-valued function and writes it in the form

$$|\mathrm{Det}\{\varphi_{\mathbf{k}}(\mathbf{r})\}|^2=\exp\left[\sum_{i<j}^{N}w_2(r_{ij})+\sum_{i<j<k}^{N}w_3(\mathbf{r}_i,\mathbf{r}_j,\mathbf{r}_k)+\cdots\right]. \quad (7)$$

The Pauli principle can be thought of as an operator that introduces many-body correlations between particles in analogy to the (pseudo)potential describing the dynamic correlations. Retaining only the two-body correlations in Eq. (7) we have approximately

$$|\mathrm{Det}\{\varphi_{\mathbf{k}}(\mathbf{r})\}|^2 \approx \exp\left[\sum_{i<j}^{N} w(r_{ij})\right], \tag{8}$$

where $w(r_{ij})$ is the two-body correlation (pseudo)potential.

In a next step of a systematic approximation scheme, one may include triplet correlation factors etc., until the required accuracy is achieved. This approximation greatly simplifies the analysis of the problem and the calculation of the radial distribution function, which is the only quantity needed to compute the interaction energy per particle. One has

$$\begin{aligned}|\Psi^{Fermi}_{\nu=1/q_e}|^2 &\approx \prod_{i<j}^{N} f(r_{ij})^2 \prod_{i<j}^{N} \exp[w(r_{ij})] \prod_{j=1}^{N} \exp(-|z_j|^2/2l_0^2) \\ &= \prod_{i<j}^{N} \tilde{f}(r_{ij})^2 \prod_{j=1}^{N} \exp(-|z_j|^2/2l_0^2),\end{aligned} \tag{9}$$

where the effective correlation factor is given by

$$\tilde{f}(r_{ij}) = f(r_{ij}) \exp\left[\frac{1}{2} w(r_{ij})\right], \tag{10}$$

and

$$f(r_{ij}) = (r_{ij})^{q_e}. \tag{11}$$

By using the Bose HNC theory one can express the radial distribution function

$$g(r_{12}) = \frac{N(N-1)}{\rho_e(\nu)^2} \times \frac{\int d^2r_3 \cdots d^2r_N \ |\Psi^{Fermi}_{\nu=1/q_e}(\mathbf{r}_1 \cdots \mathbf{r}_N)|^2}{\int d^2r_1 \ d^2r_2 \ d^2r_3 \cdots d^2r_N |\Psi^{Fermi}_{\nu=1/q_e}(\mathbf{r}_1 \cdots \mathbf{r}_N)|^2}, \tag{12}$$

as a series of cluster terms, associated with linked diagrams. The difference with respect to the case of the standard Jastrow wave function, in which the single-particle term is not present, is that the diagrams are not irreducible and each vertex brings the *uncorrelated* one-body density $\rho_0(\mathbf{r})$ as a vertex correction. It has been proved[16] that such a series can be recast into a series of irreducible diagrams with the full one-body density $\rho(\mathbf{r})$ being the vertex correction. Since the full density is a constant, then the HNC equations for the radial distribution function are exactly the same as for a Jastrow wave function without the single-particle term and at density $\rho_e(\nu) = \nu/(2\pi l_0^2)$.

To construct the potential $w(r_{12})$ for a given determinant we require that the HNC evaluation of $g(r_{12})$ recovers the *exact* radial distribution function of the noninteracting system $g_{ideal}(r_{12}) = 1 - (1/g_s)|l_{ideal}(r_{12})|^2$ where the exact statistical-exchange factor for the 2D ideal Fermi gas is defined as $l_{ideal}(\mathbf{r}_1, \mathbf{r}_2) = \hat{\rho}(\mathbf{r}_1, \mathbf{r}_2)/\rho_e(\nu)$. The (reduced) single-particle density matrix for the Slater determinant is given by

$$\hat{\rho}(\mathbf{r}_1, \mathbf{r}_2) = g_s \sum_{|\mathbf{k}| \leq k_F(\nu)} n(\mathbf{k}) \varphi^*_{\mathbf{k}}(\mathbf{r}_1) \varphi_{\mathbf{k}}(\mathbf{r}_2) \tag{13}$$

where the ground-state occupation number for a fully spin-polarized (spinless) 2D ideal Fermi gas ($g_s = 1$) is

$$n(\mathbf{k}) = \begin{cases} 1, & |\mathbf{k}| \leq k_F(\nu) \\ 0, & |\mathbf{k}| > k_F(\nu). \end{cases} \tag{14}$$

The normalized single-particle states of a 2D ideal Fermi gas of electrons occupying an area A are plane waves $\varphi_{\mathbf{k}}(\mathbf{r}) = (1/\sqrt{A}) e^{i\mathbf{k}\mathbf{r}}$. The Fermi radius that corresponds to the density $\rho_e(\nu)$ is $k_F(\nu) = (1/l_0)\sqrt{2\nu/g_s}$ and a trivial calculation gives

$$l_{ideal}(\mathbf{r}_1, \mathbf{r}_2) = 2 \ \frac{J_1[k_F(\nu) r_{12}]}{k_F(\nu) r_{12}}, \tag{15}$$

where $r_{12} = |\mathbf{r}_2 - \mathbf{r}_1|$ and $J_1(x)$ is the first-order Bessel function.

The static structure function associated with the radial distribution function $g_{ideal}(r)$ is defined as

$$S_{ideal}(k) = 1 + \rho_e(\nu)[g_{ideal}(r) - 1]^F, \tag{16}$$

where $[f(r)]^F$ denotes the 2D Fourier transform of a function $f(r)$, while $[f(k)]^{F^{-1}}$ is the inverse 2D Fourier transform. For a spin-polarized (spinless) 2D ideal Fermi gas of electrons we have

$$g_{ideal}(r_{12}) = 1 - 4 \left| \frac{J_1[k_F(\nu) r_{12}]}{k_F(\nu) r_{12}} \right|^2, \tag{17}$$

and the static-structure factor[17] is given by

$$S_{ideal}(k) = \begin{cases} \dfrac{2}{\pi} \left[\arcsin\left[\dfrac{k}{2k_F(\nu)}\right] + \dfrac{k}{2k_F(\nu)} \sqrt{1 - \left[\dfrac{k}{2k_F(\nu)}\right]^2} \right], & k \leq 2k_F(\nu) \\ 1, & k \geq 2k_F(\nu). \end{cases} \tag{18}$$

The above prescription on constructing $w(r_{12})$ leads us to a (pseudo)potential of the form

$$w(r_{12})=\ln[g_{ideal}(r_{12})]-\frac{1}{\rho_e(\nu)}\left[\frac{[S_{ideal}(k)-1]^2}{S_{ideal}(k)}\right]^{F^{-1}}, \tag{19}$$

where the radial distribution function $g_{ideal}(r_{12})$ and the associated structure function $S_{ideal}(k)$ correspond to the dynamically uncorrelated 2D ideal Fermi gas. By substituting Eqs. (17) and (18) in Eq. (19) we obtain an explicit expression for $w(r_{12})$ and consequently, due to Eqs. (9) and (10), we arrive at

$$|\Psi^{Fermi}_{\nu=1/q_e}|^2=\prod_{i<j}^{N}\exp[\tilde{u}(r_{ij})], \tag{20}$$

where $\tilde{u}(r_{ij})=2\ln f(r_{ij})+w(r_{ij})=2q_e\ln(r_{ij})+w(r_{ij})$. The radial distribution function is then found by applying the following Bose EFHNC equations:

$$X(r_{12})=\exp[\tilde{u}(r_{12})+N(r_{12})+E(r_{12})]-N(r_{12})-1, \tag{21}$$

$$N(r_{12})=\rho_e(\nu)\int d^2r_3X(r_{13})[X(r_{32})+N(r_{32})], \tag{22}$$

and

$$g(r_{12})=1+X(r_{12})+N(r_{12}). \tag{23}$$

The generation of diagrams contributing to $g(r_{12})$ must go through a self-consistent procedure. As a first approximation (and a good one) we neglect the contribution of the elementary diagrams $E(r_{12})$ and adopt the so-called EFHNC/0 approximation, where the ''0'' means neglect of elementary diagrams. The summation of the nodal diagrams $N(r_{12})$ is easily performed in the Fourier space resulting in

$$N^F(q)=\rho_e(\nu)\frac{X^F(q)^2}{1-\rho_e(\nu)X^F(q)}. \tag{24}$$

In order to handle the 2D logarithmic (pseudo)potential $\tilde{u}(r_{12})$, the standard procedure is to separate $\tilde{u}(r_{12})$, the nodal function $N(r_{12})$, and the non-nodal (composite) function $X(r_{12})$ in their short-range and long-range parts:

$$\tilde{u}(r_{12})=\tilde{u}_s(r_{12})+\tilde{u}_l(r_{12}), \tag{25}$$

$$N(r_{12})=N_s(r_{12})-\tilde{u}_l(r_{12}), \tag{26}$$

and

$$X(r_{12})=X_s(r_{12})+\tilde{u}_l(r_{12}). \tag{27}$$

The decomposition of $\tilde{u}(r_{12})$ is done as follows:

$$\tilde{u}_s(r_{12})=-2q_eK_0(Qr_{12})+\ln[g_{ideal}(r_{12})], \tag{28}$$

$$\tilde{u}_l(r_{12})=2q_eK_0(Qr_{12})+2q_e\ln(r_{12})-\frac{1}{\rho_e(\nu)}\left\{\frac{[S_{ideal}(k)-1]^2}{S_{ideal}(k)}\right\}^{F^{-1}}, \tag{29}$$

where $K_0(x)$ is the modified Bessel function, Q is the cutoff parameter of order $1/l_0$, and the relation

$$\int d^2r\ e^{i\mathbf{qr}}[\ln(r)+K_0(Qr)]=\frac{-2\pi Q^2}{q^2(q^2+Q^2)} \tag{30}$$

holds. The splitting should be done in such a way that

$$\tilde{u}(r_{12})+N(r_{12})=\tilde{u}_s(r_{12})+N_s(r_{12}), \tag{31}$$

and

$$N(r_{12})+X(r_{12})=N_s(r_{12})+X_s(r_{12}). \tag{32}$$

The final set of equations is solved by initially setting $N_s(r_{12})=0$ in the equation

$$X_s(r_{12})=\exp[\tilde{u}_s(r_{12})+N_s(r_{12})+E(r_{12})]-N_s(r_{12})-1. \tag{33}$$

Then we perform a 2D Fourier transform of $X_s(r_{12})$ to obtain $X^F_s(q)$. Later, we compute $X^F(q)=X^F_s(q)+\tilde{u}^F_l(q)$, so that $N^F(q)$ is easily computed from Eq. (24). Straightforwardly $N^F_s(q)=N^F(q)+\tilde{u}^F_l(q)$ and by applying an inverse 2D Fourier transform on it we obtain the new $N_s(r_{12})$. This procedure goes on until the desired accuracy is reached and the pair-distribution function is obtained from

$$g(r_{12})=1+X_s(r_{12})+N_s(r_{12}). \tag{34}$$

The computation of $g(r_{12})$ allows us to find several other related quantities like the interaction energy per particle and the static structure factor. In the next section we show the results we found for two even-denominator-filling states corresponding to the fractional fillings $\nu=1/q_e$ where $q_e=2$ and 4.

III. RESULTS

In this paper we applied the EFHNC method and the FHNC theory to study the even-denominator-filling state of the FQHE described by the unprojected Fermi wave function $\Psi^{Fermi}_{\nu=1/q_e}$. Since the unprojected Fermi wave function is of the Jastrow-Slater form we believe that the first choice on its study is the application of the FHNC theory, which differently from the EFHNC approach does not rely on any approximation of the wave function. A detailed description of this formalism is given elsewhere,[18] and here we limit ourselves to report the results we have obtained. Since the FHNC theory is much more complicated than its Bose counterpart, the idea to ''bosonize'' the Fermi wave function is certainly very attractive; therefore, we applied the EFHNC approach.

This approach contains an approximation of the originally unprojected Jastrow-Slater wave function since the radial distribution function is computed by approximating the squared Slater determinant of plane waves with a Bose-Jastrow wave function. The utility of this treatment is based

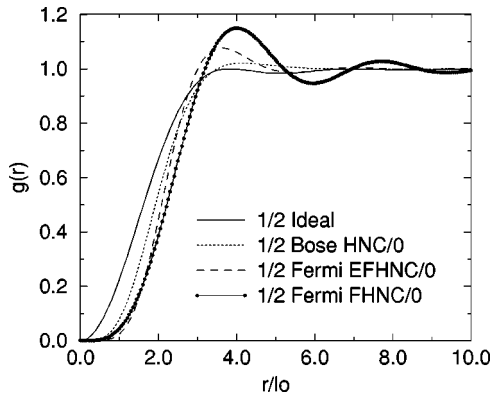

FIG. 1. We plot the radial distribution function $g(r)$ for the half-filling state $\nu=1/2$ as a function of the dimensionless distance r/l_0 for the 2D ideal Fermi gas (solid), the Bose wave function of Eq. (6) (by applying the Bose HNC/0 technique, dotted) and the unprojected Fermi wave function of Eq. (5) from the EFHNC/0 (dashed) and the FHNC/0 (solid circles) method. In all cases the calculations were performed by neglecting the elementary diagrams, and the ''0'' denotes this choice.

on the fact that many related physical quantities can now be easily computed. It has been argued[10] that this approach should be a suitable approximation for the fully projected wave function at $\nu=1/2$ since one may think that the only effect of the $\hat{P}_{LLL}$ operator is to make the wave function very Laughlin-like (but with the correct statistics). This argument looks attractive, but certainly needs reliability testing.

It is known that both the EFHNC and the FHNC techniques are intrinsically approximate because there is a set of cluster diagrams (corresponding to the so-called elementary diagrams) that cannot be fully included in any closed form. Several schemes have been devised to include such cluster diagrams at various levels of approximation. The simplest approximation of totally neglecting these terms leads to reliable results, so in this paper we adopt the so-called EFHNC/0 and FHNC/0 approximation, where ''0'' means neglect of elementary diagrams.

In both methods the interaction-energy per particle is computed from the same formula

$$u_{\nu=1/q_e}=\frac{1}{N}\frac{\langle\Psi^{Fermi}_{\nu=1/q_e}|\hat{V}|\Psi^{Fermi}_{\nu=1/q_e}\rangle}{\langle\Psi^{Fermi}_{\nu=1/q_e}|\Psi^{Fermi}_{\nu=1/q_e}\rangle}$$

$$=\frac{\rho_e(\nu)}{2}\int d^2r\ [g_{\nu=1/q_e}(r)-1]\ v(|\mathbf{r}|), \quad (35)$$

while the ''kinetic-energy'' per particle is expected to be greater than the lowest cyclotron energy

$$\frac{1}{N}\frac{\langle\Psi^{Fermi}_{\nu=1/q_e}|\hat{K}|\Psi^{Fermi}_{\nu=1/q_e}\rangle}{\langle\Psi^{Fermi}_{\nu=1/q_e}|\Psi^{Fermi}_{\nu=1/q_e}\rangle}>\frac{1}{2}\hbar\omega_c, \quad (36)$$

because the unprojected wave function $\Psi^{Fermi}_{\nu=1/q_e}$ does not entirely lie within the LLL. At present our interest is devoted only to the calculation of the interaction-energy per particle and not to the calculation of the ''kinetic-energy'' per particle.

In Fig. 1 we plot the radial distribution function corresponding to the 2D ideal Fermi gas, the Bose wave function $\Psi^{Bose}_{\nu=1/2}$, and the unprojected Fermi $\Psi^{Fermi}_{\nu=1/2}$ wave function, by employing different techniques. As seen in Fig. 1, the unprojected radial distribution function obtained from the FHNC/0 theory has more pronounced peaks than that obtained by the EFHNC/0 approach and both are not in agreement with respective results,[10] where very little oscillatory structure in $g(r)$ has been found. Finite-size calculations for $N=9$ spin-polarized electrons confined to the LLL on a spherical surface[9] confirm the existence of some sort of ''Friedel-like'' oscillations on the behavior of $g(r)$ for the filling $\nu=1/2$. The radial distribution function for the exact $N=9$ ground state has been found indistinguishable from that corresponding to the projected Fermi wave function, suggesting that the Fermi trial wave function is a good variational state. Based on the CF theory one may think of the $\Psi^{Fermi}_{\nu=1/q_e}$ wave function as the limit ($p\to\infty$) of the CF wave function due to Jain,[4]

$$\Psi^{CF}_{\nu=p/(q_ep+1)}(B^*)=\hat{P}_{LLL}[\Psi^{Bose}_{\nu=1/q_e}\Phi_p(B^*)], \quad (37)$$

where $\Phi_p(B^*)$ is the Slater determinant wave function of p-filled CF Landau levels, evaluated at the magnetic field $B^*=B/(q_ep+1)$. Fillings of the form $\nu=p/(2p+1)$ described by both the projected and the unprojected CF wave function have been extensively studied by Kamilla *et al.*[12] using Monte Carlo techniques.

In a range of filling factors in the vicinity of the half-filled state (for instance 6/13) they observed oscillations of the radial distribution function for both the projected and the unprojected[12] cases finding that the LLL projection basically reduces the magnitude of these peaks. At this stage it is natural to expect that the oscillations on the radial distribution function will persist at $\nu=1/2$ as well. The application of the FHNC/0 theory to the unprojected Fermi wave function of Eq. (5) indeed gives support to this picture. Strong oscillatory behavior in $g(r)$ is observed in agreement with our expectations. The priority of these techniques is to treat the unprojected many-body wave function $\Psi^{Fermi}_{\nu=1/q_e}$ exactly at the thermodynamic limit. It would be highly desirable to incorporate the LLL projection within these schemes in order to directly compare the results with other finite-size calculations.[9] Although unprojected, these calculations still contribute to fill the gap between finite-size simulation estimates and results exactly at the thermodynamic limit.

Both EFHNC/0 and FHNC/0 methods do not include the elementary diagrams in the solution of the respective integral equations, while the EFHNC/0 approach still contains a second approximation of the Fermi wave function in a bosonized form. A comparison between the respective radial distribution functions at filling $\nu=1/2$ suggests that ''the bosonization'' of the wave function,[10] which is the fundamental idea behind the EFHNC approach, does not approximate very well $g(r)$ and has non-negligible differences with it.

In Fig. 2 we plot the static-structure factor $S(q)$ for the half-filled state obtained from the 2D ideal Fermi gas, the Bose wave function of Eq. (6), and the unprojected Fermi wave function. One observes that the EFHNC/0 ''bosonized'' structure factor is smoother than the one ob-

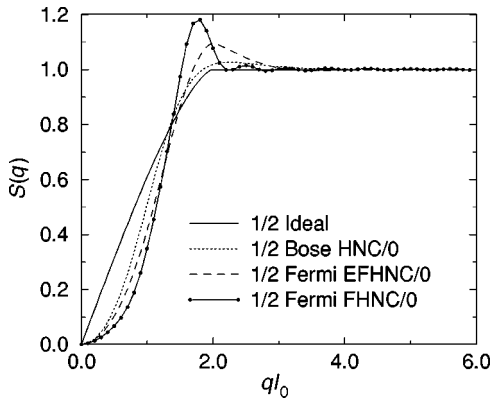

FIG. 2. We plot the static structure factor $S(q)$ for filling $\nu=1/2$ as a function of ql_0 for the 2D ideal Fermi gas (solid), the Bose wave function of Eq. (6) (dotted) and the unprojected Fermi wave function Eq. (5) employing both the EFHNC/0 (dashed) and the FHNC/0 (solid circles) technique.

tained from the FHNC/0 method. In Figs. 3 and 4 we plot the radial distribution function and the static structure factor corresponding to the unprojected Fermi wave function for the fillings $\nu=1/2$ and 1/4 obtained from the EFHNC/0 approach.

The interaction-energy per particle obtained from these wave functions is shown in Table I. The value $u_{\nu=1/2}$ for the unprojected $\Psi^{Fermi}_{\nu=1/2}$ wave function is lower than the value suggested from exact diagonalizations of small systems of up to 12 electrons[19] in the spherical geometry shown in the fourth row of Table I. We believe that the source of this discrepancy is the missing projection of $\Psi^{Fermi}_{\nu=1/q_e}$ into the LLL. As shown in Fig. 5 we found that the unprojected radial distribution function obtained from the EFHNC/0 approach has an erroneous $g_{\nu=1/2}(r\to 0)=(r/l_0)^6$ and $g_{\nu=1/4}(r\to 0)=(r/l_0)^{10}$ behavior for the respective fillings $\nu=1/2$ and 1/4.

Exact numerical diagonalizations,[20] which of course treat the electrons as fully projected, suggest a different dependence, for instance a $(r/l_0)^2$ dependence for the filling 1/2. This arises because in the fully LLL projected Hilbert space of Laughlin's Jastrow-like wave functions, the plane-wave state $\varphi_{\mathbf{k}}(\mathbf{r}_j)=\exp(i\mathbf{k}\cdot\mathbf{r}_j)$ acts as an operator; namely z_j^* acts

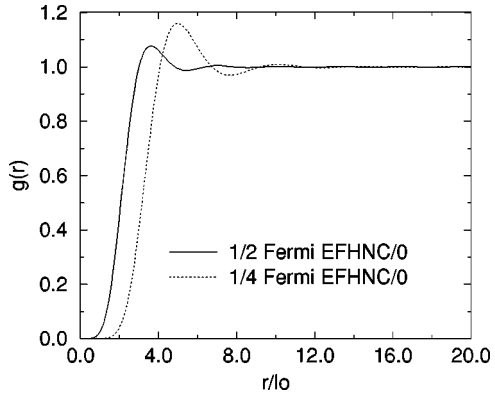

FIG. 3. We plot the radial distribution function $g(r)$ for the even-denominator-filling state $\nu=1/q_e$ where $q_e=2,4$ obtained from the unprojected Fermi wave function $\Psi^{Fermi}_{\nu=1/q_e}$ within the EFHNC/0 approximation.

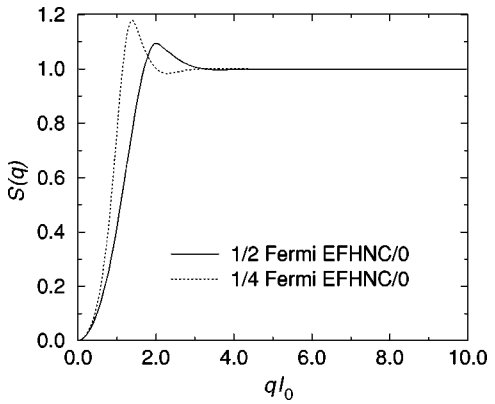

FIG. 4. We plot the static structure factor $S(q)$ for the fillings $\nu=1/q_e$ where $q_e=2,4$ obtained from the unprojected Fermi wave function $\Psi^{Fermi}_{\nu=1/q_e}$ by applying the EFHNC/0 approximation.

on a given function as $2\partial/\partial z_j$ and as a result displaces the jth electron by an amount proportional to $|\mathbf{k}|$ from the center of the vortex.

By applying the EFHNC/0 approach to the unprojected half-filled Fermi wave function we obtain a $(r/l_0)^6$ dependence of $g(r)$ for small r. With no LLL projection, the plane waves do not act as operators so the number of zeros on each electron will be three, where two zeros are coming from the Jastrow part and a single zero comes from the Slater determinant as required by the Pauli principle. In this case the plane waves of the unprojected wave function cannot displace the zeros of $\Psi^{Bose}_{\nu=1/q_e}$. In addition, the Slater determinant brings its own zero related to the antisymmetrization condition. By these arguments we could have anticipated that within the EFHNC/0 treatment, the unprojected radial distribution function should behave as $g_{\nu=1/q_e}(r\to 0)=(r/l_0)^{2(q_e+1)}$ for the corresponding fillings $\nu=1/q_e$ where $q_e=2,4$.

The LLL projection corrects this discrepancy, because the projection operator $\hat{P}_{LLL}$ makes the plane waves act as operators on the Laughlin-like wave function. The above physical arguments are expressed in mathematical form by recalling that the unprojected Fermi wave functions corresponding to $\nu=1/2$ and $\nu=1/4$ have different Jastrow factors and different Slater determinants, since the respective Fermi disk radius $k_F(\nu)$ is filling dependent.

In the small-r limit one has $g(r\to 0)\sim\exp[\tilde{u}_s(r\to 0)]$ and by using Eq. (28) and the formula $\lim_{r\to 0}K_0(Qr)=-\ln(Qr/2)-\gamma$ we obtain that

TABLE I. We show the interaction-energy per particle $u_{\nu=1/q_e}$ in $1/4\pi\epsilon_0(e^2/\epsilon l_0)$ units computed from the unprojected Fermi wave function $\Psi^{Fermi}_{\nu=1/q_e}$, where $q_e=2$ and 4. Our results were obtained within the EFHNC/0 approximation. In the fourth row we report the exact diagonalization results of Fano *et al.* (Ref. 19).

ν	Wave function	Method	u_ν
1/2	$\Psi^{Fermi}_{\nu=1/2}$	EFHNC/0	-0.4961
1/4	$\Psi^{Fermi}_{\nu=1/4}$	EFHNC/0	-0.3624
1/2	Exact diagonalization	Ref. 19	-0.469 ± 0.005

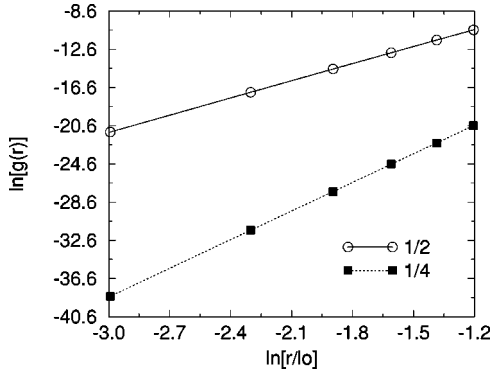

FIG. 5. The small-r behavior of the radial distribution function $g(r)$ for fillings $\nu=1/2$ (circled) and $\nu=1/4$ (squared), where $\ln[g(r)]$ versus $\ln[r/l_0]$ is plotted. Within the EFHNC/0 approximation, we have $g_{\nu=1/2}(r\to 0)\sim(r/l_0)^6$ and $g_{\nu=1/4}(r\to 0)\sim(r/l_0)^{10}$ for the respective fillings $\nu=1/2$ and 1/4.

$$g(r\to 0)\sim\exp\left[2q_e\ln\left(\frac{Qr}{2}\right)+2q_e\gamma+\ln[g_{ideal}(r\to 0)]\right], \tag{38}$$

where $\gamma=0.5772\ldots$ is the Euler constant. For states of the form $\nu=1/q_e$ where q_e is even, the behavior of $\ln[g_{ideal}(r\to 0)]$ is given by $\ln[g_{ideal}(r\to 0)]\sim 2\ln[k_F(\nu)r]-\ln(4)$, implying that

$$\ln[g(r\to 0)]\sim(2q_e+2)\ln\left(\frac{r}{l_0}\right)+c(\nu,\gamma), \tag{39}$$

where $c(\nu,\gamma)$ is an irrelevant constant that depends on ν and γ. The unprojected radial distribution function in the small-r limit has a $(r/l_0)^{2q_e}$ component coming from the Jastrow part of the Fermi wave function, while the contribution of the Slater determinant is proportional to $(r/l_0)^2$, the same for both fillings $\nu=1/2$ and 1/4, irrespective of their different Fermi radius $k_F(\nu)$ values.

Although we do not explicitly include the projection operator $\hat{P}_{LLL}$ in the Fermi wave function and instead use its unprojected counterpart, the Jastrow factor provides a good projection,[21] which should be particularly effective as far as ground-state properties are concerned. However, to achieve a deeper understanding of the even-denominator-filling state and to study other quantities, such as the excitation spectrum, these results strongly suggest that the full LLL projection is needed.

Unfortunately, such a projection leads to a wave function that cannot be treated directly within the EFHNC and FHNC formalism because the structure of a determinant of single-particle orbitals is lost. A more general projection scheme, applied to few-electron systems in a spherical geometry,[22] seems promising. Such a scheme introduces a many-body dependence on all single-particle orbitals, which, however, can be handled by introducing state-dependent correlations in the wave function in close analogy to ''backflow'' correlations[23] of liquid ^{3}He.

IV. CONCLUSIONS

In this paper we applied the EFHNC approach and the FHNC theory to study, in the thermodynamic limit, the unprojected Fermi wave function $\Psi^{Fermi}_{\nu=1/q_e}$, which describes the even-denominator-filling state $q_e=2,4$ of the FQHE.

Both EFHNC and FHNC calculations were performed by neglecting the so-called elementary (or bridge) diagrams on the cluster expansion of the radial distribution function, adopting the EFHNC/0 and FHNC/0 approximations, where ''0'' means neglect of elementary diagrams. The EFHNC approach incorporates also a second approximation since it treats the original Fermi wave function as a bosonized Jastrow function.

The unprojected radial distribution function obtained from the FHNC/0 theory has more pronounced peaks than that obtained from the EFHNC/0 approach and both are not in agreement with recent results[10] where very little oscillatory behavior has been found.

We first find numerically and then prove analytically that the unprojected radial distribution function does not have the correct short-range behavior of the projected case for the fillings $\nu=1/q_e$ where $q_e=2$ and 4. We prove that the small-r behavior of the unprojected radial distribution function $g(r)$ is given by $\ln[g(r\to 0)]\sim(2q_e+2)\ln(r/l_0)$ and we explain that this is due to the missing LLL projection of the wave function. Since there is no LLL projection, the plane waves of the Slater determinant do not act as operators to displace the electrons from the center of the vortex to give the correct small-r behavior.

The FHNC/0 study reveals strong ''oscillations'' on the unprojected radial distribution function. This oscillatory behavior has been seen[12] in a range of filling factors in the vicinity of the half-filled state (for instance filling 6/13) for both projected and unprojected wave functions and in finite-size calculations of few electrons confined to the LLL on a spherical surface,[9] and our findings are in good qualitative agreement with them.

Compared to the FHNC/0 results we see that the EFHNC/0 approach smoothes the peaks of the radial distribution function and loses part of its structure.

Although we perform our calculations by using an unprojected wave function, the Jastrow factor provides some amount of projection onto the LLL; however, a better understanding of other quantities, such as the excitation spectrum, needs the inclusion of the full LLL projection.

These results suggest that the approximation of the Slater determinant of plane waves as a Bose Jastrow wave function, namely the ''bosonization'' of the unprojected Fermi wave function (not the unprojected Fermi wave function itself) may not constitute an enough accurate approximation to give a highly reliable quantitative evaluation of the radial distribution function and related quantities.

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