PHYSICAL REVIEW B VOLUME 58, NUMBER 14 1 OCTOBER 1998-II

Skyrmion in a real magnetic film

Ar. Abanov
Department of Physics, Texas A&M University, College Station, Texas 77843-4242

V. L. Pokrovsky
Department of Physics, Texas A&M University, College Station, Texas 77843-4242
and Landau Institute of Theoretical Physics, Moscow, Russia

(Received 10 July 1998)

Skyrmions are magnetic defects in ultrathin magnetic films, similar to the bubble domains in the thicker films. Even weak uniaxial anisotropy determines their radii unambiguously. We derive equations of slow dynamics for Skyrmions. We show that the discreteness of the lattice in an isotropic two-dimensional magnet leads to a slow rotation of the local magnetization in the Skyrmion and even a small dissipation leads to decay of the Skyrmion. The radius of such a Skyrmion as a function of time is calculated. We prove that uniaxial anisotropy stabilizes the Skyrmion and study the relaxation process. [S0163-1829(98)50438-9]

Skyrmions are topological excitations of two-dimensional (2D) magnet or ultrathin magnetic films similar to the well-known bubble domains in the industrial magnetic memory materials.[1] The former play an important role as letters in magnetic records. A natural question arises whether the Skyrmions can be utilized in a similar way. To answer this question one must adjust the existing theory to a realistic condition of real films with their anisotropy, defects, dissipation, and discreteness. It is necessary also to consider how the Skyrmion can be created and destroyed, i.e., the Skyrmion dynamics, ignored in the previous studies. In this paper we address these problems.

The Skyrmion was first discovered by Skyrme[2] who considered it as a localized solution in a model of nuclear matter. Belavin and Polyakov (BP) (Ref. 3) have shown that the Skyrmion is a topologically nontrivial minimum of energy for the so-called $\vec{\mathbf{n}}$ field, the classical continuous limit of the Heisenberg model. It realizes the mapping of the plane in which the spins are placed onto sphere of the order parameter with the degree of mapping 1.

Though the $\vec{\mathbf{n}}$-field model was inspired by the studies of magnetic films, a more or less direct observation of the Skyrmions was made on the 2D electron layers under the quantum Hall effect (QHE) conditions[4] following an earlier theoretical prediction.[5] An indirect observation of Skyrmion effects in quasi-2D magnets was reported by F. Waldner[6] who found the Skyrmion energy from the heat capacity measurements in good agreement with the theoretical prediction by BP. This experimental observation is highly nontrivial since the real magnets are at least weakly anisotropic. Due to the existence of Goldstone modes in the Heisenberg magnet, even very weak anisotropy can change the excitations crucially.

Let us first approach the problem with simple dimensionality arguments. The Skyrmion is a static excitation of the homogeneous ferromagnetic state localized in a circle of the radius R. From the dimensionality consideration and from the BP results the Skyrmion energy does not depend on its size and is equal to $4\pi J|m|$ where m is the degree of mapping. Hence, the exchange energy $1/2\int J(\nabla \mathbf{S})^2 d^2x$ of a Skyrmion does not depend on its radius R. The anisotropy energy $-1/2\int \lambda S_z^2 d^2x$ of the Skyrmion is proportional to R^2 and decreases together with R. Therefore the exchange energy of the fourth order in the derivatives $1/2\int \kappa(\Delta \mathbf{S})^2 d^2x$ is crucial for the Skyrmion. It stabilizes the Skyrmion if $\kappa > 0$. Now the total energy of the Skyrmion depends on its radius and has minimum at $R \sim (\kappa/\lambda)^{1/4}$. The order of magnitude of κ is Ja^2 where a is the lattice constant, but its sign is model-dependent. It is negative for the Heisenberg model with the nearest-neighbor interaction, but it is positive for a more realistic Ruderman-Kittel-Kasuya-Yosida interaction. Thus, if there exists a stable Skyrmion, its radius R is of the order $R \sim (Ja^2/\lambda)^{1/4} \sim (l_\lambda a)^{1/2} \ll l_\lambda$, where $l_\lambda = \sqrt{J/\lambda}$ is the domain wall width. The energy of such a Skyrmion differs by about $\lambda R^2 \sim \sqrt{\kappa\lambda}$ from the classical Skyrmion energy $4\pi J \gg \sqrt{\kappa\lambda}$.

This analysis shows that a kind of perturbation approach is applied for this problem. Below we develop nonlinear perturbative theory working at rather general premises. The classical two-dimensional Heisenberg exchange ferromagnet in continuous approximation is described by the Hamiltonian

$$H_0 = \frac{1}{2} \int J(\nabla \mathbf{S})^2 d^2x \tag{1}$$

with the constraint on the vector field $\mathbf{S}(\mathbf{r})$: $\mathbf{S}^2(\mathbf{r}) = 1$. An obvious minimum of such a Hamiltonian is the homogeneous ferromagnetic configuration in which all the spins are parallel $\mathbf{S}(\mathbf{r}) = \text{const}$. The simplest topologically nontrivial minimum of the Hamiltonian (1) is given by the Skyrme solution:

$$S_{0x} = \frac{2Rr}{R^2 + r^2} \cos(\phi + \psi),$$

$$S_{0y} = \frac{2Rr}{R^2 + r^2} \sin(\phi + \psi),$$

$$S_{0z}=\frac{R^2-r^2}{R^2+r^2}. \tag{2}$$

It describes a Skyrmion of radius R with the center placed in the origin. The observation point is indicated by the polar coordinates r and ϕ; ψ is an arbitrary angle. We have already mentioned that the energy E of the Skyrmion (2) does not depend on its radius R. It also does not depend on the angle ψ.

Any static distribution of magnetization $\mathbf{S}(\mathbf{r})$ satisfies the equilibrium equation:

$$\frac{\delta H}{\delta \mathbf{S}(\mathbf{r})}=\mathbf{S}(\mathbf{r})\left(\mathbf{S}(\mathbf{r})\cdot\frac{\delta H}{\delta \mathbf{S}(\mathbf{r})}\right). \tag{3}$$

The right-hand side of Eq. (3) is added as a Lagrangian factor which ensures that $\mathbf{S}^2(\mathbf{r})=1$ at any point $\mathbf{r}$. For $H=H_0+H_1$ where H_1 is a perturbation we look for a solution in a form $\mathbf{S}=\mathbf{S}_0+\mathbf{S}_1$, where $\mathbf{S}_0(\mathbf{r})$ is determined by Eq. (2) and $\mathbf{S}_1(\mathbf{r})$ is perpendicular to $\mathbf{S}_0$ and satisfies the linearized inhomogeneous equation:

$$\hat{K}\mathbf{S}_1(\mathbf{r})=\left[\frac{\delta H_1}{\delta \mathbf{S}(\mathbf{r})}-\mathbf{S}_0(\mathbf{r})\left(\mathbf{S}_0\cdot\frac{\delta H_1}{\delta \mathbf{S}}\right)\right]_{\mathbf{S}=\mathbf{S}_0}, \tag{4}$$

where the tensor kernel $K_{ij}(\mathbf{r},\mathbf{r}')$ of the linear operator $\hat{K}$ is given by

$$K_{ij}(\mathbf{r},\mathbf{r}')=-J\delta(\mathbf{r}-\mathbf{r}')\times[\delta_{ij}\Delta_{\mathbf{r}'}-\delta_{ij}(\mathbf{S}_0\cdot\Delta\mathbf{S}_0)-S_{0i}S_{0j}\Delta_{\mathbf{r}'}].$$

The derivatives $\partial\mathbf{S}_0/\partial R$ and $\partial\mathbf{S}_0/\partial\psi$ are the zero modes and, hence, they satisfy homogeneous equations $\hat{K}\partial\mathbf{S}_0/\partial R=\hat{K}\partial\mathbf{S}_0/\partial\psi=0$. Therefore, the right-hand side of Eq. (4) must be orthogonal to the vectorial functions $\partial\mathbf{S}_0/\partial R$ and $\partial\mathbf{S}_0/\partial\psi$. The two orthogonality conditions allow us to determine both R and ψ fixed by small perturbation H_1. Considering a special perturbation Hamiltonian

$$H_1=\frac{1}{2}\int[\kappa(\Delta\mathbf{S})^2+\lambda(1-S_z^2)]d^2x, \tag{5}$$

we find from the orthogonality condition

$$R=R_0=\left(\frac{8\kappa}{3\lambda L}\right)^{1/4}, \tag{6}$$

where $L=\ln[(3\lambda/8\kappa)^{1/4}l_\lambda]$. The logarithm in Eq. (6) comes from the divergent integral $\int(1-S_{0z}^2)\partial S_{0z}/\partial R d^2x$. It was cut off at a radius $r=l_\lambda$ at which the perturbation theory fails. Due to axial symmetry, ψ remains to be zero mode. Note that the Skyrmion does not exist for $\kappa<0$. Now we are in a position to consider the dynamics of the Skyrmion.

The dynamics of the unit vector field $\mathbf{S}(\mathbf{r},t)$ is given by the Landau-Lifshitz equation:[7]

$$\dot{\mathbf{S}}(\mathbf{r},t)=-g\mathbf{S}(\mathbf{r},t)\times\frac{\delta H[\mathbf{S}]}{\delta\mathbf{S}(\mathbf{r},t)}+\nu\mathbf{S}(\mathbf{r},t)\times\left[\mathbf{S}(\mathbf{r},t)\times\frac{\delta H[\mathbf{S}]}{\delta\mathbf{S}(\mathbf{r},t)}\right]. \tag{7}$$

It can be checked that, at $\nu=0$, the equation of motion (7) conserves the magnetization and energy of the field $\mathbf{S}(\mathbf{r})$, as well as the local constraint $\mathbf{S}^2(\mathbf{r})=1$. A small dissipation term at $\nu\neq 0$ allows for the relaxation processes.

We consider only the slow Skyrmion dynamics. It means that we present again the Hamiltonian H as a sum H_0+H_1, where H_1 is a small perturbation to the exchange Hamiltonian H_0. We are looking for a solution in the form $\mathbf{S}=\mathbf{S}_0+\mathbf{S}_1$, $\mathbf{S}_0\cdot\mathbf{S}_1=0$, where $|\mathbf{S}_1|\ll|\mathbf{S}_0|$ and $\mathbf{S}_0(r;R(t),\psi(t))$ is the standard Skyrmion solution (2) with the parameters R and ψ slowly varying in time. We also consider the dissipation and the term $\dot{\mathbf{S}}$ as a perturbation. Substituting only $\mathbf{S}_0(r;R(t),\psi(t))$ in the perturbation terms in Eq. (7) and requiring the orthogonality of the perturbation terms to both zero modes $\partial\mathbf{S}_0/\partial R$ and $\partial\mathbf{S}_0/\partial\psi$, we obtain equations of motion for R and ψ:

$$\frac{\dot{R}}{R}=-\frac{\nu}{g}\omega, \tag{8}$$

$$\omega R^2\ln\frac{\tilde{R}^2}{R^2}+g\kappa\frac{16}{3}\frac{1}{R^2}-g\lambda R^2\ln\frac{\tilde{R}^2}{R^2}=0, \tag{9}$$

where $\omega=\dot{\psi}$ and $\tilde{R}$ is a scale at which the perturbation expansion breaks down, i.e., $|\mathbf{S}_0(r=\tilde{R})|\approx|\mathbf{S}_1(r=\tilde{R})|$. Equations (8) and (9) have a fixed point with $\omega=0$ and $R=R_0$, where R_0 is determined by Eq. (6).

If R deviates slightly from R_0 so that $\Delta R=R-R_0$ is still small enough, one can put $\tilde{R}=l_\lambda$ into Eq. (9) and obtain $\omega=4\lambda g\Delta R/R$. Using Eq. (8), we find

$$\Delta\dot{R}=-4\lambda\nu\Delta R. \tag{10}$$

''Small enough'' ΔR means that we still can use $\tilde{R}\approx l_\lambda$. However, if ΔR is large, the cutoff scale is determined by finite frequency: $\tilde{R}\approx l_\omega=\sqrt{gJ/|\omega|}$. Eq. (10) is valid if $l_\lambda<l_\omega$ or $\Delta R/R<1/4$. In the opposite case $l_\omega\ll l_\lambda$ and $R\gg R_0$ one can neglect the second term in Eq. (9). Then equations of motion (8) and (9) read as follows:

$$\omega=\lambda g;\quad \dot{R}=-\nu\lambda R. \tag{11}$$

For $l_\omega\ll l_\lambda$ and $R\ll R_0$ the last term in Eq. (9) can be omitted. Then

$$\omega=-\frac{16}{3}\frac{\kappa g}{R^4}\frac{1}{\ln\frac{3R^2J}{16\kappa}};\quad R^3\dot{R}=\frac{16}{3}\kappa\nu\frac{1}{\ln\frac{3R^2J}{16\kappa}}. \tag{12}$$

Thus, the easy-axis anisotropy together with the fourth-order exchange term fix the radius (6) of the Skyrmion if $\kappa>0$. In the opposite case $\kappa<0$ there is no stable configuration with nontrivial topology. We also have shown that the Skyrmion reaches its equilibrium radius within characteristic time $t_r=(\nu\lambda)^{-1}$.

Another property of a real film which should be taken into account is the lattice discreteness. We have mentioned earlier that in the continuous model the Skyrmion is a topological excitation and as such cannot dissipate. However, in the discrete lattice the continuity of the field $\mathbf{S}(\mathbf{r})$ is lost and the very notion of topological excitation becomes inconsistent. Therefore, the Skyrmion configuration in the discrete lattice is unstable. Moreover, it is sufficient to remove one plaquette in the center of the Skyrmion to make it unstable (see, for example, Ref. 8). We will imitate the discreteness effect by considering a hole in the center of the Skyrmion. In the picture due to BP the Skyrmion is described by a mesomorphic function which has a pole in the center of the Skyrmion. This pole cannot be removed by any continuous variation of the field **S**. However, the Skyrmion configuration with a hole punched in its center will dissipate.

Let us first consider the Skyrmion without anisotropy. ''Punching a small hole in the center'' means the substitution $J \rightarrow J\theta(r-r_0)$ in Eq. (1) where $r_0 \ll R$ is the radius of the hole and $\theta(x)$ is the step function: $\theta(x)=0$ for $x<0$ and $\theta(x)=1$ for $x>0$. Hence, the perturbation H_1 is given by

$$\tilde{H}_1 = -J\int \theta(r_0-r)(\nabla \mathbf{S})^2. \tag{13}$$

Employing the orthogonality condition of the right-hand side of the linearized Eq. (7) to the zero modes and neglecting all the terms of the higher order in r_0, one finds that Eq. (8) still holds, but Eq. (9) must be replaced by

$$\omega \ln \frac{\tilde{R}^2}{R^2} = g\frac{Jr_0^2}{R^4}, \tag{14}$$

where $\tilde{R} \approx l_\omega = \sqrt{gJ/|\omega|}$ is the scale where the perturbation scheme breaks down. With the logarithmic accuracy one can write $\ln(\tilde{R}^2/R^2)=\ln(R^2/r_0^2)$. Finally, substituting ω from Eq. (14) into Eq. (8), we find

$$R^3\dot{R} = -\nu\frac{Jr_0^2}{\ln \dfrac{R^2}{r_0^2}}. \tag{15}$$

From this equation one can conclude that the Skyrmion's lifetime is roughly proportional to its radius in the fourth power.

Returning to the field with the easy-axis anisotropy and the fourth-order term, let us introduce again a hole in the center. In this situation the perturbation is given by the sum of the two terms (5) and (13). Using the same orthogonality trick one gets

$$-2\frac{\omega}{g}R^2\ln\frac{\tilde{R}^2}{R^2} - \frac{32}{3}\frac{\kappa}{R^2} + 2R^2\lambda \ln\frac{\tilde{R}^2}{R^2} = -4J\frac{r_0^2}{R^2}. \tag{16}$$

After substitution $\kappa \rightarrow \tilde{\kappa} = \kappa-(3/8)Jr_0$ Eq. (16) acquires the same form as Eq. (9). It means that, as long as $\kappa > (3/8)Jr_0^2$, the Skyrmion is stable and its radius is defined by Eq. (6) with $\tilde{\kappa}$ instead of κ. Equations (10), (11), and (12) are valid as well after the same substitution. In the case $\kappa < (3/8)Jr_0^2$, however, the effective $\tilde{\kappa}$ is negative and the stable Skyrmion exists no longer.

The translation motion of a Skyrmion can be studied by the same technique as well. In order to move the Skyrmion, a nonuniform magnetic field perpendicular to the film can be applied. The perturbative part of the Hamiltonian reads

$$H_1 = \int \left(\frac{1}{2}\kappa(\Delta \mathbf{S})^2 + \frac{1}{2}\lambda(1-S_z^2) - h(\mathbf{r})S_z\right)d^2x, \tag{17}$$

where $h(\mathbf{r}) = \mu_B S H(\mathbf{r})$, μ_B is the Bohr magneton, S is the spin per unit area, and $H(\mathbf{r})$ is the magnetic field. The two new zero modes associated with the translation motion are $\partial \mathbf{S_0}/\partial X$ and $\partial \mathbf{S_0}/\partial Y$, where X and Y are the coordinates of the center of the Skyrmion. Assuming that magnetic field varies slowly, and employing the same orthogonality technique, we find equations of translational motion

$$v_x = -\frac{gR^2}{2}\partial_Y h_0 \ln\frac{\tilde{R}^2}{R^2} + \frac{\nu R^2}{2}\partial_X h_0 \ln\frac{\tilde{R}^2}{R^2},$$

$$v_y = \frac{gR^2}{2}\partial_X h_0 \ln\frac{\tilde{R}^2}{R^2} + \frac{\nu R^2}{2}\partial_Y h_0 \ln\frac{\tilde{R}^2}{R^2}, \tag{18}$$

where $v_x = \dot{X}$, $v_y = \dot{Y}$, $h_0 = h(X,Y)$, and $\tilde{R} = \min(l_\omega, l_\lambda, \sqrt{J/h_0})$. Equations (18) are valid if $|\nabla h|\tilde{R}/|h| \ll 1$.

In conclusion, we proposed a general framework for description of any slow motion in a system of Skyrmions. We have shown that in a real ultrathin ferromagnetic film with easy-axis anisotropy the Skyrmion known for the isotropic model still exists, but it acquires a definite radius R_0 given by Eq. (6). This result was obtained in Ref. 9 by other methods. By the order of magnitude $R_0 \sim \sqrt{a l_\lambda} \ll l_\lambda$ where l_λ is the domain wall width. Once we made a domain with the reversed magnetization in a ferromagnetic film, it shrinks down to the size $R_0 \sim 1nm$ (see also Ref. 9). The magnetic moment of such a Skyrmion is still rather large, $\mu_S = \mu_B \pi(l_\lambda/a)\ln(l_\lambda/a) \sim 300\mu_B$, and can be observed experimentally by the methods employed to discover the Skyrmions in QHE systems.[4] The Skyrmion stability allows us to create it with a magnetic tip.

The discreteness of the lattice in the isotropic model leads to a finite Skyrmion lifetime which is roughly proportional to the fourth power of its radius. However, anisotropy together with the higher order exchange interaction stabilizes the Skyrmion. At finite temperature it can decay through an instanton configuration. Our results allow us to understand why the activation energy found by Waldner is so close to $4\pi J$: the difference is expected to be of the relative order $a/l_\lambda \sim 10^{-2}$. The detailed thermodynamics of Skyrmions will be published elsewhere.

This work has been supported by the NSF Grant No. DMR-9705812 and by the DOE Grant No. DE-FGO3-96ER45598. We are indebted to B.I. Halperin for useful discussions and to B. Ivanov who attracted our attention to Ref. 9.

[1] See, for example, T. H. O'Dell, *Ferromagnetodynamics: The Dynamics of Magnetic Bubbles, Domains, and Domain Walls* (Holsted Press, 1981), and references therein.

[2] T. H. R. Skyrme, Proc. R. Soc. London, Ser. A **247**, 260 (1958).

[3] A. A. Belavin and A. M. Polyakov, Zh. Eksp. Teor. Fiz. Pis'ma Red. **22**, 503 (1975) [JETP Lett. **22**, 245 (1975)]; see also A. M. Polyakov, *Gauge Fields and Strings* (Harwood Academic Publishers, Chur, Switzerland, 1987).

[4] E. H. Aifer, B. B. Goldberg, and D. A. Broido, Phys. Rev. Lett. **76**, 680 (1996).

[5] S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B **47**, 16 419 (1993).

[6] F. Waldner, J. Magn. Magn. Mater. **54–57**, 873 (1986); Phys. Rev. Lett. **65**, 1519 (1990).

[7] L. D. Landau and E. M. Lifshitz, *Statistical Physics Part II* (Pergamon, New York, 1984), Vol. 9.

[8] F. D. M. Haldane, Phys. Rev. Lett. **61**, 1029 (1988).

[9] B. Ivanov, V. A. Stephanovich, and A. A. Zhmudskii, J. Magn. Magn. Mater. **88**, 116 (1990); Sov. Phys. JETP **64**, 376 (1986).