PHYSICAL REVIEW B VOLUME 58, NUMBER 14 1 OCTOBER 1998-II

Aging in a two-dimensional Ising model with dipolar interactions

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(Received 19 June 1998)

Aging in a two-dimensional Ising spin model with both ferromagnetic exchange and antiferromagnetic dipolar interactions is established and investigated via Monte Carlo simulations. The behavior of the autocorrelation function $C(t,t_w)$ is analyzed for different values of the temperature, the waiting time t_w and the quotient $\delta = J_0/J_d$, J_0 and J_d being the strength of exchange and dipolar interactions, respectively. Our results show that this nondisordered model presents a slow nonequilibrium dynamics which, depending on the values of δ, is consistent either with a linear scaling t/t_w of the correlation function or with a logaritmic scaling $\ln(t)/\ln(t_w)$ which is usually associated with activated dynamics scenarios in disordered systems. [S0163-1829(98)50138-5]

Microscopic long-ranged interactions are always of interest in different fields of physics because they can give rise to a variety of unusual macroscopic behaviors. Perhaps the better example in condensed matter are dipole-dipole interactions. In particular, the competition between long-range antiferromagnetic dipolar interactions and short-range ferromagnetic exchange interactions can give rise to several interesting magnetic phenomena. Recent works in two-dimensional uniaxial spin systems, where the spins are oriented perpendicular to the lattice and coupled with these kind of interactions, have shown a very rich phenomenological scenario concerning both its equilibrium statistical mechanics[1,2] and nonequilibrium dynamical properties.[3] Magnetization processes in these kinds of systems are of interest due to aspects related to information storage in ultrathin ferromagnetic films. Moreover, there are several contexts in which a short-ranged tendency to order is perturbated by a long-range frustrating interaction. Among others, model systems of this type have been proposed to study avoided phase transitions in supercooled liquids[4] and charge density waves in doped antiferromagnets.[5–7]

The above-mentioned systems can be described by an Ising-like Hamiltonian of the type

$$H = -J_0 \sum_{\langle i,j \rangle} \sigma_i \sigma_j + J_d \sum_{(i,j)} \frac{\sigma_i \sigma_j}{r_{ij}^3}, \qquad (1)$$

where the spin variable $\sigma_i = \pm 1$ is located at the site i of a square lattice, the sum $\Sigma_{\langle i,j \rangle}$ runs over all pairs of nearest-neighbor sites and the sum $\Sigma_{(i,j)}$ runs over all distinct pairs of sites of the lattice; r_{ij} is the distance (in crystal units) between sites i and j; $J_0 > 0$ and $J_d > 0$ are the ferromagnetic exchange and antiferromagnetic dipolar coupling parameters, respectively. For simplicity, we rewrite this Hamiltonian as follows:

$$H = -\delta \sum_{\langle i,j \rangle} \sigma_i \sigma_j + \sum_{(i,j)} \frac{\sigma_i \sigma_j}{r_{ij}^3}, \qquad (2)$$

with $\delta = J_0/J_d$. There are few numerical results concerning the equilibrium statistical mechanics, i.e., the finite temperature phase diagram, of this model. In a recent work MacIsaac, Whitehead, Robinson, and De'Bell[2] have shown that the ground state of Hamiltonian (2) is the antiferromagnetic state for $\delta < 0.85$. For $\delta > 0.85$ the antiferromagnetic state becomes unstable with respect to the formation of striped domain structures, that is, to state configurations with spins aligned along a particular axis forming a ferromagnetic strip of constant width h, so that spins in adjacent strips are antialigned, forming a superlattice in the direction perpendicular to the strips. They also showed that striped states of increasingly higher thickness h become more stable as δ increases from $\delta = 0.85$. Moreover, they showed that the striped states are also more stable than the ferromagnetic one for arbitrary large values of δ, suggesting such a phase to be the ground state of the model for $\delta > 0.85$. Monte Carlo calculations on finite lattices at low temperature[2,3] gave further support to this proposal, at least for intermediate values of δ. Furthermore, such simulations have shown that striped phases of increasingly higher values of h may become thermodynamically stable at *finite* temperatures for intermediate values of δ. This results are in agreement with other analytic ones.[1,5] For low values of δ the system presents an antiferromagnetic phase at low temperatures. At high temperatures, of course, the system becomes paramagnetic.

The dynamics of the model is characterized by the formation and growth of magnetic domains, due to the competition between the exchange and the dipolar interactions, which at low temperatures generate very large relaxation times. In a Monte Carlo study of the time evolution of the magnetization, Sampaio, de Albuquerque, and de Menezes[3] have shown the existence of two different types of relaxation, according to the value of δ. For $\delta > \delta_c \sim 2.7$ the magnetization relaxes exponentially, with a relaxation time that depends both on the temperature and δ. For $\delta < \delta_c$ the magnetization presents a power-law decay, with an exponent independent of δ. They also showed the existence of strong hysteresis effects in the presence of an external magnetic field for $\delta > \delta_c$. Since hysteresis is a typical nonequilibrium phenomenon associated with domain dynamics with large relaxation times, one also expects the presence of *aging* effects, that is, history dependence in the time evolution of the response functions after the system has been quenched into some nonequilibrium state.

Aging phenomena have been vastly studied in disordered systems such as spin glasses (see Ref. 8, and references therein), which are essentially out of equilibrium on experimental time scales. However, they appear also in the phase ordering kinetics of *ordered* systems, such as the Ising *ferromagnet*,[9–11] associated with a slow domain dynamics. Aging can be observed in real systems through different experiments. A typical example is the zero-field-cooling[12] experiment, in which the sample is cooled in zero field to a subcritical temperature at time t_0. After a waiting time t_w a small constant magnetic field is applied and subsequently the time evolution of the magnetization is recorded. It is then observed that the longer the waiting time t_w the slower the relaxation.

Although aging can be detected through several time-dependent quantities, a straightforward way to establish it in a numerical simulation is to calculate the spin autocorrelation function

$$C(t,t_w)=\frac{1}{N}\sum_i \langle \sigma_i(t+t_w)\sigma_i(t_w)\rangle, \quad (3)$$

where $\langle\cdots\rangle$ means an average over different realizations of the thermal noise and t_w is the waiting time, measured from some quenching time $t_0=0$.

In this work we present the results of Monte Carlo simulations in the two-dimensional Ising spin model defined by the Hamiltonian (2) on a $N=20\times 20$ square lattice with free boundary conditions. We chose the heat-bath algorithm for the spin dynamics and time is measured in Monte Carlo steps per site. The quantity (3) is averaged over 100 samples; for each run the system is initialized in a random initial configuration corresponding to a quenching from infinite temperature to the temperature T at which the simulation is done. We analyze the behavior of $C(t,t_w)$ as a function of the observation time t, for different values of t_w, δ, and T.

At enough high temperatures we find that the system does not present aging, that is, for any value of δ there is a temperature above which $C(t,t_w)$ is independent of t_w, as expected in a paramagnetic phase. At low temperatures we find different types of aging behaviors as δ is varied.

The typical behaviors of $C(t,t_w)$ are illustrated in Figs. 1 and 2, for $T=0.5$ and different values of δ [waiting times $t_w=5^n$ $(n=2,\ldots,6)$]. We also analyzed, for the same values of T and δ, the time evolution of the magnetization per site $m(t)$ and the staggered magnetization per site $m_s(t)$ starting from different initial conditions, in order to characterize the different relaxation regimes.

In Fig. 1 we see the typical behavior of $C(t,t_w)$ in the antiferromagnetic state. The characteristic signature of $C(t,t_w)$ in this regime is the appearance of a plateau at some intermediate value of t independent of t_w ($t\sim 10^3$ for $\delta=-1$), where $C(t,t_w)$ remains constant for a period of time that depends on T and δ; after such period $C(t,t_w)$ relaxes to zero. We also see a dependency on t_w, that is, aging. The inset shows the evolution of $m(t)$ (filled circles) and $m_s(t)$ (open circles) starting from a random initial condition. We also analyzed the evolution of the same quantities starting from a fully magnetized initial state $[m(0)=1]$. We did not find any hysteresis effect in this region, the only difference being a much more slower convergence of $m_s(t)$ towards a constant value. These kinds of behaviors appear for negative values and also for small positive values of δ at enough small temperatures, in agreement with previous calculations of the phase diagram of the model.[2] The existence of these plateaus suggests some type of quasiequilibrium state. This behavior is rather unusual in this kind of system and it is probably related to the long-range character of the interactions.

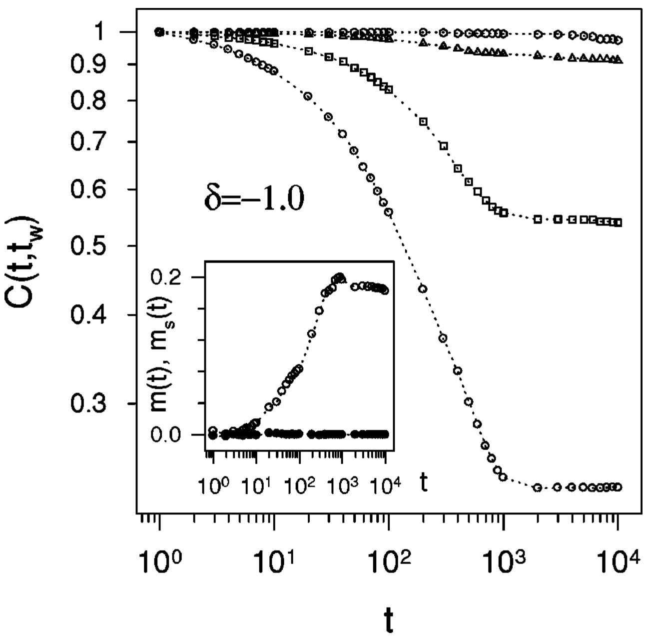

FIG. 1. Autocorrelation function $C(t,t_w)$ vs the observation time t at $T=0.5$ and $\delta=-1$, for $t_w=5^2$ (circles), $t_w=5^3$ (squares), $t_w=5^4$ (triangles), and $t_w=5^5$ (hexagons). Inset: magnetization $m(t)$ (filled circles) and staggered magnetization $m_s(t)$ vs time.

We have also studied the decay of $C(t,t_w)$ in the paramagnetic phase, for $\delta=0.5$ and $T=0.5$. Althought the system does not display aging, it is worth to note the *logarithmic* decay of $C(t,t_w)$. This is a characteristic feature of this dynamics, that is, the relaxation of $C(t,t_w)$ is always slow, even in the paramagnetic region. It would be interesting to analyze whether this behavior is characteristic of the whole paramagnetic phase.

In Fig. 2 we show $C(t,t_w)$ for different values of δ corresponding to the striped region, below[3] [Fig. 2(a)] and above [Fig. 2(b)] $\delta_c\sim 2.7$. The behavior of $m_s(t)$ in all these cases shows no traces of antiferromagnetic ordering. On the other hand, the behavior of $m(t)$ starting from a random $[m(0)=0]$ and a fully magnetized $[m(0)=1]$ states shows a clear distinction between both regions: while for $\delta>\delta_c$ strong hysteresis effects appear, for $\delta<\delta_c$ such effects are negligible, in agreement with the results of Ref. 3.

We found that $C(t,t_w)$ obeys a different type of dynamic scaling law for both striped regions, as can be seen from the data collapse of Fig. 3. For $\delta<\delta_c$ $C(t,t_w)$ obeys the dynamic scaling

$$C(t,t_w)\propto c_\delta\{\ln(t)/\ln[\tau(t_w)]\} \quad (4)$$

[see Fig. 3(a)], while for $\delta>\delta_c$ it obeys the following scaling

$$C(t,t_w)\propto c_\delta[t/\tau(t_w)], \quad (5)$$

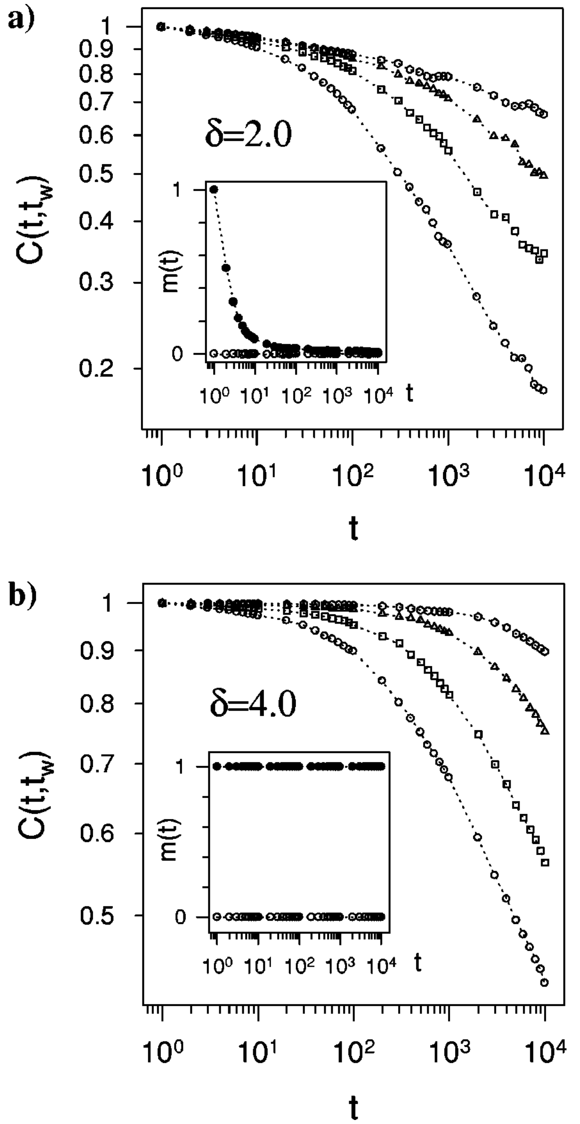

FIG. 2. Autocorrelation function $C(t,t_w)$ vs the observation time t for $t_w=5^3$ (circles), $t_w=5^4$ (squares), $t_w=5^5$ (triangles), and $t_w=5^6$ (hexagons), for different values of δ corresponding to the striped phase. Insets: magnetization $m(t)$ vs time starting from random [$m(0)=0$, open circles] and a fully magnetized [$m(0)=1$, filled circles] states.

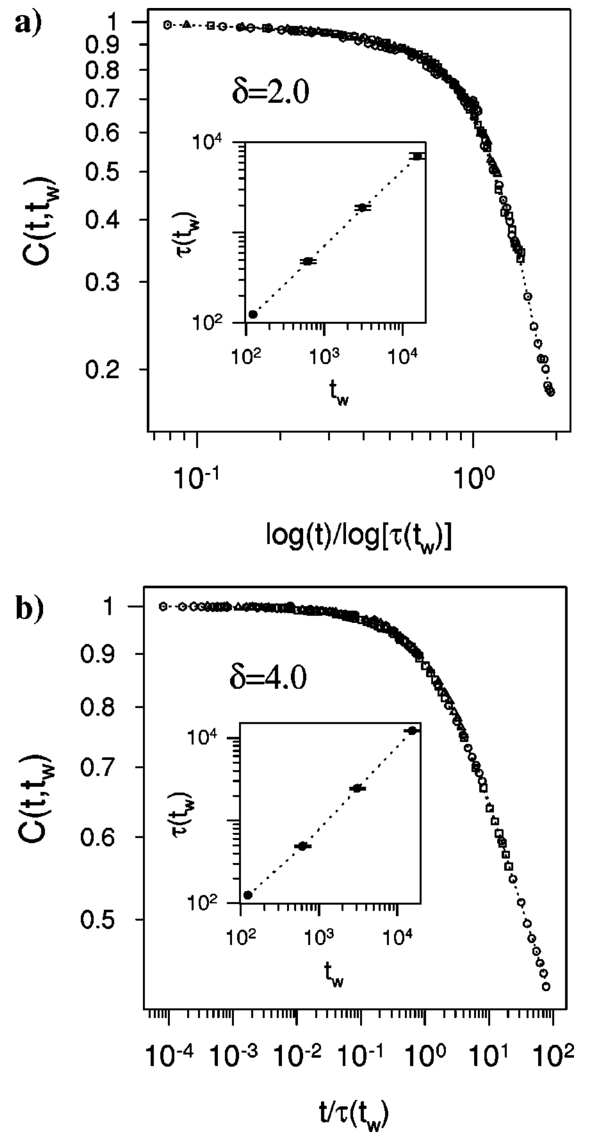

FIG. 3. Data collapse of the curves shown in Fig. 2 (the values of t_w and the corresponding symbols are the same in both figures).

[see Fig. 3(b)], where the time scale $\tau(t_w)$ (Ref. 13) is shown in the insets. We observed, for values of δ far enough of δ_c, the scaling form $\tau(t_w)\sim t_w^a$. The log-log linear fitting from Fig. 3 gives the following values: $a=0.83\pm0.02$ for $\delta=2.0$ and $a=1.00\pm0.05$ for $\delta=4.0$. We have also studied the scaling law for other values of δ in both regions, which are not presented here. In all these cases we obtained for δ well below δ_c a logarithmic scaling $C(t,t_w)\propto c[\ln(t)/\ln(t_w)]$ [Fig. 3(a)], while for $\delta>\delta_c$ we observe a scaling form $C(t,t_w)\propto c(t/t_w^a)$ [Fig. 3(b)], with a exponent a that approaches the unity as δ increases. The scaling form of $C(t,t_w)$ deviates from the previous ones as δ approaches δ_c. We also tested the linear scaling for $\delta<\delta_c$ and the logarithmic scaling for $\delta>\delta_c$. In both cases no data collapse was observed, regardless the freedom introduced by the function $\tau(t_w)$.

It has been proposed[14,11] that aging phenomena are based on a slow domain growth at low temperatures, where after certain time t a characteristic domain size $L(t)$ is reached. The time evolution of quantities like the autocorrelation function will then present a crossover from dynamical processes characterized by length scales smaller than the already achieved domain size to processes at larger scales dominated by domain growth through the movement of domain walls. In this scenario, scaling arguments[11] leads to the following expected dependency of $C(t,t_w)$:

$$C(t,t_w)\propto c[L(t+t_w)/L(t_w)], \tag{6}$$

at least for large values of t and t_w. Hence, in the aging regime $t\gg t_w$ a scaling form

$$C(t,t_w)\propto c[L(t)/L(t_w)], \tag{7}$$

is expected.[13]

Our results for $\delta>\delta_c$ are consistent with a t/t_w scaling for large values of δ, where the ferromagnetic short-range interactions are dominant. This type of scaling, which is associated with an algebraic growth of the domain size $L(t)\propto t^{\psi}$, appears in the slow domain growth dynamics of disordered systems with a ferromagnetic ground state.[9–11]

Much more interesting is the logarithmic scaling form $\ln(t)/\ln(t_w)$ when $\delta < \delta_c$. This scaling is associated with a logarithmic time dependence of the domain size $L(t) \propto (\ln t)^{\psi}$ predicted by an activated dynamics scenario proposed in the context of spin glasses,[14] in which disorder and frustration generate active droplets excitations with a broad energy distribution.

In summary we have shown that the interplay between short- and long-range competitive interactions in an ordered system give rise to different kinds of aging at low temperatures. The different behaviors appear to be related to different domain dynamics as the relative strengths of the interactions are changed. In particular, for intermediate values of δ, where the strengths are comparable, the competition between them generate a slow relaxation dynamic consistent with an activated dynamics scenario, usually associated with disordered systems like Lennard-Jones glasses[15] and disordered ferromagnets.[11] This result suggests a possible relationship between the microscopic dynamic properties of this model and that of disordered systems. In this sense, it would be of interest to investigate, for instance, the possible existence of broad energy distributions of low-lying excitations in the present model. Works along these directions are in progress and will be published elsewhere.

Fruitful suggestions from Leticia F. Cugliandolo, Silvia Urreta, and Daniel Stariolo are aknowledged. This work was partially supported by grants from Consejo Nacional de Investigaciones Científicas y Técnicas CONICET (Argentina), Consejo Provincial de Investigaciones Científicas y Tecnológicas (Córdoba, Argentina) and Secretaría de Ciencia y Tecnología de la Universidad Nacional de Córdoba (Argentina).

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