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Hall drag in correlated double-layer quantum Hall systems

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We show that in the limit of zero temperature, double-layer quantum Hall systems exhibit a phenomenon called Hall drag, namely, a current driven in one layer induces a voltage drop in the other layer, in the direction perpendicular to the driving current. The two-by-two Hall resistivity tensor is quantized and proportional to the **K** matrix that describes the topological order of the quantum Hall state, even when the **K** matrix contains a zero eigenvalue, in which case the Hall conductivity tensor does not exist. The relation between the present work and previous ones is also discussed. [S0163-1829(98)50332-3]

The experimental discovery and theoretical understanding of the fractional quantum Hall effect[1,2] (FQHE) is one of the most important advances made in condensed matter physics in the past 15 years. Recently, much attention has been focused on FQHE in multicomponent systems.[3] Such components may be the spins of electrons that are not frozen out when the external magnetic field is not too strong, or layer indices in multilayered system. Novel physics, such as even-denominator FQHE states,[4,5] spin ferromagnetism,[6] spontaneous interlayer coherence,[7–10] and canted antiferromagnetism,[11] have been discovered in these systems.

In multilayered systems, the interactions and correlations of electrons in different layers are crucial to the FQHE. Such correlations are captured by Halperin's multicomponent trial wave functions.[12] However, they are not easy to directly detect in the usual transport measurements, which are the most heavily used methods in experimental studies of FQHE.

In a drag measurement,[13] separate electric contacts are made to the electron gas in the two different layers, and the electric current is forced to flow in one layer (called the driving layer). This current will induce a measurable voltage drop in the other layer (called the drag layer), even though no current is flowing in it. In the absence of a magnetic field, the drag voltage is in the opposite direction of the driving current, and the transresistance, defined as the ratio between the drag voltage and the driving current, reflects the density fluctuations of *individual* layers, and vanishes at $T=0$ if the coupling between the layers is weak enough.[14] In the presence of a magnetic field perpendicular to the layers, a perpendicular component of the drag voltage, called Hall drag, is in principle possible.[15] In this paper we will show that this is indeed the case in correlated double-layer FQHE systems, and the trans-Hall resistivity $\rho^{xy}_{\uparrow\downarrow}$ is *finite* and *quantized* at $T=0$ even though the normal longitudinal drag voltage vanishes; the finite $\rho^{xy}_{\uparrow\downarrow}$ reflects the *interlayer electron-electron correlations* in the *ground state*. Our results may be expressed in the compact form of a 2×2 Hall resistivity tensor:

$$\rho^{xy}_{ij}=\mathbf{K}_{ij}h/e^2, \tag{1}$$

where i and j are layer indices, and **K** is a 2×2 matrix that describes the topological order of the quantum Hall state.[16] As we will see below, the above result is valid even when the **K** matrix contains a zero eigenvalue. There is no longitudinal voltage drop at zero temperature.

Using a Chern-Simons-Ginsburg-Landau type of effective theory generalized to double-layer systems, Renn[17] argued that the Hall conductivity tensor is $(e^2/h)\mathbf{K}^{-1}$. Our results are consistent with his. However, in our work we show that our results may be derived *exactly* using known *microscopic* wave functions for special types of electron-electron interaction. Also his approach formally breaks down when **K** contains a zero eigenvalue, as the inverse of **K** does not exist in this case. In our approach, however, since we calculate the *resistivity* tensor directly (in the microscopic calculation), we still obtain well defined answers. When **K** contains a zero eigenvalue, the system supports a charge neutral gapless mode. Duan[18] suggested that such a mode gives rise to a Hall drag resistivity that is not quantized. It is clear from our exact calculation below that this is *not* the case; the Hall drag resistivity is quantized even when **K** contains a zero eigenvalue.

In the rest of the paper we will start by considering a special case where the Halperin wave functions are the exact ground states of the system, in which the trans-Hall resistivity may be calculated exactly using the exact microscopic wave functions. We then formally derive the expressions for $\rho^{xy}_{\uparrow\downarrow}$ for edge currents under more general conditions, using the chiral Luttinger liquid theory. We conclude with comments on the experimental implications of our results. Throughout the paper we assume no tunneling is allowed between the layers.

We begin by considering the limit of the Landau-level spacing $\hbar\omega_c\rightarrow\infty$, so that all electrons are in the lowest Landau level and have zero kinetic energy (measured from $\frac{1}{2}\hbar\omega_c$). In this limit, the electron-electron interaction may be parametrized by Haldane's pseudopotentials[19] U_l, which are the interaction energies of a pair of electron in a state with relative angular momentum l. We assume that the intralayer pseudopotentials $U_l^{\uparrow\uparrow}=U_l^{\downarrow\downarrow}>0$ for $l<m$, and interlayer pseudopotentials $U_l^{\uparrow\downarrow}>0$ for $l<n$; all other U's are zero. We also introduce a circularly symmetric confining potential $V(r)$, which is zero in the bulk and increases smoothly with the distance from the origin r near the edge of the disk. We assume the chemical potential of the electron gas $\mu=\mu_\uparrow=\mu_\downarrow$ is much smaller than the nonzero U's, so that the elec-

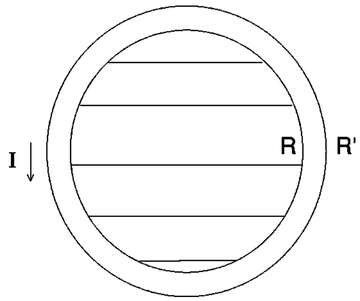

FIG. 1. Schematic illustration of a double-layer FQHE liquid and its edge. In the ground state the liquid is filled (in both layers) in the shaded region, up to $r=R$. Chiral current flows around the edge counterclockwisely. When more current is added in the upper layer, the edge in the upper layer moves outward to R'.

tron gas stays in the small V region. In this case the ground state of the system is exactly the Halperin (mmn) wave function:

$$\Psi_{mmn}=\prod_{i<j}(z_i^{\uparrow}-z_j^{\uparrow})^m(z_i^{\downarrow}-z_j^{\downarrow})^m\prod_{i,j}(z_i^{\uparrow}-z_j^{\downarrow})^n, \tag{2}$$

where $z_i^{\uparrow}$ and $z_i^{\downarrow}$ are the complex coordinates of the ith electron in the upper and lower layer, respectively. The common exponential factors are neglected in Eq. (2). The region with $V(r)<\mu$ is filled with the incompressible electron liquid described by Eq. (2), with Landau-level filling factor in individual layers $\nu_{\uparrow}=\nu_{\downarrow}=1/(m+n)$; there is a gap for all bulk excitations that is of the order of the smallest nonzero U_l.[20] Gapless excitation can only live at the edge of the incompressible liquid, which is along $r=R$ with $V(R)=\mu$ (see Fig. 1).[21]

In such a disk geometry, there is an equal amount of current flowing counterclockwisely along the edge in both layers, due to the gradient of the confining potential, while no current is flowing in the bulk. We may increase the edge current in the upper layer without changing the current in the lower layer by adding more charge to the upper layer, so that the edge in the upper layer moves from R to $R'>R$. In the region $R<r<R'$, there is no electron in the lower layer, and the filling factor in the upper layer is $1/m$, which is *bigger* than that of the bulk, due to the absence of electrons in the lower layer in the same region. The additional current is therefore

$$\delta I_{\uparrow}=\frac{1}{m}\frac{e}{h}[V(R')-V(R)], \tag{3}$$

while the change of the chemical potential in the upper layer is clearly

$$\delta\mu_{\uparrow}=V(R')-V(R)=m\frac{h}{e}\delta I_{\uparrow}. \tag{4}$$

Even though the edge of the lower layer is still at $r=R$, the charge added in the upper layer also increases the chemical potential in the lower layer. This is because if we were to add one more electron to the lower layer (at $r=R$), there would be a charge with the amount n/m moved from $r=R$ to $r=R'$ in the upper layer, because electrons in the lower layer are seen as nodal points by electrons in the upper layer, as described by the wave function (2), and electrons in the upper layer are pushed away from them. This means an additional energy cost of $(n/m)[V(R')-V(R)]$ for each electron added to the lower layer, therefore,

$$\delta\mu_{\downarrow}=n\frac{h}{e}\delta I_{\uparrow}. \tag{5}$$

From the above we find the following linear-response equation:

$$\begin{pmatrix}\delta\mu_{\uparrow}\\ \delta\mu_{\downarrow}\end{pmatrix}=\frac{h}{e}\mathbf{K}\begin{pmatrix}\delta I_{\uparrow}\\ \delta I_{\downarrow}\end{pmatrix}=\frac{h}{e}\begin{pmatrix}m & n\\ n & m\end{pmatrix}\begin{pmatrix}\delta I_{\uparrow}\\ \delta I_{\downarrow}\end{pmatrix}, \tag{6}$$

which is equivalent to Eq. (1). The edge trans-Hall resistance is nothing but the off-diagonal matrix element of the above resistance matrix:

$$\rho_{\uparrow\downarrow}^{xy}=n\frac{h}{e^2}>0. \tag{7}$$

Its positive sign is anomalous because it means if the chemical potential in the lower layer were held a constant, adding current in the upper layer induces a change of current in the *opposite* direction (back flow) in the lower layer; this is opposite to what normally happens in a drag experiment with two electron layers.[13]

So far we have been considering the special case where the Landau-level spacing is infinite and the Halperin wave functions describe the ground state exactly. In the following we show that when these conditions are released, the results derived above are not altered for edge currents.

As long as the edge is reasonably sharp so that edge reconstruction does not occur, the low-energy physics is well described by the chiral Luttinger liquid theory.[22] In this theory each edge component is described by a bosonic field ϕ_σ. In our case we have two components (upper or lower layer), and $\sigma=\uparrow$ or $\downarrow$. The edge electron density for each component is $\rho_\sigma(x)=(1/2\pi)\partial_x\phi_\sigma(x)$, and they satisfy the following commutation relation:[22]

$$[\phi_\sigma(x),\rho_{\sigma'}(x')]=i(\mathbf{K}^{-1})_{\sigma\sigma'}\delta(x-x'), \tag{8}$$

where $\mathbf{K}^{-1}$ is the inverse of the $\mathbf{K}$ matrix discussed above (here we need to assume that $\mathbf{K}$ contains no zero eigenvalue and therefore its inverse is well defined). The edge Hamiltonian is quadratic in ρ:

$$H=\frac{1}{2}\sum_{\sigma\sigma'}\int dx V_{\sigma\sigma'}\rho_\sigma(x)\rho_{\sigma'}(x), \tag{9}$$

where $V_{\sigma\sigma'}$ is a nonuniversal, positive definite interaction matrix that depends on the details of the edge confining potential as well as electron-electron interactions at the edge. From the continuity equation one obtains the edge current operator

$$\begin{aligned}I_\sigma(x)&=\frac{e}{2\pi}\dot{\phi}_\sigma(x)=\frac{e}{2\pi}\frac{1}{i\hbar}[\phi_\sigma(x),H]\\ &=\frac{e}{h}\sum_{\alpha\beta}K_{\sigma\alpha}^{-1}V_{\alpha\beta}\rho_\beta(x),\end{aligned} \tag{10}$$

and $\langle I_\sigma(x)\rangle=0$ in the ground state. Now we raise the edge electrostatic potential in layer σ by the amount δv_σ. This introduces the following perturbation to the edge Hamiltonian: $\delta H=-e\Sigma_\sigma\delta v_\sigma\int dx\rho_\sigma(x)$. Solving the new Hamiltonian one obtains $\langle\rho_\sigma(x)\rangle=e\Sigma_\beta V^{-1}_{\sigma\beta}\delta v_\beta$, and

$$\langle I_\sigma(x)\rangle=\frac{e^2}{h}\sum_{\sigma\beta} K^{-1}_{\sigma\beta}\delta v_\beta. \tag{11}$$

Interestingly, we find the result does *not* depend on V, which involves microscopic details. Reversing the matrix we obtain exactly Eq. (1). We therefore find that the edge trans-Hall resistance depends on the topological **K** matrix of the bulk FQHE state *only*, and is therefore quantized.

So far we have been focusing on the drag effect of edge current. In reality, current may flow both in the bulk and along the edge. Using continuity condition, i.e., any gain (or loss) of current at the edge must be compensated for by the current from the bulk, it is straightforward to show that the drag coefficient must be the same for bulk and edge currents.[23] Therefore our results should apply to driving current flowing both in the bulk and along the edges, and do not depend on the details of the current distribution in the sample.

We have been focusing on gapped double-layer FQHE in this paper. It is plausible, however, that Hall drag should exist in compressible double-layer systems as well, if interlayer correlation exists. In particular, by tuning some control parameters such as the layer separation d, it is possible to tune the system through a phase transition from a compressible state to a correlated double-layer quantum Hall state. Our results suggest that Hall drag is a useful way to probe such a phase transition: As d decreases from above the critical separation d^*, the Hall drag resistivity should increase (due to the interlayer correlation that is building up), and reach the quantized value at $d=d^*$. Further decreasing d should have no effect on the quantized value.

Very recently, *longitudinal* drag measurement has been performed on a double-layer system at filling factor $\nu_\uparrow=\nu_\downarrow=1/2$ for each individual layer.[24] In these systems the layer separation is larger although fairly close to the critical layer separation d^* below which the quantized double layer (111) state forms, so the system is compressible. It is found that the *longitudinal* drag resistance is much larger than that of zero magnetic field for the same system, although still much smaller than the predicted *Hall* drag resistance of the (111) state at reasonable temperatures, and appears to stay finite in the zero-temperature limit. We note that since d is larger but close to d^*, it is possible that the system has a sizable *Hall* drag resistance, which can be mixed into the *longitudinal* drag measurement in a two-terminal setup. Since the Hall drag resistance can be so large (compared to longitudinal drag resistance), even a very small mixture can lead to a very big effect. It is also observed[24] that there is a strong nonlinear current effect at low temperatures in drag signal, possibly due to sample inhomogeneity.[25] We note that since the system is close to the phase boundary at which the system becomes a bilayer quantum Hall state, it is possible that the (111) state gets stabilized in certain regions of the sample due to inhomogeneity. Since the quantum Hall state has very small longitudinal resistance, it is possible that most of the current flows through these regions when the current is very low, leading to an apparently large drag signal and nonlinear effects as the current increases (so that some of the current has to flow elsewhere).[26]

In summary, we have demonstrated the existence of Hall drag effect in correlated double-layer FQHE systems, which may be used to detect interlayer electron-electron correlation directly. The trans-Hall resistivity tensor is shown to be quantized at zero temperature.

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