Kondo resonance, Coulomb blockade, and Andreev transport through a quantum dot

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We study resonant tunneling through an interacting quantum dot coupled to normal metallic and superconducting leads. We show that large Coulomb interaction gives rise to interesting effects in Andreev transport. Adopting an exact relation for the Green's function, we find that at zero temperature, the linear response conductance is enhanced due to Kondo-Andreev resonance in the Kondo limit, while it is suppressed in the empty site limit. In the Coulomb blockaded region, on the other hand, the conductance is reduced more than the corresponding conductance with normal leads because large charging energy suppresses Andreev reflection. [S0163-1829(98)00140-4]

Electronic transport of mesoscopic devices containing superconducting electrodes has been an interesting subject in recent years.[1] Transmission of electrons through normal-metal–superconductor (N-S) interfaces requires the conversion of normal current to supercurrent, which is called Andreev reflection.[2] With the recent advances of nanofabrication techniques, quantum interference effects have been extensively studied in the mesoscopic N-S heterostructures (see, e.g., Ref. 1). In a phase coherent N-S structure, the phase of quasiparticles as well as Cooper pairs is preserved and transport properties depend strongly on the nature of the quasiparticle phase. Theoretically, the Landauer-Büttiker-type formula[3,4] has been used extensively to describe quantum transport in many kinds of N-S hybrid structures using noninteracting models. (For a review, see, e.g., Refs. 1 and 5.)

Resonant tunneling through an interacting quantum dot (QD) or Anderson impurity has been intensively investigated recently. It has been shown that large Coulomb interaction gives rise to anomalous properties in transport. An example is the Kondo-resonant transport. Kondo-resonant transport has been predicted theoretically[6–9] and verified experimentally[10–12] by conductance measurements for artificially made Anderson impurities. On the other hand, strong electron-electron interactions suppress conductance peaks where the systems are weakly coupled to the leads. It has been shown that the electron-electron interactions lead to conductance suppression due to the orthogonality catastrophe.[13–15] Stafford *et al.*[15] showed that the coherent transmission in artificial molecule structures is suppressed with increasing the system size by using the Hubbard-type model. Coulomb interaction has been found to play a crucial role in the nature of the transmission phase.[16–18] It has been shown that Coulomb interactions give rise to anomalous effects in phase evolution through a quantum dot embedded in an arm of the Aharonov-Bohm interferometer, such as an inter-resonance phase drop. Nonequilibrium transport in an interacting quantum dot where both leads are superconductors has been studied recently by using the nonequilibrium Green's-function method.[19,20] Andreev reflection has been supposed to be negligible in the weak tunneling limit because large charging energy leads to Coulomb blockade of Andreev transport. In the meanwhile, for a moderately coupled quantum dot, multiple Andreev reflections give rise to a subgap structure in the current-voltage curve due to resonant tunneling, which are quite different from those of S-S contacts.[19] Resonant Andreev tunneling in a strongly correlated quantum dot coupled to normal and superconducting leads has been investigated recently by Fazio and Raimondi.[21] Using the nonequilibrium Green's-function formalism and equation of motion technique they have shown that the Kondo-resonant transmission is enhanced in the limit of large Coulomb repulsion due to the existence of a superconducting electrode.

In this paper, we investigate coherent transport through an interacting quantum dot coupled to normal and superconducting electrodes based on the scattering matrix formulation. We consider a model as shown schematically in Fig. 1, where normal scattering and Andreev reflection are decoupled. In the QD-N_2 boundary, only normal scattering is taken into account while Andreev reflection is considered in the N_2-S boundary. It is assumed that the N_2-S boundary is perfect and the normal scattering does not occur at this boundary. This model is applicable to microjunctions where the length scale of normal scattering and Andreev reflection is well separated.[22] With this model, the Landauer-type formula for the linear response conductance has been derived by Beenakker[22] in the framework of noninteracting electron model. In the presence of interactions, this formula cannot be used, in general, because of the presence of inelastic processes. However, in the linear response regime ($V=0$) with zero temperature, there is no phase space for inelastic processes and the formula can be equally applied to the system containing interactions. The linear response conductance for the system under consideration can be written as[22]

$$G_{NS}=\frac{4e^2}{h}\sum_n \frac{T_n^2}{(2-T_n)^2}, \qquad (1)$$

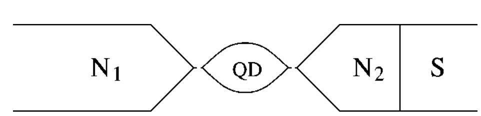

FIG. 1. Schematic diagram of the QD coupled to normal (N) and superconducting (S) leads. In the N_2-S interface, only Andreev reflection is considered.

where T_n is the transmission probability of the nth channel. This equation is valid in the absence of an applied magnetic field. We consider the single channel case where the transmission probability through the quantum dot is represented by T_{QD}:

$$G_{NS}=\frac{4e^2}{h}\frac{T_{QD}^2}{(2-T_{QD})^2}. \quad (2)$$

The corresponding formula for the normal leads is the well-known Landauer formula[3]

$$G_N=\frac{2e^2}{h}T_{QD}. \quad (3)$$

Let us consider an Anderson impurity for the quantum dot with doubly degenerate level energy ε_0 and on-site Coulomb repulsion strength $U\sim e^2/C$, C being capacitance of the dot. At zero temperature the transmission probability T_{QD} can be obtained as follows, owing to the fact that there are no inelastic processes.[23,6] Due to the absence of the inelastic scattering, the imaginary part of the self-energy for the Green's function at the Fermi energy ε_F is given by

$$\mathrm{Im}\,\Sigma(\varepsilon_F)=-\Gamma/2, \quad (4)$$

where $\Gamma=\Gamma_L+\Gamma_R$ and $\Gamma_L/\hbar$ and $\Gamma_R/\hbar$ are the tunneling rate through left and right leads, respectively. With this condition the average occupation on the dot can be written as

$$\langle n\rangle=\frac{2}{\pi}\ \mathrm{Im}[\ln G^r(\varepsilon_F)]. \quad (5)$$

At zero temperature, the transmission probability can be expressed in terms of the exact Green's function as

$$T_{QD}=\Gamma_L\Gamma_R|G^r(\varepsilon_F)|^2, \quad (6)$$

which leads to the final expression with the help of Eq. (4),

$$T_{QD}=\frac{4\Gamma_L\Gamma_R}{\Gamma^2}\sin^2\varphi, \quad (7)$$

where $\varphi=\pi\langle n\rangle/2$. ($\langle n\rangle$ is the average occupation of the dot.)

Here $\langle n\rangle$ is calculated numerically by an equation of motion method,[24] which has been shown to be quite accurate for large U. We consider a symmetric coupling of the quantum dot to leads, that is $\Gamma_L=\Gamma_R$. From the calculated values of the average occupation, we display the conductances in Fig. 2 as a function of $\varepsilon_F-\varepsilon_0$. The parameters used for calculations are $U=50\Gamma$ and $W=200\Gamma$, with W being the bandwidth of the leads. Since the transmission probability reaches one for $\varepsilon_F-\varepsilon_0\gg\Gamma$, the conductance of the normal-superconductor hybrid system goes to $4e^2/h$, which is twice the normal conductance. This is a result of perfect transmission through the quantum dot in the Kondo limit. On the contrary, the conductances are suppressed in the empty site limit because of small transparency. G_{NS} decays faster than G_N because transmission by Andreev reflection requires two particle tunneling through the quantum dot.

In real systems, nearly perfect Kondo-resonant transmission could not be realized, though it is predicted by an exact

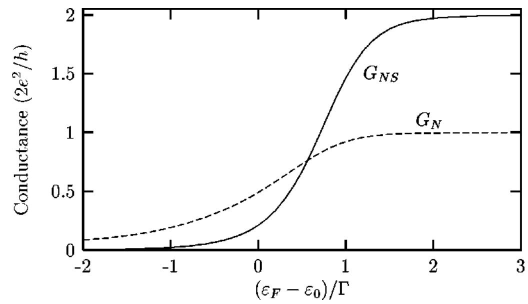

FIG. 2. Conductance G_{NS} and G_N obtained by Eq. (7) and numerical calculation of φ for $U=50\Gamma$ and $W=200\Gamma$.

relation at zero temperature. When the temperature is larger than the Kondo temperature, Kondo-resonant transmission does not occur and the transport is suppressed by Coulomb repulsion, rather than enhanced. In the case of $k_BT\ll\Gamma$, thermal broadening can be neglected and the conductances can be obtained in the following way with an approximate Green's function. If Kondo-like correlation is neglected, the transparency can be obtained by an approximate retarded Green's function,[25] which is similar to the Breit-Wigner type

$$G^r(\varepsilon)=\frac{1-\langle n\rangle/2}{\varepsilon-\varepsilon_0+i\Gamma/2}+\frac{\langle n\rangle/2}{\varepsilon-\varepsilon_0-U+i\Gamma/2}. \quad (8)$$

Note that this equation coincides with the Breit-Wigner formula for $U=0$. The self-consistent value of $\langle n\rangle$ is given by the relation

$$\langle n\rangle=-\frac{2}{\pi}\int_{-\infty}^{\varepsilon_F}\mathrm{Im}\,G^r(\varepsilon)\,d\varepsilon, \quad (9)$$

which leads to the expression[18]

$$\langle n\rangle=\frac{1+2P_1}{1+P_1-P_2}, \quad (10)$$

where

$$P_1=\frac{1}{\pi}\arctan\frac{2(\varepsilon_F-\varepsilon_0)}{\Gamma},\quad P_2=\frac{1}{\pi}\arctan\frac{2(\varepsilon_F-\varepsilon_0-U)}{\Gamma}.$$

Then we can get the conductance through Eq. (6).

Figure 3 displays the conductances obtained by Eqs. (6) and (8). As one can see, G_{NS} is suppressed more than G_N even in the ''resonance'' point. This phenomenon arises because Coulomb interactions in the dot suppress coherent transmission through the quantum dot. This would become a

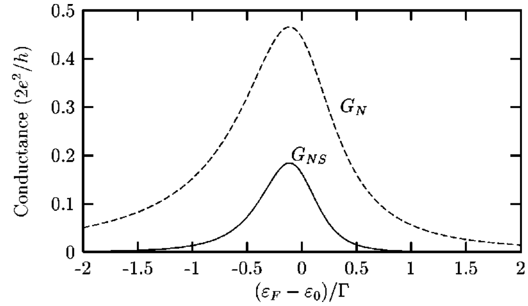

FIG. 3. Conductance G_{NS} and G_N obtained by Eq. (8) with Eq. (6) for $U=50\Gamma$.

very general feature in transport through the interacting system as the coupling to the leads are not so strong. If we consider a larger system like a coupled chain of quantum dots, transmission probability will be more reduced due to an orthogonality catastrophe. So one could say that, in general, G_{NS} will be negligible compared to G_N in strongly interacting systems weakly coupled to leads. Normal conductance suppression with increasing the system size has been studied by Stafford *et al.*[15] While the normal conductance is proportional to the transparency, G_{NS} is the second order of transmission probability. So G_{NS} will decrease faster than G_N with increasing of the system size. Even in the case of single quantum dot, we could see suppression of transmission due to the Coulomb interaction from our calculations.

For comparison, we plot the conductances of noninteracting case ($U=0$) in Fig. 4. As is well known from the Breit-Wigner formula, one can see that $G_{NS}=2G_N$ in the resonance point because of perfect transmission. Comparing Figs. 3 and 4, one can conclude that the large charging energy suppresses Andreev reflection even on resonance.

In conclusion, we have discussed resonant tunneling through a strongly interacting quantum dot coupled to normal metallic and superconducting leads. We have found that in strongly interacting quantum dots, resonant Andreev transport is qualitatively different from that of the noninteracting system. Based on the scattering matrix formalism and adopting an exact relation for the Green's function, we have shown that at zero temperature the linear response conductance is enhanced due to Kondo-Andreev resonance in the Kondo limit, while it is suppressed in the empty site limit. In the Coulomb blockaded region, on the other hand, the conductance is suppressed more than the corresponding normal conductance even in the resonance point, because large charging energy suppresses Andreev reflection.

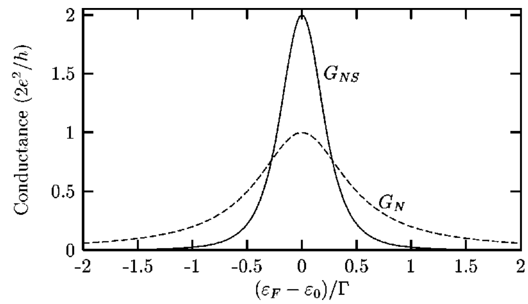

FIG. 4. Conductance G_{NS} and G_N in the noninteracting ($U=0$) limit.

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