Magnetic levitation of superconductors in the critical state

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The vertical levitation force in a cylindrically symmetric system composed of a permanent magnet and a type-II superconductor is studied assuming the latter to be in a critical state. In a first step, a constant-field-gradient approximation along the length of the superconductor is used to see the effects of the dependence of the critical current density on the internal magnetic field. In a second step, the theory is generalized in order to study the case when the field gradient is not constant, as is usually found in levitation experiments with bulk superconducting samples. The model results are discussed in relation to recent experimental data on magnetic levitation of high-T_c superconductors, explaining the observed behavior and making some predictions for future experiments. [S0163-1829(98)00426-3]

I. INTRODUCTION

When a superconductor is in the presence of a changing applied magnetic field, supercurrents are induced inside the sample producing a diamagnetic response of the material. The interaction of these supercurrents with an inhomogeneous magnetic field can produce stable levitation.[1,2]

In type-II superconductors, there is the possibility for the magnetic induction to be different from zero inside the sample in the form of quantized flux lines. In real type-II superconductors at small current densities ($J<J_c$ for a certain critical current J_c), these flux lines can be pinned by inhomogeneities in the sample. Under these conditions, when an external applied field is increased or decreased, the magnetic flux moves within the superconductor towards its interior or exterior until a critical slope is reached in the flux profile. In this critical state the current density attains its maximum value J_c.[3] Phenomenologically, this behavior has been modeled by Bean's critical-state model[4] and its extensions.[5] Brandt[6] showed qualitatively how from the pinning of flux lines some properties of levitation of type-II superconductors can be explained.

Moon *et al.*[7] measured the vertical forces on a levitating superconductor showing some of the main properties. These measurements were extended by several groups in order to study the influence of orientation,[8] material,[9,10] or shape of superconducting sample,[11] comparing, sometimes, thin films with bulk samples.[12,13]

On the other hand, few models have been developed, most of them studying the interaction between a superconductor and a permanent magnet. Hellman *et al.*[14] and Yang[15] presented models based on total flux exclusion of the sample that thus describe the behavior of type-I superconductors or type-II samples with very high critical current. Schönhuber and Moon,[16] Chan *et al.*,[17] and Torng and Chen[18] have taken into account the penetration of supercurrents, the latter with an applied field provided by a pair of oppositely wound coils. In Ref. 19, a systematic treatment of vertical levitation force for type-II superconductors based on flux penetration was done. Some papers[20,21] have considered granular structure for the superconductors when grains are completely penetrated.

All the models mentioned above assume a superconducting sample small enough to consider the magnetic-field-gradient constant along it. At the same time, the demagnetization effect due to the finite dimensions of the superconductor (SC) has been neglected. Until recently, only in numerical calculations has this effect been taken into account. Tsuchimoto *et al.*,[22] using an axisymmetric boundary element analysis, have calculated the dependence of levitation force upon different geometrical parameters of a permanent magnet-superconductor system. Portabella *et al.*[23] have used finite element calculations for the levitation force in order to estimate the value of critical current of a type-II superconductor. In a recent work,[24] we calculated the dependence of maximum vertical force upon the length of superconductor, considering both the demagnetization effect and the nonuniformity of the field gradient. In spite of these efforts, a systematic treatment of vertical levitation force for which the applied magnetic field cannot be assumed to have a constant gradient has not been developed yet.

Moreover, most of the developments consider the superconductor to be in the critical state and describe it by means of Bean's critical-state model considering the critical current J_c as constant. Hovewer, it is known that most type-II superconductors and in particular high-T_c superconductors present a dependence of its critical current upon the internal magnetic field. Some analytical expressions have been proposed and succesfully applied to describe the magnetic properties of superconductors.[25,26] It is therefore necessary to incorporate such a dependence in the study of the levitation of type-II superconductors. Although in numerical calculations by Tsuchimoto *et al.*[27] a Kim's model dependence for the critical current density was implemented, a general study of the effect of this dependence has not been done yet.

This paper is structured as follows. In Sec. II we will discuss the general characteristics of the model including the assumptions made, the starting formula for calculating the force, and the chosen values for the model parameters. Section III will deal with a simplified case assuming a constant-field gradient for the magnetic field along the volume occupied by the superconductor. This case will enable one to understand in a simpler way the effect of field dependence of the critical-current density. In Sec. IV we will consider the more general case of a nonconstant-field gradient. We will end this section with a discussion of how our calculations relate to present and future experiments on magnetic levitation of superconductors. Finally, in Sec. V we will present the main conclusions of our work.

II. MODEL

We will study a levitation system composed of a cylindrical permanent magnet (PM), of radius a and thickness b, uniformly magnetized along its axis with magnetization M, and a cylindrical type-II SC, of radius R and length L, placed at a variable distance z above it, with the same axis of symmetry. We use the usual cylindrical coordinates (ρ,θ,z) with the origin located at the center of the top face of the PM. The superconductor (zero field cooled) is moved initially far from the PM along the z axis, so that the magnetic field at every point of the SC undergoes a variation. On the axis this applied magnetic field has only a vertical component H_z given by

$$H_z(z,\rho=0)=\frac{M}{2}\left(\frac{z+b}{\sqrt{a^2+(z+b)^2}}-\frac{z}{\sqrt{a^2+z^2}}\right). \qquad (1)$$

It can be proved that close to the axis the radial component of the applied field is much smaller than the vertical one, which is almost constant with respect to ρ in that region. So, we can consider that all over a region close to the axis, the field has only an axial component given by Eq. (1). This approximation is quite good as long as $\rho<a$. Although we will use Eq. (1) for the external magnetic field, all the following treatment can be used with a different expression. If instead of a PM, we use another magnetic-field source such that it acomplishes the above assumptions, the treatment will be valid as well with the new field configuration.

Following the calculation of Landau, Lifshitz, and Pitaevskii[28] but considering a volume current instead of a linear one, it can be seen that the force that a cylindrically symmetric magnetic field produced over a region occupied by currents $\mathbf{J}=J(\rho,z)\hat{\theta}$ has only a vertical component given by

$$F_z=\frac{\pi\mu_0}{1-N}\int_{\rho',z'}J(\rho',z')\left(\frac{\partial H_z(z')}{\partial z'}\right)\rho'^2 d\rho' dz', \qquad (2)$$

where the integral has to be evaluated over the region containing currents[24] and N is the demagnetizing factor for the SC. A more detailed treatment should consider the magnetic poles that appear in the faces of SC and the magnetic field that they create. For the purposes of this work, however, it is sufficient to correct the force by the factor $(1-N)^{-1}$ in order to take into account these demagnetization effects. Demagnetizing factors for cylinders, which are dependent on the radius-length ratio of the superconductor, are calculated in Ref. 29, from which we have taken the value of N we will use.

The integral in Eq. (2) can be evaluated if we know the supercurrent distribution over the whole sample. The critical-state model[4,5] (CSM) has been widely used in order to model this distribution in different geometries. The conventional CSM assumes that, in the regions where the SC has felt a variation of magnetic field, a current of fixed value $\pm J_c$ (critical current) is induced. In this work, we shall use a more general extension of CSM that includes a dependence of critical current upon the internal magnetic field H_i. Although any other expression could be implemented, in this work we shall use an exponential dependence $[J(H_i)=K\exp(-|H_i|/H_0)]$ as studied in Ref. 26 because it was found that this field dependence succesfully describes the magnetic properties of a wide range of high-T_c superconductors.[30] It is useful to define the parameters $p=KR/H_0$ and $H_p=H_0\ln(1+p)$. p is a measurement of the degree of dependence of J_c upon H_i; the limit $p=0$ corresponds to constant critical current (Bean's limit), and the larger p is, the stronger is the dependence. H_p is called the penetration field and is the field at which supercurrents have just reached the center of the superconductor during its initial magnetization.

In order to study the effect of both the field dependence and the field-gradient variation along the sample, we fix all the parameters of the PM and the geometrical ones of the SC. Throughout all this work we will use values for those parameters encountered in typical experiments. For the PM we will take $a=12$ mm, $b=20$ mm, $M=398$ kA/m (corresponding to a remanent field of $\simeq 0.5$ T). These parameters for the PM yield a maximum applied field (at $z=0$) of $H_m\simeq 170$ kA/m. For the SC we will fix $R=6$ mm and $L=10$ mm (from which $N=0.35$ is assigned).[29] In some cases, an adequate normalization enables us to reduce the number of parameters that characterize the whole system (see Ref. 19), but in this work, we are interested in describing the behavior in absolute terms since one of our goals will be the estimation of some properties of the superconducting material from data obtained from levitation experiments.

III. CONSTANT-FIELD GRADIENT: THE EFFECT OF FIELD DEPENDENCE

We will first study only the effect of the field dependence of the critical current. So, in this section, we will consider that supercurrents fill a volume V where the field is uniform enough to enable assigning them a mean magnetization $\mathbf{M}$. In this case, Eq. (2) can be simplified yielding

$$F_z=\frac{\mu_0 V}{1-N}M_z\frac{\partial H_z}{\partial z}, \qquad (3)$$

M_z being the z component of the magnetization of the SC sample. The values of the magnetic field and its derivative (assumed as constant all over the SC in this section) are chosen as the values at the bottom face of the SC. The applied field at the SC will increase when the sample is descending (we shall call this the descending branch) and decrease when the SC is moving in the opposite sense (ascending branch).

As explained in Ref. 26, the parameters that suffice to properly describe the magnetic response of the SC in the critical state are H_m/H_p and p. In the present case, H_m corresponds to the maximum field created by the PM and therefore has a fixed value, so the variation of H_m/H_p and p will correspond exclusively to a variation of the particular $J_c(H_i)$ dependence. In Fig. 1 we show results calculated for differ-

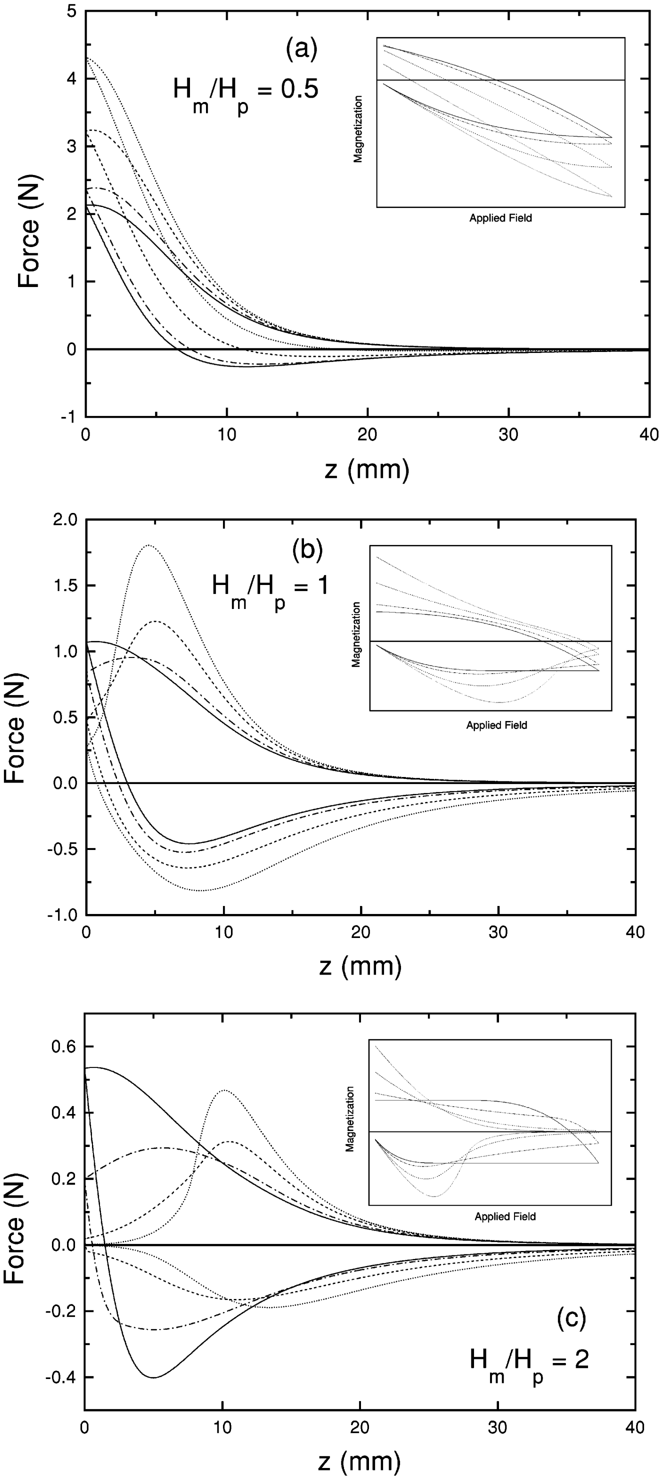

FIG. 1. Vertical levitation force using the constant-field-gradient approximation as a function of SC height above the PM, for different values of p and H_m/H_p: (a) $H_m/H_p=0.5$, (b) $H_m/H_p=1$, (c) $H_m/H_p=2$, (d) $H_m/H_p=10$. In all figures, $p=0$ (solid line), 1 (dot-dashed line), 10 (dashed line), and 100 (dotted line). The insets schematically show, in each case, the magnetization of the sample as function of the applied field (H_z) at the lower face of SC for the same parameters as above.

ent values of the parameters H_m/H_p and p. In the insets of the figures, we show the calculated magnetization of the sample with respect to the external magnetic field for each case.

In all the cases shown, the force is hysteretic because of the hysteresis in magnetization. As it can be seen in Fig. 1, when varying the ratio H_m/H_p maintaining p fixed, the force becomes more hysteretic (in the sense that there are more relative separations between the ascending and descending branches) for larger H_m/H_p ratios. This can be understood by examining the limiting cases (not shown in Fig. 1). For $H_m \ll H_p$ and for all values of p, supercurrents penetrate a very short depth into the superconductor. When reversing the movement of the sample, opposite currents also penetrate only a short distance into the sample, yielding in a nonhysteretic behavior. This limit is similar to the Meissner state and yields a maximum value for the force at a given height. In the other limit, $H_m \gg H_p$, supercurrents completely penetrate the SC at a very early stage in the descending branch (far from the magnet), and opposite currents do the same in the reverse branch when the SC is still very close to the reversing height. The result is an almost symmetric force curve with a large hysteresis.

Another general fact is that, far from the PM (low fields), the force is larger (in magnitude) in the ascending branch than in the descending one for a given height. This comes from the fact that we start from a zero-field-cooled SC so that in the descending branch, initially, there are no currents inside the SC, and thus, magnetization at low fields is almost zero. When the superconductor is far from the PM, but in the ascending branch, there are currents inside the sample that interact with the field producing a larger force with respect to the force in the descending branch.

The incorporation of H_i dependence on J_c ($p \neq 0$) results in the appearence of a minimum in the magnetization for applied fields smaller than H_p. This is understood by realizing that when $p=0$, if H_z increases the magnetization never decreases (in magnitude) reaching a saturation at $H_z=H_p$ corresponding to the field when currents have completely penetrated the sample. When $p \neq 0$, since J_c decreases with field, at a high enough applied field the magnetization would start to decrease (in magnitude), although the current would continue penetrating deeper into the SC. This produces a minimum in the magnetization at a field value H^*. H^* can be numerically calculated, being the solution of the equation

$$\frac{H^*}{H_p}[(1+p)^{H^*/H_p}+p](1+p)^{H^*/H_p}=\frac{p^2}{\ln(1+p)}. \quad (4)$$

It can be demonstrated that H^* lies between $H_p/2$ and H_p, depending on p. In the following, we shall study what the consequences are of these effects of the $J_c(H_i)$ dependence on the levitation force.

If $H_z \leq H_m < H^*(p)$ the introduction of the field dependence increases the force in the descending branch [Fig. 1(a)]. The larger p is, the larger this increase is. The reason for this is that currents penetrate a distance δ equal to

$$\delta=R\left(\frac{(1+p)^{(H_z/H_p)}}{p}-1\right), \quad (5)$$

with $H_z/H_p < H_m/H_p < H^*/H_p < 1$. So, δ is a decreasing function of p. The larger the value of p is, the faster J_c decreases but the less it penetrates. The result is, in this range of fields, a larger (in magnitude) magnetization and, thus, a larger force. At the same time, in this case the force gets less hysteretic with increasing p because of the shallower penetration of currents. In the limit $p \to \infty$ the hysteresis tends to zero since the penetration does also.

When $H^*(p) < H_z < H_m$ the field becomes high enough to produce a decrease (in magnitude) of the magnetization when the field is increasing. In the force behavior, this fact is translated into the presence of a peak in the descending branch [see Fig. 1(b)]. When p increases the peak becomes sharper because in this case currents decrease faster after an increase of applied field. It is not hard to demonstrate that when the applied field is larger than H_p, for a given magnetic field (corresponding to a given height of the SC), the larger p is, the lower the force is [see Fig. 1(c), left part]. For a given nonzero value of p, when the field is increased the value of current decays to near zero due to its dependence on field. So, if the field is high enough, depending on p the force could tend to zero even at low heights (large applied field). In fact, in the limit $p \to \infty$, $H^* = H_p$ and the value of the current is constant until the magnetic field reaches the value H_p, when it drops abruptly to zero. This produces a sharp fall of magnetization when $H_z = H_p$, and likewise the force falls sharply at a height z_p corresponding to $H_z(z_p) = H_p$. However, this fast drop in the force for the $p \to \infty$ limit appears due to the choice of the particular exponential dependence. For Kim's dependence, for instance, even in the $p \to \infty$ limit the force would not fall abruptly to zero, but it would decrease smoothly. This unphysical sudden drop to zero of the force would be rounded off when considering the more realistic case of nonconstant-field gradient even for very large values of p (see Sec. IV).

When $H_m \gg H_p$, the above behavior is accentuated. As we can see in Fig. 1(d), except for $p=0$, when the SC is close to the PM the field is large enough to make the current low enough so that the force becomes almost zero. Of course, as p increases the force becomes lower, because the larger p is, the faster is the field-induced current decay.

The approximation of constant-field gradient along the SC discussed in this section is adequate for describing levitation of SC samples when they are short or whenever the applied field does not have a strong variation. In the case of superconducting thin films, owing to their short length, this approximation is always well fulfilled. In this limit, however, the magnetization is not well described by the conventional critical-state model used here, but instead one has to use its extension to this geometry, where the demagnetization effects are implicitly included.[31] In that case, the penetration field is not given by the expression of H_p but instead depends on the SC thickness [a penetration field is defined for thin films as $H_d = J_c L/2$ (Ref. 31)]. When using realistic parameters one often encounters the case $H_m \gg H_d$, which results in a symmetric force curve as discussed above. The vertical levitation force for this particular geometry has been studied in Ref. 32.

We finally remark that the equations describing all the above stages can be calculated analytically, by means of Eq. (3) and using the results of Ref. 26.

IV. NONCONSTANT-FIELD GRADIENT

A. Formulation

In general, the dimensions of the SC and PM are such that the above approximation of constant-field gradient is not well fulfilled. In these cases, it is better to consider Eq. (2) instead of the simplified equation (3).

Following Ref. 26, the magnetization M in a cylindrical sample can be evaluated by the integral

$$M_z = \frac{2}{R^2} \int_0^R x' \left(\int_{x'}^R J(x'') dx'' \right) dx'. \tag{6}$$

Defining the function $L(x) = \int J(x) dx$ and assuming that currents completely fill the sample, it is easy to see that Eq. (6) becomes

$$M_z = L(R) - \frac{2}{R^2} \int_0^R x' L(x') dx', \tag{7}$$

which is the same result at which we would arrive at calculating the radial integral in Eq. (2). This demonstration is easily extended to the case of currents not completely penetrating the sample and also to the case for which there are different areas with different signs of the currents, just by defining different $L_i(x) = \int J_i(x) dx$ functions for each zone.

The above shows that the radial integral of Eq. (2) equals $M_z R^2$, where M_z is the magnetization calculated using the critical-state model. Equation (2) becomes

$$F_z = \frac{\mu_0 \pi R^2}{1-N} \int_z^{z+L} \left(\frac{\partial H_z(z')}{\partial z'} \right) M_z(z') dz', \tag{8}$$

or, in a simpler form,

$$F_z = \frac{\mu_0 \pi R^2}{1-N} \int_{H_z(z)}^{H_z(z+L)} M_z(H_z) dH_z, \tag{9}$$

where $M_z(H_z)$ represents, now, the z component of the magnetization of the layer placed at height z', where the field is $H_z(z')$, calculated by the CSM. As is clear from Eq. (9), the SC can be viewed as if it were composed of different layers of differential length, at different heights, each of them following the conventional CSM. The total force is evaluated by adding the contribution of all layers. As in the previous section, the demagnetization correction is done by the factor $(1-N)^{-1}$.

This integral can be analytically solved in the descending branch simply integrating the formula for magnetization. In those stages where the frozen-field profile is nonconstant for all layers (ascending branch or minor loops) the integration has to be numerically done. In the descending branch, for the case $p=0$ (Bean's limit) we have three possibles stages.

(a) When the lower layer (low face of SC) is not fully penetrated by supercurrents (consequently, none of the other layers are fully penetrated), Eq. (9) gives

$$F_z(z) = \mu_0 \pi R^2 \{ G_1[H(z+L)] - G_1[H(z)] \}. \tag{10}$$

(b) When the lower layer is fully penetrated but the upper one is not

$$F_z(z) = \mu_0 \pi R^2 \{ G_1[H(z+L)] - G_1(H_p) + G_2(H_p) - G_2[H(z)] \}. \tag{11}$$

(c) When the upper layer (and therefore, the whole SC) is fully penetrated, we have

$$F_z(z)=\mu_0\pi R^2\{G_2[H(z+L)]-G_2[H(z)]\}. \quad (12)$$

In the above we have used the definitions

$$G_1(H)=-\frac{H^2}{2}+\frac{H^3}{3H_p}-\frac{H^4}{12H_p^2}, \quad (13)$$

$$G_2(H)=-\tfrac{1}{3}H_pH. \quad (14)$$

When $p\neq 0$ analytical expressions can also be found but the resulting expressions are too cumbersome to reproduce them here.

B. Results

In order to see how the consideration of nonconstant-field gradient (NCFG) affects the results, we first show in Fig. 2 the calculations for the case $p=0$ and for some values of the parameter H_m/H_p together with the results obtained in the previous section for constant-field gradient (CFG). We incorporate in each figure an inset showing a sketch of current penetration in the NCFG and CFG approximations. The general case $p\neq 0$ shall be discussed later, although we anticipate that the main conclusions found for $p=0$ will be valid in general.

In the NCFG approximation, H_p has to be redefined since different layers of SC will not become completely filled by current at the same height. We then arbitrarily define H_p for the nonconstant-field-gradient case as the penetration field of the bottom layer (bottom face) of the SC. When considering the CFG approximation in the previous section, we have taken the values of the field and field gradient at the bottom layer of the SC and we have regarded these values as constant for all layers. Now, in NCFG, the values of field and field gradient are different in each layer, and both are largest (in magnitude) in the bottom one (since we have chosen parameters for the PM that do not provide a minimum in the derivative of field; see Ref. 19). In other words, we use in the NCFG approximation the value $M(z)dz$ instead of the values $M(z_0)dz$ used in the CFG, the z_0 being the height of the bottom layer. As a consequence, when magnetization is increasing with field [that is, when $H_z(z)<H^*$] the layer that has a larger contribution to the total force is the bottom one and thus, since the total force comes from the addition of the contributions from all layers, the force calculated in the NCFG will be lower than the one calculated in the CFG approximation.

Figure 2 shows that the general behavior when considering the NCFG is that the force is effectively an average along the length of the SC of the force calculated using the CFG, as can be also noticed from Eq. (9). In Fig. 3, we show the results for the nonconstant-field-gradient approximation for different values of the parameters p and H_m/H_p. Due to this kind of average in the force, it can be seen that if force in the CFG does not present a peak at an intermediate height (Fig. 2), the force in the NCFG will always be lower in magnitude for the descending branch and for distances far from the magnet (low fields) in the ascending branch. In fact, when using the NCFG one can never reach the maximum force calculated using the CFG. If the peak in the CFG is not very sharp, the averaging made in the NCFG makes the peak disappear [see Figs. 1(b) and 3(c), for the case $p=1$]. When the peak in the CFG is very sharp, the force in the NCFG also has a peak in the descending branch [see Figs. 1(b) and 3(c) for the case $p=100$], although the averaging tends to smooth it.

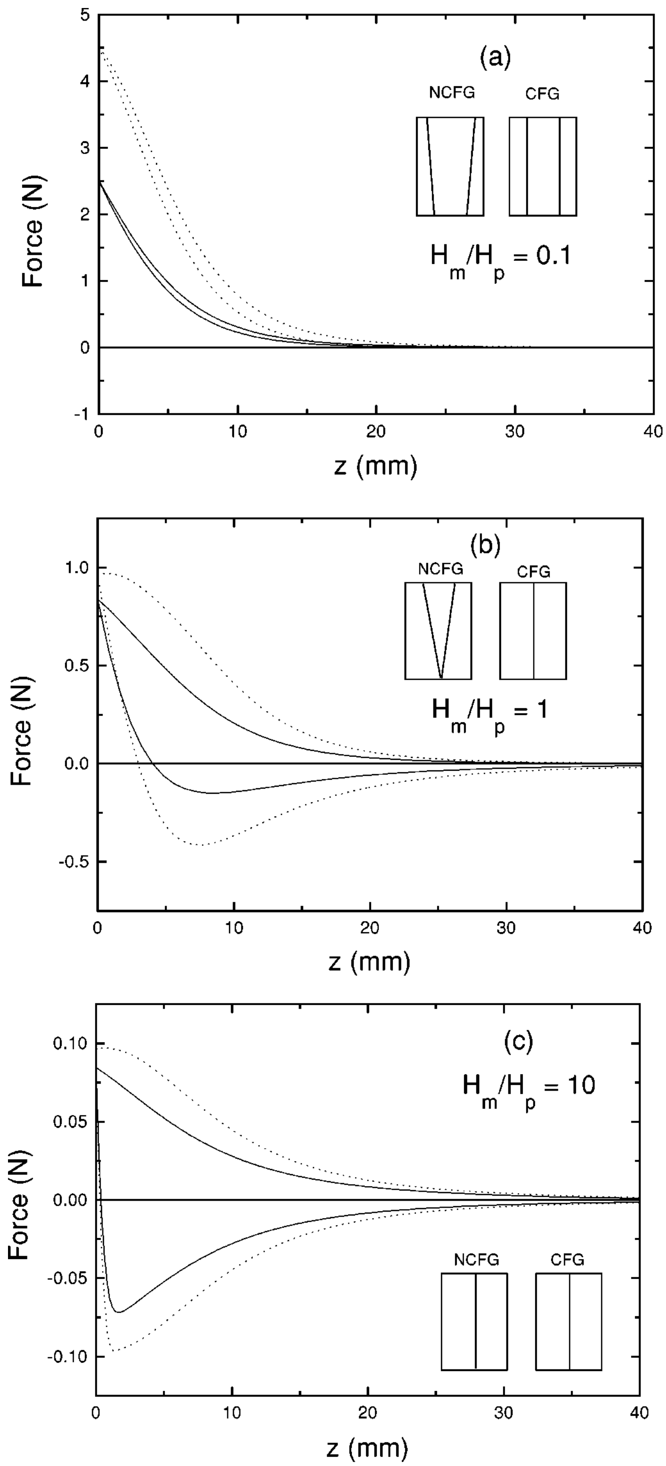

FIG. 2. Comparison of the vertical levitation force vs SC height calculated using the constant-field-gradient approximation (dotted lines) and the nonconstant-field-gradient approximation (solid lines) in the case of $p=0$ and for (a) $H_m/H_p=0.1$, (b) $H_m/H_p=1$, (c) $H_m/H_p=10$. The sketches show schematically the current penetration profile for each case.

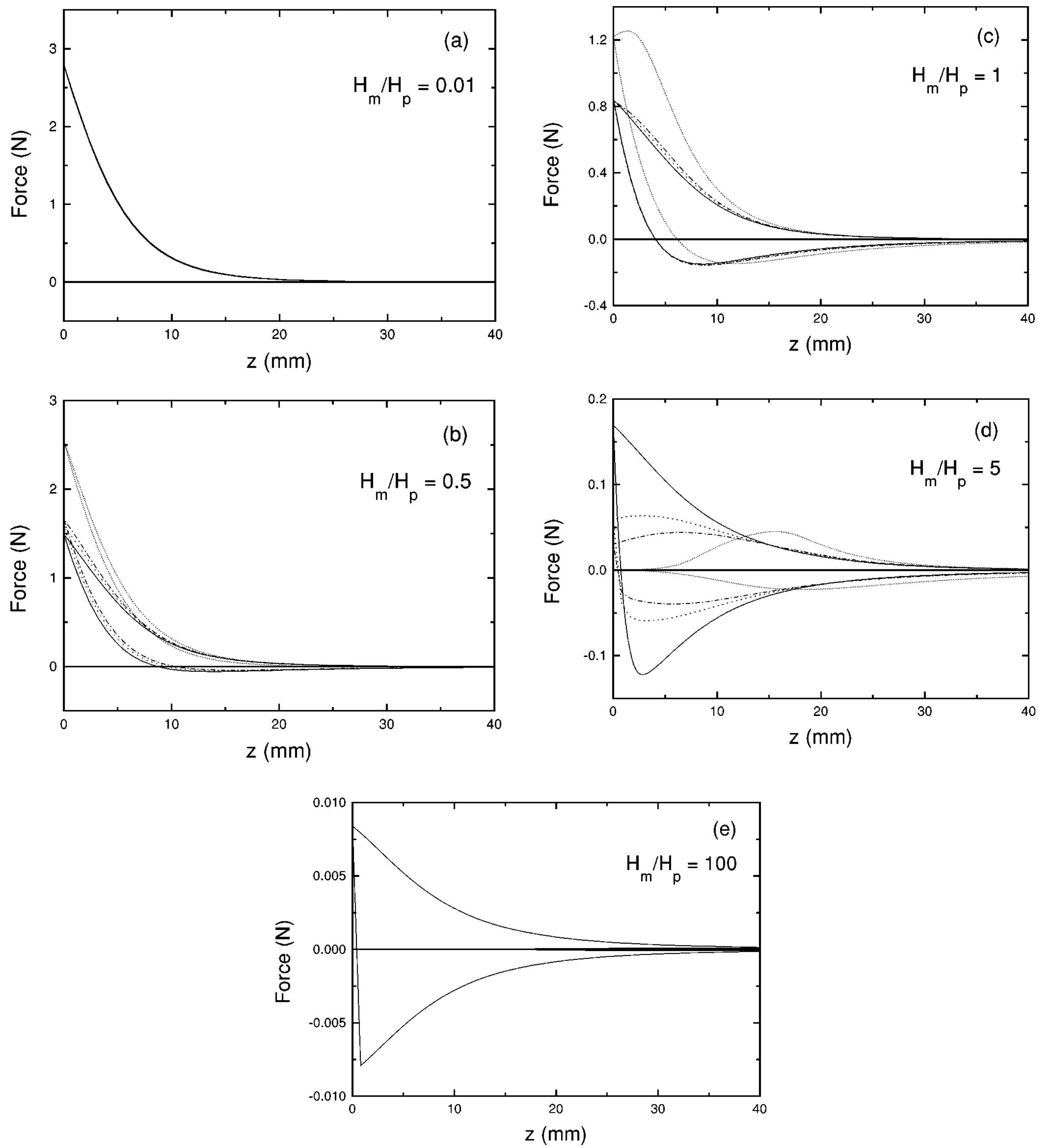

FIG. 3. Vertical levitation force as a function of SC height using the nonconstant-field-gradient approximation for different values of p and H_m/H_p: (a) $H_m/H_p=0.01$, (b) $H_m/H_p=0.5$, (c) $H_m/H_p=1$, (d) $H_m/H_p=5$, (e) $H_m/H_p=100$. The values of p are 0 (solid line), 0.5 (dotted line), 1 (dot-dashed line), and 100 (short-dotted line).

C. Comparison with experimental results

It can be seen from Fig. 3 that the levitation behavior in the full NCFG approximation depends widely on the values of the parameters p, H_m, and H_p. However, in practice the parameters vary over a narrower range than that studied here. H_p is usually of the order of H_m for typical PM of about 1 T. For a typical high-T_c superconductor, p ranges between 1 and 10.[30] For this reason, the most likely behavior should be similar to that in Fig. 3(c). The experimental results shown in Refs. 33, 17, and 12 are well represented by our calculations in this range.

If the critical current is high enough the force would be almost nonhysteretic and should have a large value. On the other hand, when the maximum field-penetration field ratio is high the descending branch of the force would be almost symmetrically opposed with the ascending one. For bulk samples, the latter case is hard to find in experiments, but this behavior is experimentally found for thin films.[12] The reasons for this have been discussed in Sec. III and in Ref. 32. The experimental data of Refs. 12 and 34 clearly show the predicted symmetric behavior of the force curve.

In Fig. 3(d), we have shown some cases in which a peak in the descending branch appears. As far as we know, no experimental results have been published showing this. We see that a significant peak in the descending branch would be observable for $H_m/H_p \simeq 5$ and $p \simeq 1$ or larger. With a typical $H_p \simeq 100$ kA/m, the peak would appear for fields $H_z \simeq 500$ kA/m, which needs a PM of $M \simeq 1.5$ T, clearly above the range used in the experiments realized until now. In this case, however, the levitation force would not be as large as in the common experiments.

D. Experimental estimation of parameters

In the previous sections we have studied how the shape of the vertical levitation force curves depends on the key parameters, and found how these curves reproduce well experimental data. The model can be used in another way as follows.

Given an actual experiment for which the assumptions are fulfilled, the model can be used to obtain a good estimation for the parameters p and H_p of the sample, which characterize the critical current of the SC and its intrinsic field dependence. The process would be as follows. The maximum applied field H_m and the dimensions of the PM and SC are known for the particular experimental setup. From the hysteresis of the experimental data the penetration field H_p can be estimated from Fig. 3. As shown in Figs. 3(a)–3(c), if $H_p \geqslant H_m$ a change in p is not very important (except when p is very high, a case seldom encountered for actual high-T_c superconductors; the estimation would become more complicated in this case); if $H_p < H_m$, the p parameter is relevant for the results and can be estimated from the position of the peak or from where an inflection in the descending branch of the force is appreciable, as can be seen in Figs. 3(d) and 3(e).

V. CONCLUSIONS

We have presented some fundamental equations describing the levitation force of type-II superconductors. The critical-state model has been used to model the penetration of currents inside the superconductors. From the resulting magnetization we have calculated the levitation force that the field created by a permanent magnet provides.

Assuming that the field gradient is constant along the length of the superconducting sample, the equation for the levitation force is simplified, which makes it easier to see the effect of the dependence of the critical current J_c on the internal magnetic field. We have seen that, when taking into account the field dependence of J_c, the force takes larger or smaller values at a given height depending on the value of the parameter p, which characterizes the dependence of the critical current on field, and the ratio between the applied and the penetration fields H_m/H_p. The limiting cases of $H_m \gg H_p$ and $H_m \ll H_p$ show that force can be almost symmetrically opposed in the descending-ascending process and almost nonhysteretic, respectively. Depending also on the value of p, a peak in the descending branch of the force can appear. The exact value of p controls the sharpness of that peak. In the limit $p=0$ we find again the result for Bean's model.

In the most general case, it is necessary to consider the variation of field and field gradient along the length of the superconductor. We have seen that in this case the force can be calculated considering the superconductor as composed of different layers at different heights, each one following the conventional critical-state model. Results for this case are similar to the constant-field-gradient approximation ones, but the force curve is now smoothed due to an integration along the sample. In particular, the peak in the descending branch would be present only for nonrealistic values of the parameters.

We believe that the present treatment is useful not only for giving a theoretical understanding of the magnetic levitation of a superconductor, but also for presenting a method to estimate material parameters of superconductors in a nondestructive way.

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