One or two transition temperatures in high-T_c cuprates: Real or complex hybrid pairings at low temperature

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We explore the number of superconducting temperatures which may be expected in high-T_c cuprates, depending on the possible attractive parts in the pairing interaction and the presence or absence of anisotropy. We also study the subsequent gap shape at 0 K. The possibility of a complex hybrid pairing $(d+is)$, at $T=0$ K, while a real one $(d+s)$ occurs near T_c, is examined for the bulk of the material and also in connection with recent experiments at the surface. [S0163-1829(98)05237-0]

I. INTRODUCTION

Twelve years after the discovery of high-temperature superconductivity[1] a somewhat general consensus has emerged, based on a variety of experiments, that the superconductivity in most of these compounds is a singlet one and essentially of d-wave type,[2] although the very source of that superconductivity is not yet completely clarified. A natural form for the pairing interaction thus consists in an expansion in terms of partial waves (d and s wave at least). A mixture of both d and s waves has been shown to be possible by group-symmetry arguments.[3] It has been suggested that the anisotropic structure of these layered compounds yields an anisotropy in the pairing interaction.[4,5] However, the superconductivity is believed to reside within the CuO_2 planes and to propagate along the **c** direction by Josephson tunneling. But even within the CuO_2 planes, some of these compounds are structurally anisotropic, the archetype being the orthorhombic YBCO(123) and YBCO(124), for which a two-dimensional $d+s$ model has been proposed[5] with a gap shape of the form

$$\Delta(\mathbf{k})=\Delta[\cos(2\phi)+r], \tag{1}$$

where ϕ is the angle between the momentum $\mathbf{k}$ and the reference axis. $r=0$ leads to the pure d-wave case studied, for instance, in Ref. 6; r represents a small amount of s-wave admixture in the $d+s$ model, assuming that no other higher order partial wave contributes. The gap shape (1) is a four leaf clover (when $|r|<1$), as in the pure d-wave one, but the size of the leaves in one direction is different from the size of the leaves in the perpendicular direction so that the nodes occur at angles slightly different from the diagonal directions. The $d+s$ model proposed in Ref. 5 followed from a two-dimensional anisotropic pairing interaction containing an attractive part in the d channel

$$V(\mathbf{k},\mathbf{k}')=(1/N_0)\{-2\lambda[\cos(2\phi)\cos(2\phi') + g(\cos(2\phi)+\cos(2\phi'))]+\mu\}. \tag{2}$$

ϕ and ϕ' are the azimuthal angles of, respectively, $\mathbf{k}$ and $\mathbf{k}'$. μ is the Coulomb repulsion in the s channel, N_0 is the density of states in the normal phase, supposed to be constant (later on, a more elaborate anisotropic density of states was considered in Refs. 7). g measures the degree of anisotropy, supposed to be small; its sign may be positive or negative.

The above interaction induces a hybrid $d+s$ pairing with a unique transition temperature. The existence of only one T_c follows straightforwardly since only one kind of attractive potential is considered in Eq. (2) in the d channel $(-2\lambda<0)$. On the other hand, the anisotropy automatically implies a mixed pairing, as was clear from Ref. 4. However recent experiments[8,9] seemed to detect the presence of a second transition below the observed T_c in YBCO compounds, possibly attributable to the chains. It is not yet clear whether these results prove the existence of a second superconducting transition or whether this is due to possible inhomogeneities or sample preparation dependence.[10] Our aim here is not to prove or disprove a separate superconductivity due to the chains in YBCO; we have presented earlier[5(b)] a discussion of various scenarios concerning that matter, from which one can only conclude that the role of the chains is still not resolved.

The purpose of this paper is rather, given an interaction more general than Eq. (2), containing or not an anisotropy and possibly more than one attractive source, to study how many transition temperatures can be expected and what would be the subsequent gap shape at the transition temperature(s) and at 0 K. This is analogous to finding what is the 0 K multicomponent order parameter of a ferrimagnetic material or of crystals which are altogether ferroelectric and ferromagnetic. Note that the two superfluid transitions found in liquid ^{3}He (Ref. 11) at elevated pressure are different from what we wish to discuss here. In contrast, our discussion may be relevant for the two superconducting transitions found in UPt_3 in zero magnetic fields.[12] We will, in particular, emphasize the fact that the presence of two T_c is only governed by the presence of two attractive potentials of different symmetries. In other words, the existence of a mixed pairing does not necessarily implies two transition temperatures. An anisotropy in the interaction potential is sufficient to induce a mixed pairing in the whole superconducting phase, even in the cases where only one T_c occurs.[4,5] In contrast, in the absence of anisotropy in the interaction and if two T_c arise, one will get a mixed pairing only below the lower T_c. We will also examine the possibility of having a

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complex hybrid pairing ($d+is$ or $s+id$) at $T=0$ K in cases for which, near T_c the pairing is real ($d+s$ or $s+d$).

II. THE MODEL INTERACTION

We consider, in two dimensions, an interaction of the form

$$V(\mathbf{k},\mathbf{k}')=-V\{a\ \cos(2\phi)\cos(2\phi')+b[\cos(2\phi)+\cos(2\phi')]+c\}, \tag{3}$$

where V is a positive constant; a, b, and c are constants of arbitrary signs, (while, in Eq. (2), a was >0 and c was <0). Then, in the simplest weak-coupling BCS-type formalism, T_c can be obtained from the normal phase, looking for the poles in the infinite ladder particle-particle correlation function

$$\Gamma(\mathbf{k},\mathbf{k}')=\Gamma^{(0)}(\mathbf{k},\mathbf{k}')+\Gamma^{(1)}(\mathbf{k},\mathbf{k}')+\cdots, \tag{4}$$

where $\Gamma^{(0)}(\mathbf{k},\mathbf{k}')\equiv V(\mathbf{k},\mathbf{k}')$ represents one line of interaction in the ladder, $\Gamma^{(1)}(\mathbf{k},\mathbf{k}')$ contains two lines of interaction, etc. The integrals over the one-electron Green's functions kinetic energies and over the angles can be decoupled if $|\mathbf{k}|=|\mathbf{k}'|=k_F$, the Fermi momentum. Then the above series (4) can be exactly summed by elementary algebra and one finds

$$\Gamma(\mathbf{k},\mathbf{k}')=-V\{A\ \cos(2\phi)\cos(2\phi')+B[\cos(2\phi)+\cos(2\phi')]+C\}. \tag{5}$$

A, B, and C are solutions of the following set of coupled equations:

$$A=[a+bYB]/[1-(aY/2)],$$
$$B=b[1+(YB/2)]/[1-cY]\equiv b[1+YC]/[1-(aY/2)], \tag{6}$$
$$C=[c+(bYB/2)]/[1-cY].$$

$Y=N_0V\ \ln[1.134\omega_0/T]$, where T is the temperature; ω_0 is an energy cutoff, and N_0 is the normal phase density of states (see the remarks below). The system of Eqs. (6) is easily solved as

$$A=\frac{1}{D}\left[\frac{a+(b^2-ac)Y_1}{Y-Y_1}-\frac{a+(b^2-ac)Y_2}{Y-Y_2}\right],$$
$$B=\frac{b}{D}\left[\frac{1}{Y-Y_1}-\frac{1}{Y-Y_2}\right], \tag{7}$$
$$C=\frac{1}{D}\left[\frac{c+(b^2-ac)Y_1/2}{Y-Y_1}-\frac{c+(b^2-ac)Y_1/2}{Y-Y_2}\right],$$

where $Y_{1,2}$ are solutions of the equation $[(a/2)-b^2]Y^2-[(a/2)+c]Y+1=0$ and read

$$Y_{1,2}=\frac{(a/2)+c\pm D}{ac-b^2} \tag{8}$$

with

$$D=\{[(a/2)-c]^2+2b^2\}^{1/2}. \tag{9}$$

One can then write Eq. (5) as

$$\Gamma(\mathbf{k},\mathbf{k}')=-\frac{V}{D}\left\{\frac{(a/2)-c-D}{Y-Y_1}[\cos(2\phi)+r_1][\cos(2\phi')+r_1]-\frac{(a/2)-c+D}{Y-Y_2}[\cos(2\phi)+r_2][\cos(2\phi')+r_2]\right\} \tag{10}$$

with $r_{1,2}$ given by

$$r_{1,2}=\frac{b}{(a/2)-c\mp D}=\frac{1}{2b}\left[c-\frac{a}{2}\mp D\right]. \tag{11}$$

The poles $Y=Y_1$ and $Y=Y_2$ will give 2, 1, or 0 transition temperatures depending whether Y_1 and Y_2 are both positive or just one of them is, or none of them. It is clear from Eq. (8) that the corresponding T_{c1} and T_{c2} contain mixed contributions of the different symmetries. (In the case where both poles contribute, the temperature where superconductivity will first appear will, of course, be given by the highest one). The gap shapes are given by the numerators in Eq. (10); they are of the form (1) with, close to T_c, the value of r, $r_{1,2}$ (corresponding to the poles $Y_{1,2}$, respectively), given by Eq. (11).

Two remarks must take place here. When T decreases below the first encountered T_c, say T_{c1}, the superconducting gap which develops below T_{c1} induces a modification in the density of states. Therefore, in principle, the next transition temperature T_{c2} should be expressed in terms of this modified density of states and not N_0. However one can check that only minor changes would result so that we will take the same density of states N_0 in the expressions of both T_{c1} and T_{c2}. Moreover the question of a common cutoff ω_0 in Y_1 and Y_2 may also be questioned. Indeed if two attractive potentials of different symmetries are present, they may involve different characteristic energies, say ω_0' and ω_0''. However, since, as clear from Eq. (8), T_{c1} and T_{c2} result from a combined effect of these two sources of attractive interactions of different symmetries, one can reasonably choose a common average cutoff ω_0.

On the other hand, one can also calculate analytically the gap at $T=0$ K. Separating again the integrals over the one-electron kinetic energy and the angular ones,[5(a)] one gets

$$\Delta(\mathbf{k})=\Delta[\cos(2\phi)+r]$$

$$=N_0V\int_0^{2\pi}\frac{d\phi'}{2\pi}\,[a\,\cos(2\phi)\cos(2\phi')+b(\cos(2\phi)$$

$$+\cos(2\phi'))+c]\Delta[\cos(2\phi')$$

$$+r]\ln\left(\frac{2\omega_0}{\Delta(r+\cos 2\phi')}\right), \tag{12}$$

which yields the set of two coupled equations

$$1=\left(\frac{\bar{a}}{2}+\bar{b}r\right)J_0+(\bar{a}r+\bar{b})J_1+\frac{\bar{a}}{2}J_2, \tag{13a}$$

$$r=\left(\frac{\bar{b}}{2}+\bar{c}r\right)J_0+(\bar{b}r+\bar{c})J_1+\frac{\bar{b}}{2}J_2, \tag{13b}$$

with

$$N_0Va=\bar{a},\quad N_0Vb=\bar{b},\quad N_0Vc=\bar{c} \tag{14}$$

and

$$J_n=\int_0^{2\pi}\frac{d\psi}{2\pi}\cos(n\psi)\ln\left(\frac{2\omega_0}{\Delta(r+\cos\,\psi)}\right),$$

$$\psi=2\phi,\quad n=1,2,3\ldots. \tag{15}$$

The integrals J_n have been computed[5(a)] (supposing that r is a real quantity), and their analytical expressions depend whether $|r|$ is smaller or larger than 1. For convenience, we recall these expressions here: for $0<|r|<1$, $J_0=\ln(4\omega_0/\Delta(T=0))$, $J_1=-r$, $J_2=r^2-1/2$; for $|r|>1$, $J_0=\ln\{[4\omega_0/\Delta(T=0)][|r|-\sqrt{r^2-1}]\}$, $J_1=-\operatorname{sgn} r\{|r|-\sqrt{r^2-1}\}$; $J_2=[1/2]\{|r|-\sqrt{r^2-1}\}^2$. Solving the system of equations (13) gives r and $[\omega_0/\Delta(T=0)]$, at $T=0$ K. We now examine specific cases.

III. RESULTS

A. In the absence of anisotropy

This is the case of tetragonal cuprates which are isotropic within the CuO_2 planes. No anisotropy means $b=0$; it then follows from Eq. (6) that $B=0$ and the system reduces to two decoupled equations: $A=a/[1-(aY/2)]$, $C=c/[1-cY]$. We then get three possible cases (we discard the case where $a<0$ and $c<0$ for which the denominators of A and C cannot vanish so that no superconductivity is induced).

(1) a<0, c>0. Only the last term in Eq. (5) is negative corresponding to the standard s-wave BCS superconductivity where T_{cs} is given by

$$1/\bar{c}=\ln(1.134\omega_0/T_{cs}) \tag{16}$$

and the gap is a constant $\Delta(\mathbf{k})=c$.

(2) a>0, c<0. The first term in Eq. (5) is the only negative one corresponding to pure d-wave superconductivity, with T_{cd} given by

$$2/\bar{a}=\ln(1.134\omega_0/T_{cd}) \tag{17}$$

and the gap has the form $\Delta(k)=a\,\cos(2\phi)$ with $r=0$. This is the case studied, for instance, in Ref. 6.

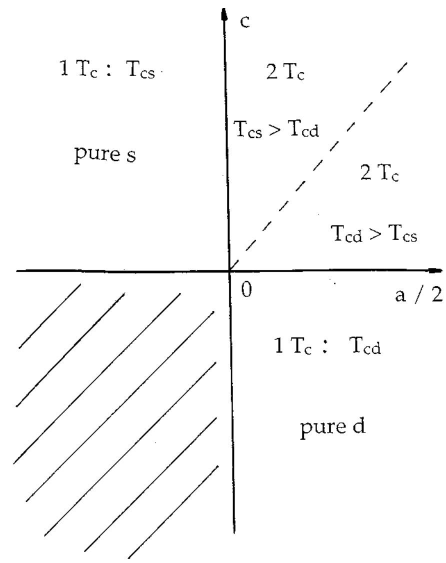

FIG. 1. In the absence of anisotropy $(b=0)$, the number of the expected T_c in the various regions of the $(a/2,b)$ plane [see Eq. (3)], near the transition temperature.

In this case, at $T=0$ K, one gets, from Eq. (17) and Eq. (13a) (with $\bar{b}=0, J_1=0$ and $J_2=-1/2$), $[\Delta(T=0)/T_{cd}]=2.139$, in agreement with formula (3) in Ref. 6. Since in the s-wave BCS case one has $[\Delta(T=0)/T_{cs}]_{\rm BCS}=1.76$, one gets here by comparison, $\Delta(T=0)/T_{cd}=1.215[\Delta(T=0)/T_{cs}]_{\rm BCS}$.[13]

We emphasize that in both cases (1) and (2), only one attractive contribution, of a given symmetry, to the pairing potential is involved either in the s or the d channel resulting into only one transition temperature.

(3) a>0, c>0. This case is new. One has two attractive contributions, of different symmetries, in the pairing potential, one in the d channel and another one in the s channel. Then two transition temperatures follow given by

$$1/\bar{c}=\ln(1.134\omega_0/T_{cs}),$$

$$s\text{-wave gap near } T_c,\quad \Delta(\mathbf{k})=c, \tag{18a}$$

$$2/\bar{a}=\ln(1.134\omega_0/T_{cd}),$$

$$d\text{-wave gap near } T_c,\quad \Delta(\mathbf{k})=a\,\cos(2\phi). \tag{18b}$$

Depending whether $\bar{c}$ is larger or smaller than $(\bar{a}/2)$, T_{cs} or T_{cd} is first encountered when T decreases. Between the higher T_c and the lower one, one has thus only one type of superconductivity and thus either a purely s or a purely d type of gap. But below the lower T_c, one will have a mixture of the two types of superconductivity, with a gap of the general shape (1), in particular at $T=0$ K. The above cases are summarized in Fig. 1.

We next examine the $T=0$ K region, with the gap of the form $\Delta(\mathbf{k})=\Delta(T=0)\,[\cos(2\phi)+r(T=0)]$ where $r(T=0)$ is finite.

(i) Let us first assume that the superconductivity is mainly of d-wave type with a small s-wave component so that $|r|<1$. Then (with the help of the corresponding values of J_0, J_1, and J_2 given above), Eq. (13) reads

$$2/\overline{a} = J_0 - r^2 - 1/2, \tag{19a}$$

$$1/\overline{c} = J_0 - 1 \tag{19b}$$

with $J_0 = \ln[4\omega_0/\Delta(T=0)]$ [noting that $r=0$ is excluded here, so that Eq. (13b) simplifies to yield Eq. (19b)]. Then, from Eq. (19), and using Eq. (18), it follows that

$$r^2(T=0) = \ln(1.649\ \ T_{cd}/T_{cs}),$$
$$\text{with}\ \ \Delta(T=0)/T_{cs} = 1.298. \tag{20}$$

Here $[\Delta(T=0)/T_{cs}] = 0.737[\Delta(T=0)/T_{cs}]_{\mathrm{BCS}}$. On the other hand, since we used to start with the J_n corresponding to $0<|r|<1$, then, this implies that $0.606<(T_{cd}/T_{cs})<1.649$. More precisely, within this range, and subtracting Eq. (19a) from Eq. (19b), one finds

$$0<r^2(T=0)<1/2 \quad \text{for}\ \ 0.606<T_{cd}/T_{cs}<1, \tag{21a}$$

$$1/2<r^2(T=0)<1 \quad \text{for}\ \ 1<T_{cd}/T_{cs}<1.649 \tag{21b}$$

[in the particular case where $T_{cd}=T_{cs}$, then $r^2(T=0)=1/2$].

When $(T_{cd}/T_{cs})<0.606$, $r^2(T=0)<0$, $r(T=0)$ is purely imaginary and one obtains at $T=0$ K, a $d+is$ pairing with a gap which cannot vanish. We will come back to this point in Sec. IV with the Appendix.

(ii) Now let us examine the case $|r|>1$. Then, here too, the gap (1) never vanishes. We solve the system (13), with $b=0$ and the appropriate values of the expressions J_n. We then find, with the help of Eq. (18)

$$\ln\left(\frac{T_{cd}}{T_{cs}}\right) = [|r| - \sqrt{r^2-1}]\left\{\frac{2r^2-1}{|r|} - \frac{1}{2}[|r| - \sqrt{r^2-1}]\right\} \tag{22}$$

and

$$\ln\left(0.283\,\frac{\Delta(T=0)}{T_{cs}}\right) = \ln[|r| - \sqrt{r^2-1}] - 1 + \frac{\sqrt{r^2-1}}{|r|}. \tag{23}$$

One verifies, in these equations, that T_{cd} is indeed larger than T_{cs} and one gets $[\Delta(T=0)/T_{cs}]<3.533$. These $T=0$ K results are summarized in Fig. 2.

To conclude on this particular case (3), we get two transition temperatures, a purely s one and a purely d one, arising in an order which depends on the ratio $2c/a$ compared to 1. But below the lower transition temperature, an hybrid pairing develops and, at $T=0$ K, the gap is a hybrid $d+s$ one, of the form (1), with a value of $|r|$ which is smaller or larger than 1/2 depending whether T_{cd} is smaller or larger than T_{cs}. In other words, it is important to note that, at $T=0$ K, a small s-wave component or a larger one is governed by the nature of the T_c which is the closest to 0 K: $0<r^2<1/2$, i.e., a hybrid $d+s$ pairing with a dominant d-wave component occurs at $T=0$ K if T_{cd} is closer to 0 K than T_{cs}; instead $r^2>1$ is accompanied by a $s+d$-wave pairing at $T=0$ K, with a dominant s-wave component, when T_{cs} arises closer to 0 K than T_{cd} and is far from it. Moreover, when $2c/a \gg 1$, i.e., T_{cs} is much higher than T_{cd}, $r(T=0)$ is imaginary and one could expect, at $T=0$ K, a complex $d+is$ pairing.

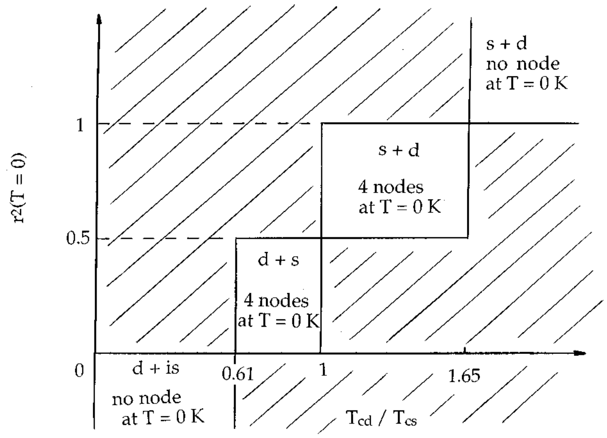

FIG. 2. In absence of anisotropy ($b=0$) and when $a>0$, $c>0$, in Eq. (3), the pairing symmetries expected at $T=0$ K in the $[r^2(T=0),\ T_{cd}/T_{cs}]$ plane, indicating also the presence or absence of nodes in the gap.

However, to better explore this region, one should reconsider the integrals (15) when r is a complex quantity (see the discussion in Sec. IV).

B. In the presence of anisotropy

This is the case of orthorhombic cuprates or, more generally, those compounds which exhibit an anisotropy within the CuO_2 planes. In this case, $b\neq 0$ and $B\neq 0$; the T_c's are given by the poles $Y=Y_{1,2}$ in Eq. (10), i.e., one does not have a purely d- or purely s-wave superconductivity, the anisotropy always yields a mixed pairing.[4] It may appear convenient to rewrite Eq. (3) as

$$V(\mathbf{k},\mathbf{k}') = -V\{a[\cos(2\phi)+b/a][\cos(2\phi')+b/a] + [ac-b^2]/a\}, \tag{24}$$

where the last term represents an ''effective'' interaction in the s channel.

Here too we examine various cases. (We discard the unphysical case of an anisotropy so strong that $|b|>|a|$ and $|b|>|c|$).

(1) $a<0$ and $c<0$ or, equivalently, $a<0$ and $(ac-b^2)/a<0$. We have no attractive contribution to the pairing interaction and thus no T_c.

(2) $a<0$ and $c>0$ with $(ac-b^2/a>0$. There is one attractive contribution (in the s channel) and thus one transition temperature.

(3) $a>0$ and $c<0$ with $(ac-b^2)/a<0$. Here one has one attractive contribution (in the d channel) and thus one transition temperature. This is the case studied in Refs. 5.

(4) $a>0$ and $c>0$ or, equivalently, $a>0$ and $(ac-b^2)/a>0$. We have two attractive parts in the pairing interaction, one in the d channel and one in the s channel. We therefore get two possible transition temperatures.

The above four cases are illustrated in Fig. 3 which separates, in the $(a/2,c)$ plane, the various regions with their pairing symmetries and the number of corresponding T_c. Note that the first bissectrix corresponds to the separatrix where $a/2=c$ and $r^2=1/2$, which separates the region $d+s$ where the d wave is dominant from the region $s+d$ where

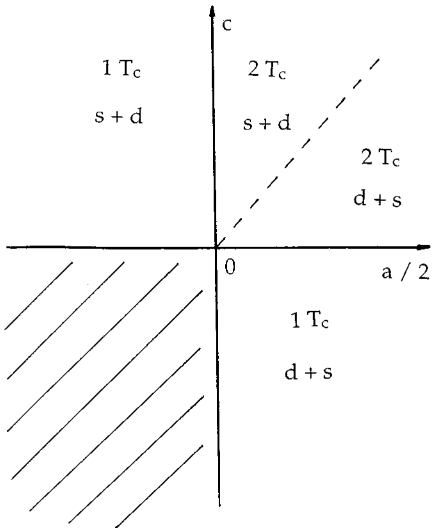

FIG. 3. In the presence of anisotropy ($b \neq 0$) and in the ($a/2$,c) plane [see Eq. (3)], the expected various pairing symmetries near the transition temperature and the corresponding numbers of T_c.

the s wave is dominant. One can indeed easily verify that, when the anisotropy b decreases toward 0, r^2 decreases in the first region with a tendency to pure d-wave behavior, while it increases in the latter one toward a pure s-wave one. However let us repeat here that, in all these various regions, we have hybrid pairings and that the corresponding transition temperatures reflect this mixture.

Now, at $T=0$ K, one gets a mixed pairing of the form (1); solving Eq. (13), we find

$$\overline{b}(1-2r^2)+2\left(\overline{c}-\frac{\overline{a}}{2}\right)r+(\overline{ac}-\overline{b}^2)[(1-2r^2)J_1-rJ_2]=0, \tag{25}$$

where J_1 and J_2 are functions of r. Equation (25) will give r as a function of the three parameters $\overline{a}$, $\overline{b}$, and $\overline{c}$. In any case r will be finite, in general, and thus the gap shape will be of the form (1); we will always have a mixed pairing both at the transition temperature and at 0 K. However, in this case, with the superconductivity first appearing at T_{c1}, there may be some sign of the existence of T_{c2}, at a temperature lower than T_{c1}, with some accident in the T dependence of the physical properties. This may be the case, for instance, if the problem contains one main source of attraction in the d channel, possibly due to antiferromagnetic spin fluctuations[14] and another one in the s channel, possibly coming from the standard electron-phonon origin, but weaker, so that the corresponding transition temperature occurs below the latter one. We do not pretend that this is the case in the cuprates; our purpose is just to show what are the consequences of such a possibility, which remains to be checked by the experiments.

One last remark: Equation (25) becomes an equation of the third degree in r if J_1 and J_2 are computed for $|r|<1$. Such an equation is known to possibly exhibit complex roots depending on the values of the parameters. However, in such a case (as in the last section), one should recompute the J_n's, assuming, to start with, that r is a complex quantity (see the Appendix). This would allow us to better explore the possibility of getting a hybrid complex pairing of the $d+is$ type, for instance, at $T=0$ K, although, as seen above, the hybrid pairing is always real and of the $d+s$ type near T_c, whatever the sign of the parameters.

IV. DISCUSSION

To conclude, with a pairing interaction expanded in terms of s and d components, containing or not an anisotropy, we have analyzed a number of situations depending on the signs of the involved parameters. We have recovered some known results and studied new ones. The number of transition temperatures is intimately linked to the number of fermion-fermion attractive parts of different symmetries involved in the pairing potential. In the presence of anisotropy, one always get a mixed pairing in the whole temperature range of the superconducting phase, independently of the number of attractions. In the absence of anisotropy and if more than one attractive interaction is involved, one also has a mixed pairing but only starting below the lower T_c; between this one and the highest one, one gets either a pure s- or a pure d-wave gap shape. In other words, the presence of anisotropy is a sufficient condition to get a hybrid pairing in the whole superconducting phase, but it is not a necessary one. The existence of two transition temperatures, in the absence of any anisotropy, also imposes a hybrid pairing, although only at very low temperature, below the lowest T_c. Note that the value of r varies between $T=0$ K and T_c and may even possibly change nature as will be discussed now.

We have found, in this paper, that the calculated parameter r near T_c is always real, resulting in a hybrid pairing $d+s$ or $s+d$ which is a real quantity, and the corresponding gap exhibits four nodes (if $|r|<1$). However, at lower temperatures and, in particular at $T=0$ K, r may either remain real or become complex; in such a case, one ends up with a complex gap of the type $d+is$ or $s+id$ with no nodes. At first sight this does not apply to the hole-doped cuprates where the gap has been shown, experimentally, to exhibit nodes;[15,16] however, this has been shown to be so only at finite T and in the bulk of the material. We now elaborate more on these two restrictions.

Indeed it has very recently been demonstrated[17] that a gap with nodes (inducing a linear T dependence in the magnetic penetration depth of the clean cuprates as experimentally observed[18]) contradicts the third law of thermodynamics; it was then suggested in Ref. 17 that a real d type of pairing, at finite temperature, may switch to a complex one near $T=0$ K. This suggestion would be in agreement with the possibility that we have encountered above. Such a change could then be checked through experiments like those in Refs. 15 and 16, performed both near T_c where four nodes have already been revealed, but also near $T=0$ K where the same experiments should not detect any node.

On the other hand, one could ask whether our finding could be related to the $d+is$ pairing observed, at low temperature, at the surface of a high-T_c compound having an otherwise d or $d+s$ pairing in the bulk, like YBCO.[19] We quote here a sentence of Ref. 19: ''Andreev scattering near the surface of a $d_{x^2-y^2}$ superconductor causes strong pair breaking. The quasiparticles may then be paired by a subdominant pairing interaction that is less sensitive to surface pair breaking than the dominant d-wave one.'' We also quote a sentence of Ref. 20: ''The surface state of any $d_{x^2-y^2}$ superconductor will exhibit a spontaneously broken time-reversal symmetry phase at sufficiently low temperature.'' Given these claims based on earlier theoretical works referred to in Refs. 19 and 20 and based also on the experimental work of Ref. 19, we have shown here that, starting with a $d+s$ pairing near T_c, we may end up at $T=0$ K, with possibly, a $d+is$ pairing depending on the values of the parameters. Modifications in these values could thus occur at the surface of the compound, compared to the bulk, because of the pair-breaking reason invoked in Ref. 19 and quoted above.

One can also wonder whether the cuprates which, so far, do not exhibit nodes in the gap at finite T, like the electron-doped Nd compounds,[21] might correspond to some of the cases studied here, where, in addition to a possible d-wave component, an s-wave one is dominant [$|r|>1$ in Eq. (1) so that $\Delta(\mathbf{k})$ never vanishes]. This could occur, for instance, if the following picture could be valid for this compound which is known to be highly disordered.[21(b)] It has been shown in Ref. 5(a), within the $d+s$ model corresponding to the pairing (2), that a sufficient amount of nonmagnetic impurities, which are pair breaking for d and $d+s$-wave superconductivity, yields the opening of a gap and a strong increase of the effective value of r at $T=0$ K; in that case the balance between the d and s components changes drastically. This occurs for an impurity concentration close to, but smaller than, the one where T_c vanishes and one must remember that the T_c in the Nd compound is about four times lower[21(a)] ($\sim$21–22 K) than those of the hole-doped cuprates, so that the above scenario may possibly apply.

Finally also, if in the future, one gets experimental strong evidence that, for instance, two different sources of attractive potential play a role, then according to our results, one should expect, below the highest T_c, some signs of the existence of a lower one in the temperature dependence of the physical properties. Whether this is the case in the experiments of Refs. 8 and 9 is still an open question.

APPENDIX: THE INTEGRALS J_1 AND J_2 WHEN THE PARAMETER r IS COMPLEX

One can easily show that these integrals are given by

$$J_1=-r+(r^2-1)K, \quad J_2=-1/2+r^2-r(r^2-1)K, \tag{A1}$$

where

$$K=\frac{1}{2\pi}\int_0^{2\pi}\frac{d\psi}{r-\cos\psi} \quad \text{with} \quad r=r'+ir''. \tag{A2}$$

K can be computed[22] and reads

$$K=\frac{1}{\sqrt{r^2-1}}=\frac{1}{X_1+iX_2} \tag{A3}$$

with

$$X_{1,2}=\frac{1}{\sqrt{2}}\{\sqrt{[r''^2+r'^2-1]^2+4r''^2}\mp(r''^2-r'^2+1)\}^{1/2}. \tag{A4}$$

With the above ingredients, one can solve Eq. (25) which gives two coupled equations yielding r' and r'' as functions of the parameters a, b, and c.

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