Attenuation factors of de Haas–van Alphen oscillations in the vortex state of layered superconductors

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We consider analytically within the Bogoliubov–de Gennes and Gor'kov approaches the magnetic oscillations due to the Landau quantization [de Haas–van Alphen (dHvA) effect] in the vortex-lattice (VL) state of layered superconductors. We found that the period of the dHvA oscillations does not change when the magnetic field H decreases below the upper critical field H_{c_2}, whereas amplitudes of the dHvA oscillations are damped by the attenuation factors. These factors appear due to (a) smearing of the Landau levels by impurities and disorder of the VL, (b) broadening of the Landau levels into dispersive bands by periodic VL, periodic external magnetic field, and periodic layered structure. In case (a) the attenuation factor is a Dingle-like exponent, $R(\Delta,\tau)=R_0(\tau)R_s(\Delta)R_{0s}(\Delta,\tau)$, where $R_0(\tau)$ is the standard Dingle factor and $R_s(\Delta)$ was calculated previously by Maki and Stephen. An extra damping is due to the interference term, $R_{0s}(\Delta,\tau)=\exp(-\pi/\Omega\tau_{\rm int})$, whose dependence on the magnetic field H is determined by the cyclotron frequency, Ω, and $\tau_{\rm int}^{-1}\sim\Delta^2/v_f l_0 H$ (v_f is the Fermi velocity, $l_0=v_f\tau$ is the mean free path). In case (b) attenuation factors differ from the simple Dingle exponent and corresponding damping of dHvA oscillations basically less than in case (a), especially for fields well below H_{c_2}. In particular, the attenuation factor due to the layered structure is determined by the one-dimensional density of states $g(\varepsilon)$, related to the electron transport across the layers. This factor is a periodic function in $1/H$ with frequencies depending on locations of the van Hove singularities in $g(\varepsilon)$ and the ones caused by the stacking faults. Competition between different attenuation mechanisms results in nonmonotonous decrease of the dHvA amplitudes and makes it possible to give a qualitative explanation of recent experiments on borocarbide YNi_2B_2C where the dHvA oscillations have been observed down to surprisingly low fields about $0.2H_{c_2}$ [T. Terashima *et al.* Phys. Rev. B **56**, 5120 (1997)]. [S0163-1829(98)04034-X]

I. INTRODUCTION

Magnetic quantum oscillations, also known as the de Haas–van Alphen (dHvA) effect, have been reported in a number of different types superconductors: layered 2H-$NbSe_2$,[1–3] the strongly coupled A-15 compounds V_3Si,[4,5] Nb_3Sn,[6] Ba(K)BiO_3,[7] the organic molecular k-$(ET)_2Cu(NCS)_2$,[8] YNi_2B_2C,[9,10] the high-T_c layered YBaCuO,[11–17] and $CeRu_2$.[18] Although an observation of the dHvA effect in superconductors was made by Graebner and Robbins[19] in 1976, it took nearly two decades before undertaking systematic explorations of this phenomenon in different types of superconductors. Currently, a considerable literature on the theory of this phenomenon exists.[20–45] All theories, in full accordance with experiments, agree that the period of dHvA oscillations below the upper critical field H_{c_2} does not change. A consent has not yet been achieved about the form and mechanisms of damping of dHvA oscillations because of the rich physics beyond this damping. A decade before the experimental observation of the dHvA effect Rajagopal and Vasudevan[35] and Gunther and Gruenberg[36] found a small correction to the critical temperature T_c periodic in the inverse field $1/H$ due to the Landau quantization. Such oscillations of the thermodynamic quantity T_c near H_{c_2} is nothing but a manifestation of the dHvA effect in superconductors. The theoretical explanation of the attenuation of dHvA oscillations by the quasiparticles scattering on a random vortex lattice (VL) was given by Maki[23] and Stephen.[24] In these papers as well as in the work of Wasserman and Springford[25] the attenuation is due to the Dingle factor, $R_s=\exp(-\pi/\Omega\tau_s)$, where Ω is the cyclotron frequency and $1/\tau_s\sim\Delta^2/H^{1/2}$ (Δ is the order parameter).

Further development of the problem of the dHvA oscillations in superconductors has required an incorporation of the self-consistency of the VL into consideration. It turned out that most conveniently this might be done in the magnetic-Bloch-state representation[44] of the microscopic BCS theory. Such a representation for the Bogoliubov–de Gennes (BdG) equations has been presented by Dukan, Andreev, and Tesanovic,[35] and was developed further in other work.[20,21,36,37] The Gor'kov-equations approach to the BCS theory in the Landau levels basis was given by Rajagopal.[22] The appropriate equations for the quasiparticle energy spectrum in the vortex state turned out to be very difficult to solve exactly. It was shown in Refs. 35, 36 that near H_{c_2} (where the VL is thick) the energy spectrum is gapless at a discrete set of points on the Fermi surface. This leads to an algebraic behavior of various low-temperature thermodynamic quantities of the system in question. Another difficulty is the off-diagonal pairing. The off-diagonal, in Landau levels index, matrix elements of the order parameter Δ_{nm}, have a very complicated dependence on indices n and m, magnetic field, and VL structure which makes analytic calculations of the dHvA oscillations possible only near H_{c_2}, where one can neglect the off-diagonal pairing due to the smallness of the parameter $\Delta/\hbar\Omega\ll 1$. Dukan and Tesanovic[29] considered the

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attenuation mechanism of the dHvA oscillations near H_{c_2} in the diagonal approximation due to the gapless portion of the quasiparticle spectrum in the magnetic Brillouin zone. Numerical calculations, done by Norman, MacDonald, and Akera[21] for a two-dimensional (2D) superconductor with a regular VL, have shown that the dHvA oscillations for H below H_{c_2} are damped due to the Landau-level broadening into bands whereas frequencies of these oscillations remain intact in the vortex state.

As we see, the damping of the dHvA oscillations in the mixed state is stipulated by a number of mechanisms acting concurrently. In view of that, one cannot write down a general expression for damping, valid for every case. Rather, before comparing between theory and experiments one has to single out a major mechanism and then only use an appropriate formula. In this connection, a good quantitative agreement between BCS-based theories and experiments on strongly coupled superconductors V_3Si and Nb_3Sn [with coupling constant $\lambda > 1$ (Ref. 46)] reported in some works, raise doubts since these A-15 compounds should be described by the Eliashberg equations rather than the BdG ones valid for $\lambda \ll 1$.

On the other hand, the current state of the art in numerical methods is still far from granting the opportunity to relate first-principles calculations with experiments since model parameters for a superconductor (usually 2D) as a rule is unrealistic. In Ref. 30, for example, $\lambda = 1$ is too big for the BCS model and so is the cutoff energy, $\omega_d = 0.5\mu$, while the Fermi energy, μ, is equal to only a few tens of $\hbar\Omega$. In addition the approximation of strongly oscillating matrix elements in the pairing blocks by the same constant may overestimate contributions from the off-diagonal terms into a secular equation, since in fact they may simply cancel each other due to oscillations.

We see therefore that both analytic and numerical approaches have their weak and strong points and have to be continued to gain a better insight into the problem in question. Recent experiments[6,9,10] indicate conclusively that dHvA oscillations in superconductors persist in the vortex state down to surprisingly low fields [equal to $H \approx 0.2 H_{c_2}$ in borocarbide YNi_2B_2C (Ref. 9)]. The oscillation amplitude is strongly suppressed in a field region immediately below H_{c_2} and recovers at lower field.[9] Such a behavior implies competition between different mechanisms of damping of dHvA oscillations due to VL, layer structure, spatial periodicity of the order parameter, and external magnetic field. We also consider the role of the off-diagonal pairing in the dHvA oscillations well below H_{c_2}. We show, in particular, that random VL damp dHvA oscillations more strongly than predicted by theories of Maki,[23] Stephan,[24] and Wasserman and Springford[25] due to the Dingle-like ''interference'' exponent. The attenuation factor caused by the regular VL is not of the Dingle-exponent form and less than the ones due to the random VL.

The paper is organized as follows. In Sec. II we calculate the spectral density of a layered superconductor in the Landau-level representation in the VL state and in the case of a one-dimensional periodicity of the external magnetic field. In Sec. III we study the free energy of a layered superconductor in the VL state and focus on the different random and periodic factors which determine amplitudes of dHvA oscillations below the H_{c_2}. Section IV contains a discussion of the main results of the paper concerning the attenuation factors damping dHvA oscillations in the VL state of a layered superconductor and gives a qualitative comparison with the recent experiments in the field. In the Appendix we present in detail calculations of the quasiparticle damping due to the electron scattering on a random VL using a combination of the Green function and de Gennes correlation function methods.

II. THE ENERGY SPECTRUM AND THE SPECTRAL DENSITY OF A LAYERED SUPERCONDUCTOR IN THE VORTEX-LATTICE STATE

Recently, a good deal of progress has been achieved in the description of the VL state in 2D superconductors within the BdG approach.[20–21,30,36,37] The problem of electron scattering on VL imperfections is easier to solve with the help of the Green functions.[23–25] In the following we will resort to both methods.

To begin with these calculations, let us consider the BdG equations

$$\begin{aligned} \hat{H}u(\mathbf{r}) + \Delta(\mathbf{r})v(\mathbf{r}) &= Eu(\mathbf{r}), \\ -\hat{H}^{*}v(\mathbf{r}) + \Delta^{*}(\mathbf{r})u(\mathbf{r}) &= Ev(\mathbf{r}), \end{aligned} \tag{2.1}$$

which in the basis of eigenfunctions of the Hamiltonian of 2D electrons in an external field of the vector potential $\mathbf{A}(\mathbf{r})$ take the form

$$\begin{aligned} \sum_m [(\varepsilon_n - E)\delta_{nm}u_m + \Delta_{nm}v_m] &= 0, \\ \sum_m [\Delta^{*}_{nm}u_m - (\varepsilon_n + E)\delta_{nm}v_m] &= 0. \end{aligned} \tag{2.2}$$

The $u-v$ functions in Eqs. (2.1) and (2.2) are related by

$$u(\mathbf{r}) = \sum_n u_n \varphi_n(\mathbf{r}), \quad v(\mathbf{r}) = \sum_n v_n \varphi^{*}_n(\mathbf{r}), \tag{2.3}$$

and the matrix element of the order parameter is equal to

$$\Delta_{nm} = \int \varphi^{*}_n(\mathbf{r})\Delta(\mathbf{r})\varphi^{*}_m(\mathbf{r})d\mathbf{r}. \tag{2.4}$$

The basis functions satisfy the eigenvalue equation $\hat{H}\varphi_n(\mathbf{r}) = \varepsilon\varphi_n(\mathbf{r})$ with the Hamiltonian

$$\hat{H} = \frac{1}{2m}\left(\mathbf{P} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2 - \mu. \tag{2.5}$$

Here μ is the Fermi energy, m and e stand for the electron mass and charge, c is the speed of light.

The specific form of the $\varphi_n(\mathbf{r})$ depends on the gauge choice, whereas the Landau spectrum, $\varepsilon_N = \hbar\Omega(N+1/2) - \mu$, is the gauge invariant ($\Omega = eH/mc$ is the cyclotron frequency). In the Landau gauge, $\mathbf{A} = (0, Hx)$, which proved to be convenient in description of the VL states,[21,30] the basis functions are

$$\varphi_n(\mathbf{r}) \equiv \Psi_{NX}(\mathbf{r}') = L_y^{-1/2} \exp\left(-i\,\frac{Xy}{l^2}\right)\Psi_N\left(\frac{x-X}{l}\right), \tag{2.6}$$

where $\Psi_N(x) = [2^N N! (\pi l)^{1/2}]^{-1/2} \exp(-x^2/2) H_N(x)$ and $H_N(x)$ is the Hermitian polynomial of the order N; $l^2 = \hbar c/eH$ is the magnetic length. In the rest of the paper we will employ a complex index $(n \equiv N, X)$ composed of the two quantum numbers: the Landau-level number N, and the coordinate of the Landau orbit center, $X = -(cp_y/eH)$.

Excluding v_n from Eqs. (2.2) we arrive at the equation for u_n

$$(E-\varepsilon)u_n - \sum_k \sigma_{nk}(E) u_k = 0, \tag{2.7}$$

which has a form of the Schrödinger equation for some fictitious ''particle on a lattice'' with the hopping integrals

$$\sigma_{nk}(E) = \sum_m \frac{\Delta_{nm}\Delta^*_{mk}}{E+\varepsilon_m}. \tag{2.8}$$

It is easy to show[47] that the Green function of Eq. (2.7),

$$\sum_m [(E-\varepsilon_n)\delta_{nm} - \sigma_{nm}(E)] G_{mk}(E) = \delta_{nk} \tag{2.9}$$

is nothing but the G function of the Gor'kov equations

$$G_E(\mathbf{r},\mathbf{r}') = \sum_{m,k} G_{mn}(E)\varphi_m(\mathbf{r})\varphi^*_n(\mathbf{r}'). \tag{2.10}$$

The corresponding F function is given by

$$F^+_E(\mathbf{r},\mathbf{r}') = -\sum_{m,n,k} \frac{\Delta^*_{nm} G_{mk}(E)}{E+\varepsilon_m}\,\varphi_n(\mathbf{r})\varphi^*_k(\mathbf{r}'). \tag{2.11}$$

One can check directly that the F-G functions of Eqs. (2.10), (2.11) satisfy the Gor'kov equations in the coordinate representation. The energy spectrum of the system under consideration is determined by poles of the diagonal matrix element of the Green function (2.9) which can be written, after some calculations[47,48] in the form

$$G_{mn} = [E-\varepsilon_n - \Sigma_n(E)]^{-1}, \tag{2.12}$$

where the self-energy is given by

$$\Sigma_n(E) = \sigma_{nn}(E) + \sum_{k\neq n} \frac{\sigma_{nk}\Gamma_{kn}}{E-\varepsilon_k-\sigma_{kk}}. \tag{2.13}$$

The function Γ_{nm} satisfies the integral equation,

$$\Gamma_{nm} = \sigma_{nm} + \sum_{k\neq n,m} \frac{\sigma_{nk}\Gamma_{km}}{E-\varepsilon_k-\sigma_{kk}}, \tag{2.14}$$

which is equivalent to an infinite series in powers of σ.

Near H_{c2} the Landau-level separation is much larger than the absolute value of the order parameter, $\hbar\Omega \gg |\Delta|$, so that according to Eq. (2.8) $|\sigma_{nm}|$ is proportional (near the Fermi level $E=0$) to the small parameter $|\Delta|/\hbar\Omega \ll 1$. Taking this into consideration one can approximate the self-energy by the first term in Eq. (2.13),

$$\Sigma_n(E) \approx \sigma_{nn}(E). \tag{2.15}$$

The spectral density is determined then by the $G_{nn}(E)$ as follows:

$$\rho_{nn}(E) = \lim_{\delta\to 0}\left[-\frac{1}{\pi}\,\mathrm{Im}\; G_{nn}(E-i\delta)\right]. \tag{2.16}$$

Substituting Eq. (2.12) into Eq. (2.16), we have

$$\rho_{nn}(E) = \frac{1}{\pi}\,\frac{\mathrm{Im}\;\Sigma_n(E)}{[E-\varepsilon_n - \mathrm{Re}\;\Sigma_n(E)]^2 + \mathrm{Im}^2\Sigma_n(E)}. \tag{2.17}$$

The specific form of the energy spectrum, given by $E-\varepsilon_n - \mathrm{Re}\;\Sigma_n(E)=0$, depends on a particular choice of a model for the system under study. If, for a while, we neglect spatial variations of the order parameter, $\Delta(\mathbf{r})\equiv\Delta$, and assume also that there is no electron scattering in the system, then $\mathrm{Im}\;\Sigma_n(E)=0$, $\mathrm{Re}\;\Sigma_n(E)=\Delta^2/(E+\varepsilon_n)$ and for $\rho_{nn}(E)$ we have

$$\rho_{nn}(E) = \delta\left(E-\varepsilon_n - \frac{\Delta^2}{E+\varepsilon_n}\right) \equiv \frac{1}{\pi}\,[u_n^2\delta(E-E_n) + v_n^2\delta(E+E_n)], \tag{2.18}$$

where $E_n = (\varepsilon_n^2 + \Delta^2)^{1/2}$ is the BCS quasiparticle energy, and $u_n^2 = 1/2(1+\varepsilon_n/E_n)$, $v_n^2 = 1/2(1-\varepsilon_n/E_n)$ denote the coherence factors.

In reality, the presence of a VL brings two major effects which should be taken into account in calculations of the density of states. First, the spatial periodicity caused by the Abrikosov lattice lifts up the degeneracy on the Landau orbit center position X, and gives rise to dispersive Landau bands $E(k)$, with k being the intraband quasimomentum. Secondly, a small disorder or random in the VL structure broadens the Landau levels due to electrons scattering. The distinction between the above-mentioned two effects is that the first is caused by the VL contribution into $\mathrm{Re}\;\Sigma_n(E)$, while the second one manifests itself through changes in $\mathrm{Im}\;\Sigma_n(E)$. Using the technique of positional averaging over the VL, introduced by Brandt, Pesch, and Tewordt[49] an inverse lifetime of quasiparticles, $1/\tau_s$, caused by the scattering of electrons on a VL, have been calculated[23–25] so that the total damping ν was obtained as a sum of the two terms

$$\nu \approx \frac{\hbar}{\tau} + \frac{\hbar}{\tau_s}, \quad \frac{\hbar}{\tau_s} \approx \Delta^2\left(\frac{\pi}{\mu\hbar\Omega}\right)^{1/2}. \tag{2.19}$$

The term $\hbar/\tau$ is due to the scattering on the crystal lattice, while $\hbar/\tau_s$ stems from the scattering on the VL.

The damping (2.19) broadens δ functions in the spectral density (2.18) into two Lorentzians and thereby reduces the amplitude of dHvA oscillations in superconductors via the Dingle exponential factor which decreases with the growth of τ_s^{-1}. The quantity τ_s^{-1} was calculated in the above papers in a pure case $(1/\tau = 0)$ so that ν in Eq. (2.19) is given by the sum of two independent terms. It is clear that taking into account the lifetime effects in calculations of τ_s^{-1} should bring in an additional (interference) term τ_{int}^{-1}, vanishing when Δ or τ^{-1} goes to zero. The interference term is calculated below in the Appendix on the basis of the de Gennes correlation function[50]

$$f_n(\mathbf{r},\mathbf{r}',\zeta)=\sum_m \langle n|\delta(\mathbf{r}-\mathbf{r}_1)\hat{k}|m\rangle \times\langle m|\hat{k}^+\delta(\mathbf{r}-\mathbf{r}_2)|n\rangle\delta(\zeta-\zeta_m), \quad (2.20)$$

which has been introduced as a tool for studies of superconducting alloys and systems without translational symmetry. ($\hat{k}$ is the complex conjugation operator.) The self-energy (2.15), as one can see from the definition (2.8), can be readily expressed in terms of this function

$$\sigma_{nn}(E)=\int d\mathbf{r}d\mathbf{r}'d\zeta\,\frac{\Delta(\mathbf{r}')\Delta^*(\mathbf{r})}{E+\zeta-i\delta}\,f_n(\mathbf{r},\mathbf{r}',\zeta). \quad (2.21)$$

In the Appendix we will show that interference term, $\hbar/\tau_{\rm int}\sim\Delta^2/v_f^2\tau H$, can be calculated after positional averaging over a VL. Thus, instead of Eq. (2.19) the total damping is

$$\nu=\frac{\hbar}{\tau}+\frac{\hbar}{\tau_{\rm int}(\Delta,\tau)}+\frac{\hbar}{\tau_s(\Delta)}. \quad (2.22)$$

Another source of damping of dHvA oscillations, which cannot be reduced to the Dingle factor, is the Landau-level broadening into bands due to the VL periodicity. Most conveniently this effect and the corresponding electronic structure of the VL state may be described in the basis (2.6) written in the Bloch representation. One can find details of calculations in Refs. 20, 21, 30, and 36. Although the off-diagonal (in Landau index) pairing makes the whole spectral picture rather nontrivial, near H_{c_2}, where $\Delta\ll\hbar\Omega$, the off-diagonal pairing is ineffective and the quasiparticle energy band is

$$E_N(k)=\sqrt{\varepsilon_N^2+|F_{k_{NN}}|^2}, \quad (2.23)$$

where k is the Bloch wave vector and $F_{k_{NN}}$ is the diagonal pairing matrix element. These bands originate from the Landau levels as a result of lifting their degeneracy by the periodic pairing potential, $\Delta(\mathbf{r}+\mathbf{a})=\Delta(\mathbf{r})$, of the VL. Note the magnetic field is assumed to be uniform in Eq. (2.23). This is a good approximation slightly below H_{c_2} where dHvA oscillations have been observed. On the other hand, for fields well below H_{c_2} and in some specific cases spatial variations of the external field are essential. For example, in layered superconductors and superlattices the periodicity of $H(\mathbf{r})$ may appear due to the intrinsic pinning[51] which can forcibly modulate a VL imposing it in a period of the superlattice. A periodic magnetic field may itself be a physical reason for dispersive broadening of Landau levels.

To illustrate this, consider a first-order correction to the energy spectrum of a uniform ($\Delta=$const) superconductor in the periodic magnetic field of the form $H(x)=H+H_1\cos qx$. The corresponding vector potential in the Landau gauge is $\mathbf{A}=[0,Hx+(H_1/q)\sin qx,0]$, where a is the period of $H(x)$ and $q=2\pi/a$.

The Hamiltonian (2.5) in this gauge can be written as a sum of the Hamiltonian $\hat{H}_0$ with the eigenfunctions (2.6) and the perturbation

$$\hat{V}(x)=m\Omega^2(x-X)\,\frac{H_1}{qH}\sin qx. \quad (2.24)$$

Finally, for the first correction to the $E_N^0=(\varepsilon_N+\Delta^2)^{1/2}$ after some standard calculations, we have

$$E_N^{(1)}(k)=\frac{m\Omega^2H_1l}{Hq}\,\frac{\varepsilon_N}{E_N^0}\exp\left(-\frac{ql}{2}\right)^2L_N\left(\frac{q^2l^2}{2}\right)\cos(2\pi l_ak), \quad (2.25)$$

where $L_N(x)$ is the Laguerre polynomial of degree N and $l_a=l^2/a$. Thus, the Landau band (2.25) is proportional to the two factors. The first one is given by the Laguerre polynomial which is typical for harmonic perturbations in nonsuperconducting conductors.[52,53] The exponential factor, $\exp(-ql/2)^2\equiv\exp(-H^*/H)$, with $H^*=\pi\Phi_0/2a^2$, is typical of Landau bands in the coherent magnetic breakdown.[54,55]

One can show that in cases of 1D harmonic perturbations of the order parameter, $\Delta(x)=\Delta+\Delta_1\cos qx$, or the scalar potential, the structure of the Landau band is similar, and it can be written as follows:

$$E_N(k)=\sqrt{\varepsilon_N^2+\Delta^2}+e^{-H^*/H}[A_N+B_N\cos(2\pi l_ak)]. \quad (2.26)$$

The coefficients A_N and B_N depend on Δ and H in a way specific to the particular choice of the perturbation. For instance, in the case of periodic $\Delta(x)$ the first-order calculations yield $A_N=0$, and

$$B_N=\frac{\Delta\Delta_1}{E_N^0}\,L_N\left(\frac{q^2l^2}{2}\right). \quad (2.27)$$

If $\Delta=0$, then $\Delta(x)=\Delta_1\cos qx$, and a first-order correction to the energy equals zero. In this case one must calculate the second-order contribution into the energy (2.26), which is given by

$$A_N=\Delta_1^2(C_N+D_N),\quad B_N=\Delta_1^2(C_N-D_N), \quad (2.28)$$

$$C_N=\frac{1}{2}\sum_{p=1}^{\infty}\left[(ql)^{2p}L_N^{2p}\left(\frac{q^2l^2}{2}\right)\right]^2[E_N^0+E_{N+2p}^0]^{-1}, \quad (2.29)$$

$$D_N=\sum_{p=1}^{\infty}\left[(ql)^{2p}L_N^{2p+1}\left(\frac{q^2l^2}{2}\right)\right]^2[E_N^0+E_{N+2p+1}^0]^{-1}. \quad (2.30)$$

Here $L_m^n(x)$ denote the associated Laguerre polynomials.

Therefore, taking into account broadening due to the electron scattering on a VL and/or crystal lattice and dispersion relations within Landau bands Eqs. (2.23) or (2.26) caused by the periodicity of a VL, one can write the spectral density in the form

$$\rho_{NN}(\omega,k,\zeta)=\frac{1}{\pi}\left[u_{Nk}^2\frac{\nu}{\nu^2+[\omega+E_N(k,\zeta)]^2}+v_{Nk}^2\frac{\nu}{\nu^2+[\omega-E_N(k,\zeta)]^2}\right]. \quad (2.31)$$

Apart from the broadening of the δ functions and the substitution E_N by the Landau bands, Eq. (2.31) differs from Eq. (2.18) in that we have included in Eq. (2.31) an additional variable ζ. This variable describes a kinetic energy associated with the electron motion along the magnetic field. Such an inclusion simply means a substitution, $\varepsilon_N\to\varepsilon_N+\zeta$, in all above 2D expressions. We will show in the next section that in the case of a layered superconductor electron motion across the layers (in field perpendicular them) yields a nontrivial factor modulating 2D dHvA oscillations appropriate to a single layer.

III. de HAAS–van ALPHEN OSCILLATIONS IN THE VORTEX-LATTICE STATE

Once the spectral density is known, the free energy of a nonuniform superconductor can be calculated as[56]

$$F=\int d\mathbf{r}\,\frac{|\Delta(\mathbf{r})|^2}{\lambda}-2T\sum_{N,\zeta,k}\int_{-\infty}^{\infty}d\omega\rho_{NN}\times(\omega,\zeta,k)\ln\left[2\cosh\left(\frac{\omega}{2T}\right)\right], \quad (3.1)$$

where λ is the BCS coupling constant. Using Eq. (2.31) for the ρ_{NN} and the integral representation[57]

$$\ln\left[\cosh\left(\frac{\omega}{2T}\right)\right]=\int_{-\infty}^{\infty}dz\,\frac{1-\cos\omega z}{z\sinh\pi Tz}, \quad (3.2)$$

one can complete integration over ω in Eq. (3.1). Then, with the help of the normalization condition

$$u_{Nk}^2+v_{Nk}^2=1, \quad (3.3)$$

and the Poisson summation rule

$$\sum_{n_0}^{\infty}\chi(n)=\int_{-\infty}^{\infty}\chi(n)dn+2\,\mathrm{Re}\sum_{p=1}^{\infty}\int_a^{\infty}\chi(n)e^{i2\pi np}dn, \quad (3.4)$$

where $n_0-1<a<n_0$, the oscillating part of the free energy can be presented in the form

$$F_{\rm osc}=4T\,\mathrm{Re}\sum_{p=1}^{\infty}\sum_{\zeta,k}\int_{-\infty}^{\infty}dz\times\int_a^{\infty}dn\,\frac{e^{i2\pi np-\nu|z|}\cos[E_n(k,\zeta)z]}{z\sinh\pi Tz}. \quad (3.5)$$

Further calculations demand a specific form of the Landau band energy, $E_n(k,\zeta)$. We first consider the case when $E_n(k,\zeta)$ is given by Eq. (2.23), which we rewrite in the form

$$E_n(k,\zeta)=\sqrt{[\hbar\Omega(n-n_\mu+\zeta)]^2+\Delta^2(k)}. \quad (3.6)$$

Here the Landau-level index at the Fermi energy, $n_\mu=\mu/\hbar\Omega-1/2$ is introduced, and $\Delta^2(k)=|F_{kn_\mu n_\mu}|^2$.

Substituting the variable n by n' according to the $\hbar\Omega n'=\hbar\Omega(n-n_\mu)+\zeta$, and introducing a one-dimensional density of states associated with the electron motion along the magnetic field

$$g(\varepsilon)=\sum_\zeta\delta(\varepsilon-\zeta), \quad (3.7)$$

after integration over ε and n, we have

$$F_{\rm osc}=4T\,\mathrm{Re}\sum_{p=1}^{\infty}\sum_k\int_{-\infty}^{\infty}G_p(k,z)I_p\,\frac{e^{-\nu|z|}}{z\sinh\pi Tz}\times\cos\left[2\pi p\left(\frac{\mu}{\hbar\Omega}-\frac{1}{2}\right)\right]dz, \quad (3.8)$$

where

$$G_p(k,z)=\int_{-\infty}^{\infty}dn\,\cos(z\sqrt{(\hbar\Omega n)^2+\Delta^2(k)})\cos 2\pi np, \quad (3.9)$$

and the factor

$$I_p=\int_{-\infty}^{\infty}g(\varepsilon)\exp\left(-i\,\frac{2\pi\varepsilon p}{\hbar\Omega}\right)d\varepsilon, \quad (3.10)$$

introduced by one of the authors in Ref. 58. Leaving aside for a while the analysis of effects related to the factor I_p, consider now the integral (3.9). It can be calculated with the help of the relationship[57]

$$J(c)=\int_0^{\infty}\frac{\sin(c\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}\cos(bx)dx=\frac{\pi}{2}J_0(y\sqrt{c^2-b^2})\,\theta(c-b), \quad (3.11)$$

where $\theta(x)$ is the Heaviside step function, and $J_0(x)$ stands for the Bessel function. In view of the fact that $\partial J(c)/\partial c$ is exactly the integral (3.9), we have

$$G_p(k,z)=\pi\delta(z\hbar\Omega-2\pi p)+\frac{\pi}{\hbar\Omega}\,\theta\left(z-\frac{\pi p}{\hbar\Omega}\right)\times\frac{\partial}{\partial z}J_0\left(\frac{\Delta(k)}{\hbar\Omega}\sqrt{(z\hbar\Omega)^2-(2\pi p)^2}\right). \quad (3.12)$$

Substitution of this equation into Eq. (3.8) yields

$$F_{\rm osc}=\frac{4\pi T}{\hbar\Omega}\sum_{p=1}^{\infty}(-1)^p\cos\left(2\pi p\,\frac{\mu}{\hbar\omega}\right)I_p\Psi(\nu,\Delta,z_p), \quad (3.13)$$

$$\Psi(\nu,\Delta,z_p)=g\Psi(\nu,z_p)+\int_{z_p}^{\infty}dz\Psi(\nu,z)\times\frac{\partial}{\partial z}\sum_k J_0\left(\frac{\Delta(k)}{\hbar\Omega}\sqrt{(z\hbar Q)^2-(2\pi p)^2}\right). \tag{3.14}$$

Here $z_p=2\pi p/\hbar\Omega$, $g=\Phi/\Phi_0$ being the degeneracy factor (Φ is the flux through a sample, Φ_0 is the flux quantum), and

$$\Psi(\nu,z)=\frac{e^{-\nu z}}{z\sinh(\pi Tz)}. \tag{3.15}$$

Using the relation $dJ_0/dz=-J_1(z)$ one can rewrite Eq. (3.14) in the form which explicitly displays the negative sign of the second term in the amplitude $\Psi(\nu,\Delta,z_p)$. This term depends on the order parameter and appears only below H_{c_2}. For $H>H_{c_2}$ the amplitude (3.14) reduces to the first term, $g\Psi(\nu,z_p)$, describing the standard picture of dHvA oscillations in normal metals, damped by the Dingle factor $\exp(-\nu z_p)$. For fields below H_{c_2}, i.e., in the VL state, the amplitudes of the oscillations decreases, while the period of dHvA oscillations, in full accordance with the experiments,[1–19] remains intact. The additional damping in the superconducting state arises owing to the two major contributions from the VL. The first one is due to the additional scattering of electrons on a VL given by Eq. (2.22). It acts via the Dingle factor which decreases with the enhancement of Δ. We will consider this mechanism of damping in detail and calculate $\nu(\Delta,\tau)$ in superconductors with impurities in the Appendix. Another mechanism of damping is determined by the last term in Eq. (3.14), which near the H_{c_2} [for $\Delta(k)\ll\hbar\Omega$] can be written in the form

$$\Psi(\nu,\Delta,z_p)=g\Psi(\nu,z_p)-\frac{1}{2}\langle\Delta^2\rangle\int_{z_p}^{\infty}dz\Psi(\nu,z)z, \tag{3.16}$$

where

$$\langle\Delta^2\rangle=\sum_k \Delta^2(k). \tag{3.17}$$

We see that even in the absence of scattering $\nu=0$ (when the Dingle factor equals unity), the dHvA oscillations are damped by the term proportional (near H_{c_2}) to the quantity $\langle\Delta^2\rangle$.

In the above considerations, as well as in every previous theoretical study of dHvA oscillations in superconductors, the external magnetic field was assumed to be spatially uniform. Since the energy spectrum of a 2D superconductor in periodic magnetic field is found, then calculations of F_{osc} with the help of Eq. (2.26) is absolutely similar to that done before. They yield again Eq. (3.13) but this time with the amplitude of oscillations different from that given by Eq. (3.14). The renormalized amplitude in this case is

$$\Psi(\nu,\Delta,z_p)=\Psi(\nu,z_p)I^*(z_p)+\int_{z_p}^{\infty}dz\Psi(\nu,z)\times\frac{\partial}{\partial z}J_0\left(\frac{\Delta}{\hbar\Omega}\sqrt{(z\hbar\Omega)^2-(2\pi p)^2}\right)I^*(z), \tag{3.18}$$

where

$$I^*(z)=J_0\left[Bz\,\exp\left(-\frac{H^*}{H}\right)\right]\cos\left[Az\,\exp\left(-\frac{H^*}{H}\right)\right]. \tag{3.19}$$

A and B equal to A_N, B_N of Eq. (2.26) with $N=\mu/\hbar\Omega$.

The amplitude (3.18) is very similar to that of Eq. (3.14). Apart from the adopted spatial uniformity of Δ, a new factor $I^*(z)$, caused by the spatial periodicity of the magnetic field appears in Eq. (3.18). This factor, depending on Δ and H, is less than unity and diminishes amplitudes of dHvA oscillations when field decreases below H_{c_2}. As in the case of a VL, the mechanism of damping of dHvA oscillations in Eq. (3.18) is determined by the formation of Landau bands. Qualitatively, the 1D periodic external magnetic field and the 2D periodic VL damp the dHvA oscillations in the same fashion. The difference is in the specific form for $E_N(k)$ and damping factors.

The dependence of dHvA oscillation amplitudes on the magnetic field in Eq. (3.13) is also related to the factor I_p, given by Eq. (3.10). This factor is a Fourier transform of a 1D density of states, $g(\varepsilon)$, associated with the electron motion along the magnetic field. It follows from Eq. (3.10) that any singularity or narrow δ peak in $g(\varepsilon)$ located at some energy ε_0 makes the factor I_p to be an oscillatory function of the reciprocal field $1/H$. Experimental detection of these oscillations provides a basis for measurements of the ε_0 and thereby to the restoration of the density of states $g(\varepsilon)$. In the case of a uniform and bulk superconductor $g(\varepsilon)$ has no singularities, but they do exist in layered superconductors among which are the high-T_c cuprates, the dichalcogenides of transitional metals and various artificial superlattices.

The prospective of restoration of the function $g(\varepsilon)$ in these materials by the dHvA measurements is a very intriguing problem.

Consider first a regular layered crystal in which

$$g(\varepsilon)=\frac{1}{\pi}(\sigma^2-\varepsilon^2)^{1/2} \tag{3.20}$$

with σ standing for the overlap integral between adjacent layers. The square-root van Hove singularities in Eq. (3.20) manifests itself in oscillations periodic in $1/H$ of the factor (3.10) which in this case is equal to

$$I_p=J_0(2z_p\sigma). \tag{3.21}$$

Any irregularity in the layer stacking (which inevitably appears in the process of intercalation, for example) results in the appearance of the local or quasilocal peaks in $g(\varepsilon)$ and thereby gives rise to additional oscillations of I_p. The frequency of these oscillations depends on the local value of the overlap integral near the stacking fault σ_f. For small concentrations of stacking faults c, the factor I_p can be written as

$$I_p=(1-c)J_0(2z_p\sigma)+c\,\delta I_p, \tag{3.22}$$

where δI_p has an exponential Dingle-like form (for $\sigma>\sigma_f$)

$$\delta I_p=\exp[-z_p(\sigma^2-\sigma_f^2)^{1/2}]\cos(2\sigma_f z_p). \tag{3.23}$$

Oscillations of the factor I_p in the reciprocal field modulate dHvA oscillations in layered crystals. They are independent of the order parameter and hence survive above T_c in the normal state. Such modulations have been observed in some intercalated layered crystals[59,60] and all the more should be observable in metallic superlattices where thicknesses of constituting layers can be varied smoothly.

IV. SUMMARY AND CONCLUSION

Theoretical problems and issues related to the dHvA oscillations in quasi-2D superconductors are extremely complex and versatile. On the other hand, the experimental fact of preservation of dHvA frequencies during the superconducting phase transition is crucial and tells that Landau-level systematics survive deep beyond H_{c_2}. It was experimentally established[6] that the well-known Lifshitz-Kosevich quasiclassical theory of the dHvA effect developed for conventional metals is inappropriate to dHvA oscillations in the mixed state. In the most pronounced and striking form the typical features of the dHvA effect in the vortex state have been observed by Goll *et al.*[10] and especially by Terashima *et al.*[9] on borocarbide YNi_2B_2C. The latter authors found that in a field immediately below H_{c_2} the oscillation amplitudes are much more strongly damped than predicted by theories of Maki,[23] Stephen,[24] and Norman and MacDonald,[30] but they recover at lower field and persist up to the field $H\approx 0.2H_{c_2}$ with amplitudes larger than theoretical predictions.[23,24,30] Thus, one of the challenges of the problem discussed is to understand why damping of dHvA amplitudes is larger than that given by the Dingle exponent of Refs. 23, 24, 30 in the pinned (disordered) vortex state and why this damping is less than the Dingle exponent for lower field where the VL is regular.[9] Physically, the disorder, the periodicity, and the temperature damp dHvA oscillations in a different fashion: disorder smears the Landau levels, while periodicity broadens them into dispersive Landau bands, and temperature smears the Fermi distribution. The most high dHvA amplitudes appear at $T=0$ in the clean normal state of a 2D metal when sharp Landau levels cross a sharp Fermi surface. Smearing of the Landau levels by the disordered (random) VL and impurities of the crystal lattice as well as broadening of Landau levels into bands by periodic layers, the external magnetic field, the order parameter and the VL, damp amplitudes of dHvA oscillations. The appropriate attenuation factors have been calculated in Sec. III. Our main result is given by Eq. (3.13) describing harmonic oscillations of the free energy modulated by attenuation factors I_p and $\Psi(\nu,\Delta,z_p)$. The factor I_p (3.10) appears due to the electron transport across the layers. It is a Fourier transform of the 1D density of states (3.7) and very sensitive to the quality of layer stacking. Even a small concentration of the stacking faults yields additional modes and results in the extra damping of dHvA oscillations compared to the case of a perfect layering as one can see from Eqs. (3.22), (3.23). In the case of small overlapping between adjacent layer wave functions, $\sigma/\hbar\Omega\ll 1$, the factor $I_p=1$ according to Eq. (3.21). If, in addition, we set in Eq. (3.13) Δ,ν,T equal to zero, the oscillating part of magnetization, $M_{\rm osc}=-dF_{\rm osc}/dH$, has the shape of a 2π periodic in a $1/H$ sawtooth function:

$$M_{\rm osc}\approx\sum_{p=1}^{\infty}\frac{(-1)^{p+1}}{p}\sin p\varphi=\frac{\varphi}{2}-\pi\left[\frac{\varphi}{2\pi}+\frac{1}{2}\right], \tag{4.1}$$

where $\varphi=2\pi\mu/\hbar\Omega$ and $[x]$ stands for the integer of x. A sawtooth magnetization in an ideal 2D metal at $T=0$ was considered by Peierls[62] in 1933 and then reexamined by Vagner, Maniv, and Ehrenfreund,[63] and Shoenberg[64] in the 1980s when 2D conductors became a popular experimental system. Temperature, quasiparticle scattering (by VL and impurities), and electron migrations across the layers smear the sawtooth profile of magnetization because factors I_p and $\Psi(\nu,\Delta,z_p)$ depend on the index p and the corresponding Fourier series for $M_{\rm osc}$ is more complicated than Eq. (4.1). The factor $\Psi(\nu,\Delta,z_p)$, according to Eq. (3.14), consists of two terms. The first one, $\Psi(\nu,z_p)$, is proportional to the total Dingle factor due to the quasiparticle scattering on crystal- and vortex-lattice imperfections: $R(\Delta,\tau)=\exp(-\nu z_p)$ where total damping ν equals Eq. (2.22). The second term in Eq. (3.14) vanishes above T_c, it describes condensate oscillation amplitudes and depends on the broadening of Landau levels into Landau bands by the VL. Near H_{c_2} it can be presented as a power series (3.16) in $\langle\Delta^2\rangle$, which is an averaged over the Brillouin zone, squared pairing matrix element $\Delta^2(k)$ (3.17). Function $\Delta(k)$, as is known, has a number of zeros which diminish integrals on k in Eqs. (3.14) and (3.16), but this effect is not so big, as that reported by Dukan and Tesanovic,[29] because of the term $\Psi(\nu,z_p)$. We see also that attenuation of dHvA oscillations caused by Landau bands is not of the exponential Dingle form. In this connection, the approximation of the Landau-band-induced damping in Ref. 21 by the Dingle factor, $R=\exp(-\pi/\Omega\tau_s)$, with $1/\tau_s\sim\lambda\hbar\Omega\Delta_0 n_\mu^{-1/4}$, simply means that the quantity $1/\tau_s$ is some effective measure of the Landau bandwidth and differs from the scattering lifetime calculated in Refs. 23–25. The total Dingle factor in our work depending both on Δ and τ is $R(\Delta,\tau)=R_0(\tau)R_s(\Delta)R_{0_s}(\Delta\tau)$, where the "interference Dingle factor" is $R_{0_s}(\Delta,\tau)=\exp(-\pi/\Omega\tau_{\rm int})$. The field dependence of $\tau_{\rm int}^{-1}\sim\Delta^2/v_f l_0 H$ is different from that found in Refs. 23–25 for $R_s(\Delta)$ in the clean limit: $\tau_s^{-1}\sim\Delta^2/v_f H^{1/2}$.

We have considered dHvA oscillations in superconductors in a nonuniform magnetic field. The periodic magnetic field lifts up the degeneracy of the Landau levels in the same fashion as the VL does and gives rise to the Landau bands. The dispersion relation, $E_N(k)$, in this case differs from that for the VL, Eq. (2.23), and is given by Eq. (2.26) corresponding to the one-dimensional periodicity of the external magnetic field and/or order parameter. Physically, this may be realized in layered crystals and superlattices due to the intrinsic pinning of vortices by periodic set of layers. The amplitude of dHvA oscillations in this case is given by Eq. (3.18) and contains a specific factor of damping, $I^*(z)$ [see

Eq. (3.19)], which depends on the field by dint of the typical exponent, $\exp(-H^*/H)$, which determines the width of the Landau bands.

Consider now briefly the connection between the results of this paper and current experiments on the dHvA effect in the mixed state. The experiments[6,9,10] show that immediately after crossing H_{c_2} there is a region, down to a some field H_m, where dHvA oscillations are damped, but then recover at lower fields $H \ll H_m$. In borocarbide YNi_2B_2C, at very low temperature $T=0.05$ K, damping within the interval $H_{c_2}<H<H_m$ is much larger than predicted by any of existing theories.[9] We can explain this by noting that according to modern view on the H-T phase diagram[51] in layered systems the VL is melted in the field region $H_{c_2}(T)<H<H_{\mathrm{melt}}(T)$ immediately below the upper critical field. In this field interval a ''vortex matter'' is in the liquid or glassy state so that the most adequate approach to the dHvA effect in this region is the one given by Maki-Stephen[23,25] which treats the VL as a random quantity. We can relate the extra damping observed in this region to the interference Dingle factor $R_{0_s}(\Delta,\tau)$ calculated above. Further decrease in H recovers a regularity of the VL and switched off the Maki-Stephen mechanism of damping when $H \ll H_m$. The periodic VL lifts up the degeneracy of the Landau levels and broadens them into the Landau bands. As far as the damping of dHvA amplitudes caused by the Landau bands Eqs. (3.14), (3.18) is less than the Dingle exponent $R(\Delta,\tau)$, the corresponding attenuation of oscillations is less than that given by theories of Refs. 23, 25, 30, in accordance with the experiments of Terashima *et al.*[9] Such a crossover from a relatively strong damping just below H_{c_2} to a weaker attenuation of dHvA amplitudes has been observed in a number of works.[6,9,10] From all these works, only the most low-temperature measurements made in Ref. 9 at $T=0.05$ K show that this crossover is closely related to the vortex pinning. Strong damping below the H_{c_2} takes place in the peak-effect region (with maximal damping at about 6.2 T), while within the interval from 2 to about 4 T both pinning and attenuation of dHvA oscillations are relatively small.[9] This picture is in a qualitative agreement with our approach.

In layered superconductors when the field is parallel to the layers intrinsic pinning[51] might create modulation of the magnetic field and the order parameter in a direction perpendicular to layers. Considering this modulation as a small perturbation, we obtained the energy spectrum (2.26) and the amplitude of dHvA oscillations (3.18) where the effect of the modulation enters through the attenuation factor $I^*(z)$ Eq. (3.19). The Landau bands (2.26) grow narrower with the field decrease because of the decrease of the $\exp(-H^*/H)$ and the factor $I^*(z)$ increases. A similar effect should be in the case of the VL which grows thinner with the field decrease and disappears at H_{c1}, but calculating it analytically is extremely difficult because even in the diagonal approximation, dependence of the quasiparticle energy (2.23) on the magnetic Brillouin zone wave vector k is unknown analytically. One can expect, nonetheless, that the above effect of narrowing of the Landau bands is not so big if one takes into account that a period of the real VL $a \approx \sqrt{\Phi_0/H}$ increases when the field goes down, so that $H^* \approx \Phi_0/a^2$ is proportional to H, and the ratio H^*/H is independent of H.

Well below H_m the diagonal approximation breaks down and off-diagonal pairing should be taken into account. Generally, it is a yet unresolved problem, but it can be solved within the model approximation implying an exponential decrease of the pairing matrix element

$$\Delta_{nm}=\Delta_0\exp(-\alpha|n-m|), \tag{4.2}$$

where α is some positive constant. Substituting Eq. (4.2) into Eq. (2.2) we find in the quasiclassical approximation ($2\pi\Delta \gg \hbar\Omega$) the energy spectrum

$$E_n(k)=\sqrt{\Delta_0^2+\zeta_n^2(k)}, \tag{4.3}$$

$$\zeta_n(k)=\hbar\Omega\left(n+\frac{1}{2}\right)-\mu+(-1)^n\frac{\hbar\Omega}{\pi}\arcsin(\rho\cos ka), \tag{4.4}$$

which yields a corresponding attenuation factor in Eq. (3.13), $\hat{I}_p(\rho)$ as a polynomial in

$$\rho=\exp\left(-\frac{2\pi\Delta\tanh\alpha}{\hbar\Omega}\right). \tag{4.5}$$

For the first three harmonics with $p=1,2,3$, we have

$$\hat{I}_1(\rho)=1-\rho^2,\quad \hat{I}_2(\rho)=1-4\rho^2+3\rho^4,$$
$$\hat{I}_3(\rho)=1-9\rho^2+18\rho^4-10\rho^6. \tag{4.6}$$

These factors appear in the dHvA oscillation of the periodic coherent magnetic breakdown system,[54] where $\rho=\exp(-H_0/H)$ is the magnetic breakdown probability. In our case the ''breakdown field'' equals $H_0=2\pi\Delta_0\tanh\alpha$. We consider in detail the above off-diagonal pairing model and the analogy with the coherent magnetic breakdown elsewhere. Now we would like to note a remarkable property of the factors (4.6): they enhance with the decrease of the field H. Physically this is because Landau bands become narrower when the VL grows thinner, as we have discussed before in the case of periodic magnetic field.

Summing up, we explain the strong attenuation of dHvA amplitudes observed by Terashima *et al.*[9] in the vortex-pinned region by strong quasiparticle scattering on the random VL, resulting in an exponential decrease of amplitudes by the Dingle factor $R(\Delta,\tau)$. Well below this region the regularity of the VL recovers and attenuation becomes much less than given by $R(\Delta,\tau)$, so that dHvA oscillations persist well down to the low fields.

We conclude by expressing our hope that methods and results obtained in this paper will be useful in the studies of the VL state in layered superconductors, in a more broad context than the problems of the dHvA oscillations.

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APPENDIX

The main effect which a perfect VL exerts on the energy spectrum of electrons is the Landau levels broadening into bands owing to the lift of their degeneracy on the orbit position. In reality, because of the vortex pinning, the VL is imperfect. Electrons scattering on VL imperfections contributes into the suppression of dHvA oscillations below H_{c_2} by dint of the Dingle factor, $\exp(-\nu z_p)$, entering the amplitude $\Psi(\nu,\Delta,z_p)$ in Eqs. (3.14) and (3.18). The corresponding contributions into ν of Eq. (2.19) was calculated in Refs. 23–25 within an approach based on the positional averaging over the VL. Following this technique, we start with Eq. (2.21) for the self-energy of the Green function $\sigma_{nn}(E)$. Averaging Eq. (2.21) and using the standard relation, $\mathrm{Im}(E+\zeta-i\delta)^{-1}=\pi\delta(E+\zeta)$, we have for $\nu(E)=\mathrm{Im}\langle\sigma_{nn}(E)\rangle$:

$$\nu(E)=\int d\mathbf{r}d\mathbf{r}'\langle\Delta^*(\mathbf{r})\Delta(\mathbf{r}')\rangle f_n(\mathbf{r},\mathbf{r}',-E). \tag{A1}$$

The damping ν at the Fermi energy determines the Dingle factor, $R=\exp(-\nu z_p)$, and enters Eqs. (3.14) and (3.18). The real part of the self-energy, $\mathrm{Re}\langle\sigma_{nn}(E)\rangle$, can be discarded since it yields a small correction to the Fermi energy μ. To calculate ν, we need a specific form for the correlation function in the right-hand side of Eq. (A1). The positional averaging over a VL for calculations of the correlator $\langle\Delta^*(\mathbf{r})\Delta(\mathbf{r}')\rangle$ was considered in detail from different points of view by Stephen[24] and we will use the results of this consideration in what follows.

The correlation function (2.20) has been introduced by de Gennes as an alternative to the Green-function method in the theory of nonuniform superconductivity of metals and alloys. In our approach, which goes back to Ref. 47, the correlation function $f_n(\mathbf{r},\mathbf{r}',-E)$ is used for calculations of the Green-function self-energy. Unfortunately, $f_n(\mathbf{r},\mathbf{r}',-E)$ cannot be calculated in a general case of arbitrary field and impurity concentrations. The calculations for the self-energy which take into account electrons scattering on the VL and ignore the finite lifetime effects caused by electrons scattering on crystal lattice imperfections, have been carried out in Refs. 23, 24. In Ref. 23, in particular, the inclusion of the electron scattering on the crystal-lattice impurities was done by the addition of the τ term into the total damping, given by Eq. (2.19). This equation is lacking the interference term which should depend on τ and Δ and vanish both in the clean limit and in the normal state. The interference term can be calculated if we take into account finite lifetime effects in $f_n(\mathbf{r},\mathbf{r}',-E)$ for calculations of Eq. (A1). It can be done in the limit when the mean free path is less than ζ, the coherence length. The correlation function in this limit satisfies, in the time representation, the diffusion equation[50]

$$\frac{\partial f(\mathbf{r},\mathbf{r}',t)}{\partial t}-D\left[\boldsymbol{\nabla}-\frac{2ie}{\hbar c}\,\mathbf{A}(\mathbf{r})\right]^2 f(\mathbf{r},\mathbf{r}',t)=L^{-2}\delta(t)\,\delta(\mathbf{r}-\mathbf{r}'). \tag{A2}$$

Here $D=v_f l_0/3$ is the diffusion constant, v_f stands for the Fermi velocity, and $l_0=v_f\tau$ is the electron free path. The right-hand side of Eq. (A2) is normalized over the square of a sample L^{-2}, as we consider a 2D superconductor. In Eq. (A2) it is assumed that $f_n(\mathbf{r},\mathbf{r}',t)$, is independent of the subscript n within the energy range relevant to superconductivity.[50]

The solution of Eq. (A2) can be written in terms of the eigenfunctions, $g_n(\mathbf{r})$, and eigenvalues Ω_n of the Schrödinger equation

$$D\left[\boldsymbol{\nabla}-\frac{2ie}{\hbar c}\,\mathbf{A}(\mathbf{r})\right]^2 g_n(\mathbf{r})=-\Omega_n g_n(\mathbf{r}), \tag{A3}$$

$$f(\mathbf{r},\mathbf{r}',t)=L^{-2}\sum_n g_n(\mathbf{r})g_n^*(\mathbf{r}')\exp(-\Omega_n(t)). \tag{A4}$$

Equation (A3) is exactly the Landau problem, provided that $\hbar^2/2m$ is replaced by the diffusion constant D. Taking this into account, the eigenfunctions $g_n(\mathbf{r})\equiv g_{NX}(\mathbf{r})$ can be written in the symmetric gauge, $\mathbf{A}=1/2(Hy,Hx)$, in the form

$$g_{NX}(\mathbf{r})=\frac{1}{\sqrt{L}}\exp\left[i\,\frac{xy}{2l^2}-i\,\frac{Xy}{l^2}\right]\Psi_N(x-X), \tag{A5}$$

where N is the number of the Landau level, X is the orbit center coordinate, and $\Psi_N(x-X)$ is given by Eq. (2.6).

Making the Fourier transform of Eq. (A4) and taking into account the relation[61]

$$\sum_X g_{NX}(\mathbf{r})g_{NX}^*(\mathbf{r}')=\frac{1}{2\pi l^2}J_{NN}\left(\frac{|\mathbf{r}-\mathbf{r}'|}{l}\right)\exp\left(i\,\frac{\mathbf{r}\times\mathbf{r}'}{2l^2}\right), \tag{A6}$$

we have

$$f(\mathbf{r},\mathbf{r}',E)=\frac{1}{2\pi^2 l}\sum_N J_{NN}\left(\frac{|\mathbf{r}-\mathbf{r}'|}{l}\right)\exp\left(i\,\frac{\mathbf{r}\times\mathbf{r}'}{2l^2}\right)\frac{\Omega_N}{E^2+\Omega_N^2}, \tag{A7}$$

where

$$J_{NN'}(x)=(-1)^{N-N'}J_{N'N}(x)=\left(\frac{N'!}{N!}\right)^{1/2}\left(\frac{x}{\sqrt{2}}\right)^{N-N'}L_N^{N-N'}\left(\frac{x^2}{2}\right)\exp\left(-\frac{x^2}{2}\right). \tag{A8}$$

The order parameter in the VL is a periodic function of the spatial variable, so that $\Delta(\mathbf{r})$ can be written in the basis of the eigenfunctions $g_{NX}(\mathbf{r})$ as a series of the form

$$\Delta(\mathbf{r})=\sum_{NX} C_{NX}g_{NX}(\mathbf{r}). \tag{A9}$$

Following the work of Stephen,[24] we will consider coefficients C_{NX} as random variables with the correlation function

$$\langle C_{NX}C_{N'X'}^*\rangle=\Delta^2 2\pi l^2\rho_N\delta_{NN'}\delta_{XX'}, \tag{A10}$$

where ρ_N is some dimensionless function of N, decreasing as N grows. Then, employing Eq. (A10), for the positional averaging of the order parameter, we have

$$\langle \Delta^*(\mathbf{r})\Delta(\mathbf{r}')\rangle = \Delta^2 \exp\left(-i\, \frac{|\mathbf{r}\times\mathbf{r}'|}{2l^2} \right) \sum_N \rho_N J_{NN}\left(\frac{|\mathbf{r}-\mathbf{r}'|}{l} \right). \tag{A11}$$

Substituting Eqs. (A11), (A7) into Eq. (A1) and making integrations over the spatial variable with the help of the relationship[57]

$$I_{NN'} = \int_0^\infty dx e^{-2x} L_N(x) L_{N'}(x) = \frac{(N+N')!}{N!N'!} \left(\frac{1}{2}\right)^{N+N'+1}, \tag{A12}$$

we arrive at

$$\nu(E) = \frac{\Delta^2}{\hbar\Omega_c} \sum_{N,N'} I_{NN'} \rho_N \frac{N+1/2}{(E/\Omega_c)^2 + (N+1/2)^2}. \tag{A13}$$

Ω_c here denotes the cyclotron frequency $\Omega_c = eDH/\hbar c$. Near the Fermi level ($E=0$) the sum in Eq. (A13) is, in essence, field independent, so that the positive constant,

$$\Gamma^2 = \sum_{N,N'} I_{NN'} \rho_N \frac{1}{N+1/2}, \tag{A14}$$

plays the role of a phenomenological parameter, renormalizing the amplitude of the order parameter: $\Delta_0 = \Delta\Gamma$. This renormalization is akin to the spatially averaged over the Fermi surface order parameter $\langle\Delta\rangle$, introduced in Ref. 25. Thus, one can estimate the electron damping at the Fermi level due to the scattering on a random VL (provided that due to the scattering on the crystal-lattice imperfections electrons have a finite free path, l_0) as follows:

$$\nu(0) = \frac{3\Delta_0^2 c}{ev_f l_0 H} = \frac{3c\Delta_0^2}{ev_f^2 \tau H}. \tag{A15}$$

Thus, $\nu(0) = \hbar/\tau_{\rm int}$ is exactly what we have called in Sec. II the ''interference term.'' In difference to the VL scattering in pure crystals which yields[23–25] $\tau_s^{-1} \sim \Delta^2/v_f H^{1/2}$, the interference term, $\tau_{\rm int}^{-1} \sim \Delta^2/v_f H l_0$, vanishes when either $\Delta\to 0$, or $l_0 \to \infty$.

Therefore, putting together all contributions into the total damping ν, we have instead of Eq. (2.19),

$$\nu = \frac{\hbar}{\tau} + \frac{\hbar}{\tau_{\rm int}(\Delta,\tau)} + \frac{\hbar}{\tau_s(\Delta)}. \tag{A16}$$

The first term in Eq. (A16) is field independent whereas the remaining ones are inversely proportional to H and $H^{1/2}$, respectively, and both vanish above H_{c_2} in the normal state. In the clean limit, $l_0 \to \infty$, the damping ν is equal to the last term in Eq. (A16) calculated in Refs. 23–25.

To conclude this section, a remark is in order. The interference term has been calculated above for $l_0 < \zeta$. On the other hand, l_0 must be not too small, since $\Omega\tau_{\rm int} \gg 1$ is the necessary condition for the dHvA effect in superconductors. Thus, the superconductors with large ζ and high H_{c_2} are those materials in which the Dingle factor $R_{0s}(\Delta,\tau) = \exp(-\pi/\Omega\tau_{\rm int})$ is yet not too small to overdamp the dHvA oscillations. In the absence of a complete analysis, which presently is very difficult to do, we hope that Eq. (A15) yields a qualitatively true dependence on τ and H beyond its formal applicability conditions. The latter, of course, can be verified only experimentally. Nonetheless, we note that the main results of Sec. III are generally independent of the particular choice of the function $\nu(\Delta,\tau)$.

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