Theoretical approach to frequency-dependent dynamics of domain walls in magnetically ordered materials

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(Received 19 December 1997; revised manuscript received 25 March 1998)

A theoretical approach to frequency-dependent dynamics of domain walls in magnetically ordered materials is presented. To carry out this approach, we solve the phenomenological equations determining the motion of Bloch walls under the influence of a magnetic field, taking into account the effect of the relaxation of the induced magnetic anisotropy due to a redistribution of the magnetic dipoles inside the material. We use harmonic fields and we obtain that wall motion amplitude is a function of time that can both increase or decrease depending on the applied field frequency. To check these conclusions, we present the experimental results obtained via the so-called permeability disaccommodation technique in the cases of a magnetite sample and a polycrystalline yttrium-iron-garnet sample. [S0163-1829(98)01137-0]

I. INTRODUCTION

In this work, we present a theoretical study of frequency-dependent dynamics of domain walls in magnetically ordered materials when magnetic aftereffects (MAE's) appear. By MAE is meant a delayed change in magnetization accompanying a change in the magnetic field.[1] We exclude from this definition the magnetization delays occasioned by eddy currents and structural changes or aging of the substance. Snoek[2] explained the nature of this phenomenon by means of a model consisting in a ball placed on a curved concrete surface covered with a mud layer of a finite thickness. From a quasistatic point of view, after the ball is displaced to a new position, it will sink into the layer of mud gradually, changing its equilibrium position. On the other hand, if we forced the ball to oscillate back and forth around the minimum position of the concrete surface (ac magnetization), we would find three possible situations. If the viscosity of mud is increased at low temperatures, the ball will move on the hardened surface of the mud with very low loss. In a similar way, if the viscosity is decreased at high temperatures, the ball will move through the unviscous layer of mud. At the intermediate temperatures, the motion of the ball is most severely damped, resulting in very large loss.

Néel[3] modified Snoek's concept of MAE's in certain respects, including detailed calculations. He introduced the idea that MAE's have their origin in the relaxation of the induced magnetic anisotropy due to a redistribution of magnetic dipoles into the lattice. The induced anisotropy does not differ from the magnetocrystalline anisotropy as far as their origin is concerned but, being related to local anisotropy arrangements in the lattice, it usually exhibits lower symmetry than that of the crystal as a whole.[4] Hence, MAE's happen to be very useful for basic research, because such studies yield information about lattice symmetry and dynamics.[5]

In accordance with the model proposed by Néel, we have considered that the motion of a Bloch wall under the influence of an applied magnetic field is then defined by the sum of different stresses: the applied magnetic field stress, an elastic term, a damping term, and a stress connected with MAE's. We have written the corresponding wall dynamics equation by means of such stresses and we have solved it using harmonic fields. As an important result, we have obtained that wall motion amplitude is a function of time that can both increase or decrease depending on the applied field frequency. Finally, we have verified this behavior experimentally for a polycrystalline yttrium-iron-garnet (YIG) sample by means of the so-called magnetic disaccommodation technique.

II. THEORETICAL ASPECTS

The motion of a Bloch wall under the influence of an applied magnetic field, possibly time dependent, is defined by the sum of different stresses. To the applied magnetic-field stress, we must add an elastic term that refers the wall to its equilibrium position and connected with lattice internal stresses, dislocations, etc., another one, proportional to its speed, to take account of damping effects that prevent a free wall oscillation when it changes its position, and finally, a stress due to the redistribution of magnetic dipoles within the lattice. Hence, we write the following dynamic equation[3] for the Bloch wall:

$$m\ddot{u}+\upsilon\dot{u}=-R(u)+\vec{H}(t)(\vec{M}_2-\vec{M}_1)+P(u,t), \qquad (1)$$

u being the wall instantaneous position, m the wall inertial mass, υ the damping coefficient, $-R(u)$ an elastic term, and $\vec{M}_1$ and $\vec{M}_2$ the magnetization on each side of the wall. The redistribution stress $P(u,t)$ is obtained by computing the instantaneous anisotropy energy associated to a unity surface of the wall considering the lattice symmetry and the directions of the magnetizations $\vec{M}_1$ and $\vec{M}_2$. This stress can be written as[6]

$$P(u,t)=-P_0\left\{\int_0^t f[u(t')-u(t)]g(t-t')dt' + o(u)\left[1-\int_0^t g(t')dt'\right]\right\}. \qquad (2)$$

The first term in the sum takes account of the wall evolution and is connected with magnetic aftereffects in the material. It refers the wall to all of its previous positions, but weighted by the function $g(t)$. This function is defined by $g(t) = \int_0^\infty p(\tau)/\tau e^{-t/\tau} d\tau$, $p(\tau)$ being the probability of a relaxation process of time constant τ, and it is strictly decreasing. As can be expected, the stress due to the inmediately preceding positions is the most important in the weighting. On the other hand, $f(U)$ depends on the wall type.[4] For example, in the case of a 90° wall of thickness d, it was found that

$$f_{90°}(U) = -\left(\frac{U}{d} + \coth\frac{U}{d} - \frac{U}{d}\coth^2\frac{U}{d}\right). \quad (3)$$

On the other hand, the second term in $P(u,t)$ takes into account the magnetic viscosity. As the $-R(u)$ term in Eq. (1), it is a stress that refers the wall to its last equilibrium position. However, its effect vanishes for large t. The function $o(u)$ gives the strength of this stress and depends on the initial state of the lattice. This function can be defined as $o(u) = f[u(0) - u]$, $u(0)$ being the wall initial position, when the material is initially in a steady state. But, if the material is in a demagnetized state, that is, when the directions of the magnetic dipoles inside the material are equally distributed, $o(u) \equiv 0$. Taking up again Snoek's model, the first situation corresponds to the case in which the ball would be totally sunk in the mud. In the second one, the mud would be uniformly distributed over the concrete. This last case allows us to study in detail the magnetic induced anisotropy relaxation and, then, the aftereffect phenomena in these materials. Hence, we consider the demagnetized state as the initial state in our study.

To simplify the solution of Eq. (1), is usually considered that the wall motion stays in the zone where the $-R(u)$ function is approximately linear. On the other hand, the magnetic-field stress can be written as proportional to its modulus and to the saturation magnetization, that is

$$m\ddot{u} + \upsilon\dot{u} = -ru + \gamma H(t) M_s + P(u,t). \quad (4)$$

Some authors[7] have proposed solutions to this equation by means of an attraction between the wall and some mobile elements called pinning centers that is used instead of the $P(u,t)$ stress. In this work, we will consider that wall displacements are also in the linear zone of the $f(U)$ function, with the same idea that we have given above for the elastic term, that is, $f(U) \simeq -fU$, where f is now a constant, and

$$m\ddot{u} + \upsilon\dot{u} = -ru + \gamma H(t) M_s - fP_0\left[u(t)G(t) - \int_0^t u(t')g(t-t')dt'\right]. \quad (5)$$

Here, we have introduced the relaxing function $G(t) = \int_0^t g(t')dt' = 1 - \int_0^\infty e^{-t/\tau} p(\tau) d\tau$. This function increases from $G(0) = 0$ to $G(\infty) = 1$. In a simple case, $G(t)$ presents multiexponential features, but it depends on the applied model for the relaxation. In some cases, a description of this relaxation in terms of the so-called stretched exponential[8] has been found useful to describe some experimental results of magnetic aftereffects in magnetite samples.[9]

A. Low-frequency motion

When the applied magnetic field has very low frequency, we can suppose that the inertial and damping terms in Eq. (5) are negligible. Hence, Eq. (5) is no longer a differential equation and

$$0 = -ru + \gamma H(t) M_s - fP_0\left[u(t)G(t) - \int_0^t u(t')g(t-t')dt'\right]. \quad (6)$$

The solution of this equation can be found if the wall is characterized by a single time constant and the field is harmonic, that is, $H(t) = H_0 \sin \omega t$, resulting in

$$u(t) = u_0 \frac{e^{-t/\tau(1+\eta)}}{[1+\eta(1-e^{-t/\tau})]^{1/1+\eta}} \times \int_0^t \frac{e^{t'\tau(1+\eta)}\left(\frac{1}{\tau}\sin \omega t' + \omega \cos \omega t'\right)}{[1+\eta(1-e^{-t'/\tau})]^{\eta/1+\eta}} dt', \quad (7)$$

where we have introduced the parameters $u_0 = \gamma M_s H_0 / r$ and $\eta = fP_0/r$. The first one represents the maximum wall amplitude in the absence of induced anisotropy relaxation, and the η value defines the ratio between the elastic constant r and another elastic term added by the induced anisotropy relaxation term. When $\eta \gg 1$, this second term prevails.

Depending on the $\omega\tau$ product, we will consider three different situations. First, Fig. 1 shows the wall response for a constant frequency given by the inverse of the time constant τ, that is, $\omega\tau = 1$, taking the η value as a parameter. So, relaxation and wall motion occur at the same speed. We can see that the wall motion is damped by the magnetic induced anisotropy relaxation and, also, how the wall motion and the applied field are out of phase as η value increases. In all cases, the displacements tend to remain near the wall equilibrium position.

On the other hand, when $\omega\tau \ll 1$, the redistribution of the magnetic dipoles is fast enough to keep the system in equilibrium. Hence, we can propose the following approximate response:

$$\frac{u(t)}{u_0} \simeq \frac{H(t)}{H_0}. \quad (8)$$

As the frequency increases, the redistribution effect will modulate the wall motion, modifying its speed. Then, the frequency spectrum of the wall response $u(t)$ spreads slightly, especially near the field frequency ω. On the other hand, if the time constants of the redistribution are greatly above the inverse of the field frequency, the $g(t-\theta)$ function acts as a low-pass filter in the convolution term $\int_0^t u(\theta) g(t-\theta) d\theta$ and, hence, this term vanishes. In this third case, an approximate solution of Eq. (6) is given by

$$u(t) \simeq \frac{\gamma M_s}{r[1+\eta G(t)]} H(t). \quad (9)$$

Hence, if $\omega\tau \gg 1$, the wall mobility decreases with time. Figure 2 presents the wall frequency response for a constant η

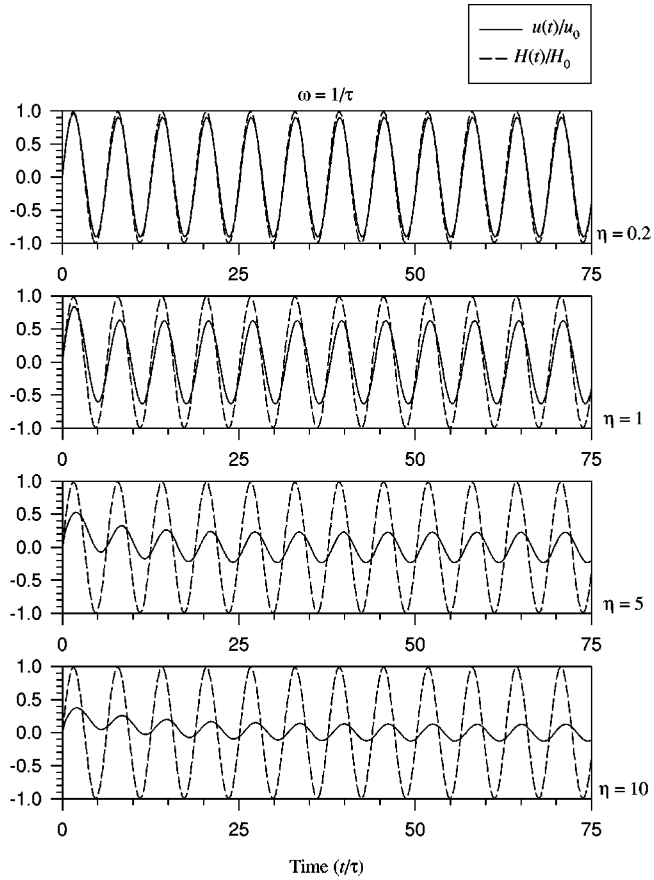

FIG. 1. Wall response under the action of a harmonic magnetic field with a frequency given by the time constant inverse. Plots correspond to different η values (see text).

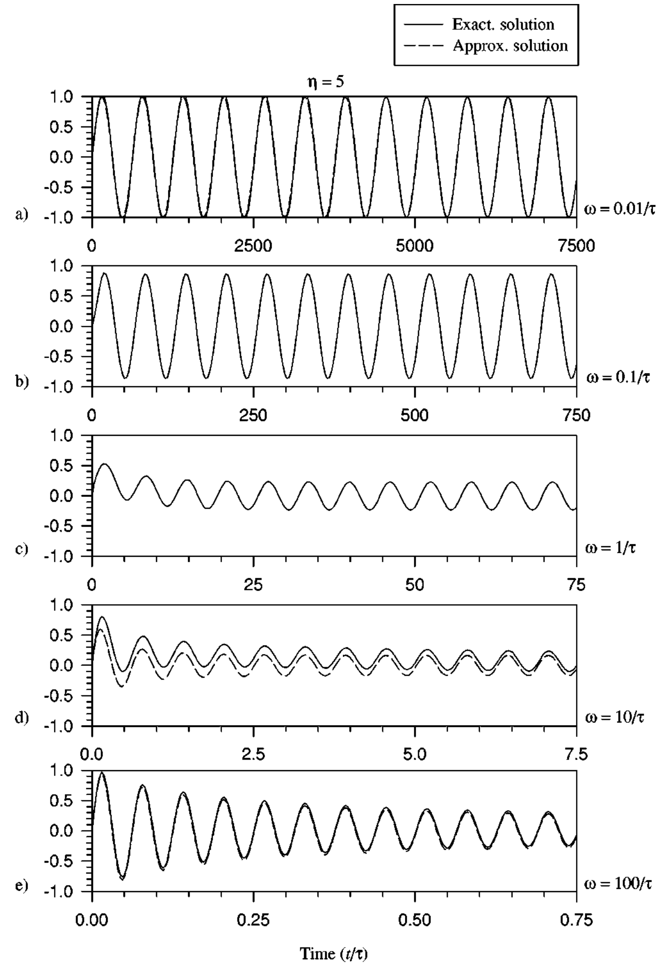

FIG. 2. Wall response under the action of a harmonic magnetic field. For a given η value, plots correspond to different frequencies. We compare the exact solutions with the corresponding approximate solutions in the cases $\omega\tau \ll 1$ (a) and $\omega\tau \ll 1$ (d) and (e).

alue, sufficiently high to take account of the induced anisotropy relaxation phenomena. We can see that both approximate responses, Eqs. (8) and (9), are in good agreement with the exact solutions.

B. High-frequency motion

Using the same approximation, as in obtaining Eq. (9), we can rewrite Eq. (5) for a high-frequency magnetic field $H(t) = H_0 \sin \omega t$ as

$$\ddot{u} + 2\delta\omega_n u + \omega_n^2[1+\eta G(t)]u = u_0\omega_n^2 \sin \omega t, \tag{10}$$

being $\omega_n = \sqrt{r/m}$ the wall natural resonance frequency and $\delta = 1/2\omega_n \upsilon/m$ the damping coefficient. This differential equation can be solved numerically. Figure 3 presents the results obtained for two illustrative cases. In the first one, we find that the wall amplitude increases with time. In the second one, we can see how the amplitude increases initially with time, passes through a maximum and then decreases. This behavior can be interpreted as a time-dependent wall resonance frequency. If we consider second-order wall dynamics, the relaxation term will increase with time, modifying the elastic term and increasing in the same way as the wall resonance frequency, that is

$$\omega_0 = \omega_n\sqrt{1+\eta G(t)}, \tag{11}$$

as shown in Fig. 4. The transients caused by this change will vanish if it is sufficiently slow. The wall response is thus approximated by

$$u(t) \simeq \frac{u_0}{\sqrt{\left[1-\dfrac{\omega^2}{\omega_n^2}+\eta G(t)\right]^2 + 4\delta^2\dfrac{\omega^2}{\omega_n^2}}} \sin[\omega t - \varphi(t)], \tag{12}$$

where

$$\tan \varphi(t) = \frac{2\delta\dfrac{\omega}{\omega_n}}{1-\dfrac{\omega^2}{\omega_n^2}+\eta G(t)}. \tag{13}$$

Transients will vanish if, for a given δ value, we have

$$\delta\omega_n \gg \frac{\eta}{\tau_m} \;\text{ if } 0 < \delta \leqslant 1, \tag{14a}$$

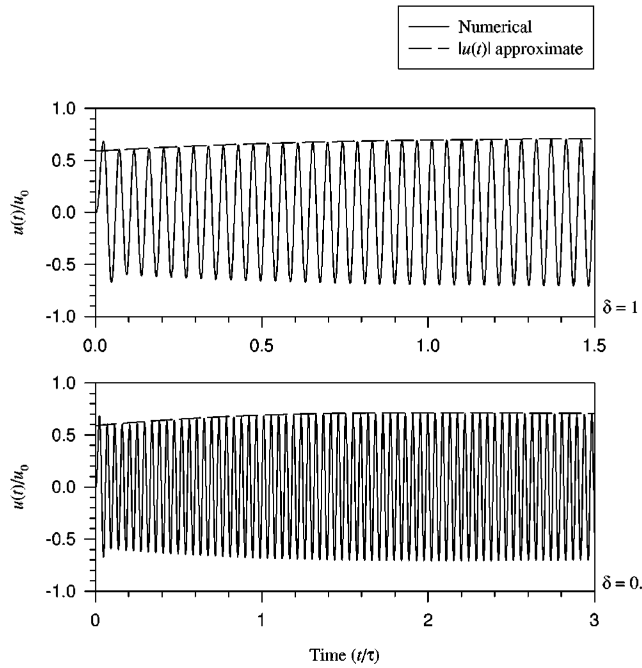

FIG. 3. High-frequency wall response in the cases $\delta=1$ and $\delta=0.5$, respectively.

$$(\delta-\sqrt{\delta^2-1})\,\omega_n \gg \frac{\eta}{\tau_m} \quad \text{otherwise,} \tag{14b}$$

where τ_m represents the lowest relaxation time constant.

Dumping effects are found to decrease with time. Figure 5 presents the evolution of the wall frequency response. Depending on the field frequency, the wall amplitude will either decrease, increase, or both with time.

III. EXPERIMENTAL RESULTS

According to our theory, in a demagnetized sample of a magnetic ordered material under the influence of an applied magnetic field, the motion amplitude of Bloch walls varies with time depending on the field frequency. We have verified this behavior experimentally. We have evaluated wall mobility by measuring reversible magnetic permeability. As a first approximation, the magnetization in the material can be considered as proportional to domain-wall displacements. If

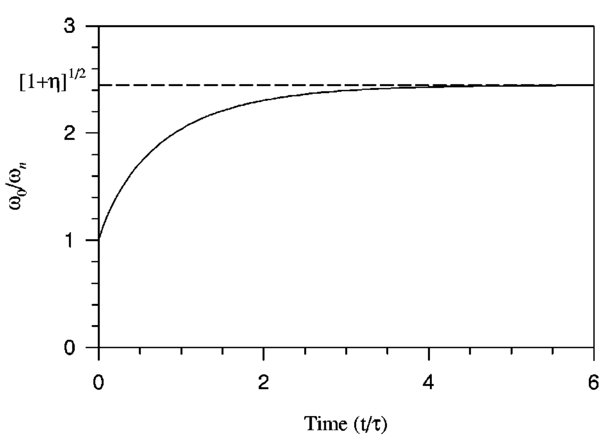

FIG. 4. Time evolution of the wall resonance frequency.

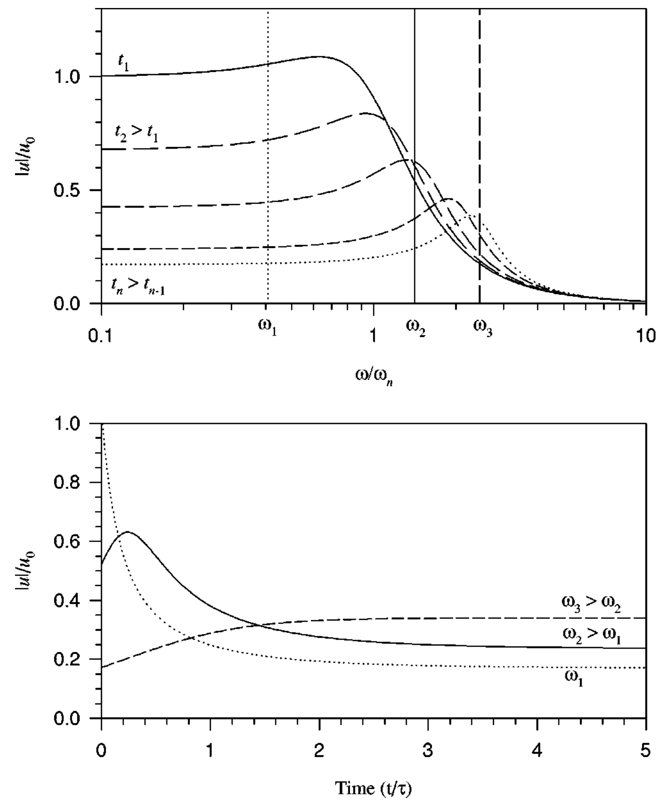

FIG. 5. Time evolution of the frequency wall response and instantaneous amplitude obtained for a given magnetic-field frequency.

these displacements are reversible, we define a magnetic susceptibility as a real magnitude that varies with time in the form $\chi(t)=|\chi'(t)-j\chi''(t)|$, where

$$\chi'(t)\simeq\chi_0\frac{1-\dfrac{\omega^2}{\omega_n^2}+\eta G(t)}{\left[1-\dfrac{\omega^2}{\omega_n^2}+\eta G(t)\right]^2+4\delta^2\dfrac{\omega^2}{\omega_n^2}}, \tag{15}$$

and

$$\chi''(t)\simeq\chi_0\frac{2\delta\dfrac{\omega}{\omega_n}}{\left[1-\dfrac{\omega^2}{\omega_n^2}+\eta G(t)\right]^2+4\delta^2\dfrac{\omega^2}{\omega_n^2}}. \tag{16}$$

A similar behavior is obtained for the magnetic permeability. Hence, a useful technique for studying aftereffect phenomena in magnetic materials consists in the measurement of the time evolution of the magnetic initial permeability of a sample after demagnetization. This technique is termed as permeability disaccommodation (DA).

In our case, magnetic DA measurements have been carried out with the help of a computer aided system based on the use of an automatic LCR bridge that records the impedance of a coil wound around the specimen, i.e., we record the value $Z=j\omega L_0(\mu'-j\mu'')$, ω being the measurement frequency and L_0 the empty coil inductance. During the measuring process, the time variation of the initial magnetic per-

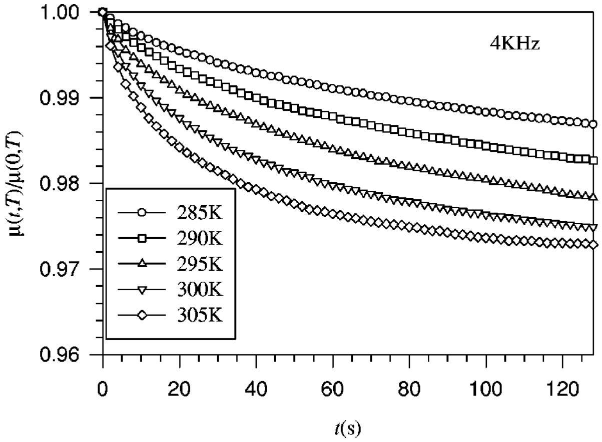

FIG. 6. Normalized magnetic permeability vs time for a polycrystalline magnetite sample at different temperatures (measurement frequency: 4 KHz).

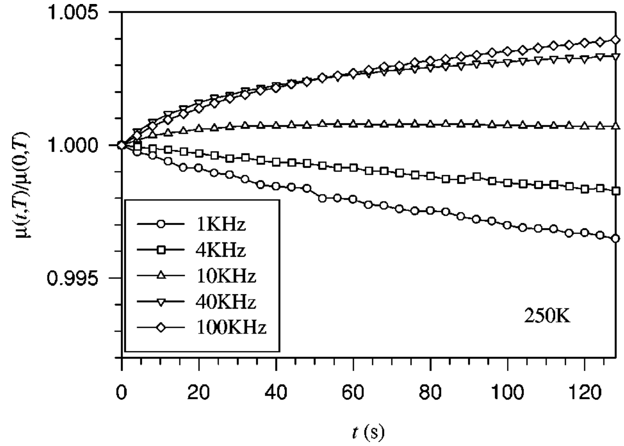

FIG. 8. Normalized magnetic permeability vs time for a polycrystalline YIG sample at different frequencies (temperature: 250 K).

meability after sample demagnetization is recorded while scanning temperature. As noted, demagnetization is very important here, because it affects DA results. In our experiments, samples are demagnetized by using a low-frequency alternating field decreasing in amplitude, as was suggested by Néel,[3] the maximum amplitude and the duration of this signal have been chosen to obtain coherent DA results. This experimental apparatus is described in detail in Ref. 10.

A great number of DA experiments have been carried out with a broad range of ferromagnetic and ferrimagnetic materials. The results of such experiments are particularly well known in the case of magnetite. In this material, a strong temperature-dependent DA is observed at room temperature with time constants of the order of minutes. Its origin has been traditionally attributed to the formation of an induced anisotropy due to thermally activated jumps of octahedral ferrous ions into octahedral spinel lattice vacancies, thus giving rise to a reorientation of the local symmetry axis.[11] Figure 6 presents, for a measurement frequency of 4 KHz, the isothermal DA results obtained for a polycrystalline magnetite sample with cation vacancies. We can see in all cases that permeability decreases with time. No frequency dependence was found.

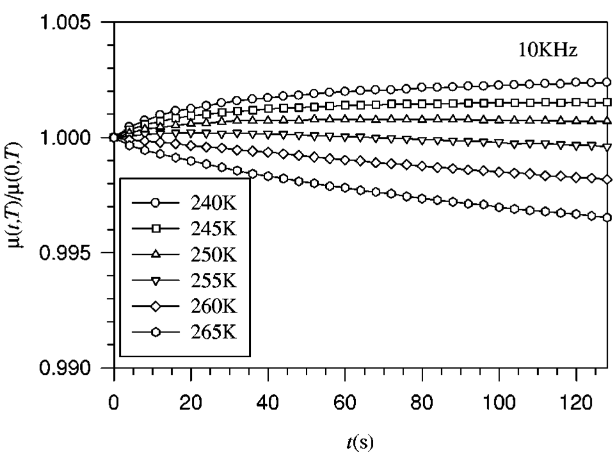

FIG. 7. Normalized magnetic permeability vs time for a polycrystalline YIG sample at different temperatures (measurement frequency: 10 KHz).

Less well known are the mechanisms that give rise to DA processes in yttrium-iron garnets (YIG). Figure 7 shows the isothermal permeability curves obtained at a measurement frequency of 10 KHz for a polycrystalline YIG sample with oxygen vacancies.[12] We observe that permeability decreases with time at high temperatures, but increases for low temperatures. Between these bounding cases we find that the permeability both increases and decreases. On the other hand, Fig. 8 presents the DA study in frequency for a temperature of 250 K. We see, in a way similar to that obtained for temperature, that high measurement frequencies

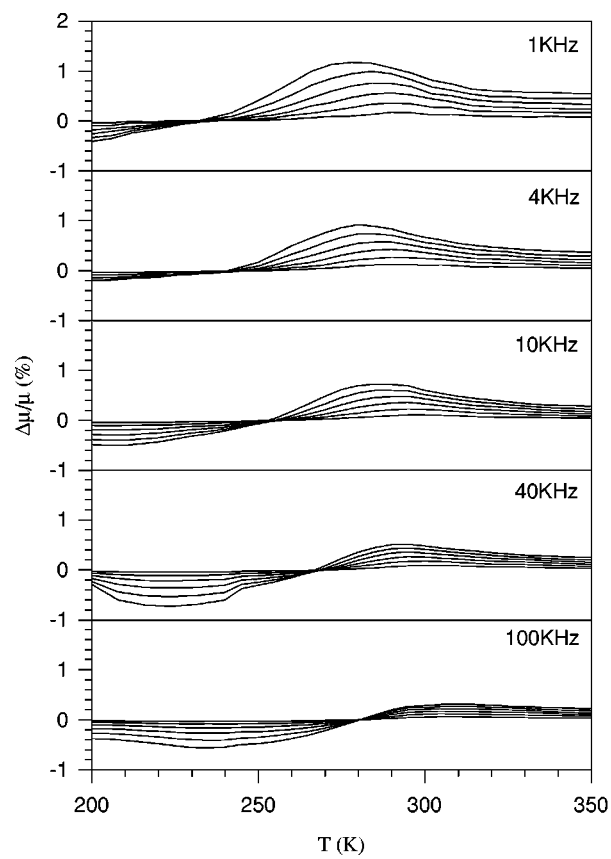

FIG. 9. Isochronal DA spectra for a YIG sample obtained at different measurement frequencies.

give rise to an increasing permeability with time and low measurement frequencies behave oppositely.

Both dependences, temperature and frequency, are shown in Fig. 9 in an alternative way. In this figure, we plot the isochronal spectra obtained for the YIG sample at frequencies from 1 to 100 KHz. From isothermal curves, the isochronal relaxation spectra are constructed in the form

$$\frac{\Delta\mu}{\mu}(\%)=\frac{\mu(t_1,T)-\mu(t_2,T)}{\mu(t_1,T)}\times 100, \tag{17}$$

where $t_1=2$ s and $t_2=4$, 8, 16, 32, 64, and 128 s. Isochronal plots are very useful in the study of thermally activated phenomena because such representations differentiate the various thermal relaxation processes that are present in a relaxing system.[9] Here, we are interested in the effect of frequency and temperature on DA. When permeability decreases with time for a given temperature, we have at this temperature a positive isochronal DA spectrum. Conversely, negative isochronal DA spectra will represent a permeability increasing with time. Hence, there must be a certain temperature for which every isochronal DA spectrum passes through zero. From Fig. 9, this temperature increases with the measurement frequency. The frequency-dependent damping and stiffness observed in the domain walls of YIG (Ref. 13) will be considered in a forthcoming paper.

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