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# Energy-dependent quasiparticle group velocity in a superconductor

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We have measured the energy dependence of the quasiparticle group velocity in a superconductor. We use a normal-insulator-superconductor tunnel junction to inject quasiparticles at known energies into a superconducting Al film. The quasiparticles diffuse throughout the superconductor and their flux is subsequently measured at a known distance from the injector junction by a thermal detector. We measure an increase in the flux of quasiparticles reaching the detector when their energy is raised and explain this result using the predictions of Bardeen, Cooper, and Schrieffer for the energy dependence of the quasiparticle group velocity. [S0163-1829(98)03238-X]

The injection and detection of quasiparticles has been used to probe nonequilibrium superconductivity for over 30 years. Particular effort has been devoted to the study of quasiparticle distributions with nonthermal densities,[1] energies,[2] and branch composition.[3,4] The field is reviewed in Ref. 5. Despite this extensive body of work, an important prediction of the theory of Bardeen, Cooper, and Schrieffer (BCS) has never been observed. According to BCS, the group velocity of a quasiparticle in a superconductor is zero when its excitation energy is equal to the energy gap, $\Delta$, of the superconductor and increases to near the Fermi velocity at a few multiples of $\Delta$.[6] This increase occurs because the energy versus wave-vector excitation spectrum in a superconductor has a minimum at the Fermi wave vector $k_F$ and, unlike in a normal metal, the derivative is equal to zero at $k_F$.

It is particularly important to verify this prediction of BCS theory for additional reasons. Recently, a class of ultrasensitive detectors that exploit the unique properties of superconducting films has emerged.[7] These detectors provide unprecedented sensitivity in applications such as x-ray spectroscopy,[8] optical photon counting,[9] mass spectrometry of biological samples,[10] and the search for weakly interacting dark matter.[11] In these detectors, an incident excitation creates quasiparticles in a superconducting absorber. The quasiparticles diffuse within the absorber until measured in a nearby sensor. It is essential to understand factors that influence quasiparticle propagation in the absorber since quasiparticle losses that occur prior to reaching the sensor can severely degrade detector performance.

In this paper, we present observations of the energy dependence of the quasiparticle velocity, and find the dependence to be in excellent agreement with the predictions of BCS theory. In what follows, we first describe our device and measurement technique and then develop a model of quasiparticle behavior which explains our data qualitatively and quantitatively. Smaller device dimensions and lower temperatures than used in previous work provide the high sensitivity to quasiparticle velocity crucial for this investigation.

A schematic of the device is shown in Fig. 1. Quasiparticles are injected into the central superconducting Al film by one of two normal-insulator-superconductor (NIS) tunnel junctions. Quasiparticles not lost to recombination diffuse into an adjoining Ag film where they thermalize to the Fermi

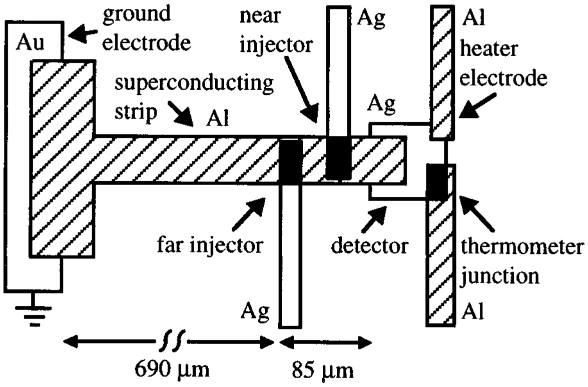

FIG. 1. Schematic of the device: superconducting films are cross-hatched, normal films are clear, and tunnel junction contacts are dark. Two NIS junctions, labeled 1 and 2 and having normal-state resistances of 70 Ω and 119 Ω, respectively, are used to inject quasiparticles into a 75 nm thick Al strip. The Al strip had a resistivity of 5.6 $\mu\Omega$ cm at 4.2 K implying a normal-state diffusion constant $D_n = 53$ cm$^2$/s. The superconducting energy gap $\Delta$ was 221 $\mu$eV. The thickness of the Ag detector was 140 nm, and its resistance as measured between the heater electrode and the Al strip was $0.040 \pm 0.004$ Ω. Not shown are additional superconducting films used to make a four-point resistance measurement of the Ag and also to reduce Joule heating in the normal electrodes of the injector junctions. All dimensions to the right of junction 2 are to scale with the 85 $\mu$m arrow. Features to the left of junction 2 are not to scale. The superconducting strip broadens to make good electrical contact with the ground electrode. The broadening occurs sufficiently far (370 $\mu$m) from junction 2 that the diffusion of quasiparticles takes place almost entirely in the narrow region. The device was fabricated on a Si substrate covered with a 1.5 $\mu$m SiO layer. The device was fabricated in a single vacuum cycle using thermal and electron beam evaporation through a micromachined Si mask and was cooled in a magnetically shielded environment.

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sea. The resulting temperature rise of electrons in the Ag is measured from the current-voltage characteristics of a third NIS junction where part of the Ag forms the normal electrode of the junction. Also shown in Fig. 1 is an additional superconducting electrode which makes metallic contact to the Ag. Current flow through this heater electrode dissipates a known Joule power directly and entirely in the electrons of the Ag and thus provides a calibration for the response of the thermometer junction to power deposited by the incident quasiparticle flux. In a separate experiment we find that the Ag film is an excellent quasiparticle detector. Trapped quasiparticles deposit more than 80% of their energy in the electrons of the Ag and this fraction is independent of temperature (and hence quasiparticle flux) over the range of interest.[12,13] As shown in Fig. 1, the circuit ground is positioned sufficiently far from the injector junctions so that a negligible fraction of the injected quasiparticles reach the ground electrode.

A typical measurement proceeds in the following manner: a current $I$ is passed through one of the two injector junctions creating $(I/e)$ quasiparticles per second in the central superconducting strip. The net power injected into the superconductor $P_i$ is thus $(I/e)\langle E\rangle$, where $\langle E\rangle$ is the average-injected quasiparticle energy which depends on the voltage $V$ across the junction and the electron temperature $T$ in the normal electrode via the relation

$$\langle E\rangle=\frac{\int_{\Delta}^{\infty}EK(E,V,T)dE}{\int_{\Delta}^{\infty}K(E,V,T)dE}. \tag{1}$$

Here $K(E,V,T)=E(E^2-\Delta^2)^{-1/2}[f(E+eV,T)+f(E-eV,T)-2f(E,T)]$ and $f(E,T)$ is the Fermi function. We calculate that self-heating in the injectors can elevate $T$ slightly above the bath temperature and include this effect when deducing $\langle E\rangle$. For our experimental conditions $\langle E\rangle$ is typically lower than $1.1\Delta$.

Due to recombination, only a fraction of the injected quasiparticles reach the detector and thus the detected power $P_d$ is lower than the injected power $P_i$. The magnitude of recombination losses will depend on the time for quasiparticles to diffuse to the detector, and hence, the quasiparticle velocity. We determine $P_d$ by finding the applied Joule power that yields the same rise in the detector temperature as the injected current $I$. In Fig. 2(a) we show the measured dependence of the detector temperature $T_d$ on the current injected through junction 1 for bath temperatures of 35, 150, and 250 mK. In Fig. 2(b) we show the detected power $P_d$ deduced from the data in Fig. 2(a) and the power calibration of the thermometer. The power injected into the superconducting strip $P_i$ is also shown. The ratio $P_d/P_i$ is less than unity at all currents which indicates that a substantial fraction of the injected quasiparticles recombine before reaching the detector. Note also that the detected power $P_d$ increases with temperature for fixed current.

To understand the measured dependence of $P_d$ on the injected current and bath temperature, we model the propagation and loss of quasiparticles in the Al strip. We solve the following one-dimensional diffusion equation for the spatially varying density $n(x)$ of injected quasiparticles in the superconductor, where $n=0$ at both ends of the strip[14]

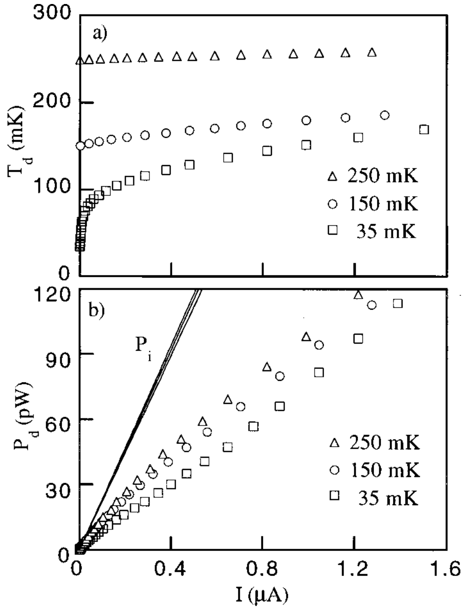

FIG. 2. (a) Dependence of the detector temperature $T_d$ on injected current through junction 1 at bath temperatures of 35, 150, and 250 mK. Detector temperatures below 100 mK probably underestimate the true value because of self-heating in the thermometer junction. (b) Values of detected power $P_d$ corresponding to the injection currents in (a). Self-heating does not affect measurements of $P_d$ because thermometer response is calibrated against power. The injected power $P_i$ is also shown and increases only slightly with bath temperature at a given current.

$$D\nabla^2 n-\Gamma n^2+(I/e)g(x)=0. \tag{2}$$

The first term in Eq. (2) describes the diffusion of quasiparticles with diffusion constant $D$. For a single quasiparticle at a given excitation energy $E$, BCS theory predicts an energy-dependent group velocity, $v(E)=v_F\sqrt{1-(\Delta/E)^2}$, which results in an energy-dependent diffusion constant $(v(E)/v_F)D_n$. Here $v_F$ is the Fermi velocity and $D_n$ is the normal-state diffusion constant calculated from the 4.2 K resistivity. Since a NIS junction injects quasiparticles at a range of energies, the diffusion constant in Eq. (2) is given by $D=(\langle v(E)\rangle/v_F)D_n$, where $\langle v(E)\rangle$ is the average group velocity calculated in a manner similar to $\langle E\rangle$ in Eq. (1). In Fig. 3 we plot calculated values of $D/D_n$ as a function of injector voltage at three experimental temperatures. It is important to note that for fixed temperature, $D$ is an increasing function of injector voltage, and that for fixed voltage, $D$ is an increasing function of temperature. Our experiment is specifically designed to test both of these dependences.

The second term in Eq. (2) describes recombination within the superconducting strip. For our experimental conditions, the density of injected quasiparticles is much larger than the density of thermal excitations.[15] Consequently, recombination occurs almost entirely within the injected population at a rate proportional to $n^2$, where $\Gamma$ in Eq. (2) is the recombination rate per unit density. The prefactor $\Gamma$ is pre-

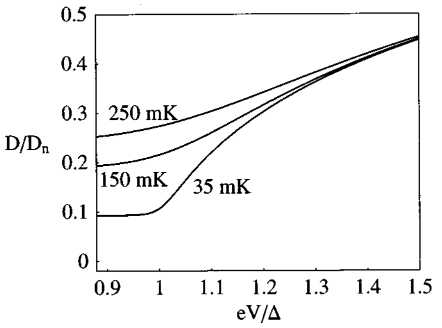

FIG. 3. Calculated values of $D/D_n$ as a function of injector voltage at 35, 150, and 250 mK. For voltages near $\Delta/e$, $D/D_n$ is a strong function of temperature.

dicted to be independent of temperature and quasiparticle energy for low temperatures and energies.[16]

The third term in Eq. (2) describes the injection of quasiparticles into the superconductor at a rate $(I/e)$ by one of the two NIS junctions. The function $g(x)$ is zero everywhere except under an injector junction where $g(x)^{-1}$ is the volume of the superconductor beneath the junction.

Once the quasiparticle density $n(x)$ is determined from Eq. (2), the detected power $P_d$ is calculated from the product $DA\langle E\rangle(dn/dx)$, where $A$ is the cross-sectional area of the superconductor and $dn/dx$ is evaluated at the detector location. We assume that thermalized quasiparticles deposit 100% of their energy in the detector. As mentioned earlier, the likely range is 80–100 %. The one free parameter in our model $\Gamma$ is varied until the predictions of the model for $P_d$ match the data at one value of current and temperature for one of the two injector junctions, and is thereafter kept fixed. We then compare our fitted value for $\Gamma$ with theoretical predictions.

In Fig. 4 we show the predicted and measured dependences of $P_d/P_i$ on injected current at bath temperatures of 35, 150, and 250 mK. Excellent agreement between experiment and model is obtained for $\Gamma=26$ $\mu\text{m}^3\,\text{s}^{-1}$, which is reasonably close to the value of $\Gamma=58$ $\mu\text{m}^3\,\text{s}^{-1}$ predicted from the theory of Kaplan.[16–18] The success of our model supports the following conclusions. For a given temperature, the ratio $P_d/P_i$ decreases as the injection current is raised because a larger fraction of the quasiparticles recombine as the quasiparticle density rises. For a given injection current, the ratio $P_d/P_i$ increases with temperature because the quasiparticle diffusion constant rises and therefore fewer quasiparticles recombine before reaching the detector. The slight upturn in $P_d/P_i$ observed for both junctions near 0.4 $\mu$A at 35 mK occurs because the increase in diffusion constant with injector voltage outweighs the effects of increasing quasiparticle density (and hence recombination) in this current range. Data sets acquired at different temperatures converge at large currents because the diffusion constant ceases to depend on temperature for injector voltages where $eV-\Delta \gg k_bT$. This convergence also supports the use of a temperature independent value for $\Gamma$. We emphasize that it is not possible to predict either the temperature or current dependence of our data without incorporating the energy dependence of the quasiparticle group velocity into the diffusion model.

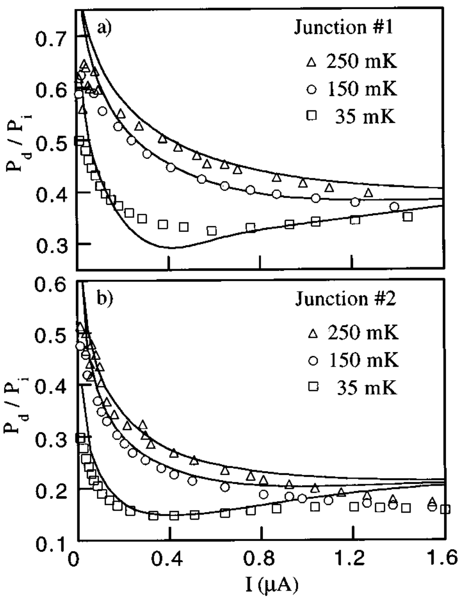

FIG. 4. (a) Theoretical and experimental dependence of $P_d/P_i$ on $I$ for junction 1 at 35, 150, and 250 mK. (b) Same as (a) but for junction 2. At 35 mK, the currents corresponding to injector voltages of $\Delta/e$ and $1.1\Delta/e$ are 0.3 and 1.0 $\mu$A, respectively for both junctions. We chose $\Gamma$ so that data and model agree at 0.64 $\mu$A through junction 1 at 150 mK, but have verified that fitting at other data points yields similar values of $\Gamma$. The quality of the fits does not depend on the assumption that quasiparticles deposit 100% of their energy in the detector.

The predictions of our model exceed the measured values of $P_d/P_i$ for large injection currents because we have neglected the scattering of quasiparticles to lower energy states. This simplification is appropriate for small currents where $\langle E\rangle \approx \Delta$ and the scattering rate is low, but it is not valid at higher currents because the scattering rate is a strongly increasing function of quasiparticle energy. Our model thus overestimates $\langle E\rangle$ and hence the diffusion constant at high currents. We calculate that the scattering time for a quasiparticle with energy $1.2\Delta$, corresponding to $I=2.3$ $\mu$A in Fig. 4, is equal to the diffusion time from injector 2 to the detector, and therefore expect the model to overestimate $P_d/P_i$ near this current.[16]

It is possible to improve the predictions of the model at very low currents by including an additional loss term $-n/\tau_D$ in Eq. (2) that accounts for recombination at defects or imperfections in the superconductor. We find that the best fit is obtained for $\tau_D=15$ $\mu$s and $\Gamma=22$ $\mu\text{m}^3\,\text{s}^{-1}$. It should be noted that if we use only the $-n/\tau_D$ loss term, rather than the $-\Gamma n^2$ term, the predictions of the diffusion model for $P_d/P_i$ increase with current which is in disagreement with the data.

In conclusion, we have directly measured the dependence of the quasiparticle group velocity on energy. This dependence was predicted over 40 years ago but has not previously been observed. Our observations are in excellent agreement with BCS theory.

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[1] D. M. Ginsberg, Phys. Rev. Lett. **8**, 204 (1962).

[2] S. B. Kaplan, J. R. Kirtley, and D. N. Langenberg, Phys. Rev. Lett. **39**, 291 (1977).

[3] J. Clark, Phys. Rev. Lett. **28**, 1363 (1972).

[4] M. Tinkham and J. Clarke, Phys. Rev. Lett. **28**, 1366 (1972).

[5] D. N. Langenberg, in *Low Temperature Physics-LT14*, edited by M. Krusius and M. Vuorio (North-Holland, Amsterdam, 1975), p. 223.

[6] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[7] N. E. Booth, B. Cabrera, and E. Fiorini, Annu. Rev. Nucl. Part. Sci. **46**, 471 (1996).

[8] C. A. Mears, S. E. Labov, M. Frank, and M. A. Lindeman, Nucl. Instrum. Methods Phys. Res. A **370**, 53 (1996).

[9] A. Peacock *et al.*, Nature (London) **381**, 135 (1996).

[10] D. Twerenbold, Nucl. Instrum. Methods Phys. Res. A **370**, 253 (1996).

[11] K. D. Irwin *et al.*, Rev. Sci. Instrum. **66**, 5322 (1995), and references therein.

[12] A detailed summary of the performance of a normal metal trap will be submitted separately for publication.

[13] We have measured the response of the detector in similar devices when the interface between the Al strip and the Ag was physically scratched and found that heating through the substrate is present but negligible compared to heating by quasiparticles when the interface is intact.

[14] The 40 $\mu$m wide Al strip overlaps the Ag detector by 30 $\mu$m. We make the simplifying assumption that $n=0$ at a distance equal to half the width of the superconducting strip beyond the Al/Ag interface.

[15] The thermal quasiparticle density is calculated to be $180/\mu m^3$ at 250 mK. In contrast, our model predicts injected quasiparticle densities greater than $900/\mu\text{m}^3$ beneath the junctions for currents as small as 19 nA.

[16] S. B. Kaplan *et al.*, Phys. Rev. B **14**, 4854 (1976).

[17] The characteristic electron-phonon time in our Al is taken to be 58 ns based on its high resistivity. This change increases $\Gamma$ by a factor of 8 from the value $\Gamma = 10\ \ \mu m^3\ \text{s}^{-1}$ in Ref. 16. C. C. Chi and J. Clark, Phys. Rev. B **19**, 4495 (1979).

[18] We deduce from Ref. 16 that the time for a phonon of energy $2\Delta$ to break a Cooper pair is 0.23 ns. In addition, we calculate that phonons emitted by recombining quasiparticles leave the Al film within 0.10 ns. [S. B. Kaplan, J. Low Temp. Phys. **37**, 343 (1979)] Hence, recombination phonons have a 30% chance of breaking a Cooper pair before they escape into the substrate. This possibility decreases $\Gamma$ by a factor of 1.4 from Ref. 16. We conclude from the measured values of $P_d/P_i$ and the pair breaking probability of 30% that the number of cycles of phonon emission and absorption before reaching the detector is of order one. Since the number of cycles, the time per cycle, and the distance traveled by a phonon during a cycle are small, we conclude that recombination phonons neither play an important role in energy transport nor alter the diffusion constant of the quasiparticle population.