Time-dependent heat diffusion in semiconductors by electrons and phonons

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In this work the electron and phonon temperature distribution functions in semiconductors are calculated. We consider the time-dependent heat flux at the surface of the sample as a boundary condition for the electron and phonon systems, and a fixed temperature at the opposite surface. Those boundary conditions reflect the usual experimental setup in some photothermal experiments where the most common mechanism for producing interacting ''thermal waves'' is the absorption of an intensity-modulated light beam by the semiconductor. The electron and phonon temperature distributions in the sample are given as a function of both, time and position valid for a wide range of the modulation frequency ω of the incident light. The response of the system in the limit of high- and low-frequency ω with respect to the characteristic time $\tau_e(\tau_p)$ of the electron (phonon) system is analyzed. [S0163-1829(98)08436-7]

I. INTRODUCTION

The study of photothermal phenomena has recently developed into a very active area of research in applied physics. Photothermal technique has become a valuable tool in measuring complex parameters of solids and for characterizing process parameters in the manufacturing of electronic devices. These techniques are versatile and nondestructive, and can be employed in a variety of experimental conditions for determining the physical parameters of solids and biological materials. Some of the different contact-type detection techniques, such as conventional gas-microphone photoacoustic detection, photopyroelectric detection, or remote sensing techniques such as photothermal reflection, etc., were revised in Ref. 1.

Being a photothermal technique, the detected signal is strongly dependent upon how the heat diffusing through the sample allows us to perform both thermal characterization of the sample (i.e., measurements of its thermal properties, such as thermal diffusivity and thermal conductivity), and carrier transport properties.[2] In the case of semiconductors, the photothermal signal can provide us with additional information regarding the heat-transport properties and electron-phonon energy relaxation by the interacting electron and phonon systems, a fact that has been recognized since the early theory of heat conduction and hot electrons in semiconductors.[3] Qualitatively this be may understood as follows: in studies of the behavior of semiconductors heated by electromagnetic waves, it is usual to assume that the phonon system remains in equilibrium, i.e., the phonon temperature T_p is equal to the temperature T_0 of the ambient medium. However, nonequilibrium carriers in the bulk of the sample can interact and transfer energy to the phonon system, producing an essential increase in the average energy of phonons, which can most conveniently be described in terms of heating of the phonon gas (increase of its temperature).[4]

Since photothermal measurements allow one to determine the diffusion length of carriers, the surface recombination velocity, and the bulk lifetime of carriers, Sablikov and Sandomirskii[5] developed a theory of the photoacoustic effect in semiconductors for the region of fundamental absorption, taking into account the generation of free electrons and holes, their diffusion, and recombination. However, these authors only considered a single-temperature approximation for carriers and phonons.

On the other hand, Sasaki, Negishi, and Inoue[6] were successful in developing a variant of that technique in which a transient thermoelectric effect (TTE) is generated by heating a sample through absorption of pulsed-laser irradiation. The absorption of optical energy generates electron-hole pairs, increases the energy (heating) of the majority carriers, and produces a thermal flux of phonon due to the energy interaction between these quasiparticles. The main advantage of the TTE method is that it is based on direct, straightforward measurements of the electron temperature as a function of position and time. Moreover, TTE also allows one to make local, instantaneous, practically inertialess measurements. Recently there has been some interest in a study of thermal waves in semiconductors. In particular,[7] the electron T_e and phonon T_p temperature distributions were calculated in the limit when the thickness of the sample is greater than the cooling length, defined as the length at which the average energy of the electron system reach equilibrium with the phonon system.[3] For typical values of the heat conductivity of electrons and phonons in semiconductors, it is possible to obtain information about physical parameters describing the diffusivity and thermal conductivity of electrons and phonons as well as the electron-phonon energy interaction in photothermal experiments. On the other hand, the phonon temperature as a function of position can be measured using a thermocouple in different points on the sample. Moreover,[8] thermal diffusion of one- and two-layer systems has been investigated using the heat-diffusion equation in the approximation when both the electron and phonon temperature distributions are equal. This approximation is only valid in the limit of infinite electron-phonon energy interaction.

In this work we further extend the earlier models to study electron and phonon thermal waves in semiconductors by taking into account the finite size of the sample. The generation and recombination of carriers by absorption of light and the diffusion process of carriers are negligible in our model.

This approximation is valid only if any of the following assumption is fulfilled.

(i) The sample is optically opaque, i.e., the incident radiation is completely absorbed at the surface and converted into heat.

(ii) Free-carrier absorption of laser light in the semiconductor, when the photon energy is less that the band-gap energy. Therefore, the initial rise in the carrier temperature due to the excess kinetic energy received during laser excitation causes a substantial increase in the diffusion heat.

(iii) Strong electron-hole recombination close to the surface of the semiconductor after photoexcitation of electron-hole pair. The carriers yield energy which is converted into heat in the sample.

Changes of the electron and phonon concentrations due to the temperature become important only if the variation of the temperature is of the same order as the electron and phonon equilibrium temperature.[9] These thermal waves generated into the material by an incident-beam radiation are analyzed in terms of the characteristic response time of the system according to the modulation frequency. In Sec. II the electron and phonon temperature distributions for semiconductors is calculated by solving the coupled heat-diffusion equations for both quasiparticle systems. In Sec. III a discussion of our results is presented, and a comparison to the predictions of the existing theories is also made. Finally, in Sec. IV we present our conclusions.

II. THEORY

It is well known that heat transport in solids is carried out by various quasiparticles systems, (electrons, holes, magnons, plasmons, etc.). Frequently the interactions between these quasiparticles systems are such that each of these systems can have its own temperature, and the physical conditions at the boundary of the sample can be formulated separately for each quasiparticle system.[10] Therefore steady-state heat conduction in an *n*-type semiconductor (for example) can be described by the following system of equations:

$$\text{div } Q_e = -P_{\text{ep}}(T_e - T_p), \quad \text{div } \vec{Q}_p = P_{\text{ep}}(T_e - T_p). \quad (1)$$

The terms $P_{\text{ep}}(T_e-T_p)$ gives the transfer of heat between electrons and phonons. Here P_{ep} is a parameter proportional to the relaxation energy frequency ν_ϵ between the electron and phonon systems ($P_{\text{ep}}=P\simeq n\nu_\epsilon$, n is electron density and $\nu_\epsilon \simeq 10^8-10^{11}$/s),[4] and the heat flux of electron $\vec{Q}_e$ and phonon $\vec{Q}_p$ systems are described by the usual relationships

$$\vec{Q}_e = -\kappa_e \nabla T_e, \quad \vec{Q}_p = -\kappa_p \nabla T_p, \quad (2)$$

where $\kappa_e(\kappa_p)$ is the electron (phonon) thermal conductivity. So far, Eqs. (1) include only the static contribution of the heat transport, i.e., the heat flux is independent of time. However, in the photothermal experiments, the incident radiation is modulated in time by the chopper, and in this case it is necessary to consider the dynamic contribution to the heat transport in the electron and phonon systems. Let us assume a one-dimensional model for the heat flux. Assuming that the sample is optically opaque to the incident light (i.e., all the incident radiation is absorbed at the surface), the electron and phonon temperature distribution functions are the solutions of[7]

$$\begin{aligned} \frac{\partial^2 T_e(z,t)}{\partial z^2} - k_e^2[T_e(z,t)-T_p(z,t)] &= \frac{1}{\alpha_e}\frac{\partial T_e(z,t)}{\partial t}, \\ \frac{\partial^2 T_p(z,t)}{\partial z^2} + k_p^2[T_e(z,t)-T_p(z,t)] &= \frac{1}{\alpha_p}\frac{\partial T_p(z,t)}{\partial t}, \end{aligned} \quad (3)$$

where $k_{e,p}^2 = P/\kappa_{e,p}$, and $k_{e,p}^{-1}$ are the electron and phonon cooling lengths.[3] The diffusivity for each system is given by $\alpha_{e,p}=\kappa_{e,p}/(\rho c)_{e,p}$, and $\rho_e(\rho_p)$ and $c_e(c_p)$ are the electron (phonon) density and specific heat, respectively.

The temperature fluctuations $T_{e,p}(z,t)$ should be supplemented by boundary conditions at the surface of the semiconductor $(z=0)$. In the photothermal experiments, the most common mechanism to produce thermal waves is the absorption by the sample of an intensity modulated light beam with frequency modulation ω. It is clear that when the intensity of the radiation is fixed, the light converted into heat at the surface of the sample can be written in general as

$$Q_{e,p}(z,t)|_{z=0} = Q_{e,p} + \Delta Q_{e,p} e^{i\omega t}, \quad (4)$$

where $Q_{e,p}$ is the average over time of the total heat flux $Q(z,t)$ transmitted to electron and phonon systems at the surface of the sample, and is proportional to the intensity of the incident light. The last term $\Delta Q_{e,p}e^{i\omega t}$ represents the modulation of the incident beam; it can be positive or negative. This means that heat flux propagates into the sample during a half-period of the modulation heat $(\sim 1/2\omega)$ when Re $\Delta Q_{e,p}e^{i\omega t}>0$, and it propagates toward the surface of the sample when Re $\Delta Q_{e,p}e^{i\omega t}<0$. The heat flux associated with the positive half-period of the modulated incident light converted into heat at the surface of the sample (Re $\Delta Q_{e,p}e^{i\omega t}>0$), can be described as a decaying solution of Eq. (3) with increasing distance from the surface. On the other hand, the growing term of $T_{e,p}$ with distance of the dynamical part of Eq. (3) represents the propagation of the flux in the opposite direction, and is associated with the negative half of the period Re $\Delta Q_{e,p}e^{i\omega t}<0$.

The temperature is not used as a boundary condition because it is usually an unknown parameter in the photothermal experiments. The general solution of the heat-diffusion equations for this one-layer system of thickness d is given by

$$T_{e,p} = A + Bz \pm \frac{k_{e,p}^2}{k^2}[C_1 e^{-kz} + C_2 e^{kz}] + \Theta_{e,p}(z,t), \quad (5)$$

where $k^2=k_e^2+k_p^2$; k represents the inverse of the cooling length. In the derivation of Eq. (5), it is assumed that the heating modulated light intensity is weak, so that we have $|T_{e,p}(z,t)-T_e| \ll T_0$, and diffusion equations can be considered linear.

In order to obtain the constants A, B, and $C_{1,2}$, and the dynamical contribution to the electron and phonon temperature distributions $\Theta_{e,p}(z,t)$, it is necessary to specify the boundary condition at $z=d$. Because the temperature is a thermodynamic parameter, the one-layer system has to be in contact with a heat reservoir at some temperature, so that it is natural to choose a continuity of the temperature distributions in the sample at $z=d$, i.e., $T_{e,p}(z,t)|_{z=d}=T_0$. In the photothermal experimental setup, T_0 represents the ambient temperature. Using this boundary conditions in Eq. (5) the constants are given by

$$A=T_0+d\,\frac{1}{k^2}\left(\frac{Q_e k_p^2}{\kappa_e}+\frac{Q_p k_e^2}{\kappa_p}\right), \tag{6a}$$

$$B=-\frac{1}{k^2}\left(\frac{Q_e k_p^2}{\kappa_e}+\frac{Q_p k_e^2}{\kappa_p}\right), \tag{6b}$$

$$C_1=-\frac{1}{2k}\left(\frac{Q_p}{\kappa_p}-\frac{Q_e}{\kappa_e}\right)\frac{e^{kd}}{\cosh kd}, \tag{6c}$$

$$C_2=\frac{1}{2k}\left(\frac{Q_p}{\kappa_p}-\frac{Q_e}{\kappa_e}\right)\frac{e^{-kd}}{\cosh kd}, \tag{6d}$$

and the function $\Theta_{e,p}(z,t)$ satisfies coupled heat-diffusion equations similar to Eqs. (3) with the following boundary conditions:

$$-\kappa_{e,p}\frac{d\Theta_{e,p}}{dz}\bigg|_{z=0}=\Delta Q_{e,p}e^{i\omega t},\quad \Theta_{e,p}(z,t)|_{z=d}=0. \tag{7}$$

The electron and phonon temperature distributions in the specimen resulting from the modulated light converted into heat in the sample are given by

$$\begin{aligned}T_e(z,t)=T_0&+\frac{1}{k^2}\left(\frac{k_p^2}{\kappa_e}Q_e+\frac{k_e^2}{\kappa_p}Q_p\right)(d-z)\\&+\frac{1}{k}\left(\frac{Q_e}{\kappa_e}-\frac{Q_p}{\kappa_p}\right)\frac{k_e^2}{k^2}\frac{\sinh k(d-z)}{\cosh kd}\\&-\left[\frac{F_1}{\cosh\sigma_1 d}\sinh\sigma_1(d-z)\right.\\&\left.+\frac{F_2}{\cosh\sigma_2 d}\sinh\sigma_2(d-z)\right]e^{i\omega t}\end{aligned} \tag{8}$$

and

$$\begin{aligned}T_p(z,t)=T_0&+\frac{1}{k^2}\left(\frac{k_p^2}{\kappa_e}Q_e+\frac{k_e^2}{\kappa_p}Q_p\right)(d-z)\\&-\frac{1}{k}\left(\frac{Q_e}{\kappa_e}-\frac{Q_p}{\kappa_p}\right)\frac{k_p^2}{k^2}\frac{\sinh k(d-z)}{\cosh kd}\\&-\left[\frac{G_1}{\cosh\sigma_1 d}\sinh\sigma_1(d-z)\right.\\&\left.+\frac{G_2}{\cosh\sigma_2 d}\sinh\sigma_2(d-z)\right]e^{i\omega t},\end{aligned}$$

where

$$F_1=-\frac{1}{\sigma_1(\sigma_2^2-\sigma_1^2)}\left[\frac{\Delta Q_p}{\kappa_p}k_e^2+\frac{\Delta Q_e}{\kappa e}(\sigma_2^2-\sigma_e^2)\right], \tag{9a}$$

$$F_2=\frac{1}{\sigma_2(\sigma_2^2-\sigma_1^2)}\left[\frac{\Delta Q_p}{\kappa_p}k_e^2+\frac{\Delta Q_e}{\kappa_e}(\sigma_1^2-\sigma_e^2)\right], \tag{9b}$$

$$G_{1,2}=-\frac{\sigma_{1,2}^2-\sigma_e^2}{k_e^2}F_{1,2}, \tag{9c}$$

and the values of $\sigma_{1,2}$ are given by

$$\sigma_{1,2}^2=\tfrac{1}{2}(\sigma_e^2+\sigma_p^2)\pm\tfrac{1}{2}[(\sigma_e^2-\sigma_p^2)^2+4k_e^2k_p^2]^{1/2}. \tag{10}$$

Equation (10) represents the condition for nontrivial solution of the coupled electron-phonon differential equations, and $\sigma_{e,p}^2$ are given by

$$\sigma_{e,p}^2=\frac{i\omega}{\alpha_{e,p}}+k_{e,p}^2. \tag{11}$$

As can be seen from Eq. (8), the decreasing exponential term associated with the dynamical part in the temperature fluctuation into the sample attenuates rapidly to zero with increasing distance from the surface, such that, at a distance $L\simeq\max(|\sigma_1|^{-1},|\sigma_2|^{-1})$, this contribution to the dynamical part of the heat-diffusion equations is effectively damped out. Physically this represents a propagation of the heat flux from the surface $z=0$ to $z=d$. Once we know the electron and phonon temperature distributions in the semiconductor, we can calculate the response of the surrounding medium due to the photothermal heating of the sample using one of the several alternative detection techniques mentioned above.

It is worth mentioning that although the dynamical contributions to the temperature distributions in Eqs. (8) have a sinusoidal dependence through the imaginary part of the exponential terms, the temperature distributions in the sample is not like a electromagnetic wave, heat conduction in solids is a diffusive process.[7]

III. ANALYSIS OF T_e AND T_p

We now turn to the discussion of the results obtained so far, and compare them with previous theories on thermal waves. For a nondegenerate semiconductor, the typical ratio of the heat conductivities of electrons and phonons satisfies $\kappa_e/\kappa_p\sim10^{-3}$. Under these circumstances, information about electron and phonon thermal parameters as well as the energy relaxation frequency can be obtained from photothermal experiments depending upon the decay length ($R_e\sigma_{1,2}$) of the dynamical component of the electron and phonon temperature. In this case expressions (8)–(10) are simplified and take the form

$$\begin{aligned}T_e(z,t)=A_e(z)&-\left[F_1\frac{\sinh\sigma_1(d-z)}{\cosh\sigma_1 d}\right.\\&\left.+F_2\frac{\sinh\sigma_2(d-z)}{\cosh\sigma_2 d}\right]e^{i\omega t},\end{aligned} \tag{12a}$$

$$T_p(z,t)=A_p(z)-G\,\frac{\sinh\sigma_2(d-z)}{\cosh\sigma_2 d}\,e^{i\omega t}, \tag{12b}$$

where $G_1\simeq0$, $\sigma_1=\sigma_e$, $\sigma_2^2=i\omega/\alpha_p$, and

$$A_e(z)=T_0+\frac{Q_e+Q_p}{\kappa_p}\,(d-z)+\frac{1}{k_e}\left[\frac{Q_e}{\kappa_e}-\frac{Q_p}{\kappa_p}\right]\frac{\sinh k_e(d-z)}{\cosh k_e d},$$

$$A_p(z)=T_0+\frac{Q_e+Q_p}{\kappa_p}\,(d-z),$$

$$F_1=-\frac{1}{\sigma_e}\left[\frac{k_e^2}{\sigma_2^2-\sigma_e^2}\,\frac{\Delta Q_p}{\kappa_p}+\frac{\Delta Q_e}{\kappa_e}\right],$$

$$F_2=\frac{k_e^2}{\sigma_2(\sigma_2^2-\sigma_e^2)}\,\frac{\Delta Q_p}{\kappa_p},\quad G=\frac{-1}{\sigma_2}\,\frac{\Delta Q_p}{\kappa_p}.$$

Because the source of the photothermal signal arise from the periodic heat flow from the semiconductor, the periodic diffusion process produces a periodic temperature variation in the semiconductor given by the sinusoidal (ac) component of Eq. (12b); In this case, only information about the phonon physical parameters are obtained from those experiments. However, in thermoelectric experiments, the important contribution to the signal comes from Eq. (12a) which gives information about the electron-phonon energy relaxation and electron and phonon thermal parameters through the following relationships:

$$\lambda_1=\frac{2\pi}{\text{Im }\sigma_1}=\frac{2\sqrt{2}\pi}{k_e\left[\left(1+\frac{\omega^2}{\omega_0^2}\right)^{1/2}-1\right]^{1/2}},$$

$$\lambda_2=\frac{2\pi}{\text{Im }\sigma_2}=2\pi\left(\frac{2\alpha_p}{\omega}\right)^{1/2},$$

$$L_1=\frac{1}{\text{Re }\sigma_1}=\frac{\sqrt{2}}{k_e}\,\frac{1}{\left[\left(1+\frac{\omega^2}{\omega_0^2}\right)^{1/2}+1\right]^{1/2}},$$

$$L_2=\frac{1}{\text{Re }\sigma_2}=\left(\frac{2\alpha_p}{\omega}\right)^{1/2},$$

where the quantity $\omega_0=\alpha_e k_e^2=[n/(\rho c)_e]\nu_\epsilon$ has the dimensions of frequency, and it coincides with ν_ϵ the electron energy relaxation frequency in steady state conditions since $(\rho c)_e\sim n$. For various semiconductors $\omega_0\sim\nu_\epsilon\sim(10^8-10^{11})\text{s}^{-1}$. Let us introduce a characteristic relaxation time of the phonon temperature fluctuation by the relationship $\tau_p=d^2/\alpha_p$, and rewrite the above equations as follows:

$$\lambda_1=2\sqrt{2}\pi l_e[\sqrt{1+(\omega\tau_e)^2}-1]^{1/2}, \tag{13a}$$

$$\lambda_2=2\sqrt{2}\pi d(\omega\tau_p)^{-1/2}, \tag{13b}$$

$$L_1=\sqrt{2}l_e[\sqrt{1+(\omega\tau_e)^2}+1]^{1/2}, \tag{13c}$$

$$L_2=\sqrt{2}d(\omega\tau_p)^{-1/2}. \tag{13d}$$

Here $l_e=k_e^{-1}$ is the electron cooling length, and $\tau_e=\nu_\epsilon^{-1}$ is the electron time of energy relaxation. It follows from the expressions (13) that characteristic parameters of the excited thermal waves in the electron and phonon systems are different, and those for the phonon system depend only on phonon parameters.

We would like to draw attention to two aspects which, in the final analysis, are important in applications of the thermoelectric and photoacoustic effects. One of these applications is a determination of a profile of the electron or phonon temperature fluctuation with depth from the surface by measuring the dependence of the voltage or the photoacoustical signal on the modulation frequency ω of the intensity of the incident light. For nondegenerate semiconductors, typical values of the density and the thermal capacity for the phonon system give that $(\rho c)_p\sim 10^{23}$ cm^{-3} (see Ref. 11), while for the electron gas $(\rho c)_e\sim n\sim(10^{14}-10^{19})$ cm^{-3}, and, because $\kappa_e/\kappa_p\sim 10^{-3}$, then the ratio of the thermal diffusivity of electrons and phonons is on the order $\alpha_e/\alpha_p\sim(10\text{–}10^6)$; therefore, from Eqs. (13), $L_1\gg L_2$ whenever $\omega\gg\nu_\epsilon$. Under this approximation, $|F_1|\gg|G|\gg|F_2|$, i.e., the time-dependent component of the phonon temperature in the semiconductor attenuates rapidly to zero with increasing distance from the surface of the solid as compared with the component of the time-dependent electron temperature distribution; in this case, information only about the electron physical parameters are carried out by the electron thermal wave through the coefficient F_1. Hence the thermoelectric experiments, which detect only the electron thermal wave, give us information about electron thermal parameters. On the other hand, information about the electron and phonon physical parameters can be obtained if $\omega\ll(\alpha_p/\alpha_e)\nu_\epsilon$; then $L_1\ll L_2$ and the amplitudes for the dynamical parts of the electron and phonon temperature distributions are given by

$$|F_1|\sim\frac{\Delta Q_e}{\kappa_e}\left[\frac{\alpha_e}{\nu_\epsilon}\right]^{1/2},\quad |F_2|=|G|\sim\frac{\Delta Q_p}{\kappa_p}\left[\frac{\alpha_p}{\omega}\right]^{1/2}.$$

If, in addition, the chopping angular frequency of the light converted into heat satisfies $\omega\ll(\alpha_p/\alpha_e)(\kappa_e^2/\kappa_p^2)\nu_\epsilon$, then $|F_2|=|G|\gg|F_1|$ and the time-dependent electron and phonon temperature fluctuations are the same; this means that phonon physical parameters can be obtained from both thermoelectric and photoacoustic experiments. Finally, for a intermediate chopping frequency i.e. $(\alpha_p/\alpha_e)(\kappa_e^2/(\kappa_p^2)\nu_\epsilon)\ll\omega\ll(\alpha_p/\alpha_e)\nu_\epsilon$, $|F_1|\gg|F_2|=|G|$, and in this limit the signal produced by the fluctuation of the electron thermal wave equation (12b) depends strongly on the thickness of the sample and the position of the detector. For example, consider the case when $\sigma_2 d\gg 1$, and the position of the detector in the sample is such that $\sigma_1 z\ll 1$ and as a consequence $T_e\sim F_1\exp(-\sigma_1 z)\gg T_p$; this, in this limit, we obtain only information about electron thermal parameters through the electron thermal waves. For $\sigma_1^{-1}\ll z\ll\sigma_2^{-1}$, $T_e=T_p+F_2\exp(-\sigma_2 z)$, and in this case the contributions of the fluctuations of the electron and phonon temperature to the photothermal signal are the same. On the other hand, for the thickness of the sample, such that $\sigma_1 d\ll 1{:}T_e\gg T_p$ and again, the signal produced in the experiment is due to the electron thermal wave. A similar analysis can be done for the intermediate values of the thickness of the sample, i.e., when $\sigma_1^{-1}\ll d\ll\sigma_2^{-1}$ (see Fig. 1).

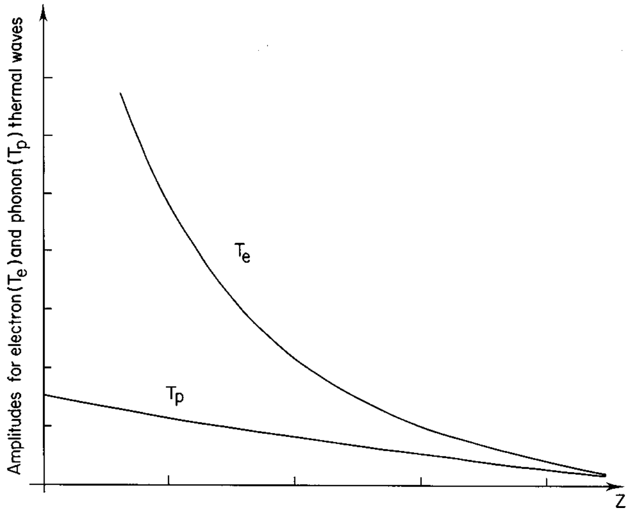

FIG. 1. Schematic representation of the spatial distribution of the amplitudes for electron (T_e) and phonon (T_p) thermal waves in the limits $|F_1| \gg |F_2|$ and $|\sigma_1| \gg |\sigma_2|$.

In the case when $k_{e,p} \to 0$ (electron-phonon interaction vanishes), it follows from Eqs. (8) that the electron and phonon temperatures reduce to

$$T_{e,p} = T_0 + \frac{Q_{e,p}}{\kappa_{e,p}}(d-z) + \frac{\Delta Q_{e,p}}{\kappa_{e,p}\sigma_{e,p}} \frac{\sinh \sigma_{e,p}(d-z)}{\cosh \sigma_{e,p} d} e^{i\omega t}, \tag{14}$$

i.e., for $P \to 0$ we have noninteracting systems of quasiparticles, and heat transport is carried out by the electrons and phonons independently through the semiconductor. In the limit when $|\sigma_{e,p}| d \ll 1$ or $\omega \tau_{e,p} \ll 1$ (low frequency) where $\tau_{e,p} = d^2/\alpha_{e,p}$ are the characteristic times at which the electron and phonon systems respond to an external perturbation, Eq. (14) reduces to

$$T_{e,p}(z,t) = T_0 + \frac{Q_{e,p}}{\kappa_{e,p}}(d-z) + \frac{\Delta Q_{e,p}}{\kappa_{e,p}}(d-z)e^{i\omega t}. \tag{15}$$

In this limit the electron and phonon temperature distributions are quasistatic and independent of the diffusivity of the systems; i.e. from any photothermal experiments only information about electron and phonon thermal conductivity can be obtained. Note that, at low modulation frequency, the thermal diffusion lengths $L_{e,p} = (2\alpha_{e,p}/\omega)^{1/2}$, are large as compared with d.

On the other hand, in the limit when $|\sigma_{e,p}| d \gg 1$ or $\omega \tau_{e,p} \gg 1$ (high modulation frequency), Eq. (14) reads

$$T_{e,p}(z,t) = T_0 + \frac{Q_{e,p}}{\kappa_{e,p}}(d-z) + \frac{\Delta Q_{e,p}}{\kappa_{e,p}\sigma_{e,p}} e^{i\omega t - \sigma_{e,p} z}. \tag{16}$$

In this limit the dynamical part of the electron and phonon temperature distributions are smaller than the static contributions, i.e., $\Delta Q_{e,p}/\sigma_{e,p}\kappa_{e,p} \ll Q_{e,p}/\kappa_{e,p}$, and they attenuate rapidly to zero with increasing distance from the surface, such that, at a distance $L \ll d$, the temperature fluctuations are effectively damped out. At frequency modulation in the range $\tau_p^{-1} \ll \omega \ll \tau_e^{-1}$, the electron system responds quasistatically to the external perturbation, while the phonon temperature fluctuation in the sample is small compared with the static contribution.

We now consider the situation represented by the strong coupling between electrons and phonons, i.e., $P \to \infty$. In this case the electron-phonon energy interaction is very efficient. Here the solutions of Eq. (10) are given by

$$\sigma_1^2 \simeq k^2; \sigma_2^2 = \frac{i\omega}{k^2}\left(\frac{k_p^2}{\alpha_e} + \frac{k_e^2}{\alpha_p}\right) = \frac{i\omega}{\kappa_e + \kappa_p}[\rho_e c_e + \rho_p c_p], \tag{17}$$

and $G_2 \simeq F_2$. Therefore, in this limit both temperatures are the same:

$$T(z,t) = T_e(z,t) = T_p(z,t) = T_0 + \frac{Q_e + Q_p}{\kappa_e + \kappa_p}(d-z) + F_2 \frac{\sinh \sigma_2(d-z)}{\cosh \sigma_2 d} e^{i\omega t}, \quad F_2 = +\frac{1}{\sigma_2}\frac{\Delta Q_e + \Delta Q_p}{\kappa_e + \kappa_p}. \tag{18}$$

Here the thermal waves generated into the semiconductor by the incident beam radiation can be analyzed (for both limits: high and low modulation frequencies), and the effective characteristic time of the total systems is

$$\tau_{\text{eff}} = \frac{d^2}{\kappa_e + \kappa_p}\left[\frac{\kappa_e}{\alpha_e} + \frac{\kappa_p}{\alpha_p}\right], \tag{19}$$

which depends upon the electron and phonon thermal parameters.

However in general, the analysis of heat flux in the semiconductor through the electron and phonon temperature distributions [Eqs. (12)] is very complicated for any modulation frequency and finite value of the electron-phonon interaction. An exhaustive study of electron and phonon thermal waves was made in Ref. 7 in the limit of very large samples.

IV. CONCLUSIONS

In conclusion, a theoretical analysis of thermal waves has been studied. Using the appropriate boundary conditions, according to the usual photothermal experiments, we obtain the electron and phonon temperature distributions in the sample. Thermal waves generated into the material by the incident-beam radiation in general can be analyzed as function of the electron and phonon thermal parameters as well as the electron-phonon energy relaxation. In particular, we have derived exact solutions for electron and phonon temperatures in semiconductors in the limit of weak and strong electron-phonon interaction for both high and low modulation frequencies of the incident radiation on the sample. The above findings tell us that the different responses of the system depend on the modulation frequency.

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[1] H. Vargas and L. C. M. Miranda, Phys. Rep. **161**, 43 (1988).
[2] E. M. Conwell, *High Field Transport in Semiconductors* (Academic, New York, 1967).
[3] F. G. Bass and Yu. G. Gurevich, Usp. Fiz. Nauk **103**, 447 (1971) [Sov. Phys. Usp. **14**, 113 (1971)].
[4] Yu. G. Gurevich and O. L. Mashkevich, Phys. Rep. **181**, 327 (1989).
[5] V. A. Sablikov and V. B. Sandomirskii, Phys. Status Solidi B **120**, 471 (1983).
[6] M. Sasaki, H. Negishi, and M. Inoue, J. Appl. Phys. **59**, 796 (1986).
[7] G. González de la Cruz and Yu. G. Gurevich, J. Appl. Phys. **80**, 1726 (1996).
[8] G. González de la Cruz and Yu. G. Gurevich, Phys. Rev. B **51**, 2188 (1995).
[9] Yu. G. Gurevich, O. Yu. Titov, G. N. Logvinov, and O. I. Lyubinov, Phys. Rev. B **51**, 6999 (1995).
[10] V. S. Bochkov and Yu. G. Gurevich, Fiz. Tekh. Poloprovodn. **17**, 728 (1983) [Sov. Phys. Semicond. **17**, 456 (1983)].
[11] *Handbook of Chemistry and Physics*, edited by Robert C. Weast (CRC Press, Boca Raton, FL, 1977), p. E101.