Conductance fluctuations of semiconductor-superconductor microjunctions in the quantum Hall regime

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We numerically study the high-magnetic-field conductance of two-dimensional electron-gas–superconductor junctions in the presence of a short-range disorder. The disorder-induced multiple Andreev reflection leads to a dip in the conductance at the depopulation thresholds of the Landau levels. The dip evolves into rapid fluctuations when increasing the magnetic field. In such a high-field regime, the correlation magnetic field of the conductance fluctuations changes enormously depending on the location of the Fermi level relative to the Landau levels. Correspondingly, the amplitude of the conductance fluctuations varies significantly as the Fermi level sweeps through the Landau levels. [S0163-1829(98)06236-5]

The conductance of normal-conductor–superconductor (NS) junctions is strongly influenced by the Andreev reflection of quasiparticles from the NS interface.[1] The single-particle excitations localize in the normal region, and so their behavior is primarily determined by the nature of the normal conductor. If a high-mobility two-dimensional electron gas (2DEG) is employed as the normal conductor, the transport of the quasiparticles can be made ballistic. Although the surface potential of semiconductors makes the interface nonideal,[2] the recent technological progress has succeeded to prepare good quality 2DEG-superconductor junctions.[3–5] The use of the 2DEG also provides a possibility to investigate the Andreev reflection in the quantum Hall regime. To this extent, more progress was made. In previous investigations, the external magnetic field B was absent or very weak as the magnetic field destroyed the superconductivity. However, a very recent experiment reported the Josephson coupling in superconductor-2DEG-superconductor junctions at $B \sim 8$ T by choosing a magnetic-field-robust material for the superconductor.[6]

Motivated by this experiment, the conductance of NS junctions in high magnetic fields was studied theoretically in Ref. 7. The edge states in the quantum Hall regime inevitably undergo multiple Andreev reflections when they travel along the NS interface. The Andreev-reflected edge state was revealed to produce a transmission resonance.[7] In intermediate magnetic fields, it was found that the conductance exhibits an oscillation that resembles the Shubnikov–de Haas (SdH) oscillation in the resistivity of 2DEG's, provided that the normal reflection probability at the NS interface is increased by a difference of the Fermi energies in the 2DEG and the superconductor.[7] An interesting feature of this oscillation is that the conductance reaches a maximum at the magnetic depopulation thresholds of the Landau levels. However, one needs to take into account the presence of disorder in comparing experimental data[6,5] with the theoretical prediction. The disorder is anticipated to suppress the conductance, in particular near the depopulation thresholds.[8] Therefore, the interplay between the Andreev reflection and the disorder needs to be clarified.

In this paper, the influence of disorder on the conductance of NS junctions is investigated in the quantum Hall regime. As expected, the conductance drops significantly at the depopulation thresholds when short-range disorder is taken into account. Consequently, the dip leads to split conductance peaks when the partial normal reflection is allowed at the NS interface. In larger fields, where the edge states produce the transmission resonance, the conductance develops rapid fluctuations around the depopulation thresholds while a smooth variation is found between the thresholds. The fluctuation amplitude is found to behave roughly as a function of the mean conductance, except that the amplitude is enhanced considerably at the thresholds.

Our model structure is schematically shown in the inset of Fig. 1. We calculate the conductance of the NS wires in the presence of a perpendicular magnetic field B. For simplicity, the magnetic field is assumed to be absent in the superconductor (hatched area). We also neglect the self-consistency of the pair potential amplitude. A constant value Δ is as-

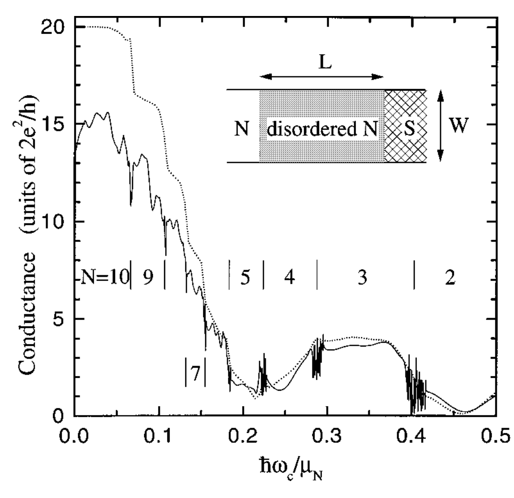

FIG. 1. Magnetoconductance of NS junction when $\mu_S = \mu_N$. The strength of disorder is $d = 0.5\mu_N$. The dotted line shows the conductance in the absence of disorder ($d=0$). Other parameters are $k_F W/\pi = 10.5$, $\Delta/\mu_N = 0.01$, and $L/W = 3$. The vertical bars indicate the depopulation threshold of the transverse modes. The number of occupied modes N is shown. The inset shows a schematic of the NS junction. Potential disorder is introduced in the shaded area. The hatched region represents the superconductor.

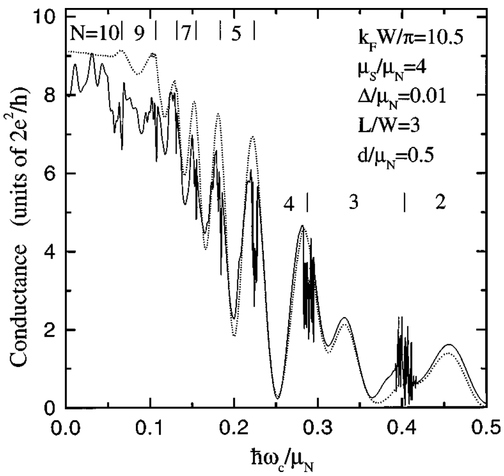

FIG. 2. Magnetoconductance of the NS junction when $\mu_S = 4\mu_N$. The solid and dotted lines show the conductance when $d/\mu_N = 0.5$ and 0, respectively. The depopulation threshold of the Landau levels in the normal wire is indicated by the vertical bars. The number of occupied modes N in the normal wire is shown.

sumed in the superconductor, and the pair potential amplitude is set to be zero in the 2DEG. The conditions in which these assumptions can be justified were discussed by Beenakker[1] for the 2DEG-based material systems. We restrict our discussion to the zero-bias conductance, and so the quasiparticles do not propagate in the superconductor. The two-terminal conductance G is hence related to the Andreev reflection amplitude s_{he} as[9]

$$G = (4e^2/h)\mathrm{Tr}[s_{he}^{\dagger} s_{he}]. \tag{1}$$

The scattering coefficients are calculated by solving the Bogoliubov–de Gennes equation using the lattice Green's-function technique.[10] In the following numerical calculations, the NS wires are simulated with 100 transverse lattice sites. A disorder potential is taken into account in the normal region adjacent to the NS interface (shaded area). The disorder is introduced as on-site potential fluctuations. The random potential is assumed to be distributed uniformly within $[-d,d]$. The superconducting wave function penetrates the 2DEG over the coherence length $\xi_N = \hbar v_F/2\Delta$, where v_F is the Fermi velocity in the 2DEG.[11] For the parameters we have assumed, $\xi_N = 3.0W$. The length L of the disordered region is chosen to be comparable to ξ_N.

The conductance is plotted in Figs. 1 and 2 as a function of $\hbar\omega_c/\mu_N$ for $\mu_S/\mu_N = 1$ and 4, respectively. Here, μ_S and $\mu_N = \hbar^2 k_F^2/2m$ are the Fermi energies respectively in the superconductor and 2DEG, and $\omega_c = eB/m$ is the cyclotron frequency. The dotted lines show the conductance in the absence of disorder. As Δ $(=0.01\mu_N)$ is very small in comparison to the Fermi energy, quasiparticles are almost perfectly Andreev reflected from the NS interface when $\mu_S = \mu_N$.[12] The nearly perfect Andreev reflection is proved in Fig. 1 by the quantization of the zero-field conductance in units of $4e^2/h$. The conductance decreases in a steplike manner in Fig. 1 whenever the one-dimensional subbands are magnetically depopulated.

The Fermi energy in metals is much larger than that in 2DEG's. The nonuniformity of the Fermi energy is taken into account in Fig. 2. Increasing μ_S relative to μ_N, the quasiparticles are partially normal reflected, resulting in a decrease of the conductance in zero and weak magnetic fields. In this case, the conductance exhibits an oscillation which resembles the SdH oscillation. Notice that the conductance becomes maximum near the depopulation thresholds.[7] Thus the oscillation is anticipated to be vulnerable against disorder.

The variation of the conductance is dominated by the Landau-level depopulation for $\hbar\omega_c/\mu_N < 0.2\sim0.3$. The high-field oscillation, however, is no longer associated with the Landau-level depopulation. When $2l_c < W$, with $l_c = 2\hbar k_F/eB$ being the cyclotron diameter which is satisfied for $\hbar\omega_c/\mu_N > 0.24$, the oscillation arises from the commensurability of the skipping orbit along the NS interface.[7] In addition, the quantum interference effect gives rise to the nearly zero conductance at the minima.

In the presence of a short-range disorder, the low-field conductance exhibits universal conductance fluctuations (UCF's). Deviations from the universality generally take place in the quantum Hall regime. The conductance shows sharp dips at the depopulation thresholds in the intermediate magnetic fields, $\hbar\omega_c/\mu_N < 0.24$. The dips are attributed to the fact that the scattering from the disorder is enhanced when the Fermi energy lies around the threshold energies because of the small kinetic energy for the topmost mode. However, these dips deserve a few comments. In normal conductors, more incident electrons are backscattered with increasing disorder strength, resulting in a lower conductance. In the NS system, however, the quasiparticles are totally reflected from the NS interface irrespective of the presence of disorder. All the incident electrons are backscattered from the system, partly as holes and partly as electrons. Therefore, the disorder-induced scattering does not necessarily suppress the conductance even in the classical limit. If the quasiparticles experience frequent reflections between the disordered region and the NS interface, the Andreev reflection probability is expected to be $\sim N/2$, while the other half of the quasiparticles leave the system as electrons.[13] Hence, the conductance is, on average, given by $2Ne^2/h$, i.e., the maximum conductance of the normal system. As some of the incident quasiparticles are directly backscattered by the disorder potential before reaching the NS interface, the mean conductance is expected to be identical to that without the superconductor segment in the strong disorder limit. It follows that the conductance may be enhanced by disorder in the strong-magnetic-field regime, where the conductance can be close to zero because of the transmission resonance. This simple argument reasonably explains the conductance values at the thresholds. Although the influence of the superconductor is obscured in the mean conductance when the transport in the normal conductor is diffusive, the existence of the superconductor is still reflected in the amplitude of the conductance fluctuations, as we will show below.

When the quantum interference effect of the edge states dominates the conductance variation, $\hbar\omega_c/\mu_N > 0.24$, the conductance fluctuates rapidly with magnetic field near the thresholds. The rapid fluctuation is strongly restricted in the vicinity of the thresholds. The conductance suddenly behaves smoothly when the magnetic field is between the thresholds. This abrupt change is actually consistent with the

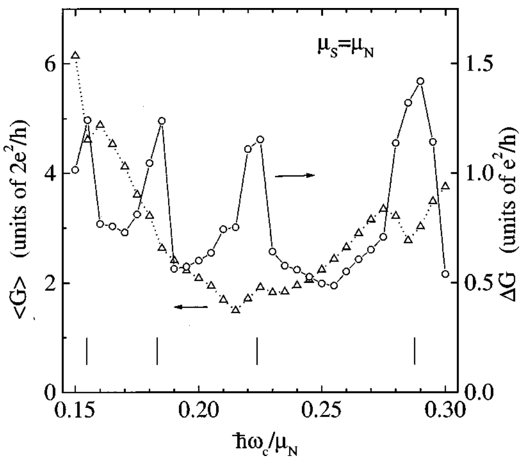

FIG. 3. The mean conductance $\langle G\rangle$ and the amplitude of the conductance fluctuations ΔG for parameters corresponding to Fig. 1. The vertical bars indicate the depopulation threshold of the transverse modes.

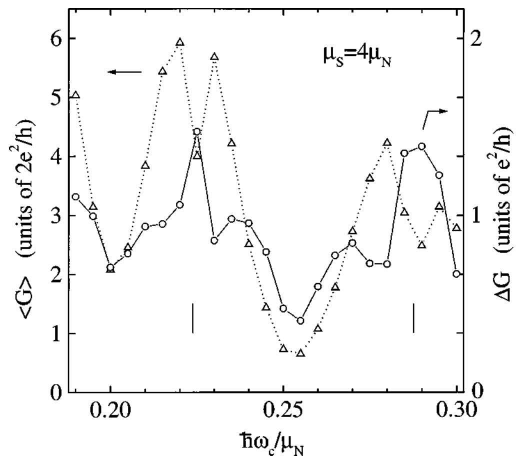

FIG. 4. The mean conductance $\langle G\rangle$ and the amplitude of the conductance fluctuations ΔG for parameters corresponding to Fig. 2. The vertical bars indicate the depopulation threshold of the transverse modes.

behavior of the conductance of normal conductors in the quantum Hall regime.[14] In normal conductors, the conductance is quantized in units of $2e^2/h$ when the edge states are well established, and so the fluctuations can be observed only near the transition between the plateaus. Unlike the normal conductors, the conductance of the NS system is modulated by the multiple Andreev reflection even when the edge states are established.[7]

To investigate the difference between the normal and NS systems further, we also examined the sample-to-sample fluctuations of the conductance. We show the amplitude of the conductance fluctuations $\Delta G=(\langle G^2\rangle-\langle G\rangle^2)^{1/2}$ in Figs. 3 and 4. The ensemble average is taken over several hundred samples. The mean conductance $\langle G\rangle$ and the fluctuation amplitude ΔG show similar variation against the magnetic field.[14] However, an exception is found at the depopulation thresholds, where $\langle G\rangle$ drops whereas ΔG grows larger. There are several studies on the UCF's of NS systems.[15,16,1] It was recognized that, due to the presence of two carriers, the conductance fluctuation in NS junction wires is enhanced in magnitude compared to that in normal-conductor wires.[15–17] The random-matrix theory predicts $\Delta G=1.46e^2/h$ when the time-reversal symmetry is broken by a magnetic field.[16] The peak values of ΔG at the thresholds are in agreement with this prediction. This can be interpreted to mean that the fluctuations at the thresholds are caused by the disorder in the bulk of the normal region. Conversely, the disorder near the interface (over the width of $\sim l_c=0.15W$ at $\hbar\omega_c/\mu_N=0.4$) which can act on the skipping orbits contributes to the fluctuations between the thresholds. The scattering is thus less effective in the latter case as the edge states propagate adiabatically. When the conductance exhibits split peaks for $\mu_S/\mu_N=4$, ΔG tends to be suppressed at the conductance maxima. Although the reason for this is not understood, it is expected to be related to the enhancement mechanism of the conductance at the thresholds.

In conclusion, the magnetoconductance of NS junctions has been calculated numerically in the presence of a short-range disorder. The disorder produces sharp dips at the depopulation thresholds of the Landau levels. Consequently, SdH-type oscillations are smeared by the disorder. In the high-magnetic-field regime, where the quantum interference effect of the edge states running along the NS interface gives rise to the transmission resonance, the conductance shows distinct features for magnetic fields in the vicinity of the thresholds and between the thresholds. In the respective magnetic-field regions, the fluctuations are caused by the bulk scattering and by the scattering near the interface. The fluctuation amplitude also reflects the difference of the fluctuation mechanism.

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