

## High-temperature magnetoconductivity of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ : Reconsideration of the Maki-Thompson contribution

J. Axnäs, B. Lundqvist, and Ö. Rapp

*Department of Solid State Physics, Kungl Tekniska Högskolan, SE-100 44 Stockholm, Sweden*

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The magnetoconductivity of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  was measured in magnetic fields  $B$  up to 12 T in temperatures up to  $2.55T_c$  for  $B\parallel c$  and up to  $1.7T_c$  for  $B\parallel ab$ . Negligible Maki-Thompson (MT) terms in the superconducting fluctuations have often been inferred from similar published data. We find that these data as well as ours can be well described by including the MT terms and the previously neglected density-of-states effects. Therefore, it cannot be concluded from magnetoconductivity alone that MT terms are negligible, and all previous such analyses must be reexamined. [S0163-1829(98)06234-1]

The effects of superconducting fluctuations, i.e., of superconducting electron pairs that exist above  $T_c$ , are particularly well studied in the electrical conductivity. It has been shown, e.g., that fluctuations can explain the well-known  $c$ -axis resistivity ( $I\parallel c$ ) peak occurring in some materials,<sup>1</sup> the deviation from linearity in the zero-field  $ab$ -plane electrical conductivity up to high temperatures,<sup>2</sup> and the sign change in the  $c$ -axis magnetoconductivity,  $\Delta\sigma = \sigma(B) - \sigma(0)$ , of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ .<sup>3</sup> Recently, it was also shown that both the  $ab$ -plane and  $c$ -axis resistivities in applied magnetic fields can be described down to temperatures well below the midpoint of the resistive transitions, into the strong-fluctuation regime.<sup>4</sup>

However, at temperatures far above  $T_c$ , the description of the magnetoconductivity by fluctuations has been problematic. Based on experiments, the Maki-Thompson (MT) contribution (an indirect fluctuation effect) has been suggested to be absent or smaller than expected in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  (Refs. 5–10) and in other materials.<sup>11,12</sup> The observed magnetoconductivity has instead been ascribed to a normal-state contribution.<sup>6,13–15</sup> This is of fundamental interest. First, negligible MT terms have been claimed to be one of the consequences of  $d$ -wave superconductivity.<sup>16</sup> Second, the temperature dependence deduced for the normal-state contribution ( $\sim T^{-4}$ ) has been noted to violate Kohler's rule and has been taken as an indication of two distinct relaxation times.<sup>13</sup>

In the present work we show that consideration of a more recently derived fluctuation contribution, the fluctuations in the normal quasiparticle density of states (DOS), significantly changes this picture. Since the DOS term may almost cancel the MT terms, one can obtain a good description, with both MT and DOS terms included, of data previously believed to demonstrate the absence of MT terms. Furthermore, our data, and the most significant published results, can be well described by fluctuations alone, without the inclusion of any normal-state contribution. It is concluded that evidence for negligible MT terms in the superconducting fluctuations cannot be obtained from magnetoconductivity experiments alone.

A twinned single crystal of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  was grown by the self-flux method,<sup>17</sup> and oxygenated at 450 K. The dimen-

sions of the crystal were  $1.8 \times 0.276 \times 0.026 \text{ mm}^3$ , and geometrical effects<sup>18</sup> in the magnetoresistance should be negligible. The width of the superconducting transition was about 100 mK. These properties enable accurate measurements close to as well as far above  $T_c$ . Figure 1 illustrates some sample-characterizing properties. The in-plane normal-state resistivity is linear and extrapolates to a value close to zero at  $T = 0 \text{ K}$ . The experiments in magnetic field were made as described previously<sup>19</sup> but with improved temperature control. The measurements covered magnetic fields up to 12 T in temperatures from close to  $T_c$  up to 230 K for  $B\parallel c$  and up to 160 K for  $B\parallel ab$ . This variation of field and temperature on a single sample is larger than in previous fluctuation studies. The results for the magnetoconductivity are illustrated in Fig. 2 as a function of temperature at 12 T for both field directions in the top panel, and as a function of  $B\parallel c$  at several temperatures in the bottom panel. The curves are calculations from theories that will now be described.

Four contributions to the fluctuation magnetoconductivity,  $\Delta\sigma_{fl}$ , were considered:

$$\Delta\sigma_{fl} = \Delta\sigma_{AL} + \Delta\sigma_{DOS} + \Delta\sigma_{MT(\text{reg})} + \Delta\sigma_{MT(\text{an})}. \quad (1)$$

The AL (Aslamazov-Larkin) and MT(an) (anomalous Maki-Thompson) terms are positive whereas the DOS (density-of-

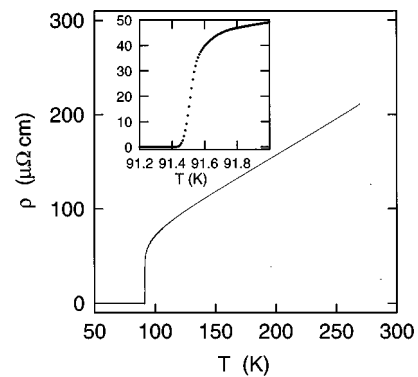


FIG. 1. Resistivity vs temperature of our single crystal  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  sample. The inset is an expansion around  $T_c$ .  $\rho(100 \text{ K}) \sim 70 \pm 30 \mu\Omega \text{ cm}$ , with a large error due to uncertainty in the sample dimensions. In this and all other figures the current is parallel to the  $ab$  planes.

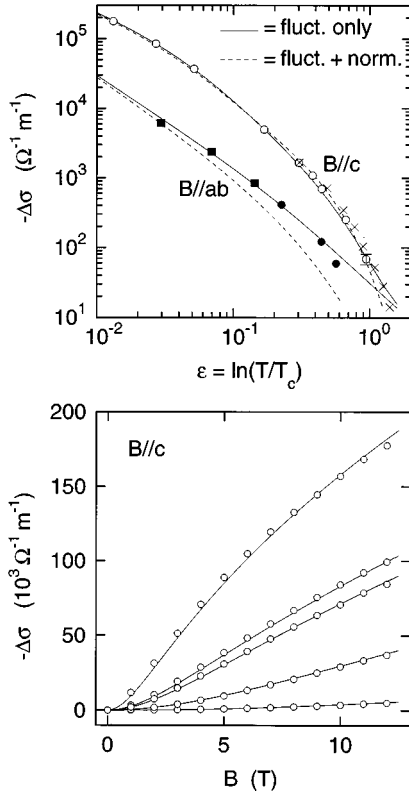


FIG. 2. Upper panel: Our results for the magnetoconductivity at 12 T. Open symbols  $B||c$ , closed symbols  $B||ab$  (circles  $B\perp I$ , squares  $B||I$ ). The solid curves are fits of fluctuation theories only, dashed curves to fluctuation theories plus a normal-state contribution. Crosses are data with  $B||c$  from Ref. 13. [Only  $\Delta\rho/\rho$  was given.  $\rho(T)=cT$  where  $c=0.5 \mu\Omega \text{ cm/K}$  was used to convert to  $\Delta\sigma \approx -\Delta\rho/\rho^2$ .] The parameters used were as follows. Solid curves:  $T_c=91.5 \text{ K}$ ,  $J=220 \text{ K}$ ,  $\tau=3.9 \text{ fs}$  [corresponding to  $\xi_{ab}(0)=1.34 \text{ nm}$  and  $\xi_c(0)=0.23 \text{ nm}$ ],  $\tau_\phi=207 \text{ fs}$  at  $100 \text{ K}$  ( $\tau_\phi \sim T^{-1}$ ), and  $C_{norm}=0$ . Dashed curves:  $T_c=91.5 \text{ K}$ ,  $J=195 \text{ K}$ ,  $\tau=4.6 \text{ fs}$  [ $\xi_{ab}(0)=1.42 \text{ nm}$  and  $\xi_c(0)=0.21 \text{ nm}$ ],  $\tau_\phi=72 \text{ fs}$  at  $100 \text{ K}$ , and  $C_{norm}=2000 \text{ K}^4\text{T}^{-2}$ . Lower panel: Field dependence for the five lowest measurement temperatures with  $B||c$  (92.7, 94.0, 96.3, 108.2, 123.8 K) together with the theoretical calculation using the parameters of the solid curves in the upper panel (no normal-state contribution).

states) and MT(reg) (regular Maki-Thompson) terms are negative. All four terms have two parts; the orbital and Zeeman contributions. There are thus eight contributions in total. The orbital parts depend on the field orientation, whereas the Zeeman parts do not. The orbital terms have only been derived for  $B||c$ , and were neglected for  $B||ab$ , as usual. The orbital contributions were taken from Dorin *et al.* (DKVBL).<sup>20</sup> The problem of a cutoff in the sum for the DOS term<sup>3</sup> and MT(reg) term was circumvented by the regularization method of Buzdin and Dorin.<sup>21</sup> The Zeeman contributions were calculated using the renormalization procedures given in Refs. 22 and 23. Since the resulting expressions have not been listed together before, they are given in an appendix.

In addition to the inclusion of the DOS term, our theoretical treatment differs from previous studies on two points. These points are of less importance to our main results, but will be discussed in this paragraph and the following one.

Most existing comparisons with experiments<sup>5–12,14,18,19</sup> have been made using the expressions presented by Aronov, Hikami, and Larkin (AHL).<sup>22,23</sup> These expressions contain no DOS term. AL and MT terms were included, but were derived in a somewhat different way, and the results of the DKVBL and AHL approaches will now be briefly compared. In both cases the results are functions of the spacing between superconducting layers  $s$ , the phase breaking time  $\tau_\phi$ , the critical temperature  $T_c$ , and magnetic field and temperature. The AHL expressions include, in addition, the in-plane and out-of-plane Ginzburg-Landau coherence lengths at zero temperature,  $\xi_{ab}(0)$  and  $\xi_c(0)$ , and, in the clean limit ( $l \gg \xi$ ), the mean free path  $l$ . The DKVBL expressions instead include the Fermi velocity  $v_F$ , an in-plane elastic scattering time  $\tau$ , and the hopping integral  $J$ , which reflects the probability of electron hopping between the layers. Both the AHL and the DKVBL results were derived assuming  $\epsilon = \ln(T/T_c) \ll 1$ . The AHL expressions were derived only for the clean ( $l \gg \xi$ ) and dirty ( $l \ll \xi$ ) limits, while the DKVBL expressions do not have this limitation. In their common ranges of validity ( $\epsilon \ll 1$  and either  $l \gg \xi$  or  $l \ll \xi$ ), the AHL and DKVBL results are almost the same, taking  $\xi_{ab}^2(0) = \eta(T_c)$  and  $\xi_c^2(0) = s^2 r(T_c)/4$  as in Ref. 20, with  $l = v_F \tau$ . For  $T \gg T_c$  ( $\epsilon > 1$ ), AHL and DKVBL may differ significantly. We observed,<sup>24</sup> however, that the AL and MT(an) terms of DKVBL are mathematically identical to the AL and MT terms of AHL when  $l \gg \xi$  or  $l \ll \xi$  if one identifies instead  $\xi_{ab}^2(0) = \eta(T)$  and  $\xi_c^2(0) = s^2 r(T)/4$ . In the limit  $\epsilon \ll 1$  these different expressions for the coherence lengths are of course equivalent.

We now discuss the scattering times. The clean-limit AHL expressions depend on  $l$  and  $\tau_\phi$  only through their product,  $l\tau_\phi$ , and thus an additional assumption is needed to extract their values from magnetoresistivity measurements. Usually one takes  $\tau_\phi = \tau_{tr}$ , where  $\tau_{tr} = l/v_F$  is the transport scattering time. This typically leads to scattering times in the range 10–100 fs. Alternatively one can consider the transport scattering time to be dominated by elastic scattering events. The elastic scattering time  $\tau$  is then much shorter than the inelastic scattering time  $\tau_{in}$ . Taking  $\tau_{tr} = \tau$  and identifying  $\tau_\phi = \tau_{in}$  we can calculate  $\tau_{tr}$  from  $\xi_{ab}^2(0)$ , which gives results of the order of 5 fs (Ref. 3 and this work).

We now return to the main line of the paper. In our analyses we calculated the fluctuation magnetoconductivity with only three adjustable parameters ( $\tau$ ,  $\tau_\phi$ , and  $J$ ). The other parameters were taken from the literature, as previously.<sup>3</sup> No adjustment for the magnitude of  $\Delta\sigma$  ( $C$  factor) was employed. When the normal-state magnetoresistivity was considered, it was assumed to be of the form  $\Delta\rho/\rho = C_{norm} B^2 T^{-4}$ , and to contribute only for  $B||c$ , as usual.<sup>6,13</sup>

The results of the analyses are shown by the curves in Fig. 2. As seen from the solid curves, data can be well described by fluctuations only, including the MT terms. Hence, our data give no grounds for excluding the MT(an) contribution from the magnetoconductivity.

To further investigate this conclusion, experiments of high quality were selected from the literature. We reanalyzed the data by Harris *et al.*,<sup>13</sup> Semba *et al.*,<sup>5,6</sup> and Lang *et al.*,<sup>7</sup> including MT and DOS terms in the fluctuation contribution.<sup>25</sup> The results are shown in Figs. 2–4 and de-

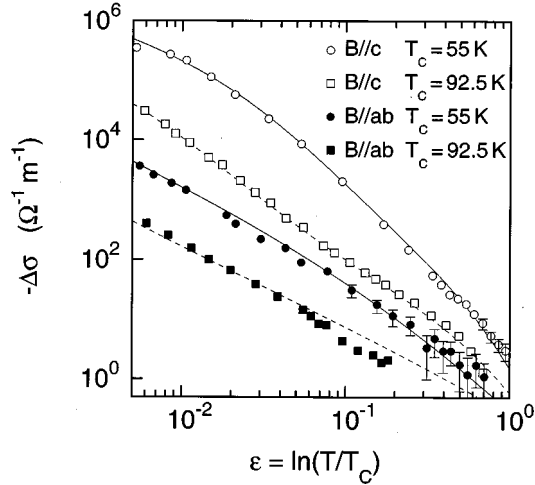


FIG. 3. Fits of theory (curves) to experimental data (symbols) from the literature (somewhat reduced data sets).  $B = 1$  T. MT terms were considered, but no normal-state contribution. Circles are data for oxygen-deficient  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  ( $T_c \approx 55$  K) from Ref. 7. Squares are data for  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  ( $T_c \approx 92.5$  K) from Ref. 6. The two upper curves include all fluctuation terms, the two lower curves only the Zeeman contributions. The parameters used are as follows. Solid curves:  $T_c = 92.5$  K,  $J = 320$  K,  $\tau = 3.8$  fs [corresponding to  $\xi_{ab}(0) = 1.32$  nm and  $\xi_c(0) = 0.32$  nm],  $\tau_\phi = 320$  fs at 100 K, and  $C_{norm} = 0$ . Dashed curves:  $T_c = 55$  K,  $J = 46$  K,  $\tau = 8.9$  fs [ $\xi_{ab}(0) = 2.4$  nm and  $\xi_c(0) = 0.09$  nm],  $\tau_\phi = 120$  fs at 100 K, and  $C_{norm} = 0$ . The quality of the fits is comparable to that in the original publications, where no MT and DOS terms were included, and where in one case (Ref. 6) a normal-state contribution was included.

scribed in the captions. In particular, in Fig. 3 the analyses include fluctuation contributions only, in Fig. 4 in addition a normal-state contribution, while in the top panel in Fig. 2, both these cases are shown. It can be seen that in all these cases observations can be well described by fluctuation contributions including MT terms. The good fits may partly be understood from the fact that a reasonable phase-breaking time,  $\tau_\phi$ , can give an MT(an) term that has almost the same magnitude as the sum of the DOS and MT(reg) terms over a wide range of temperatures, but has the opposite sign (Fig. 4). Our conclusion is thus strengthened: there seems to be no experimental basis in the literature for excluding MT contributions to the magnetoconductivity.

We now turn to the question whether there is a normal-state contribution to the magnetoconductivity. As shown by Figs. 2–4, good or excellent descriptions of data are obtained both with and without a normal-state contribution included. The apparent deterioration of the description of data for  $B\parallel ab$  in Fig. 2 when compared to an analysis with fluctuations only might not be significant, as discussed below. It is therefore concluded that the presence or absence of a normal-state contribution cannot be decisively determined from studies of magnetoconductivity alone.

Some points require further consideration. First, since MT terms become prominent at higher temperatures, all significant attempts to prove or disprove their presence in the magnetoconductivity have been made with data at rather elevated temperatures. The use of high-temperature data ( $\epsilon \sim 1$ ) may, however, be questioned. The theories discussed above were derived under the assumption  $\epsilon \ll 1$ . At higher temperatures

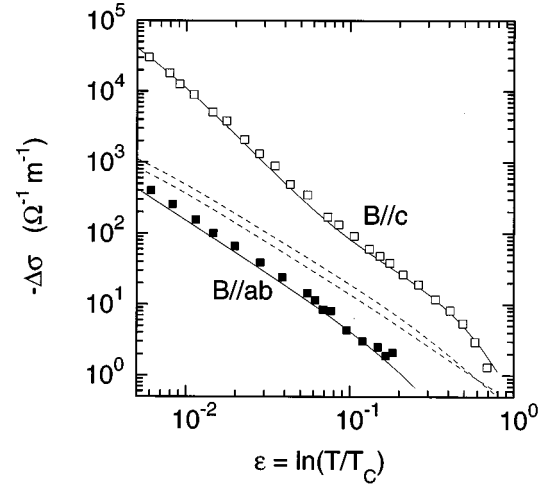


FIG. 4. Fit of theory (solid curves) to experimental data (symbols) from Ref. 6 (somewhat reduced data set for  $B\parallel ab$ ).  $B = 1$  T. MT terms as well as a normal-state contribution were considered. For  $B\parallel c$  all fluctuation terms were included, for  $B\parallel ab$  only the Zeeman terms. The normal-state contribution is only included for  $B\parallel c$ . We have used  $\tau_\phi = 78$  fs at 100 K. All other parameters were the same as in Ref. 6, i.e.,  $T_c = 92.5$  K,  $J = 287$  K,  $\tau = 3.8$  fs [corresponding to  $\xi_{ab}(0) = 1.32$  nm and  $\xi_c(0) = 0.29$  nm], and  $C_{norm} = 2100$  K<sup>4</sup>T<sup>-2</sup>. The upper dashed curve shows the magnitude of  $\Delta\sigma_{MT(an)}$  (which is negative) and the lower dashed curve the magnitude of  $\Delta\sigma_{DOS} + \Delta\sigma_{MT(reg)}$  (both of which are positive).  $|\Delta\sigma_{MT(reg)}|$  is small, less than half of each of  $|\Delta\sigma_{DOS}|$  and  $|\Delta\sigma_{MT(an)}|$  over the entire temperature interval. The quality of the fit is approximately the same as in Ref. 6, where no MT and DOS terms were included.

fluctuation effects may fall off faster than predicted by these theories. For the case of zero-field fluctuations,  $\sigma_{fl} [= \sigma_{observed} - \sigma_{normal}]$ , there are extensions to arbitrary  $\epsilon$  (i.e., including short-wavelength fluctuations).<sup>26</sup> They yield  $\sigma_{fl} \sim \epsilon^{-3}$  for  $\epsilon \gg 1$ , to be compared with  $\sigma_{fl} \sim \epsilon^{-1}$  for  $\epsilon \ll 1$ . Experiments on several materials indicate an  $\epsilon^{-3}$  behavior already at  $\epsilon \sim 0.23$ .<sup>2</sup> These observations may raise questions also about all published studies of fluctuation magnetoconductivity at high temperatures.

Second, the use of data with  $B\parallel ab$  could be questioned. The orbital contribution is usually ignored, although according to Klemm<sup>27</sup> it is always dominant in weak fields. Calculations for the orbital contribution for  $B\parallel ab$  (Ref. 28) do neither include the DOS term, nor take the layered structure into account. It should also be mentioned that for high-temperature data with  $B\parallel ab$  the differences in the literature are sometimes considerable, including different signs.<sup>6,29,30</sup> This complicates the interpretation.

In summary, magnetoconductivity measurements with  $B\parallel c$  and  $B\parallel ab$  have been made on  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  over a wider range of temperature and field than in previous studies of superconducting fluctuations. By considering the fluctuations in the normal quasiparticle density of states (DOS) it has been shown that these data can be explained by superconducting fluctuations with or without inclusion of a normal-state contribution, and that there are no experimental grounds for excluding the Maki-Thompson (MT) terms. These conclusions are confirmed by reanalyzing literature data. Further calculations of fluctuation effects seem necessary, in particu-

lar extensions to higher temperatures, in order to decisively verify which terms contribute to the magnetoconductivity.

### ACKNOWLEDGMENTS

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### APPENDIX: FULL EXPRESSIONS FOR THE MAGNETOCONDUCTIVITY

In Eq. (1) each of the four terms is a sum of two contribution, the orbital ( $O$ ) and Zeeman ( $Z$ ) contributions, i.e.,  $\Delta\sigma_{\text{AL}} = \Delta\sigma_{\text{ALO}} + \Delta\sigma_{\text{ALZ}}$ , etc.

The orbital terms were only included for the case  $B \parallel c$  axis. The following expressions were then used:

$$\Delta\sigma_{\text{ALO}} = \frac{e^2}{4s\hbar} \sum_{n=0}^{\infty} (n+1) \left[ \frac{1}{[(\epsilon_B + \beta n)(\epsilon_B + \beta n + r)]^{1/2}} \frac{2}{\{[\epsilon_B + \beta(n + \frac{1}{2})][\epsilon_B + \beta(n + \frac{1}{2}) + r]\}^{1/2}} + \frac{1}{\{[\epsilon_B + \beta(n+1)][\epsilon_B + \beta(n+1) + r]\}^{1/2}} \right] - \frac{e^2}{16\hbar s} \left( \frac{1}{[\epsilon(\epsilon+r)]^{1/2}} \right), \quad (\text{A1})$$

$\Delta\sigma_{\text{DOSO}}$

$$= \frac{e^2 \kappa \beta}{4s\hbar} \sum_{n=0}^{\infty} \left[ \frac{1}{[(\epsilon_B + \beta n)(\epsilon_B + \beta n + r)]^{1/2}} - \frac{2}{\beta} \ln \left[ \frac{[\epsilon_B + \beta(n + \frac{1}{2})]^{1/2} + [\epsilon_B + \beta(n + \frac{1}{2}) + r]^{1/2}}{[\epsilon_B + \beta(n - \frac{1}{2})]^{1/2} + [\epsilon_B + \beta(n - \frac{1}{2}) + r]^{1/2}} \right] \right], \quad (\text{A2})$$

$$\Delta\sigma_{\text{MT(reg)O}} = \frac{\tilde{\kappa}}{\kappa} \Delta\sigma_{\text{DOSO}}, \quad (\text{A3})$$

$$\Delta\sigma_{\text{MT(an)O}} = \frac{e^2 \beta}{8s\hbar(\epsilon - \gamma)} \sum_{n=0}^{\infty} \left[ \frac{1}{[(\gamma_B + \beta n)(\gamma_B + \beta n + r)]^{1/2}} - \frac{1}{[(\epsilon_B + \beta n)(\epsilon_B + \beta n + r)]^{1/2}} \right] - \frac{e^2}{4\hbar s(\epsilon - \gamma)} \ln \left( \frac{\epsilon^{1/2} + (\epsilon + r)^{1/2}}{\gamma^{1/2} + (\gamma + r)^{1/2}} \right). \quad (\text{A4})$$

In these formulas  $r = 2k_B^2 J^2 \tau^2 f_0 / \hbar^2$ ,  $\beta = 4\eta eB / \hbar$ ,  $\eta = v_F^2 \tau^2 f_0 / 2$ ,

$$f_0 = - \left[ \Psi \left( \frac{1}{2} + \frac{\hbar}{4\pi k_B \tau T} \right) - \Psi \left( \frac{1}{2} \right) - \frac{\hbar}{4\pi k_B \tau T} \Psi' \left( \frac{1}{2} \right) \right], \quad (\text{A5})$$

$$\kappa = \frac{-\Psi' \left( \frac{1}{2} + \frac{\hbar}{4\pi k_B \tau T} \right) + \frac{\hbar}{2\pi k_B \tau T} \Psi'' \left( \frac{1}{2} \right)}{\pi^2 \left[ \Psi \left( \frac{1}{2} + \frac{\hbar}{4\pi k_B \tau T} \right) - \Psi \left( \frac{1}{2} \right) - \frac{\hbar}{4\pi k_B \tau T} \Psi' \left( \frac{1}{2} \right) \right]}, \quad (\text{A6})$$

$$\tilde{\kappa} = \frac{-\Psi' \left( \frac{1}{2} + \frac{\hbar}{4\pi k_B \tau T} \right) + \Psi' \left( \frac{1}{2} \right) + \frac{\hbar}{4\pi k_B \tau T} \Psi'' \left( \frac{1}{2} \right)}{\pi^2 \left[ \Psi \left( \frac{1}{2} + \frac{\hbar}{4\pi k_B \tau T} \right) - \Psi \left( \frac{1}{2} \right) - \frac{\hbar}{4\pi k_B \tau T} \Psi' \left( \frac{1}{2} \right) \right]} \quad (\text{A7})$$

$\gamma = 2\eta / v_F^2 \tau \tau_\phi$ ,  $\epsilon = \ln(T/T_c)$ ,  $\epsilon_B = \epsilon + \beta/2$ ,  $\gamma_B = \gamma + \beta/2$ .  $T$  is the temperature,  $B$  the magnetic field,  $s$  the layer spacing,  $v_F$  the Fermi velocity parallel to the layers,  $\tau$  the in-plane elastic scattering time,  $\tau_\phi$  the phase-breaking time, and  $J$  the hopping integral (in units of K).  $\Psi = d[\ln\Gamma(x)]/dx$  is the digamma function.

The Zeeman terms are due to the pair-breaking effect of the magnetic field and are independent of the field direction. We calculated them by applying the usual renormalization procedure,<sup>22,23</sup> i.e., we replaced  $\epsilon$  in the zero-field fluctuation conductivity expressions by its value  $\epsilon' = \ln[T/T_c(B)]$  in a magnetic field. We used the usual approximation<sup>22,23</sup>

$$\epsilon' \approx \epsilon + 7\zeta(3) \left( \frac{g\mu_B B}{4\pi k_B T_c} \right)^2, \quad (\text{A8})$$

where  $\zeta$  is the Riemann zeta function,  $g \approx 2$  is the gyromagnetic ratio and  $\mu_B$  is the Bohr magneton. From the zero-field expressions for the fluctuation conductivity in Ref. 20 we then obtained:

$$\Delta\sigma_{\text{ALZ}} = \frac{e^2}{16s\hbar} \frac{1}{[\epsilon'(\epsilon' + r)]^{1/2}} - \frac{e^2}{16s\hbar} \frac{1}{[\epsilon(\epsilon + r)]^{1/2}}, \quad (\text{A9})$$

$$\Delta\sigma_{\text{DOSZ}} = -\frac{e^2 \kappa}{2\hbar s} \left[ \frac{(1 + \epsilon')^{1/2} + (1 + \epsilon' + r)^{1/2}}{\epsilon'^{1/2} + (\epsilon' + r)^{1/2}} \right] + \frac{e^2 \kappa}{2\hbar s} \left[ \frac{(1 + \epsilon)^{1/2} + (1 + \epsilon + r)^{1/2}}{\epsilon^{1/2} + (\epsilon + r)^{1/2}} \right], \quad (\text{A10})$$

$$\Delta\sigma_{\text{MT(reg)Z}} = \frac{\tilde{\kappa}}{\kappa} \Delta\sigma_{\text{DOSZ}}, \quad (\text{A11})$$

$$\Delta\sigma_{\text{MT(an)Z}} = \frac{e^2}{4\hbar s(\epsilon' - \gamma)} \ln \left[ \frac{\epsilon'^{1/2} + (\epsilon' + r)^{1/2}}{\gamma^{1/2} + (\gamma + r)^{1/2}} \right] - \frac{e^2}{4\hbar s(\epsilon - \gamma)} \ln \left[ \frac{\epsilon^{1/2} + (\epsilon + r)^{1/2}}{\gamma^{1/2} + (\gamma + r)^{1/2}} \right]. \quad (\text{A12})$$

For the DOS term an expression without the unnecessary simplification of Ref. 20 was used, as before.<sup>3</sup>

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