Quantum rotors in the presence of a random field

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We have studied the M-component quantum rotor Hamiltonian in the presence of a static random field (uncorrelated and Gaussian distributed) on each site of the lattice. This model is essentially an M-component generalization of the transverse Ising model in a random longitudinal field. We find that even the zero-temperature transition in the model from a ferromagnetic to the paramagnetic phase, is dominated by the random-field fixed point, which essentially determines the finite-temperature transition in the above model and the transition in the classical M-vector model in the presence of a random field. With the assumption that the transition is of continuous nature, we employ a standard renormalization-group method to study the effective classical action of the model and extract the exponents associated with the transition. We do also extend these renormalization-group calculations to the spherical ($M\rightarrow\infty$) limit. Finally, we develop a scaling argument that describes the zero-temperature transition and clearly indicates the occurrence of the dynamical exponents in the different scaling relations. We also qualitatively discuss the dynamic scaling scenario in the quantum model. [S0163-1829(98)03033-1]

I. INTRODUCTION

Quantum phase transitions have been attracting a great deal of attention in recent years. Especially the zero-temperature transitions in the transverse Ising models[1] and its M-component generalization, the rotors,[2] are being explored extensively. Studies have been carried out with the interaction between the rotors taken to be random and thus elucidation of properties of quantum spin glasses.[2] The random-field systems [especially the random field Ising model (RFIM)][3,4] have been investigated over the last two decades but the nature of the transition is yet to be fully understood. Whether the transition is first order or second order is still questioned[4] and the possibility of an intermediate glassy phase[5] has also been reported.

Here, we consider the M-component quantum rotors in the presence of a random external field. This is essentially an M-component generalization of the tranverse Ising system in the presence of a random longitudinal field.[6] The aforementioned problem ($M=1$) was previously studied by Aharony, Gefen, and Shapir.[7] A Langevin equation approach to the problem was adopted by Boyanovski and Cardy,[8] who could as well introduce a dynamical exponent z, which was taken to be unity by Aharony *et al.*[7] A recent paper by Senthil,[9] discusses with some generality the $M=1$ case of the problem and shows that the exponents associated with zero-temperature transition are similar to that in classical RFIM. Our calculations are completely in agreement with Ref. 9 in this regard.

We here assume the transition of the model to be of continuous nature. In Sec. II, we start with quantum rotor Hamiltonian in the presence of random field, and write down the zero-temperature ''partition function'' (with soft-spin consideration) in a general form within the static framework. To take account for the quenched randomness, we employ a replica trick and the Fourier-transformed replicated Hamiltonian clearly shows that the random field couples to the static ($\omega=0$) part of the order parameter. Introducing the connected and disconnected correlator in the replica language, we find the replica-symmetric Gaussian propagators and it is readily shown that the disconnected correlator does not depend upon the frequency ω. The critical dimensionalities in the quantum problem are shown to remain the same as in the classical case. Further, we carry out a systematic perturbation renormalization treatment, namely the ϵ expansion around the upper critical dimension[10] (within a replica-symmetric framework) and write down the flow equations. As shown in Ref. 8, the exponents come out to be the same as in the classical case. For example, for the correlation length exponent ν, we find to the $O(\epsilon=6-D)$

$$\nu=1/2\left(1+\frac{(M+2)\epsilon}{2(M+8)}\right),$$

which shows the dependence of the exponents on components M and matches perfectly with the exponent ν in the classical case.[10] The zero-temperature transition is found to be governed by the random-field fixed point which also determines the transition in the classical random-field system and the finite-temperature transition in the present model. Up to $O(\epsilon^2)$, it has been shown using the self-energy diagram, that the dynamical exponent z (which determines growth of temporal correlation $\xi_\tau\sim\xi^z$ near the quantum critical point) is given by $z=1+\eta$, which is in agreement with Ref. 8 where supersymmetric methods were used. We will argue that the dimensional reduction in the hyperscaling relation[11] will be by a factor of 2 but will be a combination of two effects, a change of $d\rightarrow d+z$ in the hyperscaling relation and a change of the transmutation exponent from $\theta=-2$ to $\theta=-(2+z)$, up to the first order in ϵ expansion. It is in this changed θ that the quantum effect shows up. Our calculation will support the traditionally held view (coming from the Harris criteria[12]) that random-field fluctuations mask the quantum fluctuations but we will be able to point out certain features of the various characteristic quantities—mainly the connected correlation function where the quantum fluctuations play a role.

In Sec. III we extend our calculations to the spherical ($M\to\infty$) (Ref. 13) limit and derive exponents associated with the zero-temperature transition.

In Sec. IV, we present scaling relations associated with the corresponding zero-temperature transitions using the ''dangerous irrelevance'' of the quantum fluctuations. It is shown that the quantum effects manifest in the connected correlation function which is ω dependent and thus has a crossover at finite temperature and its scaling function is different from the classical case. In a recent work,[9] strong arguments have been provided to conjecture that like the classical random-field Ising system, quantum random-field Ising ($M=1$) systems also exhibit ''activated dynamical scaling.''[14] In our work, we qualitatively argue that for rotor systems with $M>1$, the dynamical scaling is expected to be conventional rather than ''activated.''

II. RENORMALIZATION GROUP CALCULATIONS

The Hamiltonian for the quantum rotors can be written as[2]

$$H=\frac{g}{2}\sum_i \hat{L}_i^2-\sum_{ij} J_{ij}\hat{x}_i\hat{x}_j, \quad \hat{x}_i^2=1,$$

where $\hat{x}_i$ is a unit length rotor sitting at the site i with M components $x_{i\mu}$, N is the number of sites, $L_{i\mu\nu}$ ($\nu,\mu=1,2\dots,M$) are the $M(M-1)/2$ components of the angular momentum generator L_i in the rotor space. We now switch on an external field f_i (random in space) which couples to the components of the rotors x_i, producing a term $-\vec{f}_i\cdot\hat{x}_i$ in the Hamiltonian, so that

$$H(f)=\frac{g}{2}\sum_i \hat{L}_i^2-\sum_{ij} J_{ij}\hat{x}_i\hat{x}_j-\sum_i \vec{f}_i\cdot\hat{x}_i. \tag{1}$$

The field f_i is random and has the Gaussian probability distribution

$$P(f_i)=\frac{1}{(\pi\Delta)^{1/2}}e^{-(f_i-\overline{f})^2/2\Delta}. \tag{2}$$

We shall here consider the case where the mean of the random field at each site vanishes, i.e., $\overline{f}=0$. Adopting a classical statistical mechanical point of view, we shall work with the action

$$\mathcal{A}=\int_0^\beta d\tau\left(\mathcal{L}_0-\sum_{ij} J_{ij}x_i(\tau)x_j(\tau)-\sum_i f_i x_i(\tau)\right), \tag{3}$$

where

$$\mathcal{L}_0(\tau)=\frac{1}{2g}\sum_i (\partial_\tau x_i)^2+\frac{r}{2}\sum_i x_i^2+\frac{u}{4}\sum_i (x_i^2)^2.$$

In passage from Eqs. (1), (2) to Eq. (3), we have relaxed the rigid-spin constraint and cast the action with the soft-spin consideration in a form suitable for working around the upper critical dimension. We will also assume short-ranged interactions among the rotors and the interaction term $-\Sigma J_{ij}x_ix_j$ will contribute $\int(\nabla x_\mu)\cdot(\nabla x_\mu)d^Dr$. We shall henceforth drop the coefficient $1/2g$. The action is consequently

$$\mathcal{A}=\int_0^\beta\int d^Dr\left[\frac{r}{2}x_\mu x_\mu+\frac{1}{2}[\partial_\tau x_\mu(\tau)]^2+\frac{1}{2}(\nabla x_\mu)\cdot(\nabla x_\mu)+\frac{u}{4}(x_\mu x_\mu)^2-f_\mu x_\mu\right]. \tag{4}$$

As usual, the aim here is to calculate the quenched averaged partition function

$$\mathcal{Z}=\int\left[\mathcal{D}[x_\mu(\tau)]\exp(-\mathcal{A})\prod_i df_iP(f_i)\right], \tag{5}$$

where because of the quenched disorder, the quantum fluctuations are to be handled first under the field frozen and then an averaging has to be done over the field distribution given as $P(f_i)$. This calls for the replica trick,[15] whereby we introduce the n replicas and write the density of the action as (with replica indices α and β)

$$\frac{r}{2}x_\mu^\alpha x_\mu^\alpha+\frac{1}{2}[\partial_\tau x_\mu^\alpha(\tau)]^2+\frac{1}{2}(\nabla x_\mu^\alpha)\cdot(\nabla x_\mu^\alpha)+\frac{u}{4}(x_\mu^\alpha x_\mu^\alpha)^2-f_\mu\sum_{\alpha=1}^n x_\mu^\alpha. \tag{6}$$

Performing the average over the magnetic field $f_\mu(\vec{r})$ at this stage leads to the replicated Hamiltonian

$$\mathcal{A}^{(n)}=\int_0^\beta\int d^Dr\left[\frac{r}{2}x_\mu^\alpha x_\mu^\alpha+\frac{1}{2}(\partial x_\mu^\alpha(\tau))^2+\frac{1}{2}(\nabla x_\mu^\alpha)\cdot(\nabla x_\mu^\alpha)+\frac{u}{4}(x_\mu^\alpha x_\mu^\alpha)^2\right]-\Delta/2\int_0^\beta d\tau_1\int_0^\beta d\tau_2\left[\sum_\alpha x_\mu^\alpha(\tau_1)\right]\left[\sum_\beta x_\mu^\beta(\tau_2)\right]. \tag{7}$$

One is required to take the limit of $n\to 0$ at the end of the calculation. It is convenient to rewrite the above action in terms of Fourier components $\phi_\mu^\alpha(\vec{k},\omega)$ of the $x_\mu^\alpha(\vec{r},\tau)$ fields so that the action now becomes

$$\mathcal{A}^{(n)}=\int\frac{d\omega}{2\pi}\frac{d^Dk}{2\pi}[(r/2+\omega^2/2+k^2/2)\phi_\mu^\alpha(\vec{k},\omega)\times\phi_\mu^\alpha(-\vec{k},-\omega)]-\Delta/2\sum_\alpha\int\frac{d\omega}{2\pi}\frac{d^Dk}{(2\pi)^D}\phi_\mu^\alpha(k,\omega)\times\sum_\beta\phi_\mu^\beta(-k,-\omega)\delta(\omega). \tag{8}$$

Clearly, one finds that the random-field fluctuations couple to the static $(\omega=0)$ part of the order parameter.

The Gaussian propagator in the replica space can be written as

$$G_{\alpha\beta}(k,\omega)=\frac{1}{(k^2+\omega^2+r)}\delta_{\alpha\beta}+\frac{\Delta\,\delta(\omega)}{(k^2+\omega^2+r)(k^2+\omega^2+r-n\Delta)}, \qquad (9)$$

where $n\rightarrow 0$ limit gives the replica-symmetric Gaussian propagator. We now define the connected and disconnected correlation functions[16] (it is to be noted that the static ($\omega=0$) part of the connected correlation function decays as $k^{-2+\eta}$ while the disconnected correlator decays as $k^{-4+\bar{\eta}}$, at the critical point) given as

$$G_{\rm con}(k,\omega)\,\delta^{\mu\nu}=\overline{\langle\phi_k^{\mu}(\omega)\phi_{-k}^{\nu}(-\omega)\rangle}-\overline{\langle\phi_k^{\mu}(0)\rangle\langle\phi_{-k}^{\nu}(0)\rangle}, \qquad (10)$$

$$G_{\rm dis}\delta^{\mu\nu}=\overline{\langle\phi_k^{\mu}(0)\rangle\langle\phi_{-k}^{\nu}(0)\rangle} \qquad (11)$$

where the overhead bar, as usual denotes the configurational average over the random field. These two types of correlations are related to the replica correlation functions as[5]

$$G_{\rm dis}(k)=\lim_{n\rightarrow 0}\frac{1}{n(n-1)}\sum_{\alpha\neq\beta}G_{\alpha\beta}(k),$$

$$G_{\rm con}=\lim_{n\rightarrow 0}\frac{1}{n}\sum_{\alpha}G_{\alpha\alpha}(k,\omega)-G_{\rm dis}.$$

In the Gaussian case, we can readily find the explicit forms of the above correlations in the $n\rightarrow 0$ limit

$$G_{\rm dis}(k)=\frac{\Delta\,\delta(\omega)}{(k^2+r)^2},$$

$$G_{\rm con}(k,\omega)=\frac{1}{k^2+\omega^2+r}.$$

As mentioned previously, the static random field couples to the static part of the order parameter, the disconnected correlation function is always independent of the Matsubara frequency ω. On the other hand the connected part of the propagator incorporates the net ω-dependent part of the propagator thus showing a crossover to the classical value at finite temperature when the Matsubara frequencies renormalize to zero.[17] Using this above form of the Gaussian propagator, we evaluate the Feynman diagram shown in Fig. 1(b), given as

$$\int\frac{d^Dk}{(2\pi)^D}\int\frac{d\omega}{2\pi}\frac{\Delta\,\delta(\omega)}{(k^2+\omega^2+r)(k^2+r)^2}=\int\frac{d^Dk}{(2\pi)^D}\frac{\Delta}{(k^2+r)(k^2+r)^2}.$$

FIG. 1. (a) Diagram which renormalizes r and Δ, (b) diagram which renormalizes u. The thin line corresponds to the propagator with $\Delta=0$ and the circled straight line stands for the Δ-dependent part of the propagator.

This shows that the infrared divergence occurs above the dimensionality $D=6$. Hence, $D=6$ which comes out to be the upper critical dimension in the quantum-mechanical case for all M showing that the upper critical dimensionality remains the same as that in the classical case. We shall see it shortly using the renormalization-group flow equations.

Let us now address the question of lower critical dimension of the zero-temperature transition in the quantum random-field system. For both $M=1$ and $M\geqslant 2$ quantum systems, one can look at the equivalent $(D+1)$-dimensional classical system[18] with randomness correlated in the ($D+1$)-th (Trotter) direction. To extend the Imry-Ma argment[3] to the present case, we imagine that the equivalent $(D+1)$-dimensional classical model is deep in the ordered phase. The stability to a weak random field is determined by balancing the typical energy gain because of the random field. Deep in the ordered phase, the typical energy cost for a domain of the linear dimension $L\sim L^{(D+1)-1}$ for $M=1$ and $\sim L^{(D+1)-2}$ for systems with $M\geqslant 2$, while the typical energy gain due to random field $\sim L^{D/2}$ [not $L^{(D+1)/2}$, since the randomness is correlated in $(D+1)$th direction]. Thus for $D>2$ for Ising systems and $D>4$ for systems for $M\geqslant 2$, large domain formation is not favorable. Thus, as in the classical case, here as well the lower dimensionality is 2 (for $M=1$) and 4 (for $M\geqslant 2$), which are marginally stable to the random field.

With $u=0$, a standard renormalization-group treatment yields for the Gaussian fixed point

$$\zeta(\text{field rescaling factors})=b^{(D+z+2)/2},$$

$$r'=rb^2,$$

$$\Delta'=\Delta b^{2+z},$$

$$z=1.$$

The terms r and Δ are both relevant and have to be controlled to observe the critical behavior. The quartic perturbation u to the Gaussian fixed point shows the scaling

$$u'=ub^{4-D-z},$$

which shows that the variable is certainly irrelevant (more strictly speaking the quantum fluctuations are ''dangerously irrelevant'') for $D>4$. Under a change of length scale by a factor b, the parameter u scales as $b^{-\theta}$, this is how we define the exponent θ in the present case. However, the combination $w=u\Delta$ has the scaling

$$w'=wb^{6-D},$$

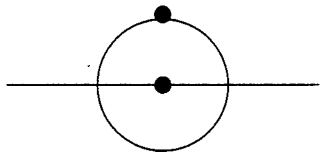

FIG. 2. The self-energy diagram up to the second order in ϵ with two cirles (Δ) and two u's.

and is a relevant perturbation for $D<6$. Consequently, the possible deviation from the Gaussian fixed point will be brought about by the combination $w=u\Delta$.

Using the above Gaussian propagator, we can now carry out a perturbative renormalization-group analysis for $4<D<6$ (the restriction $4<D$ ensures that there is no divergence in Δ-free part). Correct to $O(\epsilon)$, where $\epsilon=6-D$, our flow equations (diagrams as shown in Fig. 1) with scaling variables r, u and $w=u\Delta$, are constructed.

Keeping the replicas at a finite value of n and taking the limit $n\rightarrow 0$, we obtain the flow equations given as

$$\frac{\partial r}{\partial l}=\left[2-(M+2)\frac{S_6}{(2\pi)^6}(u\Delta)\right]r+(M+2)\Lambda^2(u\Delta), \tag{12}$$

$$\frac{\partial u}{\partial l}=[2-z]u-u\Delta^2(M+8)\frac{S_6}{(2\pi)^6}, \tag{13}$$

$$\frac{\partial(u\Delta)}{\partial l}=\epsilon(u\Delta)-(M+8)\frac{S_6}{(2\pi)^6}(u\Delta)^2. \tag{14}$$

Clearly for $\epsilon>0$ (i.e., $D<6$), the fixed point $w=u\Delta=w^*=0$ (Gaussian) is unstable, and flows to the random-field fixed point

$$w^*=\frac{\epsilon}{K_6(m+8)}, \tag{15}$$

$K_6=S_6/(2\pi)^6$. At this fixed point,

$$\frac{du}{dl}=-(2+z)u \tag{16}$$

leading to

$$u'=b^{-(2+z)}u,$$

showing the irrelevence of u and the exponent θ as

$$\theta=2+z, \tag{17}$$

which is independent of M. All other exponents, as already discussed in the Introduction, depend on M and are found to be the same as the classical exponents.[10]

At this order, we have not picked up any correction to the $k^2\phi_k\phi_{-k}$ part of the Hamiltonian and hence $\eta=0+O(\epsilon^2)$. The first contribution to η comes from the diagram shown in Fig. 2, and we can readily establish from the analysis of the corresponding integral of the self-energy that $z=1+\eta$. The self-energy corresponding to Fig. 2 can be written as

$$\Sigma(k,\omega)=\int\frac{d^Dp}{(2\pi)^D}\frac{d^Dq}{(2\pi)^D}\frac{1}{p^4}\frac{1}{q^4}\frac{1}{(k-p-q)^2+\omega^2}. \tag{18}$$

For $\omega\rightarrow 0$, the behavior of $\Sigma(k,\omega)$ is expected to be $-\eta k^2\ln b$, while for $k\rightarrow 0$, $\Sigma(0,\omega)-\Sigma(0,0)$ is expected to scale as $\omega^2\eta_\tau\ln b$. By studying the two differences, we can conclude that

$$\eta_\tau=-\frac{D}{4-D}\eta=-[3+O(\epsilon)]\eta=-3\eta. \tag{19}$$

To keep the coefficient of $\omega^2\phi(k,\omega)\phi(-k,-\omega)$ (i.e., $1/2g$) unaltered in the action we need to set the dynamical exponent $z=1+\eta+O(\epsilon^3)$ (see the Appendix). This result was also derived for the transverse Ising case in Ref. 8 using supersymmetric techniques.

III. SPHERICAL LIMIT

In this section, we shall extend the above renormalization group calculation in the $M\rightarrow\infty$ (Ref. 19) with $u=O(1/M)$ and derive the exponents for the zero-temperature transition in the quantum random-field model below the upper critical dimension 6, which come out the same as the classical exponents.[13] To evaluate γ, one considers the scaling of the mass renormalization term (we need not consider the integral over the Matsubara frequencies since we consider the random-field part of the Gaussian propagator which contains a δ function in the frequency)

$$\int\frac{d^Dp}{(2\pi)^D}\left(\frac{1}{(r+p^2)^2}-\frac{1}{p^4}\right)\sim r^{(d-4)/2},$$

which yields $\gamma=2/(d-4)$ and with $\eta=0$, $\nu=1/(d-4)$ as in the classical case.[13] To calculate α, we use[19]

$$\chi(0)=M\prod(r,0)\left[1+Mu\prod(r,0)\right]^{-1}\sim u^{-1}-Cr^{3-d/2}.$$

As $r\sim(r_0-r_{0c})^{(3-d/2)\gamma}$, we find the specific-heat exponent $\alpha=(d-6)/(4-d)$. (In a zero-temperature quantum phase transition the exponent α denotes the scaling of the singular part of the ground-state energy density as the quantum critical point is approached $E_{\text{sing}}\sim\delta^{2-\alpha}$, where δ is the distance from the quantum critical point.)

To evaluate the exponent β we consider the disconnected correlation function and using the argument that the disconnected correlation scales as the square of the configuration-averaged magnetization[20]

$$2\beta=\nu(d-4)=1.$$

We thus derive the exponents in the spherical limit using the form of Gaussian propagator. Clearly, above the upper critical dimension, the exponents are the usual mean-field exponents.

IV. THE ZERO-TEMPERATURE SCALING RELATIONS

In this section we shall derive the scaling relations, associated with the quantum phase transition in the random-field

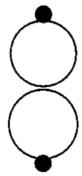

FIG. 3. Typical graph for the ensemble-averaged ground-state energy with two Δ's and one u.

systems, involving the exponents ν, η, $\bar{\eta}$, and the dynamical exponent z. This scaling relation is quite general and is expected to hold for all M. Eventually we shall need following assumptions:

(1) The fluctuations induced by the random-field dominates over the quantum fluctuations arising due to the presence of noncommuting terms in the Hamiltonian.

(2) In the renormalization-group sense, we need to consider the flow of three parameters r, u, and $w=\Delta u$. As shown earlier r and w are relevant parameters. The parameter u is ''dangerously irrelevant''[10] as will be shown below. It decays as $b^{-\theta}$ under a change of length scale by a factor of b, which as mentioned earlier, defines the exponent θ.

To study the zero-temperature transition, we shall consider the renormalization of the singular part of the disorder-averaged ground-state energy density (not the free-energy density as in Ref. 10) under a change of length scale by a factor of b,

$$E(u_0,\delta r_0,\delta w_0)=b^{-(d+z)}E(u_0 b^{\lambda_u},\delta w_0 b^{\lambda_w},\delta r_0 b^{\lambda}), \tag{20}$$

where u_0 (infinitesimally small), δr_0, and δw_0 [deviations from the nontrivial fixed point r^* and u^* given in Eq. (19)] are the scaling fields of the linearized renormalization-group equations. If we set $\delta w_0=0$, i.e., we work at the random-field fixed point, we have under a renormalization-group transformation by a factor of length scale b

$$E(u_0,\delta r_0)=b^{-(d+z)}E(u_0 b^{\lambda_u},\delta r_0 b^{\lambda}). \tag{21}$$

Here, δr_0 denotes the deviation from the nontrivial zero-temperature fixed point which we denote as t and the eigenvalue λ is the inverse of the exponent ν associated with the zero-temperature transition. Hence, we have

$$E(u_0,t)=b^{-(d+z)}E(u_0 b^{\lambda_u},t b^{\lambda}). \tag{22}$$

Identifying, $b=t^{-\nu}$, we have

$$E(u_0,t)=t^{\nu(d+z)}E(u_0 b^{\lambda_u}). \tag{23}$$

If $f(x)$ tends to a constant value (as in the quantum phase transition in the pure system) we obtain the usual hyperscaling relation[21]

$$2-\alpha=\nu(d+z)\,. \tag{24}$$

From the Hamiltonian (1), if we expand in power series of Δ, the term contributed by the diagram shown in Fig. 3 is of the order $u_0\Delta^2$ (one vertex u_0 and two dots indicating two disorder averaging yielding $\Delta^2=w^2/u_0$ (written in terms of scaling variables). Thus as the system is driven towards the nontrivial fixed point (with $u_0=0$), this term goes as u_o^{-1}, hence $f(x)\sim 1/x$ and clearly diverges as $x\to 0$ indicating that we find the parameter u is not only irrelevant, but is clearly ''dangerously irrelevant.'' We thus find

$$E\sim t^{\nu(d+z-\theta)}, \tag{25}$$

so that we obtain the hyperscaling relation

$$2-\alpha=\nu(d+z-\theta). \tag{26}$$

Up to the first order in ϵ expansion $\theta=2+z$ (with z being unity up to the first loop order), we have $\theta=3$,[8] yielding

$$2-\alpha=\nu(d-2), \tag{27}$$

which is the same dimensional reduction as obtained in the classical case in the replica-symmetric framework. But instead of θ being 2 in the present case $\theta=2+z$, as has been shown previously. It should be noted that in the classical RFIM (Ref. 4) $\theta\neq 2$ in general.

We have already defined the exponents η and $\bar{\eta}$ through the connected and disconnected correlation functions. As mentioned previously, as the random field couples to the static part of the order parameter, the disconnected part is a function q only. We propose a scaling form for the connected correlation function

$$G_{\rm con}(q,\omega)\sim \xi^{2-\eta}f(q\xi,\omega\xi^z), \tag{28}$$

where ξ and $\xi_\tau(\sim\xi^z)$ are spatial and temporal correlation lengths, respectively. At the criticality, both ξ and ξ_τ diverge and we find a simpler scaling form

$$G_{\rm con}(q,\omega)\sim q^{-2+\eta}\tilde{f}\left(\frac{\omega}{q^z}\right) \quad \text{for} \quad \omega,\ q\to 0. \tag{29}$$

Using the fluctuation-dissipation theorem for the quantum systems,[21] we find the relation between the wave-vector-dependent susceptibility $\chi(q)$ and $G_{\rm con}$ given as

$$\chi(q)=\int \frac{d\omega}{2\pi}\frac{G_{\rm con}(q,\omega)}{\omega}. \tag{30}$$

Defining, $\chi(q=0)\sim t^{-\gamma}$ and using the scaling form of $G_{\rm con}$, we obtain

$$\gamma=(2-\eta)\nu. \tag{31}$$

For the disconnected correlation function, we have at the criticality

$$G_{\rm dis}(x)=\overline{\langle\phi(x,\tau_1)\rangle\langle\phi(0,\tau_2)\rangle}\sim x^{-(d-4-\bar{\eta})}, \tag{32}$$

so that the Fourier transform goes as $q^{-4+\bar{\eta}}$ and the correlator is independent of τ_1 and τ_2.[2] Clearly the scaling dimension of $\phi(x,\tau)$ is $(d-4+\bar{\eta})/2$. Again the disconnected correlation function scales as the square of the configuration-averaged magnetization[20] and thus scales as $\xi^{2\beta}$, where ξ is the correlation length, yielding

$$2\beta=\nu(d-4+\bar{\eta}). \tag{33}$$

Again, under renormalization-group transformation the connected correlation function $G_{\rm con}(q,\omega)$ should scale as

$$u\times \text{ scaling dimension of } G_{\rm dis}\sim x^{-(d-4+\bar{\eta}+\theta)},$$

so that we get

$$\theta = 2 + z + \eta - \overline{\eta}, \tag{34}$$

in contrast to the classical case, where $\theta = 2 + \eta - \overline{\eta}$. The hyperscaling relation thus becomes

$$2 - \alpha = \nu(d - 2 - \eta + \overline{\eta}).$$

We thus obtain the scaling relation $\alpha + 2\beta + \gamma = 2$ satisfied.

The scaling relation thus obtained does not depend upon the dimensionality of the order parameter. We have essentially used the ''dangerous irrelevence'' of the quantum fluctuations at the zero-temperature transition. The Schwartz-Soffer inequality[16] holds between η and $\overline{\eta}$ even when the disorder is static in time[2] (see also the Appendix) so one expects $\overline{\eta} \geqslant 2\eta$. It should be noted that recent extensive series studies for the classical model have shown that this inequality is satisfied as an equality with $\overline{\eta} = 2\eta$.[22]

This is also to be noted here in the present case, that since the disorder is static, the quantum effect does not manifest in the disconnected correlation functions. It really shows up in the connected correlation function which has a weaker divergence in comparison to the connected part and the scaling functions f and $\tilde{f}$ are different from the corresponding classical scaling functions. In the finite-temperature classical case, the disconnected correlation function does not show a crossover whereas the connected part changes to the classical value because at the finite temperature the Matsubara frequency ω renormalizes to zero.[17]

We shall now briefly discuss the dynamical scaling aspect of a quantum random-field system. Strong arguments are provided in Ref. 9 to establish that the dynamical scaling in the quantum Ising system in the random field is activated like other random quantum Ising transitions.[23] For $M > 1$, the argument will be modified in the following way. As argued in Ref. 9, the contribution to the dynamics will essentially come from the effect of quantum fluctuations on ''large rare'' blocks which are locally ordered. Neglecting the coupling to the environment, the fluctuations of this block spin can be described by a one-dimensional (corresponding Trotter dimension) classical, M-component spin chain with ferromagnetic coupling $K_\tau \sim L^D$. This chain has a finite correlation length (time) ξ_τ which scales for large $K_\tau(\sim L^D)$ for $M > 1$ as[2]

$$\xi_\tau \sim K_\tau \sim L^D, \tag{35}$$

whereas in the Ising case ($M = 1$) we have $\xi_\tau \sim \exp(cL^D)$, where c is constant. This shorter correlation time for continuous spins shows that quantum dynamics is not activated, rather Eq. (35) suggests that the dynamical scaling is expected to be conventional (power law).

V. CONCLUSION

We have studied the quantum rotor Hamiltonian in the presence of quenched Gaussian random field. The zero-temperature transition is governed by the random-field fixed point and the critical dimensionalities are found to be the same as in the corresponding classical case. The exponent θ and the connected correlation function incorporates the quantum effects whereas the disconnected part is independent of the Matsubara frequencies ω. A standard renormalization technique within the static framework provides us the renormalization group flow equations showing the irrelevence of the parameter u. We extract the exponents associated with the zero-temperature phase transitions in the model up to the first order in ϵ. Exploiting the nature of the self-energy diagram we have shown that up to the second order in ϵ, the dynamical exponents of the quantum Hamiltonian are related to the exponent η given as $z = 1 + \eta$. We have also derived the exponents in the spherical limit.

In the concluding section, we derive the scaling relations associated with the zero-temperature transition in the model indicating clearly the ''dangerous irrelevence'' of the parameter u. These scaling relations are quite general and are expected to hold for any M. We also argue that for $M > 1$, dynamical scaling is expected to be conventional.

We must mention here that we have assumed the transition to be second order and have employed a replica-symmetric theory. But, the possibility of replica symmetry breaking will be considered later to see whether an intermittent glassy phase occurs.

We just note in conclusion that there is the strong possibility of the occupance of ''Griffiths-McCoy''-type singularities[24] associated with the quantum phase transition in the present model, which is being currently explored.

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APPENDIX

1. In this section, we shall establish the relationship between the exponents z and η up to the second order in ϵ. As pointed out in Ref. 8, this correspondence is really an outcome of the symmetry of the propagators up to this order. We here consider Fig. 2, which is the only relevant diagram up to this order of ϵ expansion. The corresponding contribution to the free energy can be written as

$$\Sigma(\vec{p},\omega) = \int \frac{d^D p_1}{(2\pi)^D} \int \frac{d^D p_2}{(2\pi)^D} \frac{1}{p_1^4} \frac{1}{p_2^4} \frac{1}{(\vec{p} - \vec{p_1} - \vec{p_2})^2 + \omega^2}. \tag{A1}$$

In the $\omega \to 0$ limit, the vortex function scales as $\Gamma^2(p) \sim p^{2-\eta} \approx p^2(1 - \eta \ln p)$. We find in the $\omega \to 0$ limit,

$$\Sigma(p,0) - \Sigma(0,0) = \int_{\Lambda/b}^{\Lambda} \frac{d^D}{(2\pi)^D} \int_{\Lambda/b}^{\Lambda} \frac{d^D}{(2\pi)^D} \times \left[\frac{1}{p_1^4} \frac{1}{p_2^4} \left(\frac{1}{(\vec{p} - \vec{p_1} - \vec{p_2})^2} - \frac{1}{(p_1 + p_2)^2} \right) \right] = -\eta p^2 \ln b. \tag{A2}$$

Once again, in the $p \to 0$ limit,

$$\Sigma(\omega,0)-\Sigma(0,0)=\omega^2\eta_\tau \ln b \tag{A3}$$

$$=\int \frac{d^D}{(2\pi)^D}\frac{d^D}{(2\pi)^D}\frac{(-\omega^2)}{p_1^4 p_2^4(\vec{p_1}+\vec{p_2})^2}. \tag{A4}$$

From Eq. (A2), we find

$$\Sigma(p,0)-\Sigma(0,0)$$

$$=\int_{\Lambda/b}^{\Lambda}\frac{d^D p_1}{(2\pi)^D p_2}\int_{\Lambda/b}^{\Lambda}\frac{d^D}{(2\pi)^D}\frac{1}{p_1^4}\frac{1}{p_2^4}\frac{1}{(\vec{p_1}+\vec{p_2})^2}$$

$$\times\left[1-\frac{\vec{p}\cdot(\vec{p_1}+\vec{p_2})}{(\vec{p_1}+\vec{p_2})^2}+\frac{p^2}{(\vec{p_1}+\vec{p_2})^2}\right]. \tag{A5}$$

A few lines of algebra yield

$$\Sigma(p,0)-\Sigma(0,0)=\int\int\frac{1}{p_1^4}\frac{1}{p_2^4}\frac{p^2(4\cos^2\theta-1)}{(\vec{p_1}+\vec{p_2})^4}. \tag{A6}$$

Inserting the angular average of $\cos^2\theta=1/d$, we find from Eq. (6),

$$\int \frac{1}{p_1^4}\frac{1}{p_2}\frac{{}^4p^2(4/d-1)}{(\vec{p_1}+\vec{p_2})^4}. \tag{A7}$$

Comparing Eqs. (A4) and (A7), we find

$$\eta_\tau=-\left(\frac{d}{4-d}\right)\eta. \tag{A8}$$

With $d=6$, we find $\eta_\tau=-3\eta$. Hence, we find the scaling of $(1/2g)\omega^2$ under renormalization group transformation as $(1/2g)\omega^2 b^{2-2z-\eta+3\eta}$. For the fixed-point behavior of the coefficient g, we find $z=1+\eta+O(\epsilon^3)$.

2. In this section we shall indicate how the Schwartz-Soffer inequality can be extended to the case where the disorder is static. The average of the mth component of ϕ_k can be set in the form

$$\langle\phi_k^m(\omega=0)\rangle=\frac{\mathrm{tr}\left\langle\phi_k^m(0)\exp\left[-\int_{k,\omega}\mathcal{L}_0(k,\omega)-\int_k\sum_{m=1}^{M}f_k^m\phi_k^m(\omega=0)\right]\right\rangle}{\mathrm{tr}\exp\left[-\int_{k,\omega}\mathcal{L}_0(k,\omega)\right]}, \tag{A9}$$

where by tr, we represent the functional integral over the fields $\phi_k(\omega)$ and the effective classical action $\mathcal{L}_0$ is defined previously from Eqs. (3) and (8). We here consider the static part of the average of ϕ_k^m because the random-field couples to the static part. The $\omega=0$ part of the connected correlation function is obtained as

$$\frac{\overline{\partial\langle\phi_k(0)\rangle}}{\partial h_{-k}}=-\overline{\langle\phi_k(0)\phi_{-k}(0)\rangle}-\overline{\langle\phi_k(0)\rangle\langle\phi_{-k}(0)\rangle}. \tag{A10}$$

Now, identical steps in Ref. 16, essentially based on the assumption of the Gaussian nature of the randomness, lead us to the inequality

$$\overline{\langle\phi_k(0)\phi_{-k}(0)\rangle}-\overline{\langle\phi_k(0)\rangle\langle\phi_{-k}(0)\rangle}\leqslant\Delta^{-1}(\langle\phi_k(0)\rangle\langle\phi_{-k}(0)\rangle)^{1/2}. \tag{A11}$$

As mentioned previously, the $\omega=0$ part of the connected correlator diverges as $k^{-2+\eta}$ whereas the disconnected part diverges as $k^{-4+\bar{\eta}}$ at the criticality. Hence, Eq. (A11) readily shows that the Schwartz-Soffer inequality, i.e., $\bar{\eta}\geqslant 2\eta$ is valid even in the present case.

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