Two-dimensional electron gas under a spatially modulated magnetic field: A test ground for electron-electron scattering in a controlled environment

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We have studied the electronic transport in a two-dimensional electron gas (2DEG) subjected to a spatially modulated magnetic field and electrostatic potential. Independent control of the magnetic field components parallel and perpendicular to the 2DEG plane allows us to manipulate the amplitude of the magnetic modulation independently of the normal field component relevant to the magnetotransport in the 2DEG. The amplitudes of the magnetic and electrostatic modulations can be evaluated from an analysis of the commensurability oscillation of magnetoresistance. The increase of resistivity in the presence of a modulated magnetic field at a zero uniform magnetic field is found to be quadratic in the modulation amplitude and to vary as $AT^2 + C$ at low temperatures. This is a signature of the electron-electron scattering in the presence of a spatially modulated magnetic field. The present system provides an ideal test ground for theories of electron-electron-scatteringinduced resistivity. [S0163-1829(98)07431-1]

I. INTRODUCTION

A two-dimensional electron gas (2DEG) at the GaAs/Al_xGa_{1-x}As heterointerface constitutes one of the cleanest electron systems in solids. It allows us to investigate various effects of artificial potential that can be realized, for example, by placing a microstructured gate electrode on the surface. The simplest type of artificial potential involves sinusoidal modulation of a electrostatic potential. It gives rise to oscillatory magnetoresistance known as a Weiss oscillation.¹⁻³ The phenomenon has its physical origin in the commensurability between the electronic cyclotron motion and the spatial periodicity and manifests itself as 1/B-periodic oscillations in the resistivity measured with the current parallel to the direction of the potential modulation. The condition for the occurrence of resistivity minima is given by²⁻⁶

$$\frac{2R_c}{a} = n - \frac{1}{4} \quad (n = 1, 2, 3, \dots).$$
(1)

Here $R_c = \hbar k_F / eB$ is the cyclotron radius of an electron at the Fermi surface and *a* is the modulation period.

Recently, a magnetic analog of the Weiss oscillation predicted earlier^{7,8} has been experimentally demonstrated.^{9–11} In that case, a spatially modulated magnetic field is imposed on a 2DEG by placing a microstructured ferromagnet or superconductor on the surface. The magnetoresistance exhibits 1/B-periodic oscillations similar to the ordinary (electrostatic) Weiss oscillation except that the minima and maxima exchange their positions. The condition for the resistivity minima in the magnetic Weiss oscillation is given by

$$\frac{2R_c}{a} = n + \frac{1}{4} \quad (n = 1, 2, 3, \dots).$$
 (2)

In both cases of electrostatic and magnetic modulations, rather detailed information on the nature of modulation imposed on the 2DEG can be gained from the analysis of the commensurability oscillations.⁸ This kind of system thus offers a unique opportunity to explore the electronic transport in a highly controlled artificial environment. In this paper we demonstrate how we can control and evaluate the electrostatic and magnetic modulations at the 2DEG plane and show that the present system is a good candidate for quantitative studies of the combined effect of the electron-electron interaction and artificial periodicity. Part of this work has been reported in Ref. 12.

II. EXPERIMENTAL METHOD

Samples used in the present study were fabricated from a GaAs/Al_xGa_{1-x}As single heterojunction wafer grown by molecular beam epitaxy. The 2DEG in the wafer had the density $n_e = 1.9 \times 10^{15}$ m⁻² and mobility $\mu = 43$ m² V⁻¹ s⁻¹ at 1.5 K. The depth of the heterointerface from the top surface was 90 nm. A standard Hall-bar pattern was defined by photolithography and wet etching. An array of 60-nm-thick nickel stripes with periodicity *a* (*a*/2 wide and *a*/2 apart) was fabricated on the top surface by electron-beam lithography, vacuum deposition, and a lift-off process. The nickel stripes were all connected so that a gate bias could be applied to them. The experimental results presented in this paper were obtained from intensive measurements on one particular sample with periodicity *a* = 500 nm.

One of the experimental obstacles encountered in the study of the magnetic Weiss oscillation is the fact that placing the microstructured gate generally induces strain and produces a strain-induced electrostatic potential modulation.⁷ In our previous studies we applied a gate bias to counteract and cancel the strain-induced potential.⁹ Recently, it has been shown that the strain-induced potential is mostly via piezoelectric coupling and that it can be minimized by setting the direction of modulation parallel to the [100] direction.^{14–16} Accordingly, we patterned the Hall bar with the current direction aligned along the [100] axis.

Transport measurements were carried out using a standard

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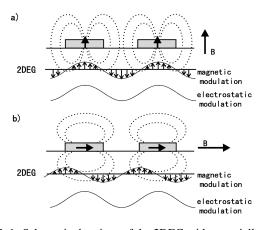


FIG. 1. Schematic drawings of the 2DEG with a spatially modulated magnetic field. (a) When the ferromagnetic stripes are vertically magnetized, the resultant magnetic modulation at the 2DEG plane is in phase with the electrostatic modulation. (b) When they are horizontally magnetized, the magnetic and electrostatic modulations are out of phase.

low-frequency ac technique. As discussed below, the precise alignment of the magnetic field direction with respect to the 2DEG plane and to the stripe pattern was essential. The cross-coil magnet system used in the present work consisted of a 6-T split-coil superconducting magnet in combination with a small homemade solenoid and enabled us to independently control the horizontal and vertical components of the magnetic field. A rotating sample holder was used to adjust the angle of the sample-mounting stage so as to make the 2DEG plane horizontal. The azimuthal angle φ between the horizontal magnetic field and the direction of the modulation (see the inset of Fig. 2) was varied by turning the sample holder around its vertical axis. For each setting of φ , the horizontal alignment was readjusted. The misalignment component that was not accessible by the sample rotation stage was compensated by applying a small bias field in the vertical direction.

The horizontal magnetic field served to control the magnetization of the ferromagnetic gate and the vertical field was used for measurements of magnetotransport in the 2DEG. This scheme allowed us to keep the ferromagnetic stripes fully polarized while working with low (or even zero) perpendicular field for the 2DEG.

III. RESULTS AND DISCUSSION

Prior to the analysis of the experimental data, we give a brief account of the Weiss oscillation in coexisting electrostatic and magnetic modulations. Equations (1) and (2) give the positions of resistance minima for the electrostatic potential modulation and for the magnetic field modulation, respectively. The Weiss oscillation in the presence of both types of modulation is predicted to show distinct behavior depending on the relative spatial phase of the two types of modulation.⁸ The key parameter is

$$\delta = \frac{2\pi m^* V_0}{ak_F \hbar eB_0},\tag{3}$$

which represents the ratio of the electrostatic potential modulation amplitude V_0 and the magnetic field modulation am-

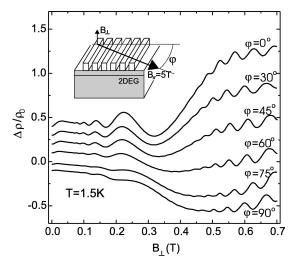


FIG. 2. Magnetoresistance traces for different settings of the azimuthal angle φ of the parallel magnetic field defined as shown in the inset. The traces are vertically shifted for clarity.

plitude B_0 . If the two types of modulation are spatially in phase [i.e., $V(x) = V_0 \cos(2\pi x/a)$ and $B_z(x) = B_0 \cos(2\pi x/a)$], the Weiss oscillation pattern shifts horizontally as a function of δ in such a way that the positions of the resistance minima become

$$\frac{2R_c}{a} = n + \frac{1}{4} - \frac{1}{\pi} \arctan \delta \quad (n = 1, 2, 3, \dots).$$
(4)

This sort of phase shift as a function of δ has been experimentally verified.¹³ In that experiment, the ferromagnetic stripes were magnetized by a vertical magnetic field so that the resulting magnetic modulation at the 2DEG plane was in phase with the electrostatic modulation as depicted in Fig. 1(a). In contrast, if the two types of modulation are out of phase [i.e., $V(x) = V_0 \cos(2\pi x/a)$ and $B_z(x) = B_0 \sin(2\pi x/a)$], the oscillatory part of the magnetoresistance is given by

$$\Delta \rho(B) \propto (1 - \delta^2) \sin^2 \left(\frac{2\pi R_c}{a} - \frac{\pi}{4} \right), \tag{5}$$

so that the Weiss oscillation pattern is inverted without a phase shift.

In the present experimental configuration, the ferromagnetic stripes are magnetized by the horizontal magnetic field, so that the resulting magnetic field modulation at the 2DEG plane is expected to be out of phase with the electrostatic modulation as depicted in Fig. 1(b). This is seen in Fig. 2, which shows the magnetoresistance as a function of the perpendicular magnetic field B_{\perp} with zero gate bias and under a constant parallel field $B_{\parallel} = 5$ T. This parallel magnetic field is much higher than the saturation field $B \approx 0.3$ T for nickel, so that the magnetization of the nickel stripes are fixed irrespective of the change in B_{\perp} . The different traces are taken with the parallel field applied at different azimuthal angles φ . The oscillations observed in the field range $B_{\perp} < 0.3$ T are the Weiss oscillations, while the shorter period oscillations at higher B_{\perp} are the Shubnikov-de Haas effect. No hysteresis appears because the magnetization of the nickel stripes is fixed by the constant horizontal field $B_{\parallel} = 5$ T. Note that

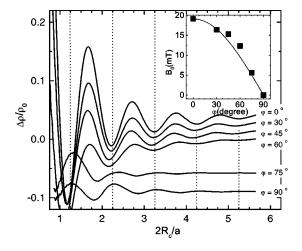


FIG. 3. Data shown in Fig. 2 replotted as a function of $2R_c/a$. The traces are vertically shifted for clarity. The inset shows the $\cos \varphi$ dependence of the amplitude B_0 of the magnetic field modulation at the 2DEG plane.

only the magnetization component parallel to the modulation direction is effective in generating the magnetic field modulation at the 2DEG plane.

Figure 3 is a replot of the data in Fig. 2 as a function of $2R_c/a$ calculated by using the value of n_e as determined from the Shubnikov-de Haas effect. The traces for $\varphi \leq 60^\circ$ have their minima at the positions in agreement with Eq. (2) (vertical dotted lines in the figure). However, the minima turn to maxima in traces for higher values of φ , namely, the Weiss oscillation diminishes in amplitude with increasing φ and is eventually inverted without any phase shift. This peak-valley inversion corresponds to the behavior for the mutually out-of-phase electrostatic and magnetic modulations mentioned above.

We can evaluate the modulation amplitude by fitting the Weiss oscillation with the theoretical formula given by Peeters and Vasilopoulos.⁸ Let us begin with the case φ $=90^{\circ}$. For this field orientation, the nickel stripes are magnetized along their length so that the magnetic field modulation produced at the 2DEG plane is minimal.¹⁷ The Weiss oscillation observed in this configuration is mostly due to the strain-induced electrostatic modulation. (The nature of the electrostatic modulation will be discussed later.) The amplitude of the strain-induced electrostatic modulation extracted from this data is $V_0 = 0.10$ meV. Now that V_0 is determined, the values of B_0 for different φ settings can be extracted from these data. The results are plotted in the inset of Fig. 3, which clearly shows the expected $\cos \varphi$ dependence. The maximum amplitude of the magnetic field modulation at the 2DEG plane in this sample is found to be $B_0 = 19.1$ mT.

The peak-valley inversion of the Weiss oscillation can be also achieved by changing the gate bias. Figure 4 shows the magnetoresistance traces for different values of gate bias V_g ranging from -200 to 200 mV. For this series of measurements, the magnetic modulation is fixed at B=12 mT by setting $B_{\parallel}=5$ T at $\varphi=55^{\circ}$. The amplitude V_0 of the electrostatic potential modulation as determined from the analysis of the Weiss oscillation amplitude is plotted as a function of the applied gate bias V_g in the inset of this figure. The electrostatic modulation amplitude in the present system can be

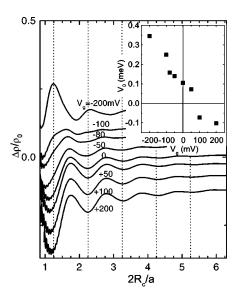


FIG. 4. Magnetoresistance as a function of $2R_c/a$ for different values of the gate bias V_g with the magnetic modulation amplitude fixed at $B_0=12$ mT by setting $B_{\parallel}=5T$ at $\varphi=55^{\circ}$. The traces are vertically shifted for clarity. The inset shows the change in the electrostatic potential modulation amplitude V_0 as a function of V_g . The V_0 at $V_g=0$ represents the strain-induced component.

approximately written as $V_0 \approx -1.2 \times 10^{-3} V_g + 0.10$ mV. The first term represents the gate-bias-induced component, which is roughly proportional to V_g . The coefficient 1.2×10^{-3} is the reduction factor that depends on the distance of the 2DEG from the patterned gate and on the details of screening. The value $V_0 = 0.10$ meV at $V_g = 0$ represents the strain-induced component. It is noted that the straininduced component in the present sample is about an order of magnitude smaller than those for samples with the modulation structure made along the [110] direction.^{13,18} Since the strain-induced potential via piezoelectric coupling is minimized in the present system, the one that plays the role here may be attributed to deformation potential coupling.¹⁵

We have thus demonstrated that a spatially modulated magnetic field can be imposed on the 2DEG at the GaAs/Al_xGa_{1-x}As heterointerface and that the modulation amplitude can be precisely determined from the analysis of the Weiss oscillation. It is then of great interest to explore the effect of such artificial superperiodicity on electronic transport. We learn in elementary solid state physics textbooks that electron-electron scattering does not contribute to the resistance in a translationally invariant system, but can play a role in a translational-symmetry-broken system. In the case of periodic symmetry breaking, the relevant process is called the umklapp process.

The present system allows us to break the translational symmetry in a controlled fashion so that it may provide an excellent test ground for this problem. This issue was recently addressed by Messica *et al.*²¹ for electrostatic modulation. They have measured the low-temperature resistance of a 2DEG for different gate bias settings and found an excess resistance with a quadratic temperature dependence, which is taken as a signature of the electron-electron scattering. In the present work, we have used magnetic field modulation as the artificial periodicity. In addition to the interest in the possible difference associated with the different types of

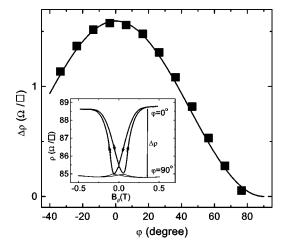


FIG. 5. Inset: resistivity as a function of B_{\parallel} , with $B_{\perp}=0$. The two traces correspond to $\varphi=0^{\circ}$ (maximum magnetic modulation) and 90° (null modulation). Main panel: excess resistivity $\Delta \rho$ as a function of the azimuthal angle φ of the parallel magnetic field. The observed $\cos^2 \varphi$ dependence verifies the relation $\Delta \rho \propto B_0^2$.

modulation (magnetic as opposed to electrostatic), there is also a good reason for the preference of magnetic over electrostatic modulation. Namely, use of magnetic modulation has a distinct advantage that it allows us to change the modulation amplitude without affecting the density of the 2DEG. This is a crucial point in conducting experiments because the electron-electron interaction is anticipated to be sensitive to the electron density.

The inset of Fig. 5 shows the resistivity as a function of the parallel magnetic field B_{\parallel} with $B_{\perp} = 0$. Two traces correspond to $\varphi = 0^{\circ}$ and 90°, respectively. Note that the magnetic field is aligned exactly parallel to the 2DEG plane, so that there is no uniform field component normal to the 2DEG. For $\varphi = 90^{\circ}$, the resistivity is found to be independent of the applied magnetic field except for a small peak in the low field region where the magnetization reversal occurs. As mentioned earlier, the magnetic field modulation vanishes for this field orientation, so that the observed fieldindependent resistivity can be taken as the reference value at zero modulation amplitude. The origin of the small peak can be interpreted as follows. During the magnetization reversal, small domains with different magnetization directions are formed and they produce a certain degree of magnetic field nonuniformity at the 2DEG plane that contributes to the carrier scattering. For $\varphi = 0^{\circ}$ at which the magnetic modulation can be maximized, a resistance change by about 5% is observed. The constant resistivity for B > 0.3 T gives the value under the maximum magnetic field modulation, i.e., B_0 = 19.1 mT as determined from the Weiss oscillation. We define the excess resistivity $\Delta \rho$ by the difference between the values for the two field orientations, as indicated in the figure.

The main panel of Fig. 5 shows the φ dependence of the excess resistivity $\Delta \rho$. As seen earlier in the inset of Fig. 2, the amplitude of magnetic field modulation changes as $B_0 \propto \cos \varphi$. Since the excess resistivity associated with the modulation should be proportional to the square of the transition matrix element, $\Delta \rho \propto B_0^2 \propto \cos^2 \varphi$ is expected. The data

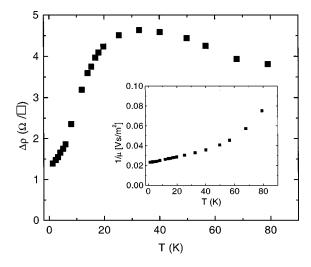


FIG. 6. Inset: temperature dependence of the inverse mobility μ^{-1} . Main panel: temperature dependence of the excess resistivity $\Delta \rho$ for $B_0 = 19.1$ mT.

shown here corroborate this dependence as demonstrated by the good fit of the $\cos^2\varphi$ curve.

Next we turn to the temperature dependence. The inset of Fig. 6 shows the temperature dependence of the inverse mobility μ^{-1} calculated from $\rho(T)$. The behavior at low temperatures is expressed as $\mu^{-1} = \mu_0^{-1} + \alpha T$. The *T*-linear term is attributed to acoustic phonon scattering with deformation potential coupling. For the present sample, $\alpha = 3.2 \times 10^{-4}$ V s/m² K, which is in agreement with those reported earlier for plain 2DEG samples with similar density and mobility.^{19,20}

The main panel of Fig. 6 shows the temperature dependence of the excess resistivity $\Delta \rho$ for $B_0 = 19.1$ mT. The excess resistivity increases with *T* at low temperatures, reaches a maximum at $T \approx 30$ K, and decreases with *T* at higher temperatures. The low-temperature part of the excess resistivity is found to increase with increasing temperature. The main panel of Fig. 7 shows the excess resistivity plotted against T^2 for four different φ settings. It is seen that for *T* <10 K, the excess resistivity can be expressed as $\Delta \rho$

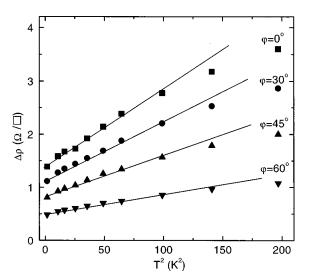


FIG. 7. Excess resistivity $\Delta \rho$ plotted against T^2 for four different values of φ , which set the magnetic modulation amplitude.

 $=AT^2+C$. It is stressed that the quantity $\Delta\rho$ only picks up the change induced by the application of the magnetic field modulation, while keeping the other parameters unchanged.²²

The temperature dependence of resistivity can be summarized as follows. The total resistivity is written as

$$\rho = \frac{m^*}{ne^2 \tau_{\text{total}}} = \frac{m^*}{ne^2} (\tau^{-1} + \tau_{\text{MM}}^{-1}). \tag{6}$$

Here τ^{-1} represents the electron scattering processes unrelated to the magnetic modulation and $\tau_{\rm MM}^{-1}$ those associated with the magnetic modulation.

For T < 30 K, $\tau^{-1} = 5.5 \times 10^{10} + 5.8 \times 10^8 T$ sec⁻¹. The first term represents the elastic scattering rate of the 2DEG including additional scattering associated with the non-magnetic (strain-induced) part of the effect of the striped gate. The second term represents the acoustic phonon scattering as stated above with possible minor modification due to the presence of the striped gate.

The scattering rate $\tau_{\rm MM}^{-1}$ in Eq. (6) represents the magnetic part of the effect of the striped gate. This term is proportional to B_0^2 , as shown in Fig. 5. For T < 10 K, it can be expressed as $\tau_{\rm MM}^{-1} = AT^2 + C$. The coefficient of the T^2 term is A = 1.1×10^7 K⁻² sec⁻¹ for the maximum magnetic modulation amplitude $B_0 = 19.1$ mT, or A = 3.0 $\times 10^4 B_0^2$ (mT) K⁻² sec⁻¹. The constant term is C = 1.0 $\times 10^9$ sec⁻¹ for $B_0 = 19.1$ mT. A similar T^2 -dependent excess resistance is recently reported by Overend *et al.*²³ for their device with cobalt stripes.

The AT^2 term represents the electron-electron scattering that contributes to momentum relaxation by lifting of the translational invariance by the magnetic modulation. The constant term C represents additional elastic scattering associated with the imposed magnetic modulation. It would be absent in an ideal case of perfect periodicity, but reflects the deviation therefrom in a real sample. We note that the small value of $\tau_{\rm MM}^{-1}$ as compared to τ^{-1} warrants that the modulation-induced effect can be treated in a perturbation scheme. This should make a detailed comparison between theory and experiment quite feasible. The AT^2 term is basically attributed to the electron-electron umklapp process in the presence of a periodic magnetic field. Considering the presence of the constant term C, however, the electronelectron scattering term may contain not only the umklapp process arising from the periodic magnetic modulation but also some contribution attributable to accompanying disorder.

An important point to address next is how this T^2 term changes at higher temperatures. As seen in Figs. 6 and 7, with increasing temperature $\Delta\rho$ deviates from the T^2 dependence, takes a broad maximum around $T \sim 30$ K, and eventually decreases at higher temperatures. One possible source of such behavior may be sought in the electron-electron scattering itself. Since the T^2 term is the lowest-order term of the electron-electron scattering, it is conceivable that the higherorder (T^4) term becomes important at higher temperatures. Unfortunately, to our knowledge, there exist no theoretical models that can give a quantitative estimate of the higherorder terms. Another possible source of the deviation from the T^2 dependence is the diminishing effectiveness of the imposed periodicity at high temperatures. For example, if the mean free path becomes much shorter than the period, the effect of imposed periodicity should disappear altogether. However, as seen in the inset of Fig. 6, a significant decrease in the mobility (and hence the mean free path) only occurs at high enough temperatures where the optical phonon scattering comes into play. The mean free path only changes from $\ell \approx 3.1$ to $\approx 2.2 \ \mu$ m between T = 1.3 and 30 K. Therefore, the decrease in the mean free path cannot explain the observed behavior of $\Delta \rho(T)$.

A magnetic field modulation with amplitude B_0 is seen as a potential energy modulation with amplitude $U_0 = eav_F B_0$ by an electron at the Fermi level.²⁴ When the temperature exceeds U_0 , the thermal smearing will wash out the effect of the artificial periodicity. Numerically, $U_0 = 1.8$ meV at B_0 = 19.1 mT for the present system, which is in a right order of magnitude. We note, however, an experimental fact that cannot be readily understood in this line of reasoning. According to this picture, the deviation from the T^2 dependence should occur at a lower temperature for a smaller B_0 . The data shown in Fig. 7 do not exhibit such behavior. Thus this point remains as an unsolved problem to be explored in the future.

IV. SUMMARY

We have demonstrated that the spatially modulated magnetic field can be imposed on a 2DEG at the GaAs/Al_xGa_{1-x}As heterointerface in a controlled fashion. Independent control of the field components parallel and perpendicular to the 2DEG plane allowed us to fix the magnetization of the striped ferromagnetic gate, while we measure the low (or zero) magnetic field transport in the 2DEG. The amplitudes of the magnetic and electrostatic modulations can be determined from the analysis of the Weiss oscillation.

The 2DEG under a modulated magnetic field with zero uniform component is of particular interest in the context of the combined effect of the electron-electron interaction and artificial periodicity. Here the use of a magnetic field modulation in the place of an ordinary electrostatic modulation has a distinct advantage in that the modulation amplitude can be varied without affecting the 2DEG density. The excess resistivity associated with the imposed magnetic modulation is found to vary as $AT^2 + C$ at low temperatures. The T^2 dependence is a signature of the electron-electron scattering. The present system offers an ideal ground for a quantitative comparison between theory and experiment for this fundamental issue of solid state physics. We hope that the present work may stimulate efforts on the theoretical side.

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- 22 In addition to the orbital (Lorentz force) effect considered here, the magnetic field modulation also produces a potential modulation via the Zeeman effect. However, it is quite negligible in the present field range because of the small g value of GaAs.
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- ²⁴ This can be understood by recalling that the Lorentz force exerted by the modulated magnetic field is equivalent to an effective electric field modulation with amplitude $v_F B_0$.