# Role of inhomogeneous widening in radiofrequency coherent superradiation from highly polarized spin systems

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We discuss the coherent behavior of a highly polarized, nuclear or electron, spin system for which the magnetic dipole radiation, emitted in the radio-frequency region, has a quadratic dependence on the number of spins. Phenomenological theories are unable to completely predict many of the features of low-temperature spin systems. Computer simulations provide a very effective and accurate method of describing such systems. Our investigation is based on a microscopic model, which makes it possible to accurately analyze the role of dipole interactions and inhomogeneous widening. Inhomogeneous widening complicates the behavior of the system leading to a longer delay in the appearance of the superradiation pulse and a decrease in its intensity. If the inhomogeneity is large enough, it destroys the coherence in a spin system at times shorter than the characteristic dipole relaxation time. [S0163-1829(98)03622-4]

## INTRODUCTION

Superradiation phenomena are characterized by a quadratic dependence of the radiation intensity on the number of radiators $N$. The properties of such coherent effects in optics, for example, superfluorescence, superluminescence, collective induction, and photon echo, have been extensively studied. A detailed description of them may be found in many books as well as more recent studies.[1] An effect similar to optical superfluorescence but in the radio-frequency region is much less well known. It is this superradiation (SR) phenomenon that is the topic of this study.

Although this phenomenon is very similar to optical superfluorescence, the physics governing SR in nuclear or electron spin systems is very different, thus requiring a completely different mathematical approach.[2] From a physical standpoint the main difference between the present and optical cases is that the radio-frequency wavelength is much longer than the size of the system so that propagation effects play no role. A theoretical investigation of SR for a system of nuclear spins using this technique was performed by Blombergen and Pound.[3] Their results indicated that it might be experimentally possible to obtain a measurable SR signal and described the important features of the process. However, it was not until 1977 that a successful experiment to observe radio-frequency superradiation was performed.[4]

It is well known in magnetic resonance theory that, despite the fact that the Bloch equations (involving a phenomenological relaxation time) are very useful, they lead to certain incorrect predictions for low-temperature highly polarized spin systems. The SR phenomenon can only be observed in such spin systems. In Refs. 5 and 6 the computer modeling of SR from a homogeneous spin system based on a microscopic description is described. This analysis allowed the investigation of the role of dipole interactions, but it is known that in real systems inhomogeneous widening due to irregularities in the spin system can be at least as important as the homogeneous widening caused by dipole interactions.

Our analysis takes into account two types of spin damping: transverse damping due to dipole-dipole interactions and damping due to inhomogeneous widening. Longitudinal damping due to spin-lattice interactions plays no role on the time scale for which SR is observed. At very high polarizations the spin-lattice relaxation time ranges from minutes to hours, whereas the spin relaxation time is a few orders of magnitude shorter.[2] In this article we present the results of an investigation into the role of homogeneous widening in the phenomenon of SR.

## I. MICROSCOPIC SPIN MODEL

Consider a system of $N$ spins whose sites in real space are enumerated by the index $i=1,2,\dots,N$. The spin operator $\vec{S}_i$ corresponds to the effective spin $S=1/2$ of the paramagnetic ions (or nuclei) at the nodes of a regular rigid simple cubic lattice. The dipole interaction of spins has the form

$$\mathcal{H}_{ij}=\frac{\vec{\mu}_i\vec{\mu}_j}{r_{ij}^3}-\frac{3(\vec{\mu}_i\cdot\vec{r}_{ij})(\vec{\mu}_j\cdot\vec{r}_{ij})}{r_{ij}^5}, \tag{1}$$

where the magnetic moments are $\vec{\mu}_i=\mu\vec{S}_i$, $\mu=\gamma\hbar$, $\vec{r}_{ij}=\vec{r}_i-\vec{r}_j$, $r_{ij}=|\vec{r}_{ij}|$ and $\gamma$ is the gyromagnetic ratio ($\gamma>0$ for protons, $\gamma<0$ for electrons).

The system is under the influence of a constant external magnetic field $\vec{H}_0$. If there is a passive electric resonant circuit that is tuned to the Zeeman frequency $\omega_0=\gamma H_0$, the system ''feels'' an additional magnetic field $\vec{H}_{res}$ induced by

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the rotating spins of the system. The coupling with the circuit produces a coherent pulse in the spin system. This problem was investigated in Ref. 3 using the Bloch equations with a phenomenological ''dipole'' relaxation time $T_2$. With the coil axis along $\vec{e}_x$, the field induced by the resonance circuit is given by

$$\vec{H}_{res} = -\frac{2g}{\gamma N}\,\vec{e}_x \sum_i \dot{S}_i^x = H_{res}\vec{e}_x. \tag{2}$$

In this equation $g$ defines the strength of the coupling between the spin system and the resonance circuit $g = 2\pi\eta\gamma\rho\mu Q/\omega_0$; where $Q = \omega_0 L/R$ is the quality factor of the circuit, $\omega_0 = (LC)^{-1/2}$ is the natural frequency of the circuit, $\eta \simeq V/V_c$, with $V$ the sample volume and $V_c$ the coil volume, and $\rho = N/V$ is the density of the system.

If an alternating magnetic field $\vec{H}_1(t)\cos(\omega t)$ along the $x$ axis is also applied to the system, as in the case of magnetic resonance, the total Hamiltonian has the form

$$\hat{\mathcal{H}} = -\mu H_0 \sum_i S_i^z - \frac{1}{2}\mu H_x \sum_i (S_i^+ + S_i^-) + \hat{\mathcal{H}}_d + \hat{\mathcal{H}}_\Delta, \tag{3}$$

where $H_x = H_1(t)\cos(\omega t) + H_{res}$ and $\hat{\mathcal{H}}_d$ is the dipole-interaction part. The term $\hat{\mathcal{H}}_\Delta$ describes the inhomogeneous widening, for example, due to the variations of the magnetic $g$ factors of the effective spins. This term is usually written in the form

$$\hat{\mathcal{H}}_\Delta = -\mu \sum_i \Delta_i \vec{S}_i^z, \tag{4}$$

where $\Delta_i$ is the corresponding random inhomogeneous field at the $i$th spin. Using the mean-field approach, the effective dipole Hamiltonian is given by

$$\hat{\mathcal{H}}_d = \frac{1}{2}\sum_{i\neq j} \{a_{ij}(S_i^z\langle S_j^z\rangle - \tfrac{1}{2} S_i^+\langle S_j^-\rangle) + 2c_{ij}S_i^+\langle S_j^z\rangle + 2c_{ij}^* S_i^z\langle S_j^-\rangle + e_{ij}S_i^+\langle S_j^+\rangle + e_{ij}^* S_i^-\langle S_j^-\rangle\}, \tag{5}$$

where the angular brackets represent the statistical average, corresponding to mean polarization of the spin system, which corresponds to its temperature.[7] The coefficients $a_{ij}, c_{ij}, e_{ij}$ are given by

$$a_{ij} = \frac{\gamma^2\hbar^2}{r_{ij}^3}(1 - 3\cos^2\theta_{ij}),$$

$$c_{ij} = -\frac{3\gamma^2\hbar^2}{4r_{ij}^3}\sin 2\theta_{ij}\,\exp(-i\phi_{ij}), \tag{6}$$

$$e_{ij} = -\frac{3\gamma^2\hbar^2}{4r_{ij}^3}\sin^2\theta_{ij}\,\exp(-2i\phi_{ij}).$$

The usual way to proceed in magnetic resonance applications is by the use of perturbation theory in the small quantity $|H_{loc}/H_0|$,[2] where $H_{loc}$ represents the local magnetic fields. However, coherent effects are described by essentially nonlinear equations for which this method is not applicable. The equations of motion are to be written using the total Hamiltonian (5) rather than only its secular part as in perturbation theory.

With the notation $\omega_0 = \gamma H_0$, $\omega_1 = \gamma H_1$, and $\omega_i = \gamma\Delta_i$ the Heisenberg equations of motion for the spin operators are

$$i\frac{dS_i^z}{dt} = \frac{\omega_1}{2}(S_i^- - S_i^+)\cos\omega t - \frac{g}{2N}(S_i^- - S_i^+)\frac{d}{dt}\sum_j (S_j^- + S_j^+) + \frac{1}{\hbar}\sum_{j(\neq i)}\left\{\frac{a_{ij}}{4}(S_i^- S_j^+ - S_i^+ S_j^-) + (c_{ij}S_i^+ - c_{ij}^* S_i^-)S_j^z + e_{ij}S_i^+ S_j^+ - e_{ij}^* S_i^- S_j^-\right\}, \tag{7}$$

$$i\frac{dS_i^-}{dt} = -(\omega_0 + \omega_i)S_i^- + \omega_1 S_i^z\cos\omega t - \frac{g}{N}S_i^z\frac{d}{dt}\sum_j (S_j^- + S_j^+) + \frac{1}{\hbar}\sum_{j(\neq i)}\left\{\frac{a_{ij}}{2}(S_i^z S_j^- + 2S_i^- S_j^z) + c_{ij}(S_i^- S_j^+ - 2S_i^z S_j^z) + c_{ij}^* S_i^- S_j^- - 2e_{ij}S_i^z S_j^+\right\}, \tag{8}$$

and the conjugate equation to Eq. (8). It is not feasible to diagonalize the quantum Hamiltonian for spin systems of a size necessary for this type of modeling. Numerical simulations for ''classical'' spin systems have been used by many authors to investigate such topics as magnetic resonance line shapes and free induction decay,[8] spin diffusion[9] and spin glasses (see Ref. 10 and references therein). In this case the solution is obtained from $3N$ differential equations describing the classical spins. In those situations where a quantum phenomenological approach is reasonable, such as the method of moments or the truncated dipolar interaction approximation, the results of these methods are qualitatively similar to the classical methodology and both are in agreement with experiment. There is particularly good agreement over the time scales involved in this study.

Using this method the spins are considered as classical vectors whose initial distribution, defining the initial conditions for the differential equations, is determined by a Monte Carlo technique similar to the procedure of Metropolis *et al.*[11] in the thermodynamic averaging over a canonical ensemble at a given temperature. A random configuration of spins $\{\vec{S}_i\}_{i=1}^N$ is taken as the first member of a Gibbs ensemble of spin arrays and its polarization $p_{init}$ is evaluated. A new, random direction is chosen for an arbitrary spin and the new total polarization $p'$ is calculated. If $\Delta p = |p' - p_0|$ is less than $\Delta p_{init} = |p_{init} - p_0|$ (where $p_0$ is the desired initial polarization of the system) then the array with the changed spin is chosen as the second member of the Gibbs ensemble; otherwise, it is rejected. This procedure is then repeated to obtain the third member of the ensemble and so on until we obtain an array whose polarization is very close to $p_0$. This array is taken as the initial distribution of our system.

The differential equations describing the classical spin vectors are solved numerically via the Runge-Kutta method. The equations are made dimensionless by means of the factor $\omega_L = T_2^{-1} = a^3/\gamma^2\hbar^2$, where $a$ is the lattice constant and we use the notation $T_2$ to facilitate comparison with the dipole spin-spin relaxation time in the phenomenological equations. The summations in Eqs. (7) and (8) are limited to the $N-1$ nearest spins.

To take into account inhomogeneous widening the set of frequencies $\{\omega_i\}_{i=1}^{N}$ must be modeled. Their random nature requires a statistical methodology. $\omega_i$ is modeled using a normally distributed random value with zero average and dispersion $\sigma$. The frequencies $\omega_i$ are expressed in units of $\omega_L$ and the dispersion $\sigma$ in realistic situations varies from zero to a few $\omega_L$. In our simulations $\omega_0$ is in the range $10^2-10^3\omega_L$ and $\omega_1$ is of order $\omega_L$, corresponding to actual values in magnetic resonance experiments.

In order to ensure that each spin in our cubic lattice has the same number of neighbors we adhere to the common practice in the modeling of paramagnetic materials[12] of requiring that the system obey periodic boundary conditions. For a cube of side $L$, centered at the origin, with vectors $\vec{L}_1$, $\vec{L}_2$, and $\vec{L}_3$ defined by $\vec{L}_1=(L,0,0)$, $\vec{L}_2=(0,L,0)$, and $\vec{L}_3=(0,0,L)$ the boundary conditions are given by

$$\vec{S}(\vec{x})=\vec{S}(\vec{x}\pm\vec{L}_\alpha),\quad \alpha=1,2,3,$$

for any point $\vec{x}$. The results of the modeling of spin systems over short time periods is known to be insensitive to the number of spins,[8–10] thus reducing the possible effect of requiring periodic boundary conditions. For our analysis the validity of using periodic boundary conditions was verified by observation of only minimal differences in the shape of the curves in Figs. 2–6 for spin systems of 125 and 343 nodes.

## II. RESULTS OF CALCULATIONS

In the phenomenon of SR a coherent addition of radiating magnetic moments creates a macroscopic magnetic moment proportional to the number of spins. The charactersitic time in which this takes place is called the delay time. Coherence is created by the common field in the resonator or the field of the external pumping.

We have calculated the time dependence of the main characteristics: radiation intensity, absorbed power, coherence coefficients, and $z$ component of the average spin. The intensity of magnetodipole radiation is

$$I=\frac{2}{3c^3}|\ddot{\vec{M}}|^2, \tag{9}$$

where $\vec{M}=\mu\Sigma_i\langle\vec{S}_i\rangle$ is the total magnetic moment of the system. The intensity (9) can be presented as a sum of incoherent and coherent terms

$$I=I_{inc}+I_{coh}=\frac{2\mu^2}{3c^3}\sum_i |\langle\ddot{\vec{S}}_i\rangle|^2+\frac{2\mu^2}{3c^3}\sum_{i\neq j}\langle\ddot{\vec{S}}_i\ddot{\vec{S}}_j\rangle. \tag{10}$$

The latter quantity is similar to the sum of off-diagonal elements of the density matrix, essential for the coherent states. The relative contribution of coherent radiation to the total intensity can be characterized by the coherence coefficient $C_{coh}=I_{coh}/I_{inc}$.

The other observable quantity describing the process of radiation is the power absorbed by the coil $P=\langle U_{ind}^2\rangle/2R$, in which $U_{ind}$ is the voltage induced in the coil and $R$ is the resistance of the circuit. In our situation this power is given by the sum of incoherent and coherent terms

$$P=P_{inc}+P_{coh}=\frac{g\hbar}{N}\sum_i\left(\mathrm{Re}\left\langle\frac{dS_i^-}{dt}\right\rangle\right)^2+\frac{g\hbar}{N}\sum_{i\neq j}\mathrm{Re}\left\langle\frac{dS_i^-}{dt}\right\rangle\mathrm{Re}\left\langle\frac{dS_j^-}{dt}\right\rangle. \tag{11}$$

The absorbed power $P$ contains an additional factor $Q\lambda^3/V$ and is always larger than the radiation intensity $I$. We have also calculated the $z$ component of the mean spin

$$p_z(t)=\sum_{i=1}^{N}\langle S_i^z\rangle/N. \tag{12}$$

To illustrate the influence of inhomogeneous widening, we have performed calculations with $\sigma=0$ (no widening, $\hat{\mathcal{H}}_\Delta=0$) and with different values of $\sigma$. All quantities are given in dimensionless units. The frequencies $\omega_0,\omega_1,\omega_i$, and $\omega$ are measured in units of $\omega_L=T_2^{-1}$ and time is measured in units of $\omega_L^{-1}$, so that $T_2=1$. The radiation intensity is presented in units of $2\omega_L^4\mu^2/3c^3$, becoming $I(t)=\Sigma_{i,j}\ddot{\vec{S}}_i\ddot{\vec{S}}_j$. In such units we use $\omega_0\sim10^2-10^3$, $\omega_1\sim1$, and $0<g<0.1$, reflecting the usual ratios in magnetic resonance and SR experiments.

Initially our spin system is created in a state characterized by the large polarization of spins along the $z$ axis. Due to the coupling with a passive coil or external resonance pumping, the transverse spin components gradually begin to grow. The speed of this process reaches a maximum at $t=t_0$ when $p_z(t)$ is changing most rapidly. The phenomenon of superradiation, in which the maximum value of $I$ or $P$ is proportional to $N^2$, is observed to take place at $t\sim t_0$. At this time the coherence is proportional to $N$ and, assuming that the initial polarization $p_z^{(0)}$ is large enough, the spins in the system are moving coherently. In order to show that the radiation intensity is indeed proportional to the square of the number of spins Fig. 1(a) includes results for $N=27$, 125, and 343, in the case of the presence of a resonance circuit. In each case we use $\omega_0=200.0$, $\omega_1=\omega=0.0$ (no external pumping), $g=0.0005$, and $p_z^{(0)}=0.475$. The ratio of the maximum values of the $I$ is found to be approximately $27^2{:}125^2{:}343^2$, as expected. From Fig. 1(a) it can also be seen that the time at which the maximum radiation intensity is obtained is approximately proportional to $1/N$. It is the $1/N$ dependence that leads to the $N^2$ superradiation phenomenon. It is important to realize that the radiation at its maximum value is coherent despite the fact that the initial spins had different phase displacements. Figure 1(b) shows clearly the linear dependence of the coherence coefficient $C_{coh}$ on the number of spins $N$.

As indicated in Sec. I, we use a statistical methodology to take into account the effect of inhomogeneous widening. When $\sigma=0$ the $\omega_i$ for each spin is zero. If $\sigma=1.0$ then the $\omega_i$ are assigned such that the distribution of the $\omega_i$ is Gauss-

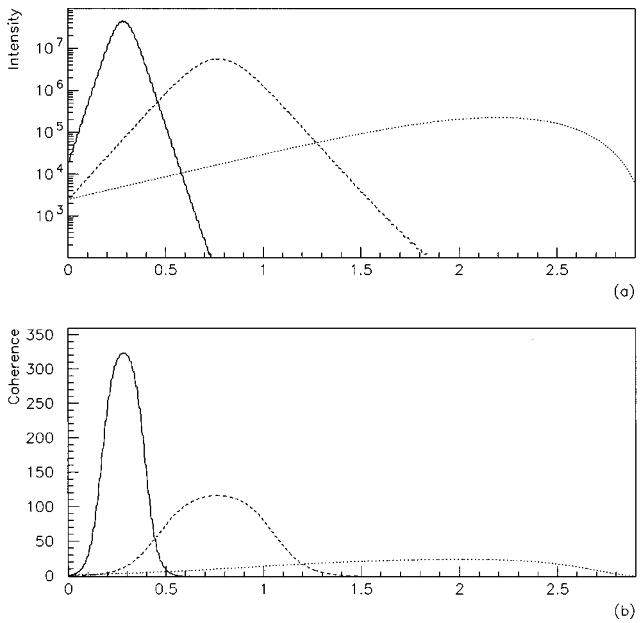


FIG. 1. Radiation intensity $I$ and coherence coefficient $C_{coh}$ as functions of time in the case of the spin system coupled with a resonance circuit for the parameters $p_z^{(0)}=0.475$, $\omega_0=200$, $g=0.0005$, $\omega_1=\omega=0$, and no inhomogeneous widening ($\sigma=0.0$). The solid line is for $N=343$, the dashed line for $N=125$, and the dotted line for $N=27$.

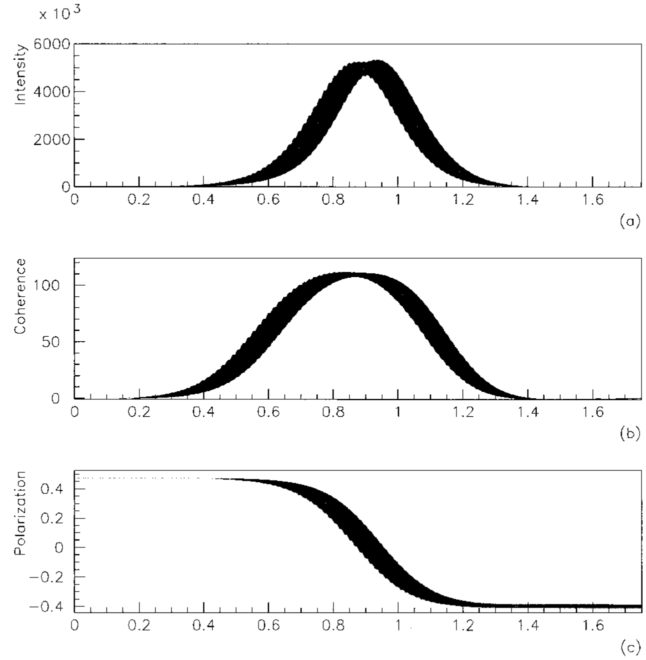


FIG. 2. Radiation intensity $I$, coherence coefficient $C_{coh}$, and $z$ projection of polarization $p_z$ as functions of time in the case of the spin system coupled with a resonance circuit for the parameters $N=125$, $p_z^{(0)}=0.475$, $\omega_0=200$, $g=0.0005$, and $\omega_1=\omega=0$. The inhomogeneous widening parameter $\sigma=1.0$ and the ''bands'' correspond to the broadening caused by different initial assignments of the frequencies $\omega_i$.

ian with zero average and dispersion (standard deviation) equal to 1.0. There is an infinite number of ways in which the assignments of $\omega_i$ can be made for each spin in order to reproduce the required Gaussian distribution for $\omega_i$. Each different set of $\omega_i$ assignments will produce a unique spin system with a unique time evolution. In Figs. 2(a), 2(b), and 2(c) we have chosen to represent this effect for $\sigma=1.0$ ($\omega_0=200.0$, $\omega_1=\omega=0.0$, $g=0.0005$, and $p_z^{(0)}=0.475$) by means of broadening the curves for intensity, intensity coherence coefficient, and the $z$ component of the mean spin, respectively. Each curve is centered on the average over multiple sets of $\omega_i$, the width of the ''band'' corresponding to twice the standard deviation of the set of values at each $t$. Similar ''bandwidths'' are obtained for different values of $\sigma$.

When considering the effect of different magnitudes of inhomogeneous widening (different $\sigma$'s) the fact that each value of $\sigma$ is represented by a band, rather than a single curve must be taken into account. While this phenomenon reduces our ''resolution'' in differentiating small changes in intensity, power, coherence, and $p_z$ it does not alter the overall conclusions of this work. In Figs. 3–6, we represent results for different $\sigma$'s by single curves rather than bands. This is done in order not to overly complicate the figures with multiple bands. However, in every case there is an underlying band structure with characteristic width indicated in Fig. 2.

Figures 3 and 4 display the evolution of the intensity, intensity coherence coefficient, and $z$ component of the mean spin of the system for various values of $\sigma$ and for two different values of the coupling constant $g$. In each case $N=125$, $\omega_0=200.0$, $\omega_1=\omega=0.0$, and $p_z^{(0)}=0.475$. For $g=0.0005$ (Fig. 3) the time at which the maximum value of the intensity and coherence occurs $t_0$ is less than $T_2(=1.0)$, except for $\sigma=5.0$. When $g=0.00025$ (Fig. 4) $t_0$ is larger than $T_2$. Since $T_2$ represents the dipole relaxation time it is clear that at times significantly smaller than $T_2$ dipole interactions will have only a small effect on the behavior of the

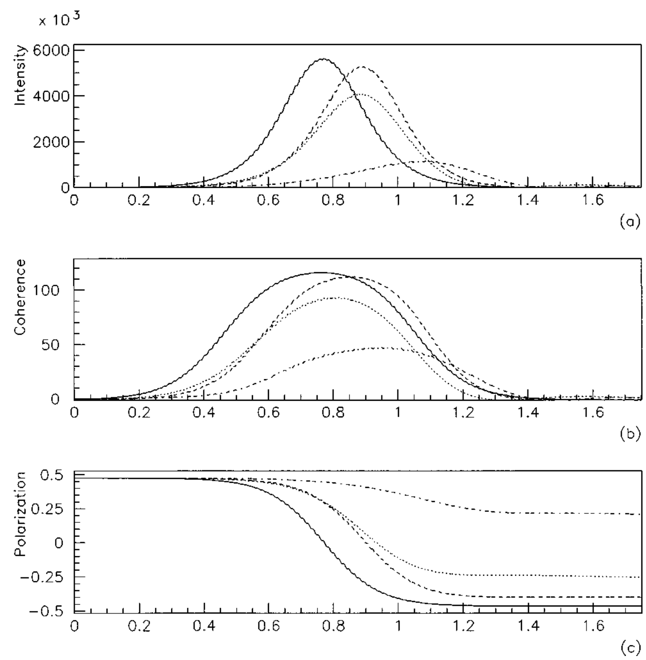


FIG. 3. Radiation intensity $I$, coherence coefficient $C_{coh}$, and $z$ projection of polarization $p_z$ as functions of time under the same conditions as in Fig. 2. The solid line is for $\sigma=0$, the dashed line for $\sigma=1.0$, the dotted line for $\sigma=2.0$, and the dot-dashed line for $\sigma=5.0$.

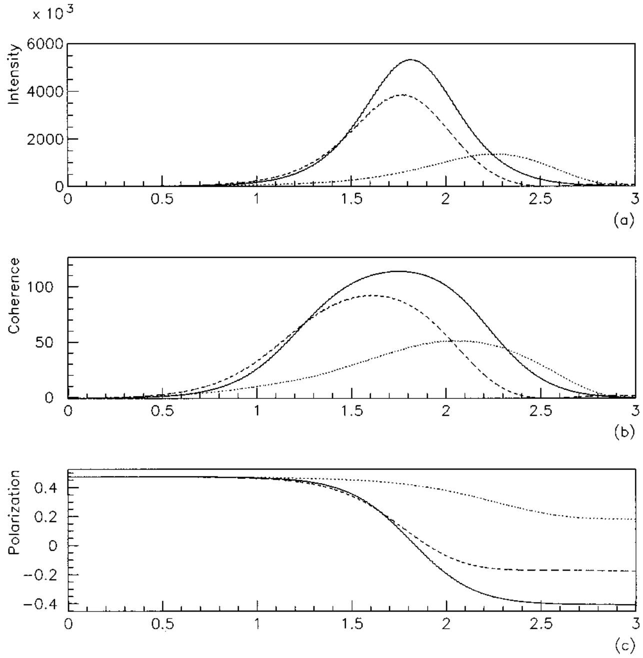


FIG. 4. Radiation intensity $I$, coherence coefficient $C_{coh}$, and $z$ projection of polarization $p_z$ as functions of time under the same conditions as in Fig. 2, except $g=0.000\,25$. The solid line is for $\sigma=0$, the dashed line for $\sigma=1.0$, and the dotted line for $\sigma=2.0$.

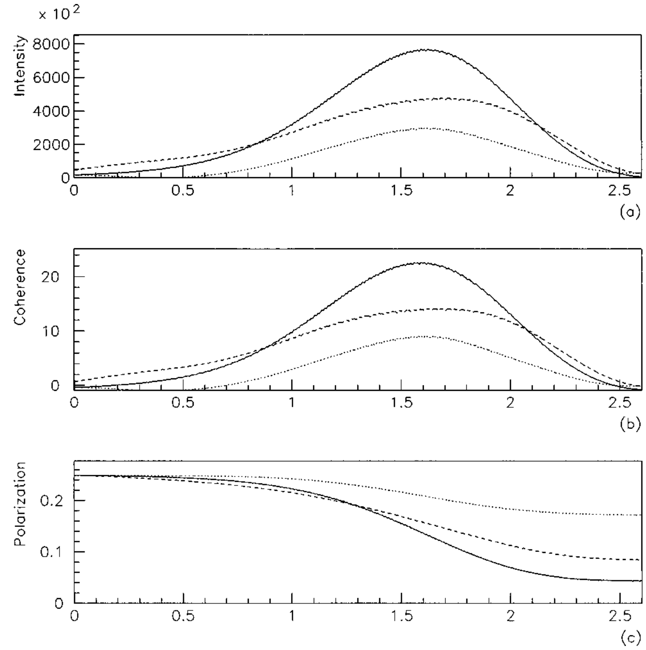


FIG. 5. Radiation intensity $I$, coherence coefficient $C_{coh}$, and $z$ projection of polarization $p_z$ as functions of time under the same conditions as Fig. 4, except $p_z^{(0)}=0.25$. The solid line is for $\sigma=0$, the dashed line for $\sigma=1.0$, and the dotted line for $\sigma=2.0$.

system. The effect of the inhomogeneous widening is also small. The results presented in Figs. 3 and 4 are consistent with this conclusion. With $g=0.0005$ [Fig. 3(a)] when $\sigma=1.0$ the intensity relative to $\sigma=0.0$ is reduced by no more than 5%. Due to the necessity to describe each value of $\sigma$ by a band rather than a single curve, such a reduction is not statistically significant. However, with $g=0.00025$ [Fig. 4(a)] the $\sigma=1.0$ intensity is reduced by at least 30% relative to $\sigma=0.0$, a reduction much larger than the $\sigma=1.0$ bandwidth. Similarly, a reduction of the intensity to about 20% of the $\sigma=0.0$ value requires $\sigma=5.0$ for $g=0.0005$, but only $\sigma=2.0$ for $g=0.00025$. The same qualitative behavior is observed in the intensity coherence coefficient [Figs. 3(b) and 4(b)]. The effect of increasing $\sigma$ on the time dependence of the $z$ component of the spin of the system [Figs. 3(c) and 4(c)] is to inhibit the spin inversion observed when $\sigma=0.0$. It is also worth noting that, even allowing for the bandwidth at each $\sigma$, the time before the appearance of the SR pulse $t_0$ gradually increases with increasing $\sigma$. If $t_0$ is smaller than $T_2$ and the widening is not large, the final $p_z \approx -p_z(0)$, corresponding to the conservation of the length of the Bloch vector.[3]

When the initial polarization $p_z^{(0)}$ is smaller (0.25), with $g=0.00025$ and all other parameters the same as in Figs. 3 and 4, we observe a broadening and reduction in the intensity and coherence of the superradiation pulse [Figs. 5(a) and 5(b)]. Even when $\sigma=0.0$ the spin system does not undergo spin inversion [Fig. 5(c)]. The introduction of inhomogeneous widening (nonzero $\sigma$) leads to results qualitatively similar to the $p_z^{(0)}=0.475$ case.

Consider now a spin system without a passive resonance electric circuit $g=0$. In this case the coherence can be caused by external pumping as illustrated in Fig. 6. To produce coherence in the spin system the pumping must be strong and, more importantly, resonant ($\omega=\omega_0$). In Fig. 6, $\omega_0=\omega=200.0$, $\omega_1=5.0$,and $p_z^{(0)}=0.475$ for four different values of the inhomogeneous widening. When $\sigma=0$, $\omega_1=5.0$ is large enough to almost completely invert $p_z$ after a time $\sim T_2$. The inclusion of inhomogeneous widening can

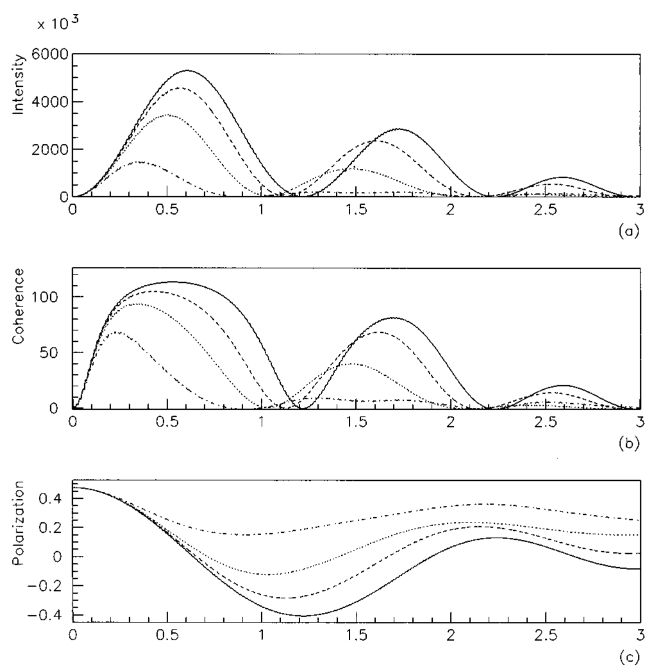


FIG. 6. Radiation intensity $I$, coherence coefficient $C_{coh}$, and $z$ projection of polarization $p_z$ as functions of time in the case of the spin system without a resonator ($g=0.0$), for the parameters $N=125$, $\omega_1=5.0$, $p_z^{(0)}=0.475$, $\omega_0=\omega=200$, and $g=0.000\,25$. The solid line is for $\sigma=0$, the dashed line for $\sigma=1.0$, the dotted line for $\sigma=2.0$, and the dot-dashed line for $\sigma=5.0$.

be seen to significantly reduce the magnitude of the oscillations in intensity, coherence, and $p_z$. In fact, when $\sigma = 0.5$, the external pumping is unable to invert the polarization $p_z$.

The time evolution of the absorbed power has also been investigated for the situations described in Figs. 3–6. In each case the power behaves in a manner almost identical to the intensity as we vary the widening parameter $\sigma$.

## CONCLUSION

In this paper we have presented the results of computer simulations of the coherent behavior of a polarized spin system for which the magnetic dipole radiation has a quadratic dependence on the number of spins. The simulation is based on a microscopic model, which makes it possible to accurately analyze the role of dipole interactions and inhomogeneous widening. The latter effect cannot be ignored in most physical situations and, as we have demonstrated, plays an important role in the phenomenon of superradiation. If the widening is large enough the coherence in a spin system is destroyed after a time of order $T_2$.

For many of the problems of spin systems at low temperatures, computer simulations provide the only adequate method of investigation. Phenomenological models based on the Bloch-type equations have played an outstanding role in the understanding of coherent processes. However, it is well known[2] that at low temperatures such equations can lead to certain incorrect predictions. In the SR process, for example, such models always lead to a delay time less than $T_2$. Other models that reduce very complicated multiparticle spin systems to a sort of single-particle system, due to their many simplifications, lead to essentially a phenomenological description. These models do not allow the possibility of investigating the time evolution dependence on initial polarization, initial spin distribution, parameters of homogeneous and inhomogeneous widenings, etc., and can even lead to incorrect conclusions regarding the role of dipole interactions at times corresponding to the beginning of SR. Further, these models must produce results that are consistent with microscopic simulations.

The situation becomes even more complicated when describing a disordered system having random, irregular, distributions of spins in the lattice. This problem, which we intend to discuss in a forthcoming paper, is impossible to formulate in the framework of any single-particle approximation. In this case the only possible avenue of investigation is the microscopic approach, via computer simulation, described in this article.

## ACKNOWLEDGMENT

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