Josephson plasma resonance in superconducting multilayers

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We derive an analytical solution for the Josephson plasma resonance of superconducting multilayers. This analytical solution is derived mainly for low-T_c systems with magnetic coupling between the superconducting layers, but many features of our results are more general, and thus an application to the recently derived plasma resonance phenomena for high-T_c superconductors of the $Bi_2Sr_2CaCu_2O_x$ type is discussed. Our approach allows us to give full details of the different plasma resonance excitations, and we also predict the existence of new nonlinear effects, so far only identified in single junctions.
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I. INTRODUCTION

The phenomenon of plasma resonance has been known in low-T_c single junctions for many years.[1,2] The measurement of this resonance in low-T_c junctions has been an important diagnostic tool leading, for example, to the verification of the so-called cos φ term.[3] The plasma resonance has also been utilized in the construction of parametric amplifiers.[4]

In this paper we extend the theory of the plasma resonance from the single Josephson junction to a stack of long Josephson junctions. The motivation for this is the considerable interest in the Josephson stacks that has been seen recently. An important step for the theoretical understanding of Josephson stacks was a theoretical model published by Sakai, Bodin, and Pedersen (SBP) in which the system is represented by a set of coupled sine-Gordon equations including losses and biases.[5] The most interesting case occurs when the thickness of the superconducting layer is smaller than the London penetration depth, such that a strong inductive coupling is expected. For low-T_c Nb AlO_x stacks the SBP model accounted very well for experimental observation works of the flux-flow mode and Fiske-like resonances.[6–8]

As was pointed out by SBP (Ref. 5), the inductively coupled model may also be applicable to stacked intrinsic high-T_c Josephson junctions of the $Bi_2Sr_2CaCu_2O_x$ (BSCCO) type. In fact, equivalent theoretical models using different notations appeared lately,[9,10] and were used for the interpretation of measurements on stacks of BSCCO intrinsic Josephson junctions. The observed overall features depending on the applied magnetic field, the flux flow velocities, and the microwave emission are excellently explained by the SBP or equivalent models.[11,12]

Recently the plasma resonance was ''reinvented'' in connection with layered high-T_c superconductors of the BSCCO type. Microwave absorption experiments[13–15] have been reported, and a model considering a charging effect in the superconducting layers in high-T_c intrinsic stacks has been recently proposed by Tachiki and his group.[16] This model results in a modified second Josephson relation, where the time derivative of the phase difference of a junction is represented not only by the voltage of this junction but also by the voltages of adjacent junctions.[16] They propose a longitudinal plasma resonance (a collective excitation transverse to the layers) due to this coupling mechanism.

In the SBP model, the coupling in the longitudinal direction occurs inductively through the screening current for the magnetic field. In this paper, we clarify which kind of plasma resonance may appear within the range of applicability of the SBP model. We demonstrate the existence of a variety of plasma resonance excitations, and derive the corresponding equations, which in some cases are an extension of the well-known single junction case.

The paper is organized in the following way. Section II contains the derivation of the plasma resonance for an N stack. In Sec. III possible resonance modes are discussed, and a comparison is given between our results and the results derived from other models. Section IV discusses other nonlinear phenomena connected with the plasma resonance. Our results are summarized in Sec. V.

II. DERIVATION OF THE PLASMA RESONANCE IN SUPERCONDUCTING MULTILAYERS

For a single small area Josephson junction with the equivalent diagram shown in Fig. 1, the derivation of the plasma resonance was made in Refs. 1–3. Kirchhoff's law for the equivalent circuit in Fig. 1 leads to

$$C_J dV/dt + V/R + i_0 \sin\varphi = i_{dc} + i_{rf} \sin \omega t \tag{1}$$

with $\partial\varphi/\partial t = (2e/\hbar)V$, where all variables and parameters have their standard meaning. Assuming that $i_{dc} < i_0$ and i_{rf}/i_0 are small, we may assume $\varphi = \varphi_0 + \varphi_1$, with $\sin \varphi_0 = i_{dc}/i_0$ and $|\varphi_1| \ll 1$. After some algebra we find[1–3,17]

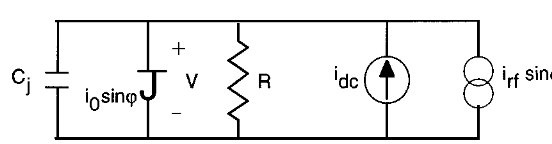

FIG. 1. Equivalent circuit for a single Josephson junction with dc and rf bias current sources.

$$\varphi_1 = \frac{2ei_{\mathrm{rf}}}{\hbar C_J\sqrt{(\omega^2-\omega_{pS}^2)^2+(\omega/C_JR)^2}}\sin(\omega t+\Theta) \quad (2)$$

and the rf-voltage amplitude V_{rf} is given by

$$V_{\mathrm{rf}} = (\hbar\omega/2e)\varphi_1 . \quad (3)$$

The plasma frequency ω_{pS} in Eq. (2) is given by

$$\omega_{pS} = \sqrt{2ei_0\cos\,\varphi_0/\hbar C_J}. \quad (4)$$

The Q of the plasma resonance is given by $Q=\omega_{pS}C_JR$. We note that the plasma oscillation is a longitudinal oscillation of Cooper pairs across the barrier. Alternatively we may describe it as an inductance-and-capacitance resonance between the Josephson inductance $L_J=\hbar/(2ei_0\cos\varphi_0)$ and the capacitance C_J. It is typically in the microwave frequency range.

For an N-layer stack of long Josephson junctions as shown in Fig. 2(a), we will use the equations derived by Sakai *et al.* in Refs. 5 and 6. We will also adopt the notation from these papers. Figure 2(b) shows details of the superconducting layers i and $i-1$. To clarify the problem and avoid confusion, the coupled equations to be solved are written here again:

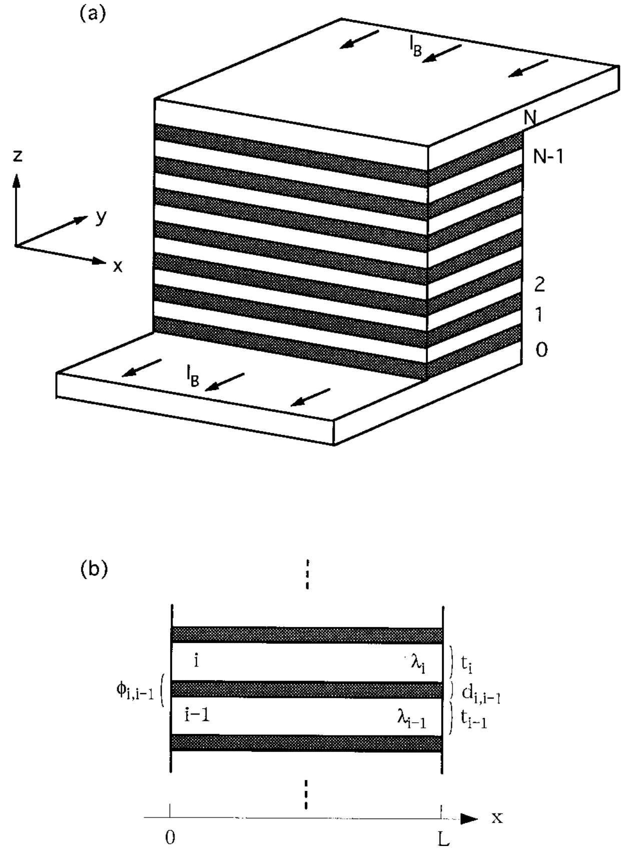

FIG. 2. (a) Schematic picture of a vertically N-stacked Josephson junction. (b) Enlarged diagram for showing the parameter notation in this paper.

$$\frac{\hbar}{2e\mu_0}\partial_{xx}\begin{bmatrix}\phi_{1,0}\\ \vdots\\ \vdots\\ \phi_{i,i-1}\\ \vdots\\ \vdots\\ \phi_{N,N-1}\end{bmatrix}=\begin{bmatrix}d'_{1,0} & s_1 & & & & & \\ s_1 & d'_{2,1} & s_2 & & 0 & & \\ & \ddots & \ddots & \ddots & & & \\ & & s_{i-1} & d'_{i,i-1} & s_i & & \\ & & & \ddots & \ddots & \ddots & \\ & 0 & & & \ddots & \ddots & s_{N-1}\\ & & & & & s_{N-1} & d'_{N,N-1}\end{bmatrix}\begin{bmatrix}\Delta J^Z_{1,0}\\ \vdots\\ \vdots\\ \Delta J^Z_{i,i-1}\\ \vdots\\ \vdots\\ \Delta J^Z_{N,N-1}\end{bmatrix}, \quad (5)$$

where

$$\Delta J^Z_{i,i-1} = J^Z_{i,i-1} - I_B , \quad (6)$$

$$J^Z_{i,i-1} \equiv \frac{\hbar}{2e}C_{i,i-1}\partial_{tt}\phi_{i,i-1} + \frac{\hbar}{2e}G_{i,i-1}\partial_t\phi_{i,i-1} + J_{i,i-1}\sin\,\phi_{i,i-1}, \quad (7)$$

$$d'_{i,i-1} = d_{i,i-1} + \lambda_{i-1}\coth\left[\frac{t_{i-1}}{\lambda_{i-1}}\right] + \lambda_i\coth\left[\frac{t_i}{\lambda_i}\right], \quad (8)$$

and

$$s_i = -\frac{\lambda_i}{\sinh(t_i/\lambda_i)}. \quad (9)$$

Here $J^{Z}_{i,i-1}$ is the total current flowing in the ith junction and I_B is the bias current per unit length which is supplied to the Nth S layer and extracted from the 0th S layer. $d'_{i,i-1}$ may be called the effective thickness of the junction that includes the shielding current effect for the magnetic flux density in the ith insulating layer. s_i, which expresses the shielding current effect of the magnetic flux density in the adjacent insulating layer, may be called the coupling parameter. To simplify the matrix expression in Eq. (5) we have defined $s_i<0$. $d_{i,i-1}$ is the barrier thickness of the ith junction, λ_i is the magnetic penetration depth of the ith superconducting layer, $C_{i,i-1}$ and $G_{i,i-1}$ are the capacitance and quasiparticle tunnel conductance, respectively, per unit length. $J_{i,i-1}$ is the dc maximum Josephson current.

If the magnetic flux density B_a is applied in parallel to the y direction defined in Fig. 2, the boundary conditions to be satisfied are, at both $x=0$ and $x=L$,

$$-\frac{\hbar}{2e}\,\partial_x\phi_{i,i-1}=B_a(s_{i-1}+d'_{i,i-1}+s_i)=B_a\left\{d_{i,i-1}+\lambda_{i-1}\tanh\left(\frac{t_{i-1}}{2\lambda_{i-1}}\right)+\lambda_i\tanh\left(\frac{t_i}{2\lambda_i}\right)\right\}\quad\text{for } i=1,2,\ldots,N. \tag{10}$$

In this paper, we discuss the case of no applied magnetic field, $B_a=0$.

Let us consider a small rf bias current in addition to the dc bias current, similarly to the single junction case, Eq. (1). In the present paper, however, we will assume a spatially dependent rf current such that

$$I_B=I_{\rm dc}+I_{\rm rf}\cos(kx-\omega t). \tag{11}$$

A spatially dependent bias is physically realistic. For instance, a current concentration near the edges may occur in a superconducting current lead. In such cases, the rf bias may be expanded in its Fourier components. When the rf term is small and the system becomes linear, the solution of the system is a superposition of the solution for each Fourier component and thus considering the case of Eq. (11) is general and sufficient.

For the phase differences $\phi_{i,i-1}$ between the superconducting layers i and $i-1$ we will—in analogy with the derivation in Ref. 17—assume the form

$$\phi_{i,i-1}=\phi^{(0)}_{i,i-1}+\phi^{(1)}_{i,i-1}, \tag{12}$$

where $\phi^{(0)}_{i,i-1}$ is a dc term and $|\phi^{(1)}_{i,i-1}|\ll 1$. Accordingly we find

$$\sin\phi_{i,i-1}\cong\sin\phi^{(0)}_{i,i-1}+\cos(\phi^{(0)}_{i,i-1})\,\phi^{(1)}_{i,i-1}. \tag{13}$$

Inserting the bias current, Eq. (11), into Eqs. (5) and (6) and applying the approximation Eq. (13) we obtain equations for the terms at zero frequency and the terms at the angular frequency ω.

The equation for the dc terms gives

$$I_{\rm dc}=J_{i,i-1}\sin\phi^{(0)}_{i,i-1} \tag{14}$$

which determines the dc part of the phases $\phi^{(0)}_{i,j-1}$. For the remaining terms at angular frequency ω we get the following coupled linear equations:

$$\frac{\hbar}{2e\mu_0}\,\partial_{xx}\begin{bmatrix}\phi^{(1)}_{1,0}\\ \vdots\\ \vdots\\ \phi^{(1)}_{i,i-1}\\ \vdots\\ \vdots\\ \phi^{(1)}_{N,N-1}\end{bmatrix}=\begin{bmatrix}d'_{1,0} & s_1 & & & & & \\ s_1 & d'_{2,1} & s_2 & & 0 & & \\ & \ddots & \ddots & \ddots & & & \\ & & s_{i-1} & d'_{i,i-1} & s_i & & \\ & & & \ddots & \ddots & \ddots & \\ & 0 & & & \ddots & \ddots & s_{N-1}\\ & & & & & s_{N-1} & d'_{N,N-1}\end{bmatrix}\begin{bmatrix}\Delta J^{Z(1)}_{1,0}\\ \vdots\\ \vdots\\ \Delta J^{Z(1)}_{i,i-1}\\ \vdots\\ \vdots\\ \Delta J^{Z(1)}_{N,N-1}\end{bmatrix}, \tag{15}$$

where

$$\Delta J^{Z(1)}_{i,i-1}=\frac{\hbar}{2e}\,C_{i,i-1}\partial_{tt}\phi^{(1)}_{i,i-1}+\frac{\hbar}{2e}\,G_{i,i-1}\partial_t\phi^{(1)}_{i,i-1}+J_{i,i-1}(\cos\phi^{(0)}_{i,i-1})\,\phi^{(1)}_{i,i-1}-I_{\rm rf}\cos(kx-\omega t). \tag{16}$$

For mathematical convenience, $I_{\rm rf}\cos(kx-\omega t)$ is here replaced with the complex form $(I_{\rm rf}/2)\exp[j(kx-\omega t)]$, where $j^2=-1$. As is well known, the final physical quantities can be obtained by a summation of the solution in this case and the solution in the case where the bias has its complex conjugate form. Then the solution to Eq. (15) is $\phi^{(1)}_{i,i-1}=A_{i,i-1}\exp[j(kx-\omega t)]$, where

$$\begin{bmatrix} d'_{1,0}\Omega_{1,0}-k^2 & s_1\Omega_{2,1} & & & & & \\ s_1\Omega_{1,0} & d'_{2,1}\Omega_{2,1}-k^2 & s_2\Omega_{3,2} & & & 0 & \\ & \ddots & \ddots & \ddots & & & \\ & & s_{i-1}\Omega_{i-1,i-2} & d'_{i,i-1}\Omega_{i,i-1}-k^2 & s_i\Omega_{i+1,i} & & \\ & & \ddots & \ddots & \ddots & & \\ & 0 & & & & \ddots & \ddots \\ & & & & & \ddots & d_{N,N-1}\Omega_{N,N-1}-k^2 \end{bmatrix}$$

$$\times \begin{pmatrix} A_{1,0} \\ \vdots \\ \vdots \\ A_{i,i-1} \\ \vdots \\ \vdots \\ A_{N,N-1} \end{pmatrix} = -\frac{2e\mu_0}{\hbar}\frac{I_{\rm rf}}{2} \begin{pmatrix} d'_{1,0}+s_1 \\ d'_{2,1}+s_1+s_2 \\ \vdots \\ d'_{i,i-1}+s_{i-1}+s_i \\ \vdots \\ \vdots \\ d'_{N,N-1}+s_{N-1} \end{pmatrix}, \tag{17}$$

$$\Omega_{i,i-1}\equiv\mu_0 C_{i,i-1}\omega^2-\frac{2e\mu_0}{\hbar}J_{i,i-1}\cos\phi^{(0)}_{i,i-1} + j\omega\mu_0 G_{i,i-1}. \tag{18}$$

In the following we will simplify the problem by assuming identical layers. This permits us to drop unnecessary subscripts in the equation above, and by introducing the notation $H=d'\Omega$ and $S=s/d'$, we get

$$\begin{pmatrix} H-k^2 & SH & & & & & \\ SH & H-k^2 & & & 0 & & \\ & \ddots & \ddots & \ddots & & & \\ & & SH & H-k^2 & SH & & \\ & & & \ddots & \ddots & \ddots & \\ & 0 & & & \ddots & \ddots & SH \\ & & & & & SH & H-k^2 \end{pmatrix}$$

$$\times \begin{pmatrix} A_{1,0} \\ \vdots \\ \vdots \\ A_{i,i-1} \\ \vdots \\ \vdots \\ A_{N,N-1} \end{pmatrix} = -\frac{1}{\lambda^2_{\rm exc}} \begin{pmatrix} 1+S \\ 1+2S \\ \vdots \\ 1+2S \\ \vdots \\ 1+2S \\ 1+S \end{pmatrix}, \tag{19}$$

where

$$\lambda^{-2}_{\rm exc}=\frac{2e\mu_0 d'}{\hbar}\frac{I_{\rm rf}}{2} \tag{20}$$

defines a length scale for the applied rf bias current.

First we will find the solution to Eq. (19) with no applied rf current, i.e., having the right-hand side of Eq. (19) equal to zero. In that case we find, in a similar way as in Ref. 6, but using the opposite ordering definition with respect to m,

$$k^2=H\left\{1-2S\cos\left(\frac{m\pi}{N+1}\right)\right\} \quad \text{for } m=1,2,\ldots,N, \tag{21}$$

where N is the number of stacks and m is the mode number. The solution $A_{i,i-i}$ for mode number m is

$$A^m_{i,i-1}=\sqrt{\frac{2}{N+1}}\sin\left[\frac{i(N+1-m)\pi}{N+1}\right]. \tag{22}$$

From Eq. (21), we get explicitly the $k-\omega$ plasma dispersion relation for each mode m,

$$\omega^2-\omega_p^2-[c^{(N)}_m]^2k^2+j\omega G/C=0, \tag{23}$$

where

$$\omega_p=\sqrt{2eJ\cos\phi^{(0)}/\hbar C} \tag{24}$$

and

$$c^{(N)}_m=\frac{c_0}{\left[1-2S\cos\left(\frac{m\pi}{N+1}\right)\right]^{1/2}}, \tag{25}$$

with $c_0=(\mu_0 d' C)^{-1/2}$. Note that the plasma frequency [Eq. (23)] has the same form as that [Eq. (4)] in the case of the single barrier Josephson junction. We find from Eq. (23) that k is imaginary due to the presence of losses, and thus a mode which is once excited is damped with time. In the lossless limit, we have $k-\omega$ dispersion curves, $\omega^2=\omega_p^2+[c^{(N)}_m]^2k^2$. Figure 3 shows the dispersion curves for the case of $N=3$. At $\omega\gg\omega_p$, $d\omega/dk$ approaches asymptotically the Swihart-type velocities $c^{(N)}_m$ in the case of an N stack, i.e., the characteristic velocities of electromagnetic waves in the N-stack system with no tunnel currents. As will be shown below, in

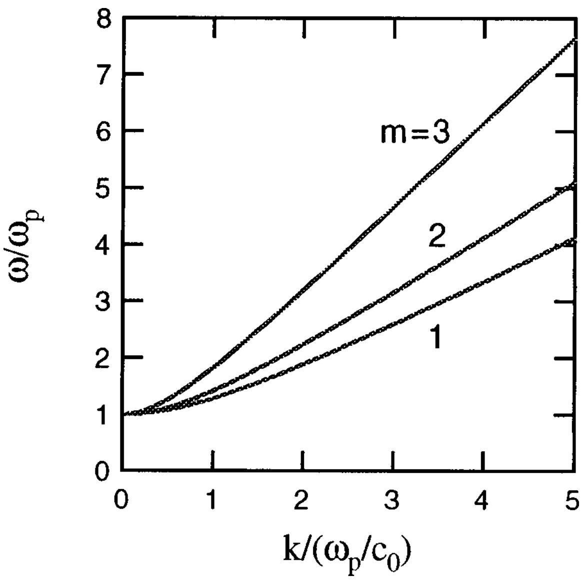

FIG. 3. Plasma dispersion relation curves for $N=3$. $S=-0.4$ is used in this calculation.

the presence of an rf bias current, plasma resonances may appear on some selected modes of these $k-\omega$ dispersion curves.

As the simplest nontrivial case let us first study the case $N=2$ with an rf excitation of the form $I_{\mathrm{rf}}\cos(kx-\omega t)$. Equation (19) gives a solution of the form

$$\begin{Bmatrix}\phi_{1,0}^{(1)}\\ \phi_{2,1}^{(1)}\end{Bmatrix}=\frac{2eI_{\mathrm{rf}}}{\hbar C}\frac{\cos(kx-\omega t+\theta)}{\sqrt{(\omega^2-\omega_p^2-c_+^2k^2)^2+(\omega G/C)^2}}\begin{Bmatrix}1\\1\end{Bmatrix} \tag{26}$$

with

$$\tan\theta=\frac{\omega G/C}{(\omega^2-\omega_p^2-c_+^2k^2)}\quad\text{and}\quad c_2^{(2)}\equiv c_+=\frac{c_0}{\sqrt{1+S}}.$$

We notice that Eq. (26) has a form analogous to Eq. (2) for the single junction case. Note also that $\phi_{1,0}^{(1)}$ and $\phi_{2,1}^{(1)}$ are in phase and identical, and their magnitudes are enhanced on the plasma dispersion curve corresponding to the c_+ mode, i.e., the plasma resonance takes place on the c_+ dispersion curve. On the other hand on the c_- dispersion curve with the characteristic velocity $c_1^{(2)}\equiv c_-=c_0/\sqrt{1-S}$, and where the opposite phase relation ($\phi_{1,0}^{(1)}=-\phi_{2,1}^{(1)}$) is found, a resonance does not appear, because, for reasons of symmetry, it is not being exited by I_{rf}, applied in series through the two junctions.

In the case of a three junction stack ($N=3$), the solution to Eq. (19) becomes

$$A_{1,0}=A_{3,2}=-\frac{1}{\lambda_{\mathrm{exc}}^2}\frac{(1+S)(H^2-k^2)-(1+2S)SH}{[(1+\sqrt{2}S)H-k^2][(1-\sqrt{2}S)H-k^2]}, \tag{27}$$

$$A_{2,1}=-\frac{1}{\lambda_{\mathrm{exc}}^2}\frac{(1+2S)(H^2-k^2)-2(1+S)SH}{[(1+\sqrt{2}S)H-k^2][(1-\sqrt{2}S)H-k^2]}. \tag{28}$$

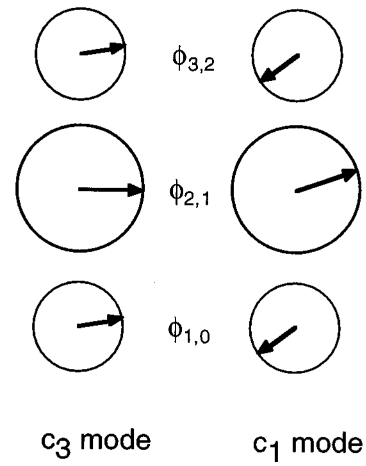

FIG. 4. Schematic drawing on a complex number domain of possible plasma resonance modes for $N=3$. The phase and voltage (the time derivative of the phase) are shown as vectors rotating with time as $e^{-j\omega t}$. Real quantities are the projection to a major axis.

We change the terms appearing in the denominators in Eqs. (27) and (28) to the more explicit forms

$$(1+\sqrt{2}S)H-k^2=c_1^{-2}\{(\omega^2-\omega_p^2-c_1^2k^2)+i\omega G/C\} \tag{29}$$

and

$$(1-\sqrt{2}S)H-k^2=c_3^{-2}\{(\omega^2-\omega_p^2-c_3^2k^2)+i\omega G/C\} \tag{30}$$

with

$$c_1^{(3)}\equiv c_1=\frac{c_0}{\sqrt{1-\sqrt{2}S}}\quad\text{and}\quad c_3^{(3)}\equiv c_3=\frac{c_0}{\sqrt{1+\sqrt{2}S}}.$$

These equations show that resonances take place on the plasma dispersive curves of the c_1 and c_3 modes. (See Fig. 3.) The c_2 mode does not appear as a resonance, which is also understandable from symmetry considerations. Equation (27) also assures $\phi_{1,0}^{(1)}=\phi_{3,2}^{(1)}$ in any case, i.e., the oscillation of the first and third junctions are completely in phase and identical.

In order to determine the relationship between A_{10} and A_{21} we take the ratio A_{21}/A_{10}. Let us first consider the case where the $k-\omega$ relationship is on the c_3 dispersion curve, i.e., $\omega^2=\omega_p^2+c_3^2k^2$. In this case we find

$$\frac{A_{2,1}}{A_{1,0}}=\frac{-\sqrt{2}k^2c_0^2(1+\sqrt{2})S+j\omega(1-2S^2)G/C}{-k^2c_0^2(1+\sqrt{2})S+j\omega(1-2S^2)G/C}. \tag{31}$$

From this equation and recalling $S<0$, we find that A_{21} and A_{10} changes with time almost in phase and $|A_{21}|>|A_{10}|$. This case is shown in Fig. 4(a). Due to the losses, the phase of A_{10} is slightly delayed with respect to the phase of A_{21}. In the lossless limit, we see that A_{21} and A_{10} are perfectly in phase and $|A_{21}|/|A_{10}|=\sqrt{2}$.

Similarly, at the resonance condition of the c_1 mode ($\omega^2=\omega_p^2+c_1^2k^2$), we obtain

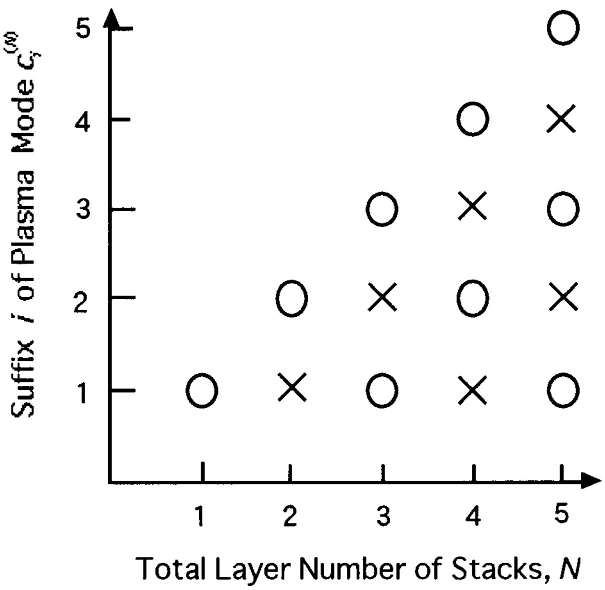

FIG. 5. Possible resonance modes under a bias current application as in Fig. 2(a). The open circles indicate possible modes and the crosses are prohibited modes.

$$\frac{A_{2,1}}{A_{1,0}}=\frac{-\sqrt{2}k^2c_0^2(\sqrt{2}-1)S+j\omega(1-S^2)G/C}{k^2c_0^2(\sqrt{2}-1)S+j\omega(1-S^2)G/C}. \quad (32)$$

Again we find that $|A_{21}|>|A_{10}|$, but A_{21} and A_{10} changes with time almost in an antiphase manner as shown in Fig. 4(b). In the lossless limit, $|A_{21}|/|A_{10}|=\sqrt{2}$ and A_{21} has the perfectly opposite phase of A_{32}. From Fig. 4, a large voltage amplitude across the stack may be expected as for the c_3 resonance condition.

For the case of N stacks we find that the highest phase velocity $c_N^{(N)}$ mode always corresponds to all junctions oscillating in phase at the plasma resonance. The general picture may be understood from Fig. 5. Possible dispersion modes with stack plasma resonances are found for every second mode obtained by counting downwards from the $c_N^{(N)}$ mode, which has the highest velocity.

III. DISCUSSION

Our model implies that the coupling between the junction is a magnetic field coupling through the superconducting layers. We note from Eq. (5), that in order for such a coupling to take place we must have a second order spatial derivative, which is different than zero. Thus for the uniform (in the x direction) solution case the junctions are completely decoupled in the z direction. Nevertheless there will be a plasma resonance in the usual sense, since each of the stacked junctions has a plasma frequency given by Eq. (24), as can be seen by setting $\Delta J_{i,i-1}^{Z(1)}=0$ in Eq. (16). On the other hand this resonance is hardly a longitudinal collective excitation in the sense of for example Ref. 16, since all the junctions are decoupled.

In our inductively coupled junction model, the coupling between the layers may take place under various conditions. (i) If fluxons are present ϕ_{xx} is different than zero and a coupling takes place. (ii) The finite size of the sample necessarily introduces a spatial dependence of the applied $I_{\rm rf}$ [Eq. (11)] and thus φ itself will have spatial frequencies commensurate with the sample dimension in the x direction. (iii) If a magnetic field is applied in the y direction the necessary spatial variation for a coupling between layers is introduced.

We see from the form of the equations derived in the previous section that we have the so-called transverse plasma excitations with the transverse wave numbers given by Eq. (23). We also note that the dispersion relation contains the coupling coefficients and this is a collective excitation for the whole system. For the highest velocity excitation (corresponding to the velocity $c_N^{(N)}$) we find from Eqs. (26) and (31) that, although a slight phase delay for $N>2$ exists due to the losses, all the rf voltage across the layers are almost in phase and thus adds to a large total voltage in the z direction.

The nature of this excitation was shown in Fig. 5 for arbitrary N cases. For the lowest velocity $c_1^{(3)}$ of $N=3$ we find, as shown in Fig. 4, that voltage of the neighboring junctions are out of phase, and thus we get a spatial dependence of the voltage in the z direction. For solutions with intermediate velocities such as $c_2^{(4)}$ and $c_3^{(5)}$ we find a spatial dependence in the z direction intermediate to the situations shown in Figs. 4(a) and 4(b). Such a spatial dependence in the z direction that is suggested by Eq. (22) may be relevant to the z-direction Fiske modes found previously by Kleiner.[18]

Recently a model introduced by Tachiki *et al.*,[16] where the coupling between layers take place by a charging effect (CE) in the superconducting electrodes, has been used in explaining plasma resonance experiments in BSCCO samples. We find it relevant here to discuss the differences between our inductive coupling (IC) model and the CE model in Ref. 16. In most cases the CE model has been used to interpret experiments on BSCCO samples[13–15] and the IC model to interpret experiments with stacks made of traditional superconductors.[6–8] However the IC model has also been used to successfully interpret experiments on BSCCO samples.[11–12] In the charging effect model,[16] one finds that purely collective longitudinal plasma excitations exist even with $k_x=0$ but, as for transverse plasma excitations with $k_x\neq 0$, only all-in-phase $k_z=0$ excitations are discussed.

In the IC model,[5] there is no restriction for the thickness of superconducting layers, and thus within the range of the inductive coupling the zero thickness limit case such as that in Ref. 16 is correctly included. Furthermore, since the model takes spatial x dependence into account, the presence of plural propagation modes based on their characteristic velocities [Eq. (25)] has been demonstrated.[6,18] Inevitably these plural modes result in the presence of longitudinal (z direction) states for the Fiske modes[18] and the plasma mode in this paper.

IV. SUBHARMONIC GENERATION

The results for a two stack and an n stack has the same appearance as for the case of a single junction, at least when we use the largest wave velocity and consequently the voltages over all the layers are added in phase. From a comparison with Ref. 17 it is then clear that just in the same way as derived there, half harmonic generation at the plasma resonance frequency in a stack may be done. We have not explicitly written the relevant equations down since it is a trivial extension of the results in Ref. 17. We note that a singly degenerate parametric amplifier of the same type as in Refs. 4 and 17 can be made. However, because of the stacked structure the serious problem of impedance matching may be easier to handle.

For intrinsic stacks experimental attempts have already been made,[19] however, so far no more results have been reported. Other nonlinear results from single junctions may, by analogy, also be found in the *n* stack with the in phase excitation corresponding to the highest wave velocity.

V. SUMMARY

We derived analytic equations for describing plasma resonances in vertically stacked Josephson junction systems. When a bias current consisting of dc and rf terms were introduced to the top superconducting layers and extracted from the bottom layer, possible resonance modes in collective excitations for the whole system were discussed. The highest velocity mode always exists as a stable mode. The phase behavior of this mode is all in phase among junctions in the lossless limit and thus large rf voltage outputs are expected.

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