Quantum magnetoresistance

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An explanation is proposed of the unusual magnetoresistance, linear in magnetic field and positive, observed recently in nonstoichiometric silver chalcogenides. The idea is based on the assumption that these substances are basically gapless semiconductors with a linear energy spectrum. Most of the excess silver atoms form metallic clusters which are doping the remaining material to a very small carrier concentration, so that even in a magnetic field as low as 10 Oe, only one Landau band participates in the conductivity.
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I. INTRODUCTION

Recently an interesting phenomenon has been observed by Xu *et al.*[1] in two doped silver chalcogenides $Ag_{2+\delta}Se$ and $Ag_{2+\delta}Te$, with $\delta \sim 0.01$, the magnetoresistance depended linearly on magnetic field starting from approximately 10 Oe and going on to 55 kOe without any sign of saturation. This took place at temperatures ranging from 4.5 to 300 K. The slope of the linear field dependence decreased with increasing temperature in this range approximately three times. The Hall constant did not depend on temperature at $T < 100$ K and decreased at higher temperatures.

With ideal stoichiometry, $\delta = 0$, or small nonstoichiometry, at low temperatures these substances are intrinsic semiconductors with a narrow direct gap.[2,3] The gap is sample dependent and lies in the range of several tens of meV. The effective mass of carriers is of the order of $10^{-2}m_0$, where m_0 is the free electron mass.[4] At higher temperatures ($T = 133$ °C for Ag_2Se) these substances undergo a phase transition into a phase which behaves more similar to a metal. Contrary to that, for nonstoichiometric (or doped) compounds, from the finite value of the Hall constant we can conclude that at any temperature they are more similar to the high-temperature phase of the stoichiometric compounds.

According to conventional theory (see, e.g., Ref. 5) the magnetoresistance in a metal with nonequal densities of electrons and holes behaves as

$$\Delta\rho \sim \begin{cases} \rho_0(\Omega\tau)^2, & \Omega\tau \ll 1 \\ \rho_0, & \Omega\tau \gg 1 \end{cases} \qquad (1)$$

where $\Omega = eH/(m_1 c)$ is the Larmor frequency, m_1 is the cyclotron mass, τ is the collision time, and ρ_0 is the resistance at zero field. This means that first it grows quadratically with field and then reaches saturation. With equal densities of electrons and holes (this can hardly correspond to the present case), the limit at $\Omega\tau \gg 1$ is quadratic in Ω. None of these laws resembles the observations. There exists a certain possibility of a linear magnetoresistance, namely, a polycrystalline metal with an open Fermi surface (see in Ref. 5) but this is definitely not the present case. Therefore it is clear that the conventional approach, based on the quasiclassical treatment of electrons, cannot explain the phenomenon. Exotic mechanisms, such as constant and variable range hop-

ping in semiconductors (see Ref. 6) also do not lead to a linear dependence of the resistivity on magnetic field. Besides, the substances in question behave as metals, and not as semiconductors.

In an old and thoroughly forgotten (including the author himself) paper,[7] published almost 30 years ago, the present author developed a more general approach to the theory of galvanomagnetic phenomena in metals, based on the exact quantum description of electrons in a magnetic field. The real goal of that paper was to establish the truth concerning the amplitude of the Shubnikov-deHaas oscillations (this problem caused great confusion in the scientific community of those days), however, for generality, the limiting quantum case, with electrons only in one Landau band, was also considered. The formulas obtained in that work were

$$\rho_{xx} = \rho_{yy} = \frac{N_i H}{\pi n^2 ec}, \quad \rho_{xy} = RH = \frac{H}{nec}, \qquad (2)$$

where N_i is the density of scattering centers, n is the electron density, R is the Hall constant, and it was assumed that $T = 0$. To my knowledge, this was the only possible case of a linear positive magnetoresistance.

In formula (2) $\rho_{xx} = \rho_{yy}$ is the total resistance, whereas the experimental result[1] can be described as

$$\rho = \rho_0 + aH, \qquad (3)$$

and this formula is valid not only at magnetic fields of several T but down to 10 Oe. According to what was said before [formula (1)], there is no way to explain such a behavior assuming that the metal is homogenous. Indeed the condition for only one Landau level to participate is (we include $\hbar$ in the numerical estimates; otherwise, units are used with $\hbar = 1$)

$$n < (eH/c\hbar)^{3/2}. \qquad (4)$$

If we substitute the electron concentration obtained from the Hall measurements $n \sim 10^{17}$ cm^{-3}, we get $H > 2$ T. The only possibility is to assume that the real samples used in the experiments are highly inhomogeneous, so that they contain small regions with a large concentration of excess silver atoms and, correspondingly, higher electron concentration, imbedded into regions with a much smaller electron concentration where the extremal quantum situation takes place. For

such a structure one could get the formula (3), however, one would have to assume that the concentration in the poorly doped regions has the order of 10^{12} cm^{-3}, i.e., $\sim 10^{-10}$ per atom, or less. This can be obtained from the condition (4) and the experimental result that the dependence of the magnetoresistance deviates from linearity only at $H < 10$ Oe. Since the atomic concentration of excess silver atoms is of the order of 1%, it seems that these atoms form metallic clusters, which are doping the rest of the material with electrons.

For an inhomogenous system the calculation of the Hall constant requires a more precise characterization of the material, and therefore the experimentally obtained ''effective concentrations'' of the order of 10^{17} cm^{-3} cannot be calculated unambiguously. However, even these concentrations corresponding to $\sim 10^{-5}$ electrons per atom are much less than the concentration of the excess silver atoms; this shows the trend.

There are, however, other concerns. At small fields the Landau level spacing becomes smaller than T. For a quadratic energy spectrum at 10 Oe it is (in K)

$$\frac{\hbar e H}{m_1 c k_B} \sim 10^{-3} \left(\frac{m_0}{m_1} \right) \quad \text{K},$$

where k_B is the Boltzmann constant and m_0 is the mass of the free electron. If one uses the value of $m_1/m_0 \sim 10^{-2}$ given in the literature,[4] one gets 10^{-1} K, which is too small, since the linear magnetoresistance at 10 Oe was observed at $T = 4.5$ K. The other assumption in Ref. 7, namely, that the electrons are degenerate, is even more difficult to achieve at the required densities.

II. MODEL

Since the problem seems completely hopeless for an ordinary approach, some very unusual path has to be taken. In Ref. 3 an idea was proposed about the high-temperature α phase of Ag$_2$Se. Due to the increased mobility of silver ions a substantial disorder appears which creates ''tails'' in both the conduction band and the valence band. Eventually the bands overlap, and the substance becomes metallic. It should be mentioned that according to experimental data the gap is direct. In Ref. 8 it was concluded that in Ag$_2$Te with increasing temperature a phase transition from a narrow-gap semiconductor to a gapless semiconductor takes place. One could guess that since the main reason for this change of the spectrum is disorder, the same could happen as result of doping and not of increased mobility (the ions move slowly, and they are always static from the viewpoint of electrons); this conclusion is supported by the finite Hall constant. Under these conditions the nonstoichiometric compound cannot be treated as a semiconductor with carriers in the bands resulting from doping such as, e.g., Ge and Si, but the start must be done from a different phase, which is closer to a gapless semiconductor.[9,10] The latter is a substance where at $T = 0$ a completely filled valence band matches an empty conduction band. From the small value of the effective mass[4] it looks more likely that the energy spectrum in both bands is linear. Indeed, since a small effective mass can appear only in some restricted regions of the momentum space, it has to grow

with energy (momentum). If the valence band contacts the conduction band, or slightly hybridizes with it, the spectrum becomes even closer to linearity.

The possibility of a linear spectrum was analyzed in Ref. 11 (type I). It can be a consequence of cubic symmetry without an inversion center and can also happen at some random point in the reciprocal space at proper ''tuning'', e.g., by pressure or doping. An example of the latter was analyzed in Ref. 12.

We will consider here the simplest spectrum of this kind, just in order to have an example of what can happen in such a substance. The spectrum corresponds to a double representation of cubic group T or O, and its Hamiltonian can be written as

$$H = \int \psi^+ v \left[\sigma \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right) \right] \psi \, dV, \tag{5}$$

where σ are the Pauli matrices and $\mathbf{p}$ is the momentum operator. The velocity v can be assumed of the usual order of magnitude $v \sim 10^8$ cm/sec. In the absence of magnetic field we get two branches of the spectrum with energies $\varepsilon = vp$, $-vp$. Suppose that the magnetic field is along z, and we chose the vector potential $A_y = Hx$. The electronic wave functions will have two components satisfying the equations

$$-i \frac{\partial}{\partial z} \psi_1 + \left(-i \frac{\partial}{\partial x} - \frac{\partial}{\partial y} + i \frac{eH}{c} x \right) \psi_2 = \frac{\varepsilon}{v} \psi_1,$$

$$\left(-i \frac{\partial}{\partial x} + \frac{\partial}{\partial y} - i \frac{eH}{c} x \right) \psi_1 + i \frac{\partial}{\partial z} \psi_2 = \frac{\varepsilon}{v} \psi_2. \tag{6}$$

Since the equations contain explicitly only x, we will search for solutions of the usual form

$$\psi_{1,2} = \psi_{1,2}(x) e^{(ip_y y + ip_z z)}. \tag{7}$$

The eigenvalues of Eqs. (6) are

$$\varepsilon_n^{(+)} = v \left(p_z^2 + \frac{2eHn}{c} \right)^{1/2},$$

$$\varepsilon_n^{(-)} = -v \left(p_z^2 + \frac{2eHn}{c} \right)^{1/2}. \tag{8}$$

Corresponding normalized eigenfunctions have the form

$$\psi_{n1}^{(+)} = \frac{1}{\sqrt{2}} \left(1 + \frac{p_z}{(p_z^2 + 2neH/c)^{1/2}} \right)^{1/2} \psi_n,$$

$$\psi_{n2}^{(+)} = -\frac{i}{\sqrt{2}} \left(1 - \frac{p_z}{(p_z^2 + 2neH/c)^{1/2}} \right)^{1/2} \psi_{n-1},$$

$$\psi_{n1}^{(-)} = \frac{1}{\sqrt{2}} \left(1 - \frac{p_z}{(p_z^2 + 2neH/c)^{1/2}} \right)^{1/2} \psi_n,$$

$$\psi_{n2}^{(-)} = \frac{i}{\sqrt{2}} \left(1 + \frac{p_z}{(p_z^2 + 2neH/c)^{1/2}} \right)^{1/2} \psi_{n-1}, \tag{9}$$

where ψ_n are the usual normalized eigenfunctions of a free electron in a magnetic field:

$$\psi_n = (2^n n!)^{-1/2} (\beta/\pi)^{1/4} e^{-(\beta/2)[x-(p_y/\beta)]^2}$$

$$\times H_n[\sqrt{\beta}(x - p_y/\beta)]. \qquad (10)$$

Here $\beta = eH/c$ and H_n are Hermite polynomials.

Formulas (9) describe the eigenfunctions for $n \neq 0$. In the case $n = 0$ the eigenfunctions are

$$\psi_{01}^{(+)} = \theta(p_z)\psi_0,$$

$$\psi_{02}^{(+)} = 0,$$

$$\psi_{01}^{(-)} = \theta(-p_z)\psi_0,$$

$$\psi_{02}^{(-)} = 0. \qquad (11)$$

We assume that in the undoped substance at zero temperature all the negative bands are filled and the positive bands are empty (see Ref. 9). We will assume also that at the temperatures and levels of doping under consideration only the bands $\varepsilon_0^{(+)} = v|p_z|$ and $\varepsilon_0^{(-)} = -v|p_z|$ will contain charge carriers. The necessary condition will be established later. Imagine that the doping corresponds to the electron density n_0. This means that the density of electrons in the band $\varepsilon_0^{(+)}$ minus the density of holes in the band $\varepsilon_0^{(-)}$ must be equal to n_0:

$$\frac{eH}{\pi c} \int_0^\infty ([e^{(vp_z - \mu)/T} + 1]^{-1} - [e^{(vp_z + \mu)/T} + 1]^{-1}) \frac{dp_z}{2\pi} = n_0. \qquad (12)$$

Here we took into account two projections of the electron spin and the fact that the spin splitting $\mu_B H$ can be neglected. It is also important to remember that according to Eq. (11) only electron states with $p_z > 0$ and hole states with $p_z < 0$ are available. From condition (12) we obtain

$$\mu = \frac{2\pi^2 n_0 c v}{eH}. \qquad (13)$$

We see that the chemical potential does not depend on temperature, which means that at all temperatures the electron system is described by the Fermi distribution.

Now we can obtain the condition for one Landau level to participate in kinetics. The first and rather evident condition is (in ordinary units)

$$T < v\sqrt{eH\hbar/c}. \qquad (14)$$

If we substitute $H \sim 10$ Oe, we get $T < 10$ K, which corresponds to the measurement conditions of Ref. 1. With larger H the lower temperature boundary also rises, so that at 1 T we get $T < 300$ K, which is also quite satisfactory. The contribution of the Landau band with $n = 1$ to the electron density is

$$n^{(1)} = \frac{eH}{2\pi^2 c} \int_0^\infty [e^{[v(p_z^2 + 2eH/c)^{1/2} - \mu]/T} + 1]^{-1} dp_z.$$

The contribution will be small if condition (14) is fulfilled, provided that $\mu < (eH/c)^{1/2}$. Using formula (13), we get the condition (4) for n_0.

We must be aware that for the model under consideration this quantity is only the difference between densities of electrons and holes, entering, as we will see below, the Hall constant. Both types of carriers contribute to the conductivity, and their densities can be considerably larger than n_0. For example, the density of electrons is

$$n^{(0)} = \frac{eH}{2\pi^2 c} \int_0^\infty [e^{(vp_z - \mu)/T} + 1]^{-1} dp_z$$

$$= \frac{eHT}{2\pi^2 cv} \ln(1 + e^{\mu/T}).$$

At low temperatures, when $\mu \gg T$, the density is equal to n_0 but at higher temperatures and magnetic fields it becomes much larger.

III. HALL CONSTANT

For the Hall constant we need only ''bare'' Green functions of the type defined in Ref. 7:

$$G_{\alpha\beta}^{(0)}(p_z, p_y, x, x', \omega_m)$$

$$= \sum_n \frac{\psi_{n\alpha}(x - cp_y/eH)\psi_{n\beta}^*(x' - cp_y/eH)}{i\omega_m + \mu - \varepsilon_n(p_z)}, \qquad (15)$$

where the summation over n means different energy levels, positive and negative, and $\alpha, \beta = 1, 2$. We use the temperature technique since we want to consider finite temperatures.

The current operator can be obtained from the Hamiltonian (1) and the relation

$$\delta H = -\frac{1}{c} \int \mathbf{j} \delta \mathbf{A} dV. \qquad (16)$$

From here we obtain

$$\mathbf{j}(\mathbf{r}, t) = e\psi^+(\mathbf{r}) v \sigma \psi(\mathbf{r}, t). \qquad (17)$$

In the usual way we get for the linear response (see Ref. 13)

$$j_i(\omega_0) = -Q_{ik}^R(\omega_0) A_k(\omega_0), \qquad (18)$$

where $Q_{ik}^R(\omega_0)$ is the analytical continuation from the upper half plane of complex frequencies of the corresponding thermodynamic average

$$Q_{ik}(i\omega_0) = \frac{2e^2 v^2}{c} T \sum_m \int \frac{dp_y}{2\pi} \int \frac{dp_z}{2\pi} \int dx'$$

$$\times \text{Tr}[\sigma_i G(p_y, p_z, x, x', \omega_m + \omega_0)$$

$$\times \sigma_k G(p_y, p_z, x', x, \omega_m)], \qquad (19)$$

where the trace is taken over the components 1,2 and two spin projections were taken into account. Performing the analytical continuation we get

$$Q_{xy}^R(\omega_0) = \frac{2ie^2v^2}{c} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{dp_y}{2\pi} \int \frac{dp_z}{2\pi} \int dx' \left\{ \tanh \frac{\omega}{2T} \left[G_2^R(p_y,p_z,x,x',\omega+\omega_0) \text{Im } G_1^R(p_y,p_z,x',x,\omega) \right. \right.$$

$$\left. - G_1^R(p_y,p_z,x,x',\omega+\omega_0) \text{Im } G_2^R(p_y,p_z,x',x,\omega) \right] + \tanh \frac{\omega+\omega_0}{2T} \left[\text{Im } G_2^R(p_y,p_z,x,x',\omega+\omega_0) \right.$$

$$\left. \left. \times G_1^A(p_y,p_z,x',x,\omega) - \text{Im } G_1^R(p_y,p_z,x,x',\omega+\omega_0) G_2^A(p_y,p_z,x',x,\omega) \right] \right\}. \tag{20}$$

Now we substitute the Green functions (15) and use the eigenvalues and eigenfunctions (8)–(11). Here Im G^R means that the imaginary part of the factor $[\omega+\mu-\varepsilon_n(p_z)+i\delta]^{-1}$ in Eq. (15) has to be taken equal to $-\pi\delta[\omega+\mu-\varepsilon_n(p_z)]$. We can limit ourselves with the states $\varepsilon_0^{(\pm)}$, $\varepsilon_1^{(\pm)}$ since the Green function of the state $n=0$ has no large magnetic field in the denominator, and the orthogonality of the eigenfunctions permits the connection between n's differing not more than by 1. Then every one of the Green functions contains the product $\psi_0(x-cp_y/eH)\psi_0(x'-cp_y/eH)$. Integrating in Eq. (20) over x' and p_y, we obtain the factor $eH/(2\pi c)$. After that, instead of the eigenfunctions we can substitute only the p_z-dependent prefactors in Eq. (9). Taking into account that every Green function can correspond to a $(+)$ state, as well as to a $(-)$ state, we obtain for small ω_0

$$Q_{xy}^R(\omega_0) = \frac{2ie^2v^2}{c} \frac{eH}{2\pi c} \frac{c\omega_0}{2eHv^2} \int_0^{\infty} \frac{dp_z}{2\pi} \left(\tanh \frac{vp_z-\mu}{2T} \right.$$

$$\left. - \tanh \frac{vp_z+\mu}{2T} \right). \tag{21}$$

The connection between $Q_{xy}^R(\omega_0)$ and the static conductivity σ_{xy} is

$$\sigma_{xy} = \left[\frac{ic}{\omega_0} Q_{xy}^R(\omega_0) \right]_{\omega_0 \to 0} \tag{22}$$

From here we obtain

$$\sigma_{xy} = \frac{e^2}{\pi} \int_0^{\infty} ([e^{(vp_z-\mu)/T}+1]^{-1} - [e^{(vp_z+\mu)/T}+1]^{-1}) \frac{dp_z}{2\pi}.$$

Comparing with Eq. (12), we obtain

$$\sigma_{xy} = \frac{n_0ec}{H}, \quad \rho_{xy} = \frac{H}{n_0ec}, \quad R_H = \frac{\rho_{xy}}{H} = \frac{1}{n_0ec}, \tag{23}$$

i.e., the usual result for the Hall constant R_H.

IV. CONDUCTIVITY

To obtain the conductivity we use the same approach except that we calculate this time the diagonal component of the Q matrix. We obtain

$$Q_{xx}^R(\omega_0) = Q_{yy}^R(\omega_0) = \frac{2e^2v^2}{c} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{dp_y}{2\pi} \int \frac{dp_z}{2\pi} \int dx' \times \left\{ \tanh \frac{\omega}{2T} \left[G_2^R(p_y,p_z,x,x',\omega+\omega_0) \text{Im } G_1^R(p_y,p_z,x',x,\omega) \right. \right.$$

$$\left. + G_1^R(p_y,p_z,x,x',\omega+\omega_0) \text{Im } G_2^R(p_y,p_z,x',x,\omega) \right] + \tanh \frac{\omega+\omega_0}{2T} \left[\text{Im } G_2^R(p_y,p_z,x,x',\omega+\omega_0) G_1^A(p_y,p_z,x',x,\omega) \right.$$

$$\left. \left. + \text{Im } G_1^R(p_y,p_z,x,x',\omega+\omega_0) G_2^A(p_y,p_z,x',x,\omega) \right] \right\}. \tag{24}$$

The connection between $Q_{xx}^R(\omega_0)$ and σ_{xx} is similar to Eq. (22), and hence we need the imaginary part of $Q_{xx}^R(\omega_0)$. From Eq. (24) we obtain

$$\text{Im } Q_{xx}^R(\omega_0) = \frac{2e^2v^2}{c} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{dp_y}{2\pi} \int \frac{dp_z}{2\pi} \int dx' \left(\tanh \frac{\omega}{2T} - \tanh \frac{\omega+\omega_0}{2T} \right) [\text{Im } G_2^R(p_y,p_z,x,x',\omega+\omega_0)$$

$$\times \text{Im } G_1^R(p_y,p_z,x',x,\omega) + \text{Im } G_1^R(p_y,p_z,x,x',\omega+\omega_0) \text{Im } G_2^R(p_y,p_z,x',x,\omega)] \tag{25}$$
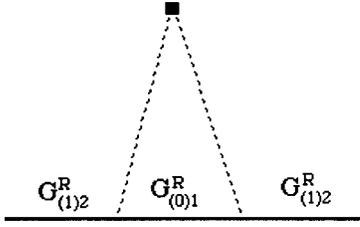
FIG. 1. Correction to the Green function due to scattering, entering the expression (28).

(we used here that Im $G^R = -$Im G^A). Since, according to Eq. (22), we are interested in small ω_0, we expand to the lowest order and obtain

Im $Q_{xx}^R(\omega_0)$

$$
= -\frac{e^2 v^2 \omega_0}{cT} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{dp_y}{2\pi} \int \frac{dp_z}{2\pi} \int dx'
$$

$$
\times \cosh^{-2} \frac{\omega}{2T} [\text{Im } G_2^R(p_y, p_z, x, x', \omega)
$$

$$
\times \text{Im } G_1^R(p_y, p_z, x', x, \omega)
$$

$$
+ \text{Im } G_1^R(p_y, p_z, x, x', \omega) \text{Im } G_2^R(p_y, p_z, x', x, \omega)].
$$

Again, since all the Green functions contain products $\psi_0(x - cp_y/eH)\psi_0(x' - cp_y/eH)$, the integrations over x' and p_y can be performed resulting in a factor $eH/(2\pi c)$, and we are left with the prefactors in formulas (9) and energy denominators. We can write this as

Im $Q_{xx}^R(\omega_0)$

$$
= -\frac{2e^2 v^2 \omega_0}{cT} \frac{eH}{2\pi c} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \cosh^{-2} \frac{\omega}{2T}
$$

$$
\times \text{Im } G_{(1)2}^R(p_z, \omega) \text{Im } G_{(0)1}^R(p_z, \omega), \tag{26}
$$

where $G_{(1)2}^R$, $G_{(0)1}^R$ mean that in the sums over n [see Eq. (15)] only terms with $n=1$ or $n=0$ are left.

Taking the scattering into account we can conclude that the product of Green functions contains

$$
\frac{1/(2\tau_0)\,\theta(\pm p_z)}{(\omega + \mu \mp v|p_z|)^2 + (1/2\tau_0)^2} \frac{1/(2\tau_1)}{[\omega + \mu \mp \varepsilon_1(p_z)]^2 + (1/2\tau_1)^2}. \tag{27}
$$

Both products are close to δ functions. Since, however, the second factor would be $\pi\delta[\omega + \mu \mp \varepsilon_1(p_z)]$ and $\varepsilon_1(p_z)$ contains the large magnetic field, this would substantially reduce the contribution to Eq. (26) due to the factor $\cosh^{-2}(\omega/2T)$ in the integrand. Therefore, we can replace only the first factor by a δ function and get from Eq. (27)

$$
\frac{\pi\delta(\omega + \mu \mp v|p_z|)\,\theta(\pm p_z)(1/2\tau_1)}{[\omega + \mu \mp \varepsilon_1(p_z)]^2}. \tag{28}
$$

Here $1/(2\tau_1)$ is $-$Im Σ, where Σ is the self-energy due to scattering. The expression (28) contains, actually, the first order scattering correction to the Green function, which is presented in Fig. 1. The imaginary part of Σ comes from

integration along the half circle around the pole in the central Green function, and hence, it will impose a limitation $\omega > \min[\varepsilon(p_z') - \mu]$. Again, in order not to reduce the contribution to the integral over ω, the central Green function has to belong to $n=0$.

The impurity lines contribute after averaging over positions a factor

$$
N_i \int d^3\mathbf{k}(2\pi)^{-3}|U(\mathbf{k})|^2 \int \psi_{12}^*(x_1 - cp_y/eH)e^{ik_x x_1}
$$

$$
\times \psi_{0j}[x_1 - c(p_y - k_y)/eH]dx_1
$$

$$
\times \int \psi_{0j}^*[x_2 - c(p_y - k_y)/eH]e^{-ik_x x_2}
$$

$$
\times \psi_{12}(x_2 - cp_y/eH)dx_2. \tag{29}
$$

We have included here integrals coming from the "envelope" ψ functions. $U(\mathbf{k})$ is the Fourier component of the electron-impurity interaction potential. It is natural to assume that it is a screened Coulomb interaction

$$
U(\mathbf{k}) = \frac{4\pi e^2}{\varepsilon_\infty(k^2 + \kappa^2)}, \tag{30}
$$

where ε_∞ is the background dielectric constant, which may be large; κ is the reciprocal Debye screening radius

$$
\kappa^2 = \frac{8\pi e^2}{\varepsilon_\infty} \frac{eH}{2\pi c} T\sum_m \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \frac{1}{(i\omega_m + \mu - vp_z)^2}
$$

$$
= \frac{2e^3 H}{\pi\varepsilon_\infty cv}. \tag{31}
$$

Here we took into account both branches $+$ and $-$ at $n=0$ and integrated in the same way as before.

Since $e^2/\hbar v \sim 1$ for $v \sim 10^8$, $\kappa^2 \ll eH/c$, if $\varepsilon_\infty \gg 1$. We will assume this, since in semimetals, where the situation is close to the one under consideration, $\varepsilon_\infty \sim 100$. According to this, in the integral over $d^3\mathbf{k}$ in Eq. (29) small $\mathbf{k}$'s are the most important ($k \sim \sqrt{eH/c}$ is the upper limit). However, if we put $k_x = k_y = 0$ in Eq. (29), the integral will vanish, since $\psi_{02} = 0$, and there are no nondiagonal matrix elements. Therefore we must expand the matrix elements in powers of k_x and k_y. In this way we get

$$
\int \psi_{12}^*(x_1 - cp_y/eH)e^{ik_x x_1}\psi_{0j}[x_1 - c(p_y - k_y)/eH]dx_1
$$

$$
\approx ik_x(\hat{x})_{0j}^{12} + (c/eH)k_y(\partial/\partial\hat{x})_{0j}^{12}. \tag{32}
$$

The matrix elements, entering here we can obtain from two relations. Since the group velocity in the current is $\hat{v} = v\sigma$, we can write

$$
\hat{v}_x = \partial\hat{x}/\partial t = i[H, \hat{x}]; \quad (\hat{x})_{0j}^{12} = \frac{(\hat{v}_x)_{0j}^{12}}{i(\varepsilon_1 - \varepsilon_0)}. \tag{33}
$$

Similarly, the matrix elements of $\partial/\partial\hat{x}$ can be obtained from the commutation relation $[\partial/\partial\hat{x}, \hat{x}] = 1$. One must also take into account that every one of the three Green functions entering the diagram in Fig. 1 can be $+$ or $-$, and the matrix elements depend on that. So we obtain

$$(\hat{x})_{01+}^{12+}=(\hat{x})_{01+}^{12-}=(\hat{x})_{12+}^{01+}=(\hat{x})_{12-}^{01+}=\frac{\theta(p_z)}{\sqrt{2}(p_z^2+2eH/c)^{1/2}},$$

$$(\hat{x})_{01-}^{12+}=(\hat{x})_{01-}^{12-}=(\hat{x})_{12+}^{01-}=(\hat{x})_{12-}^{01-}=\frac{\theta(p_z)}{\sqrt{2}(p_z^2+2eH/c)^{1/2}},$$

$$(34)$$

$$(\partial/\partial\hat{x})_{12+}^{01+}=(\partial/\partial\hat{x})_{12-}^{01+}=-(\partial/\partial\hat{x})_{01+}^{12+}=-(\partial/\partial\hat{x})_{01+}^{12-}$$

$$=\frac{\theta(p_z)(p_z^2+2eH/c)^{1/2}}{2\sqrt{2}},$$

$$(\partial/\partial\hat{x})_{12+}^{01-}=(\partial/\partial\hat{x})_{12-}^{01-}=-(\partial/\partial\hat{x})_{01-}^{12+}=-(\partial/\partial\hat{x})_{01-}^{12-}$$

$$=\frac{\theta(-p_z)(p_z^2+2eH/c)^{1/2}}{2\sqrt{2}}.$$

$$(35)$$

The matrix elements do not depend on the $+$ or $-$ index of external Green functions. Therefore from the internal Green function we obtain

$$\int_0^\infty \frac{dp_z'}{2\pi}\frac{1}{\omega+\mu-vp_z'+i\delta}+\int_{-\infty}^0 \frac{dp_z'}{2\pi}\frac{1}{\omega+\mu+v|p_z'|+i\delta}$$

$$=\int_{-\infty}^\infty \frac{dp_z'}{2\pi}\frac{1}{\omega+\mu-vp_z'+i\delta}=-\frac{i}{2v}.$$

In the last factor of Eq. (29) the operator $\partial/\partial\hat{x}$ is acting on the internal function, and so integration by parts has to be performed.

We will see below that under condition (14) the p_z, entering the matrix elements will be much less than $\sqrt{eH/c}$ and can be neglected. Then the terms with k_x and k_y will be equal, and since the $k_z=p_z-p_z'$ is small, the integral over **k** becomes

$$N_i\int d^2\mathbf{k}(2\pi)^{-2}(k_x^2+k_y^2)|U(\mathbf{k})|^2=N_i\frac{4\pi e^4}{\varepsilon_\infty^2}\ln\varepsilon_\infty$$

[we have substituted formulas (30) and (31)]. Actually, this result is true with logarithmic accuracy. Higher order powers of k_x and k_y would add a term of the order of unity to $\ln\varepsilon_\infty$.

Collecting all the factors together, using Eqs. (22) and (28), and taking into account that

$$\delta(\omega+\mu-v|p_z|)\theta(p_z)+\delta(\omega+\mu+v|p_z|)\theta(-p_z)$$

$$=\delta(\omega+\mu-vp_z)$$

we obtain finally

$$\sigma_{xx}=\sigma_{yy}=\frac{1}{2\pi}\left(\frac{e^2}{\varepsilon_\infty v}\right)^2\ln\varepsilon_\infty\frac{ecN_i}{H}.$$

$$(36)$$

The corresponding component of the resistivity tensor is

$$\rho_{xx}=\frac{\sigma_{xx}}{\sigma_{xx}^2+\sigma_{xy}^2}$$

and σ_{xy} is given by Eq. (23). It is most likely that $(e^2/\varepsilon_\infty v)^2 N_i\ll n_0$, and hence

$$\rho_{xx}=\frac{1}{2\pi}\left(\frac{e^2}{\varepsilon_\infty v}\right)^2\ln\varepsilon_\infty\frac{N_i}{ecn_0^2}H.$$

$$(37)$$

V. DISCUSSION

As we argued in the Introduction, the resistivity (37) is only a part of the total resistivity and is due to regions with a very small concentration of electrons [term aH in Eq. (3)]. The resistivity indeed depends linearly on H. The interesting feature is that it formally does not depend on temperature. From experimental data[1] we see that it, actually, has to depend on temperature, decreasing approximately three times, as the temperature varies from 4.5 to 300 K. This rather small decrease of ρ, instead of a marked increase, demonstrates that it is not due to the appearance of phonon scattering but is most likely associated with the change of doping (n_0). This is qualitatively confirmed by the fact that the decrease with temperature of the observed Hall constant and of the resistivity starts at the same temperature: 60–70 K in $Ag_{2+\delta}Se$, and 100–130 K in $Ag_{2+\delta}Te$. As we argued already, in an inhomogeneous sample with an unknown internal structure the "effective" Hall constant cannot be calculated. The electron concentration per atom obtained from it has to be somewhere between $\delta\sim10^{-2}$ and the limiting value from our estimate (4), i.e., $\sim10^{-10}$. As we mentioned, the experimental value was $\sim10^{-5}$.

The fact that both the resistivity and the Hall constant decrease with temperature correspond qualitatively to formulas (23) and (37). Since the doping mechanism is rather unusual (doping of pure regions by electrons from metallic clusters) it is difficult to predict the temperature dependence of n_0. Most likely, it will grow with temperature but much slower than the familiar exponent; this corresponds to the observations. Since the resistance depends not only on n_0 but also on N_i, it is hard to compare at this stage samples with different δ's.

In our model we made several unusual assumptions. The gapless spectrum with a linear momentum dependence in both bands, making the electrons similar to charged neutrinos, can be checked by diamagnetic (cyclotron) resonance. The main feature of this spectrum is that contrary to more conventional cases, the quantum condition persist to rather small magnetic fields and rather high temperatures. In order to distinguish this phenomenon from other kinds of unusual magnetiresistance phenomena (they all lead to negative magnetoresistance) I suggest that it be called *quantum magnetoresistance*.

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[9] This reminds me of the situation in semimetals. The theory can be constructed, if we start not from a simple cubic metal but from a specific type of gapless semiconductor, which looks quite artificial and definitely has a larger energy. However, it becomes energetically favorable at finite deformations (see Ref. 10).

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