Dynamic dielectric response of an asymmetric double quantum well near the bounding surface of a semi-infinite dynamic plasmalike host medium

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We have examined the dynamic, nonlocal dielectric response function of a pair of asymmetric quantum wells embedded in a semi-infinite dynamic (local) host plasmalike medium, near its bounding surface, by carrying out a closed form inversion of the dielectric function of this system explicitly. The resulting random-phase-approximation (RPA) inverse dielectric function $K(z,z';\mathbf{q},\omega)$, which depends on lateral wave vector $\mathbf{q}$ and frequency ω, is obtained analytically in position representation with z,z' describing distances into the medium from the bounding surface. In this, we neglect intersubband transitions in the two quantum wells (assumed to be thin) and ignore tunneling effects. The frequency poles of $K(z,z';\mathbf{q},\omega)$, describe the collective modes resulting from the coupling of the double quantum well quasi-two-dimensional intrasubband plasmons with the bulk and surface plasmons of the host medium, and the residues at these poles provide the oscillator strengths of such coupled collective modes. [S0163-1829(98)03528-0]

I. RPA INTEGRAL EQUATION FOR A DOUBLE QUANTUM WELL EMBEDDED IN A SEMI-INFINITE MEDIUM

The collective electrostatic normal modes of layered electron gas systems have been the subject of both experimental[1–3] and theoretical studies for some time. In particular, the spectra of double quantum well (DQW) plasmons have been carefully analyzed both with and without a magnetic field.[4–13] Our considerations here are focused on the interaction of such DQW plasmons with the collective modes of a semi-infinite plasmalike host medium in which the quantum-well system is embedded. In this, we will examine the coupling of the DQW plasmons with bulk plasmons of the host and with surface/interface plasmons that are active in the vicinity of the host's bounding surface, on the other side of which lies a different dielectric medium. To carry out this study, we perform a closed form inversion of the dielectric function of the combined system (DQW and semi-infinite host plasma) in position representation. Its frequency poles explicitly show the preferential coupling of the DQW plasmons to the host bulk and surface plasmons as a function of distance from the bounding surface.

The determination of the inverse dielectric function of an asymmetric double quantum well system embedded near the bounding surface of a semi-infinite dynamic host medium is addressed here in terms of the random phase approximation (RPA) integral equation. The system is illustrated in Fig. 1. The quantum wells are in the x-y plane and have center-to-center separation a and widths b,b' in the z direction. The two quantum wells may have different polarizabilities, and we assume that $a \gg b,b'$ so that both overlap and intersubband transitions may be neglected. The RPA integral equation for the inverse dielectric function $K(1,2) = \delta V(1)/\delta U(2)$, which describes the effective potential $V(1)$ at space-time point 1 due to an impressed potential $U(2)$ at space-time point 2, may be written in the form ($1=\vec{\mathbf{r}}_1,\ t_1=\vec{\mathbf{r}}_1,z_1,t_1$, etc.)

$$K(1,2)=\delta(1-2)-\int d3K(1,3)4\pi\alpha(3,2). \tag{1a}$$

Here, $4\pi\alpha(3,2)=-\int d1' v(3,1')R(1',2)$ is the joint polarizability of the combined system of two quantum wells lodged in a semi-infinite plasmalike medium, and $v(3,1')$ is the Coulomb potential and $R(1',2)=G(1',2)G(2,1')$ is the ring diagram density perturbation response function [$G(1',2)$ being the free one-particle thermodynamic Green's function of the system]. Fourier transforming in the translationally invariant x-y plane and in time, $\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2\rightarrow\mathbf{q}$, and $t_1-t_2\rightarrow\omega$, we have (supressing $\mathbf{q},\omega$)

$$K(z_1,z_2)=\delta(z_1-z_2)-\int dz_3K(z_1,z_3)4\pi\alpha(z_3,z_2), \tag{1b}$$

and

$$4\pi\alpha(z_3,z_2)=-\int dz_1 v(|z_1-z_3|)R(z_1,z_2). \tag{2}$$

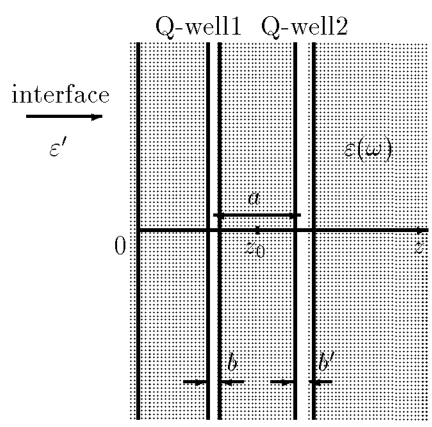

FIG. 1. A pair of asymmetric quantum wells having their centers at distances $z_0 \pm a/2$ from the interface at $z=0$ of the semi-infinite (local) host plasmalike medium of dielectric function $\varepsilon(\omega)$ with an adjoining medium of dielectric constant ε'. The widths of the quantum wells are b and b'.

The local polarizability for a semi-infinite medium was determined in Ref. 14 as (we allow here that the adjoining medium has dielectric constant ε' instead of the vacuum value 1):

$$4\pi\alpha^{\text{semi}}(z_3,z_2)=\delta(z_3-z_2)[\eta_+(z_3)-1]+\delta(z_2)(\varepsilon'-\varepsilon)\eta_-(z_3)e^{-q|z_3|}/2, \quad (3)$$

where we have defined $\eta_-(z)=\theta(z)-\theta(-z)=1,0,-1$ for $z>0,z=0,z<0$, and $\eta_+(z)=\varepsilon\theta(z)+\varepsilon'\theta(-z)=\varepsilon,(\varepsilon'+\varepsilon)/2,\varepsilon'$ for $z>0,z=0,z<0$, respectively. [$\theta(z)$ denotes the Heaviside unit step function, $\theta(z)=1$ for $z>0$, 1/2 for $z=0$, and 0 for $z<0$.] The first, local [$\sim\delta(z_3-z_2)$] term of $4\pi\alpha^{\text{semi}}(z_3,z_2)$ on the right-hand side of Eq. (3) is naively expected on the basis of the differing dielectric properties of the medium across the plane $z_3=0$. The second term [$\sim\delta(z_2)$] assures correct dynamic imaging due to the interface.

Designating coordinates relative to the centers of the quantum wells as $z_\sigma=z-z_0-\sigma a/2$ ($\sigma=\pm1$), the polarizability of the quantum wells may be expressed in terms of the Green's function for a quantum well with n subbands, $G(1,2)=\Sigma_{\alpha=1}^{n}\Sigma_{\sigma=\pm1}\xi_\alpha^\sigma(z_{1\sigma})\xi_\alpha^\sigma(z_{2\sigma})e^{iE_\alpha^\sigma(t_1-t_2)}g_\alpha^\sigma(\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2)$, where ξ_α^σ denotes a real quantum-well subband wave function in z direction across the σ well, E_α^σ denotes the corresponding subband energy. $g_\alpha^\sigma(\vec{\mathbf{r}}_1-\vec{\mathbf{r}}_2)$ is the Schrödinger Green's function for motion on the plane of the quantum well with chemical potential ζ_σ. This yields the ring diagram density perturbation response function for the double quantum well system as $R(z_1,z_2)=\Sigma_{\alpha,\beta}\Sigma_{\sigma,\sigma'}R_{\alpha\beta}^{\sigma\sigma'}\xi_\alpha^\sigma(z_{1\sigma})\xi_\beta^{\sigma'}(z_{1\sigma'})\xi_\alpha^\sigma(z_{2\sigma})\xi_\beta^{\sigma'}(z_{2\sigma'})$ where the matrix element $R_{\alpha\beta}^{\sigma\sigma'}$ of the response function with subband indicies α,β and well indicies σ,σ' is given by

$$R_{\alpha\beta}^{\sigma\sigma'}(\mathbf{q},\omega)=2\sum_{\mathbf{k}}\frac{f_0(\epsilon_{\mathbf{k}}+E_\alpha^\sigma-\zeta_\sigma)-f_0(\epsilon_{\mathbf{k-q}}+E_\beta^{\sigma'}-\zeta_{\sigma'})}{\omega+\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}}+E_\alpha^\sigma-E_\beta^{\sigma'}}. \quad (4)$$

Here, f_0 is the Fermi distribution function $f_0=[e^{(\epsilon-\zeta)/kT}+1]^{-1}$ with chemical potential ζ and $\epsilon_{\mathbf{k}}=\hbar^2\mathbf{k}^2/2m$ refers to the part of single-electron kinetic energy along the quantum-well plane, etc. The DQW polarizability, $4\pi\alpha^{\text{DQW}}$, is thus

$$4\pi\alpha^{\text{DQW}}(z_3,z_2)=-\sum_{\alpha,\beta}\sum_{\sigma,\sigma'}R_{\alpha\beta}^{\sigma\sigma'}V_{\alpha\beta}^{\sigma\sigma'}(z_3)\xi_\alpha^\sigma(z_{2\sigma})\xi_\beta^{\sigma'}(z_{2\sigma'}), \quad (5)$$

where $V_{\alpha\beta}^{\sigma\sigma'}$ is a matrix element of the Coulomb potential,

$$V_{\alpha\beta}^{\sigma\sigma'}(z_1)=\frac{2\pi e^2}{q}\int dz_2 e^{-q|z_1-z_2|}\xi_\alpha^\sigma(z_{2\sigma})\xi_\beta^{\sigma'}(z_{2\sigma'}). \quad (6)$$

Within the framework of the RPA the joint polarizability $4\pi\alpha$ of the combined system is given by the sum of the polarizabilities of the constituent parts (DQW and semi-infinite medium), $4\pi\alpha=4\pi\alpha^{\text{semi}}+4\pi\alpha^{\text{DQW}}$, whence

$$4\pi\alpha(z_3,z_2)=\delta(z_3-z_2)[\eta_+(z_3)-1]+\delta(z_2)(\varepsilon'-\varepsilon)\eta_-(z_3)e^{-q|z_3|}/2-\sum_{\alpha,\beta}\sum_{\sigma,\sigma'}R_{\alpha\beta}^{\sigma\sigma'}V_{\alpha\beta}^{\sigma\sigma'}(z_3)\xi_\alpha^\sigma(z_{2\sigma})\xi_\beta^{\sigma'}(z_{2\sigma'}). \quad (7)$$

Employing this result in Eq. (1b), the RPA integral equation for $K(z_1,z_2)$ takes the form

$$K(z_1,z_2)=\frac{1}{\eta_+(z_2)}\left\{\delta(z_1-z_2)-\delta(z_2)(\varepsilon'-\varepsilon)K(z_1,q)/2+\sum_{\alpha,\beta}\sum_{\sigma,\sigma'}R_{\alpha\beta}^{\sigma\sigma'}\xi_\alpha^\sigma(z_{2\sigma})\xi_\beta^{\sigma'}(z_{2\sigma'})K_{\alpha\beta}^{\sigma\sigma'}(z_1)\right\}, \quad (8)$$

where we have defined $K(z_1,q)$ and $K_{\alpha\beta}^{\sigma\sigma'}(z_1)$ as $K(z_1,q)=\int dz_3\eta_-(z_3)e^{-q|z_3|}K(z_1,z_3)$ and $K_{\alpha\beta}^{\sigma\sigma'}(z_1)=\int dz_3V_{\alpha\beta}^{\sigma\sigma'}(z_3)K(z_1,z_3)$. Our analysis of the integral equation yields $K(z_1,q)$ in terms of $K_{\alpha\beta}^{\sigma\sigma'}(z_1)$ as

$$K(z_1,q)=[\eta_-(z_1)/\eta_+(z_1)]e^{-q|z_1|}+\sum_{\alpha,\beta}\sum_{\sigma,\sigma'}R_{\alpha\beta}^{\sigma\sigma'}I_{\alpha\beta}^{\sigma\sigma'}K_{\alpha\beta}^{\sigma\sigma'}(z_1), \quad (9)$$

where

$$I_{\alpha\beta}^{\sigma\sigma'}=\int dz_2e^{-q|z_2|}\xi_\alpha^\sigma(z_{2\sigma})\xi_\beta^{\sigma'}(z_{2\sigma'})[\eta_-(z_2)/\eta_+(z_2)]=\frac{1}{\varepsilon}\int dz_2e^{-q|z_2|}\xi_\alpha^\sigma(z_{2\sigma})\xi_\beta^{\sigma'}(z_{2\sigma'}), \quad (10)$$

(the DQW subband states are taken to be wholly confined within the bounding surface). Therefore, the integral equation may be rewritten in the form

$$K(z_1,z_2)=\frac{1}{\eta_+(z_2)}\Big\{\delta(z_1-z_2)-\delta(z_2)(\varepsilon'-\varepsilon)\times[\eta_-(z_1)/2\eta_+(z_1)]e^{-q|z_1|}+\sum_{\alpha,\beta}\sum_{\sigma,\sigma'}R_{\alpha\beta}^{\sigma\sigma'}K_{\alpha\beta}^{\sigma\sigma'}(z_1)[\xi_\alpha^\sigma(z_{2\sigma})\xi_\beta^{\sigma'}(z_{2\sigma'})-\delta(z_2)(\varepsilon'-\varepsilon)I_{\alpha\beta}^{\sigma\sigma'}/2]\Big\}. \quad (11)$$

Further analysis of this integral equation yields a matrix equation for $K_{\alpha\beta}^{\sigma\sigma'}(z_1)$ as

$$K_{\mu\nu}^{\sigma''\sigma'''}(z_1)=[1/\eta_+(z_1)]M_{\mu\nu}^{\sigma''\sigma'''}(z_1)+\sum_{\alpha,\beta}\sum_{\sigma,\sigma'}R_{\alpha\beta}^{\sigma\sigma'}N_{\alpha\beta\mu\nu}^{\sigma\sigma'\sigma''\sigma'''}K_{\alpha\beta}^{\sigma\sigma'}(z_1), \quad (12)$$

where we have defined

$$M_{\mu\nu}^{\sigma''\sigma'''}(z_1)=V_{\mu\nu}^{\sigma''\sigma'''}(z_1)-\Gamma\,\eta_-(z_1)e^{-q|z_1|}V_{\mu\nu}^{\sigma''\sigma'''}(0), \tag{13a}$$

$$N_{\alpha\beta\mu\nu}^{\sigma\sigma'\sigma''\sigma'''}=J_{\alpha\beta\mu\nu}^{\sigma\sigma'\sigma''\sigma'''}-\Gamma I_{\alpha\beta}^{\sigma\sigma'}V_{\mu\nu}^{\sigma''\sigma'''}(0), \tag{13b}$$

with

$$J_{\alpha\beta\mu\nu}^{\sigma\sigma'\sigma''\sigma'''}=\int dz_2\,\xi_\alpha^\sigma(z_{2\sigma})\xi_\beta^{\sigma'}(z_{2\sigma'})V_{\mu\nu}^{\sigma''\sigma'''}(z_2)/\eta_+(z_2)$$
$$=\frac{2\pi e^2}{q\varepsilon}\int dz_1\int dz_2\,e^{-q|z_1-z_2|}\xi_\alpha^\sigma(z_{2\sigma})\times\xi_\beta^{\sigma'}(z_{2\sigma'})\xi_\mu^{\sigma''}(z_{1\sigma''})\xi_\nu^{\sigma'''}(z_{1\sigma'''}), \tag{13c}$$

and $\Gamma=(\varepsilon'-\varepsilon)/(\varepsilon'+\varepsilon)$.

II. SOLUTION OF RPA INTEGRAL EQUATION NEGLECTING INTERSUBBAND TRANSITIONS AND TUNNELING

Our considerations here are directed at solving the RPA integral equation in circumstances where we may neglect intersubband transitions in the two quantum wells (assumed to be thin) and tunneling effects are small. The neglect of intersubband transitions may be expressed as $R_{\alpha\beta}^{\sigma\sigma'}\to\delta_{\alpha\beta}R_{\alpha\alpha}^{\sigma\sigma'}$, and the elimination of tunneling effects corresponds to $\xi_\alpha^\sigma(z_\sigma)\xi_\beta^{\sigma'}(z_{\sigma'})\to\delta_{\sigma\sigma'}\xi_\alpha^\sigma(z_\sigma)\xi_\beta^\sigma(z_\sigma)$. With these replacements, Eq. (11) reduces to

$$K(z_1,z_2)=\frac{1}{\eta_+(z_2)}\Big\{\delta(z_1-z_2)-\delta(z_2)(\varepsilon'-\varepsilon)[\eta_-(z_1)/2\eta_+(z_1)]e^{-q|z_1|}+\sum_\alpha\sum_\sigma R_{\alpha\alpha}^{\sigma\sigma}K_{\alpha\alpha}^{\sigma\sigma}(z_1)[\xi_\alpha^\sigma(z_{2\sigma})\xi_\alpha^\sigma(z_{2\sigma})-\delta(z_2)(\varepsilon'-\varepsilon)I_{\alpha\alpha}^{\sigma\sigma}/2]\Big\}, \tag{14}$$

and Eq. (12) becomes

$$K_{\nu\nu}^{\sigma'\sigma'}(z_1)=M_{\nu\nu}^{\sigma'\sigma'}(z_1)/\eta_+(z_1)+\sum_\mu\sum_\sigma R_{\mu\mu}^{\sigma\sigma}N_{\mu\mu\nu\nu}^{\sigma\sigma\sigma'\sigma'}K_{\mu\mu}^{\sigma\sigma}(z_1). \tag{15}$$

The dimension of this matrix equation is twice the number of occupied subbands and it may be solved numerically. However, we can obtain an analytic solution for thin quantum wells, such that $qb,qb'\ll1$. Since σ,σ' each takes values ±1, we distinguish two cases in the evaluation of the integral $J_{\alpha\beta\mu\nu}^{\sigma\sigma'\sigma''\sigma'''}$ [Eq. (13c)] for thin quantum wells: (i) σ and σ' are confined to the same quantum well: Because of thinness, $z_1-z_2\cong b,b'\ll1/q$, the integrals of Eq. (13c) are determined by orthonormality of the subband wave functions (note that the condition $qb,qb'\ll1$ assumes that the exponential $e^{-q|z_1-z_2|}$ varies slowly even if subband wave functions vary rapidly across the quantum well), whence

$$J_{\mu\mu\nu\nu}^{\sigma\sigma\sigma'\sigma'}=\frac{2\pi e^2}{q\varepsilon}e^{-qb}\approx\frac{2\pi e^2}{q\varepsilon}\quad(\sigma=\sigma'),$$

(ii) σ and σ' are different: Translating integration variables to the quantum well centers, ($\tilde z_1=z_1-z_0-\sigma'a/2$; $\tilde z_2=z_2-z_0-\sigma a/2$), "thin" now means that the four wave functions in the integrand of Eq. (13c) amount to $\delta(\tilde z_1)\delta(\tilde z_2)$, whence

$$J_{\mu\mu\nu\nu}^{\sigma\sigma\sigma'\sigma'}=\frac{2\pi e^2}{q\varepsilon}e^{-q|\sigma a/2-\sigma'a/2|}=\frac{2\pi e^2}{q\varepsilon}e^{-qa}\quad(\sigma\neq\sigma').$$

For the two cases jointly we have

$$J_{\mu\mu\nu\nu}^{\sigma\sigma\sigma'\sigma'}=\frac{2\pi e^2}{q\varepsilon}e^{-q|\sigma a/2-\sigma'a/2|}. \tag{16}$$

Similar considerations yield

$$I_{\mu\mu}^{\sigma\sigma}=\frac{1}{\varepsilon}e^{-q(z_0+\sigma a/2)},\quad V_{\nu\nu}^{\sigma'\sigma'}(z_1)=\frac{2\pi e^2}{q}e^{-q|z_1-z_0-\sigma'a/2|}. \tag{17}$$

Within the scope of these approximations we write $M_{\nu\nu}^{\sigma'\sigma'}$ and $N_{\mu\mu\nu\nu}^{\sigma\sigma\sigma'\sigma'}$ in Eq. (13) as

$$M_{\nu\nu}^{\sigma'\sigma'}(z_1)\equiv M^{\sigma'\sigma'}(z_1)=\frac{2\pi e^2}{q}[e^{-q|z_1-z_0-\sigma'a/2|}-\Gamma\,\eta_-(z_1)e^{-q|z_1|}e^{-q|z_0+\sigma'a/2|}], \tag{18a}$$

$$N_{\mu\mu\nu\nu}^{\sigma\sigma\sigma'\sigma'}\equiv N^{\sigma\sigma\sigma'\sigma'}=\frac{2\pi e^2}{q\varepsilon}[e^{-q|\sigma a/2-\sigma'a/2|}-\Gamma e^{-q(z_0+\sigma a/2)}e^{-q|z_0+\sigma'a/2|}]. \tag{18b}$$

It should be noted that, in the thin quantum well limit, neither $M_{\nu\nu}^{\sigma'\sigma'}(z_1)$ nor $N_{\mu\mu\nu\nu}^{\sigma\sigma\sigma'\sigma'}$ depends on subband indicies μ,ν. Subsitution of $M_{\nu\nu}^{\sigma'\sigma'}(z_1)\equiv M^{\sigma'\sigma'}(z_1)$ and $N_{\mu\mu\nu\nu}^{\sigma\sigma\sigma'\sigma'}\equiv N^{\sigma\sigma\sigma'\sigma'}$ of Eq. (18) into Eq. (15) yields

$$K^{\sigma'\sigma'}_{\nu\nu}(z_1)=M^{\sigma'\sigma'}(z_1)/\eta_+(z_1)-\frac{1}{\varepsilon}\sum_{\mu}\sum_{\sigma}4\pi\alpha^{\sigma\sigma}_{\mu\mu}K^{\sigma\sigma}_{\mu\mu}(z_1)[e^{-q|\sigma a/2-\sigma' a/2|}-\Gamma e^{-2qz_0}e^{-q(\sigma a/2+\sigma' a/2)}], \quad (19)$$

where we have defined the dynamic nonlocal 2D polarizability of subband μ in quantum well σ (with chemical potential measured relative to E^{σ}_{μ}) as

$$4\pi\alpha^{\sigma\sigma}_{\mu\mu}\equiv 4\pi\alpha^{\sigma\sigma}_{\mu\mu}(\mathbf{q},\omega)=-\frac{2\pi e^2}{q}R^{\sigma\sigma}_{\mu\mu}(\mathbf{q},\omega). \quad (20)$$

Considering each of the two values of $\sigma'=\pm'1$, the sum over $\sigma=\pm 1$ on the right hand side of Eq. (19) yields

$$K^{++}_{\nu\nu}(z_1)=M^{++}(z_1)/\eta_+(z_1)-c_1\sum_{\mu}4\pi\alpha^{++}_{\mu\mu}K^{++}_{\mu\mu}(z_1)-c_2\sum_{\mu}4\pi\alpha^{--}_{\mu\mu}K^{--}_{\mu\mu}(z_1), \quad (21a)$$

$$K^{--}_{\nu\nu}(z_1)=M^{--}(z_1)/\eta_+(z_1)-c_2\sum_{\mu}4\pi\alpha^{++}_{\mu\mu}K^{++}_{\mu\mu}(z_1)-c_3\sum_{\mu}4\pi\alpha^{--}_{\mu\mu}K^{--}_{\mu\mu}(z_1), \quad (21b)$$

where we have defined

$$c_1=(1-\Gamma e^{-2qz_0}e^{-qa})/\varepsilon, \quad c_2=(e^{-qa}-\Gamma e^{-2qz_0})/\varepsilon, \quad c_3=(1-\Gamma e^{-2qz_0}e^{qa})/\varepsilon. \quad (22)$$

Furthermore, we define a total dynamic, nonlocal 2D polarizability for the quantum well σ by summing Eq. (20) over subband index μ,

$$4\pi\alpha^{\sigma\sigma}_{2\mathrm{D}}\equiv 4\pi\alpha^{\sigma\sigma}_{2\mathrm{D}}(\mathbf{q},\omega)=\sum_{\mu}4\pi\alpha^{\sigma\sigma}_{\mu\mu}(\mathbf{q},\omega). \quad (23)$$

Neglecting intersubband transitions, it is clear that each subband contributes like an independent 2D plasma, populated by a Fermi function having the chemical potential measured relative to E^{σ}_{μ}. To solve Eqs. (21a,b), we multiply Eq. (21a) by $4\pi\alpha^{++}_{\nu\nu}$ and Eq. (21b) by $4\pi\alpha^{--}_{\nu\nu}$ and sum over index ν, obtaining two simultaneous equations for $\Sigma_{\nu}4\pi\alpha^{\pm\pm}_{\nu\nu}K^{\pm\pm}_{\nu\nu}(z_1)$, which have the solutions,

$$\sum_{\nu}4\pi\alpha^{++}_{\nu\nu}K^{++}_{\nu\nu}(z_1)=\frac{4\pi\alpha^{++}_{2D}}{\Delta\,\eta_+(z_1)}[(1+c_3 4\pi\alpha^{--}_{2D})M^{++}(z_1)-c_2 4\pi\alpha^{--}_{2D}M^{--}(z_1)], \quad (24a)$$

$$\sum_{\nu}4\pi\alpha^{--}_{\nu\nu}K^{--}_{\nu\nu}(z_1)=\frac{4\pi\alpha^{--}_{2D}}{\Delta\,\eta_+(z_1)}[(1+c_1 4\pi\alpha^{++}_{2D})M^{--}(z_1)-c_2 4\pi\alpha^{++}_{2D}M^{++}(z_1)]. \quad (24b)$$

Here, Δ is given by

$$\Delta=(1+c_1\,4\pi\alpha^{++}_{2\mathrm{D}})(1+c_3\,4\pi\alpha^{--}_{2\mathrm{D}})-c_2^2\,4\pi\alpha^{++}_{2\mathrm{D}}\,4\pi\alpha^{--}_{2\mathrm{D}}. \quad (25)$$

Combining Eqs. (24a,b) into a single expression, we have

$$\sum_{\nu}4\pi\alpha^{\sigma\sigma}_{\nu\nu}K^{\sigma\sigma}_{\nu\nu}(z_1)=\frac{4\pi\alpha^{\sigma\sigma}_{2\mathrm{D}}}{\Delta\,\eta_+(z_1)}[M^{\sigma\sigma}(z_1)+(c_1\delta_{\sigma-}+c_3\delta_{\sigma+})4\pi\alpha^{-\sigma-\sigma}_{2\mathrm{D}}M^{\sigma\sigma}(z_1)-c_2 4\pi\alpha^{-\sigma-\sigma}_{2\mathrm{D}}M^{-\sigma-\sigma}(z_1)], \quad (26)$$

which we employ in Eq. (19) and Eq. (14) to finally obtain $K(z_1,z_2)$ as

$$K(z_1,z_2)=\frac{1}{\eta_+(z_2)}\Bigg\{\delta(z_1-z_2)-\delta(z_2)(\varepsilon'-\varepsilon)[\eta_-(z_1)/2\eta_+(z_1)]e^{-q|z_1|}+\sum_{\alpha=1}^{n}\sum_{\sigma=\pm 1}4\pi\alpha^{\sigma\sigma}_{\alpha\alpha}\{[\xi^{\sigma}_{\alpha}(z_2-z_0-\sigma a/2)]^2$$
$$-\delta(z_2)(1/2\varepsilon)(\varepsilon'-\varepsilon)e^{-q(z_0+\sigma a/2)}\}\frac{-q}{2\pi e^2}\frac{1}{\eta_+(z_1)}\Bigg(M^{\sigma\sigma}(z_1)-\sum_{\sigma'=\pm 1}\frac{4\pi\alpha^{\sigma'\sigma'}_{2\mathrm{D}}}{\varepsilon\Delta}(e^{-q|\sigma a/2-\sigma' a/2|}$$
$$-\Gamma e^{-2qz_0}e^{-q(\sigma a/2+\sigma' a/2)})[M^{\sigma'\sigma'}(z_1)+(c_1\delta_{\sigma'-}+c_3\delta_{\sigma'+})4\pi\alpha^{-\sigma'-\sigma'}_{2D}M^{\sigma'\sigma'}(z_1)$$
$$-c_2 4\pi\alpha^{-\sigma'-\sigma'}_{2D}M^{-\sigma'-\sigma'}(z_1)]\Bigg)\Bigg\}, \quad (27)$$

where $M^{\sigma\sigma}(z_1)$ is given by Eq. (18). This result reduces properly to known results for the case of a single quantum well $(\varepsilon'\neq\varepsilon)$ (Ref. 14) and for the case of a DQW in a uniform background $(\varepsilon'=\varepsilon)$.[15]

III. COUPLED-MODE DISPERSION RELATION

The dispersion relation for the coupled plasma oscillations of the system illustrated in Fig. 1 is given by the poles of $K(z_1,z_2)$ as

$$\frac{\varepsilon\Delta}{\Gamma}=\frac{1}{\Gamma}\left[\varepsilon+4\pi\alpha_{2D}^{++}+4\pi\alpha_{2D}^{--}+\frac{1}{\varepsilon}4\pi\alpha_{2D}^{++}4\pi\alpha_{2D}^{--}(1-e^{-2qa})\right]-\left[4\pi\alpha_{2D}^{++}e^{-qa}+4\pi\alpha_{2D}^{--}e^{qa}+\frac{2\sinh(qa)}{\varepsilon}4\pi\alpha_{2D}^{++}4\pi\alpha_{2D}^{--}\right]e^{-2qz_0}=0, \tag{28}$$

where a is the separation between the two quantum wells, z_0 is the distance between the center of the two quantum wells and the bounding surface of the host medium. For the local cold plasma limit $4\pi\alpha_{2D}^{\pm\pm}\to-\omega_{2D(\pm)}^2/\omega^2$, where $\omega_{2D(\pm)}^2=2\pi n_\pm e^2q/m_\pm$ is the 2D classical plasma frequency of a single plane sheet of plasma having 2D density $n_\pm$. Also, $\varepsilon=\varepsilon_0-\omega_p^2/\omega^2$, where ε_0 is the background dielectric constant, and $\omega_p^2=4\pi e^2\rho/m$ is the 3D classical plasma frequency of the semi-infinite host plasma, and $\Gamma=(\varepsilon'-\varepsilon)/(\varepsilon'+\varepsilon)$.

In some cases, the roots of Eq. (28) can be obtained analytically (approximately):

(i) For z_0 large, the coupled plasma oscillation roots of Eq. (28) show that the 2D DQW plasmons couple preferentially to the bulk plasmon of the host medium and the surface plasmon is approximately uncoupled. Writing the 2D DQW plasmon dispersion relation in the absence of the background host plasma and in the absence of a bounding surface as (we define $\omega_{2D}^2=\omega_{2D(+)}^2+\omega_{2D(-)}^2$)

$$\omega_\pm^2=\frac{1}{2\varepsilon_0}\{\omega_{2D}^2\pm[(\omega_{2D}^2)^2-4\omega_{2D(+)}^2\omega_{2D(-)}^2(1-e^{-2qa})]^{1/2}\}, \tag{29}$$

the coupling of the modes described by Eq. (28) for large z_0 yields weakly perturbed roots $\Omega_\pm^2$ close to $\omega_\pm^2+\omega_p^2/\varepsilon_0$ and another root Ω_s^2 close to the surface plasmon $\omega_s^2=\omega_p^2/(\varepsilon'+\varepsilon_0)$ as follows (we employ the definition $\omega_{II}^2=\omega_{2D(+)}^2e^{-qa}+\omega_{2D(-)}^2e^{qa}$):

$$\Omega_\pm^2=\left\{\omega_\pm^2+\frac{\omega_p^2}{\varepsilon_0}-e^{-2qz_0}\frac{(\varepsilon_0\omega_\pm^2)[\gamma\omega_\pm^2+(\varepsilon'/\varepsilon_0)\omega_s^2]}{[\omega_\pm^2+(\varepsilon'/\varepsilon_0)\omega_s^2](\omega_\pm^2-\omega_\mp^2)}\left[\omega_{II}^2-2\frac{\omega_{2D(+)}^2\omega_{2D(-)}^2}{\varepsilon_0\omega_\pm^2}\sinh(qa)\right]\right\}, \tag{30a}$$

$$\Omega_s^2=\omega_s^2\left\{1+e^{-2qz_0}\frac{(\varepsilon'\omega_s^2)[2\varepsilon'/(\varepsilon'+\varepsilon_0)]}{[\omega_+^2+(\varepsilon'/\varepsilon_0)\omega_s^2][\omega_-^2+(\varepsilon'/\varepsilon_0)\omega_s^2]}\left[\omega_{II}^2+2\frac{\omega_{2D(+)}^2\omega_{2D(-)}^2}{\varepsilon'\omega_s^2}\sinh(qa)\right]\right\}, \tag{30b}$$

where $\gamma=(\varepsilon'-\varepsilon_0)/(\varepsilon'+\varepsilon_0)$, independent of frequency.

(ii) For z_0 small, near the bounding surface ($z_0=a/2$ is as close as the DQW system can come to the bounding surface without expelling part of the quantum well from the host), the 2D DQW plasmons couple preferentially to the surface plasmon. For $z_0=a/2$, these coupled modes are given by

$$\tilde{\omega}_\pm^2=\frac{1}{2\varepsilon_0}\left\{(\omega_p^2+\omega_{2D(+)}^2)+\left(\frac{\varepsilon_0}{\varepsilon'+\varepsilon_0}\right)(\omega_p^2+2\omega_{2D(-)}^2)-\gamma\omega_{2D(+)}^2e^{-2qa}\pm\left(\left[(\omega_p^2+\omega_{2D(+)}^2)+\left(\frac{\varepsilon_0}{\varepsilon'+\varepsilon_0}\right)(\omega_p^2+2\omega_{2D(-)}^2)-\gamma\omega_{2D(+)}^2e^{-2qa}\right]^2-4\left(\frac{\varepsilon_0}{\varepsilon'+\varepsilon_0}\right)[(\omega_p^2+\omega_{2D(+)}^2)(\omega_p^2+2\omega_{2D(-)}^2)+\omega_{2D(+)}^2(\omega_p^2-2\omega_{2D(-)}^2)e^{-2qa}]\right)^{1/2}\right\}, \tag{31}$$

and there is a third mode at $\tilde{\omega}_p^2=\omega_p^2/\varepsilon_0$. For z_0 near $a/2$, the coupling of the modes described by Eq. (28) for small z_0 yields weakly perturbed roots $\tilde{\Omega}_\pm^2$ close to $\tilde{\omega}_\pm^2$ and another root $\tilde{\Omega}_p^2$ close to the classical plasma frequency of the semi-infinite host plasma as follows:

$$\tilde{\Omega}_\pm^2=\tilde{\omega}_\pm^2\left\{1+(e^{-qa}-e^{-2qz_0})\frac{(\varepsilon'-\varepsilon_0+\omega_p^2/\tilde{\omega}_\pm^2)}{(\tilde{\omega}_\pm^2-\tilde{\omega}_\mp^2)}\left[\omega_{II}^2-2\frac{\omega_{2D(+)}^2\omega_{2D(-)}^2}{\varepsilon_0(\tilde{\omega}_\pm^2-\omega_p^2/\varepsilon_0)}\sinh(qa)\right]\right\}, \tag{32a}$$

$$\tilde{\Omega}_p^2=\frac{\omega_p^2}{\varepsilon_0}\left\{1-(e^{-qa}-e^{-2qz_0})\frac{2(\varepsilon'/\varepsilon_0)\omega_{2D(+)}^2\omega_{2D(-)}^2}{(\tilde{\omega}_+^2-\omega_p^2/\varepsilon_0)(\tilde{\omega}_-^2-\omega_p^2/\varepsilon_0)}\sinh(qa)\right\}. \tag{32b}$$

In both cases (i) and (ii) much of the bulk is located at large distances from both the double-quantum-well system and the bounding surface, and the associated bulk plasmon given by $\varepsilon=0,\omega^2=\omega_p^2/\varepsilon_0$, is clearly evident in a denominator factor ε in the structure of the inverted dielectric function $K(z,z';\mathbf{q},\omega)$, Eq. (27). Moreover, the relative oscillator strengths of these various modes, and their dependencies on z, z', and z_0 may be obtained from the residues of $K(z,z';\mathbf{q},\omega)$ at the corresponding frequency poles.

The explicit analytic dispersion relations Eq. (30) and Eq. (32) are approximate for z_0 large and z_0 small ($z_0 \approx a/2$), respectively. More generally, the exact dispersion relation, Eq. (28), is cubic in ω^2 in the local cold plasma limit,

$$(\omega^2)^3 + b(\omega^2)^2 + c(\omega^2) + d = 0\,, \tag{33}$$

where the coefficients b, c, and d are given by

$$b = -\frac{1}{\varepsilon_0}\{\varepsilon_0\omega_s^2 + 2\omega_p^2 + \omega_{2\mathrm{D}}^2 - \gamma\omega_{\mathrm{II}}^2 e^{-2qz_0}\}, \tag{34a}$$

$$c = \frac{1}{\varepsilon_0^2}\{\varepsilon_0\omega_s^2(2\omega_p^2 + \omega_{2\mathrm{D}}^2) + [\omega_p^2(\omega_p^2 + \omega_{2\mathrm{D}}^2) + \omega_{2\mathrm{D}(+)}^2\omega_{2\mathrm{D}(-)}^2(1 - e^{-2qa})] + [\varepsilon_0\omega_s^2\omega_{\mathrm{II}}^2 - \gamma(\omega_p^2\omega_{\mathrm{II}}^2 + 2\sinh(qa)\,\omega_{2\mathrm{D}(+)}^2\omega_{2\mathrm{D}(-)}^2)]e^{-2qz_0}\}, \tag{34b}$$

$$d = -\frac{\omega_s^2}{\varepsilon_0^2}\{\omega_p^2(\omega_p^2 + \omega_{2\mathrm{D}}^2) + \omega_{2\mathrm{D}(+)}^2\omega_{2\mathrm{D}(-)}^2(1 - e^{-2qa}) + [\omega_p^2\omega_{\mathrm{II}}^2 + 2\sinh(qa)\,\omega_{2\mathrm{D}(+)}^2\omega_{2\mathrm{D}(-)}^2]e^{-2qz_0}\}, \tag{34c}$$

where ω_s^2, $\omega_{2\mathrm{D}}^2$, and ω_{II}^2 were defined above. The three roots of this cubic equation for ω^2 can be obtained either analytically or numerically for arbitrary z_0. The results shown in the figures are based on GaAs/Al$_x$Ga$_{1-x}$As parameter values, $\rho = 2\times 10^{18}$ cm^{-3}, $n_+ = 2\times 10^{12}$ cm^{-2}, $n_- = 1\times 10^{12}$ cm^{-2}, $m_\pm = 0.063 m_e$, $m = 0.084 m_e$, $\varepsilon' = 1.00$, $\varepsilon_0 = 10.33$; $a = 100$ Å; and we normalize q to units of $Q = 0.1\times(2\pi n_-)^{1/2} = 2.5066\times 10^5$ cm^{-1}.

Figure 2 illustrates the dependencies of the three local collective mode frequencies $\omega/(\omega_p/\varepsilon_0^{1/2})$ on z_0 in Å for $q = Q$. The lowest mode changes character from being a decoupled bulk plasmon $\omega^2 \sim \omega_p^2/\varepsilon_0$ near the interface ($z_0 = a/2 = 50$ Å) to being a decoupled surface plasmon $\omega^2 \sim \omega_s^2 = \omega_p^2/(\varepsilon' + \varepsilon_0)$ when the DQW is deep in the host medium ($z_0 \to \infty$). The highest mode is a DQW plasmon coupled to a surface plasmon $\tilde{\omega}_+$ in the vicinity of the interface ($z_0 = a/2 = 50$ Å) and it changes character deep inside for large z_0 ($\to\infty$) to become a DQW plasmon coupled to a bulk plasmon Ω_+. The middle mode represents the other DQW plasmon coupled to the surface plasmon $\tilde{\omega}_-$ at the interface, changing as z_0 becomes large to represent the coupling of a DQW plasmon with the bulk plasmon Ω_-. The middle mode is relatively unperturbed by the variation of z_0, remaining close to bulk plasmon frequency for the parameter ranges considered here.

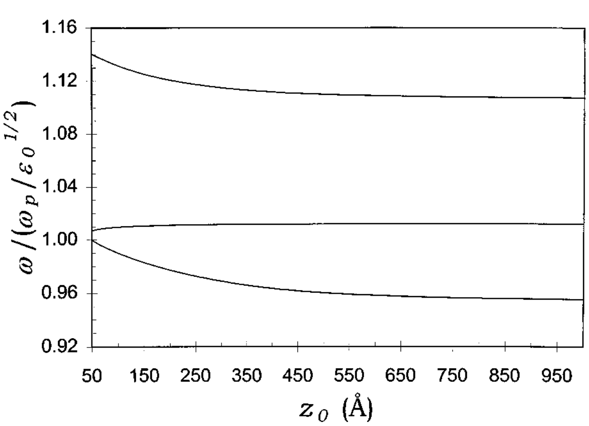

FIG. 2. $\omega/(\omega_p/\varepsilon_0^{1/2})$ for a DQW system as a function of its distance z_0 from the bounding surface of the host plasmalike medium, for $q = Q$. (The parameters are given in the text.)

In Fig. 3, the dispersion of the three modes, $\omega/(\omega_p/\varepsilon_0^{1/2})$, is shown as a function of wave number q/Q for the fixed value $z_0 = 100$ Å. For $q/Q = 1.0$, on the right, the three mode frequencies are just those of Fig. 2 for $z_0 = 100$ Å. On the left, for $q/Q \to 0$, the upper two modes merge to the bulk plasmon frequency, and the lowest mode approaches the surface plasmon frequency, all decoupled from the nonexistent DQW modes, which vanish for $q/Q \to 0$. Mode repulsion is clearly in evidence in the vicinity of $q/Q = 0.15$.

In Figs. 4 and 5, we exhibit the two modes which occur if a perfect metal ($\varepsilon' \to -\infty$) replaces the adjoining medium. The surface plasmon and its possible couplings are eliminated since $\omega_s^2 = \omega_p^2/(\varepsilon' + \varepsilon_0) \to 0$, so there are just two local modes that can be obtained as the roots of a quadratic equation (to which the cubic reduces) as

$$\omega_\pm^2 = \frac{1}{2\varepsilon_0}\{2\omega_p^2 + \omega_{2\mathrm{D}}^2 - \omega_{\mathrm{II}}^2 e^{-2qz_0} \pm [(\omega_{2\mathrm{D}}^2 - \omega_{\mathrm{II}}^2 e^{-2qz_0})^2 - 8\omega_{2\mathrm{D}(+)}^2\omega_{2\mathrm{D}(-)}^2 \sinh(qa)(e^{-qa} - e^{-2qz_0})]^{1/2}\}. \tag{35}$$

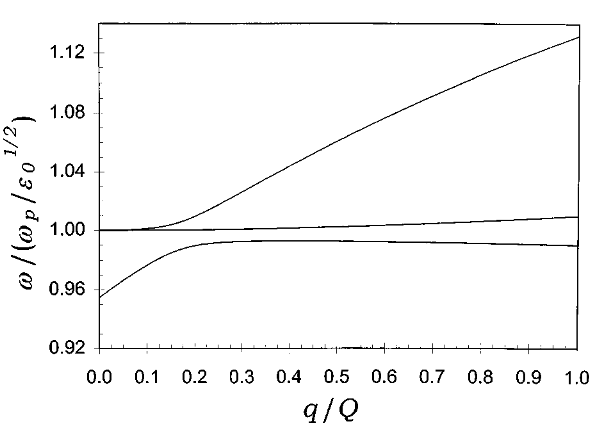

FIG. 3. $\omega/(\omega_p/\varepsilon_0^{1/2})$ for a DQW system as a function of lateral wave number q/Q for $z_0 = 100$ Å.

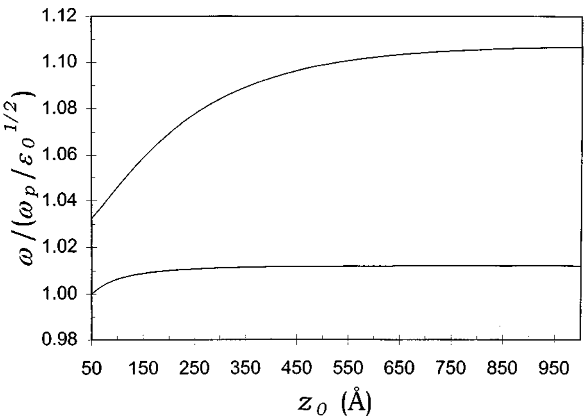

FIG. 4. $\omega/(\omega_p/\varepsilon_0^{1/2})$ for a DQW system as a function of its distance z_0 from the interface of the host plasmalike medium with a perfect metal ($\varepsilon' \to -\infty$), for $q=Q$.

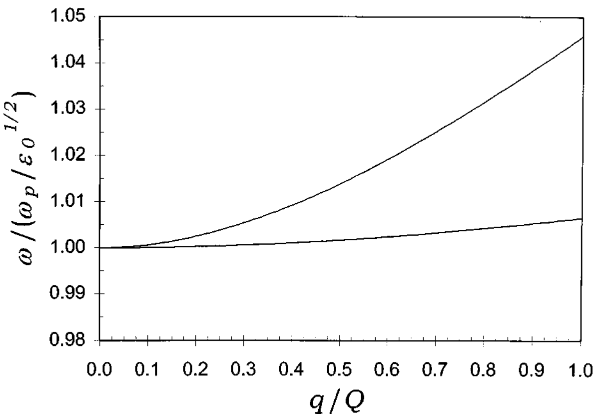

FIG. 5. $\omega/(\omega_p/\varepsilon_0^{1/2})$ for a DQW system as a function of lateral wave number q/Q for $z_0=100$ Å, when the adjoining medium is a perfect metal ($\varepsilon' \to -\infty$).

These modes represent the interaction of the modes of the DQW system, including its image across the perfect metal interface, with the bulk plasmon. In Fig. 4, they are exhibited as functions of z_0 for $q/Q=1$. Deep in the medium ($z_0 \to \infty$), the lower mode is $\Omega_-^2 = \omega_-^2 + \omega_p^2/\varepsilon_0$ (coupling of the lower DQW plasmon to bulk plasmon), and as z_0 approaches the interface ($z_0 \to a/2$) the bulk plasmon decouples, as one should expect. The upper mode, similarly, is $\Omega_+^2 = \omega_+^2 + \omega_p^2/\varepsilon_0$ far from the interface, where the image is not felt: Near the interface it becomes $\tilde{\omega}_+^2$, adjusted for $\varepsilon' \to -\infty$ and $\gamma \to 1$. The coupling of the imaged DQW plasmons with the bulk mode is given explicitly in Fig. 4 for intermediate z_0 values. Finally, the dependence of these mode frequencies on dispersion is shown as a function of wave number in Fig. 5, for $z_0 = 100$ Å. For $q/Q \to 0$, the DQW plasmon frequencies vanish, so both modes converge to the decoupled bulk plasma frequency. For $q/Q=1$, the two modes in Fig. 5 take the values shown in Fig. 4 for $z_0=100$ Å.

IV. SUMMARY

This analysis of the DQW mode spectrum as a function of z_0 explicitly exhibits the transference of DQW mode coupling from the bulk plasmon deep in the medium ($z_0 \to \infty$), to the surface plasmon where the DQW approaches the bounding surface ($z_0 \to a/2$). Our explicit inversion of the inverse dielectric function [Eq. (27)] permits further determination of the relative excitation amplitudes of the various modes as functions of the distance of the DQW system from the interface. This determination can be carried out by simply evaluating the residues of $K(z,z';\mathbf{q},\omega)$ at the frequency poles corresponding to the coupled modes, and can be readily executed for comparision with experimental data as it becomes available.

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