Lattice-gas model for electron-hole coupling in disordered media

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We study an effective lattice-gas model for electron-hole coupling in disordered semiconductor structures. Despite its simplicity, the model turns out to be quite rich. It possesses several crossover regimes between phases of bound electron-hole pairs (excitons) and unbound electrons and holes. It has been shown that a sufficiently strong disorder promotes dissociation of bound electron-hole pairs and may decrease considerably the range of existence of exciton gas. [S0163-1829(98)05948-7]

I. INTRODUCTION

There is considerable research interest in systems of excitons in semiconductor quantum wells (QW's). This interest is connected with the important role excitons play in light-matter interaction processes in light-emitting devices and semiconductor microcavities. Another reason for the research interest is related to a possibility of Bose-Einstein condensation of excitons in QW's.[1] In this respect, special attention is attracted to spatially indirect excitons formed by spatially separated electrons (e) and holes (h).[2] Due to a strongly enhanced annihilation time of spatially indirect excitons, these systems are especially attractive with respect to the search for collective phenomena. There is a rapidly increasing amount of publications devoted to both theoretical[3–6] and experimental[7,8] investigations of indirect e-h coupling; see also references 4–6 and 8. Some experimental evidence has been reported[8] for a stable excitonic ground state in a strong magnetic field, which favors the stability of the excitonic phase.[9]

One of the intrinsic physical problems one meets when dealing with excitons in semiconductor nanostructures is the presence of a disorder that hinders manifestation of collective or coherent effects of exciton-exciton or exciton-light interaction. The disorder corresponds to structure imperfections unavoidable in the course of fabrication. For thin QW's and, especially, in the case of indirect excitons, the disorder is mainly determined by an interface roughness and a thickness variation of QW's. The presence of a moderate disorder results in scattering of excitons and is responsible for relatively small values of the exciton diffusion coefficient. With an increase of the disorder excitons may become localized. [With respect to two-dimensional (2D) systems a more correct statement is that the exciton localization length is becoming small as compared to the system size.] However, even in this ''strong-localization'' regime, typical variations V_0 of the random potential $V(\mathbf{r})$ may be still small with respect to the exciton binding energy E_0, so that the internal structure of excitons is weakly affected by the disorder.

The question addressed in this paper relates to the opposite case when the random potential variation V_0 is greater than the exciton binding energy E_0. This situation may occur in sufficiently thin QW's. For instance, as small as 0.1 nm thickness variation δL in a GaAs QW of an average thickness $L=3$ nm (this corresponds to the experiment[8]) would result in the variation of the electron confinement energy $\pi^2\hbar^2\delta L/(m_e L^3)\sim 30$ meV, which is considerably greater than the exciton binding energy. In general, electrons and holes in semiconductors are subject to different variations of their potential energies, $V_e(\mathbf{r})$ and $V_h(\mathbf{r})$, respectively. Moreover, the correlation between $V_e(\mathbf{r})$ and $V_h(\mathbf{r})$ may be rather weak. The difference in the electron and hole random potentials influences the internal degrees of freedom of an e-h pair. Our task is to investigate under which circumstances this may lead to the exciton dissociation.

An exciton created at an arbitrary place is not in the most favorable—lowest potential energy—state of the e-h pair. At large (as compared to the exciton binding energy) and noncorrelated random-potential variations there is a good chance for the electron and the hole to find lower energy positions that may be quite far from each other, so that at first glance, the e-h pair should dissociate (see Fig. 1). On the other hand, for finite and finite-size correlated potentials, there is always a possibility for the e-h pair to find a position that corresponds to the minimum of both $V_e(\mathbf{r})$ and $V_h(\mathbf{r})$. Electron-hole coupling in the vicinity of this point and the formation of the exciton state leads to an additional lowering of the energy. Thus, speaking about thermodynamically equilibrium states one might come to the opposite conclusion that the disorder may have only a little influence on the existence of excitons. However, the latter reasoning relates to a single exciton. But a single exciton cannot survive even in an ordered macroscopic sample at thermal equilibrium with a nonzero temperature T, it will unavoidably dissociate into an electron and a hole. At $T>0$, only a finite-density gas of bound e-h pairs coexisting with a gas of free electrons and holes may be in the ''ionization equilibrium.''[10] The subject of the present study is the influence of a random potential on the ionization equilibrium condition for e-h pairs. We shall restrict our consideration to the case of low electron and hole densities when we may neglect screening effects.

The quantum-mechanical problem of a particle subjected

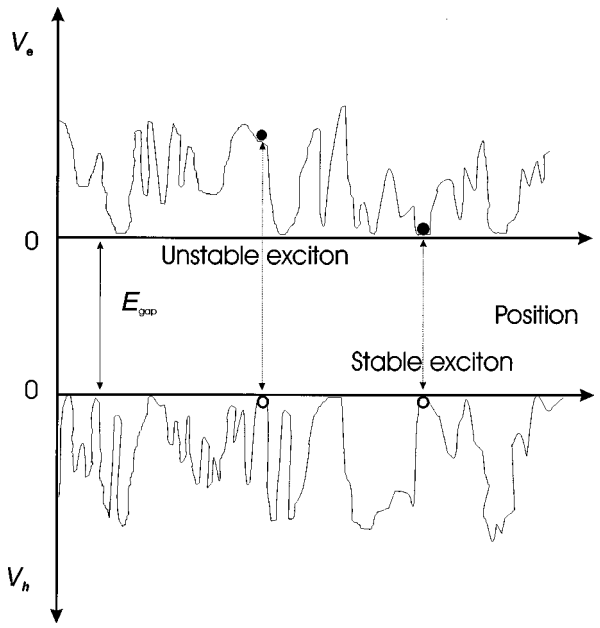

FIG. 1. Electrons and holes in random potentials.

to a strong random disorder is very complicate even for noninteracting particles. The presence of an interaction that is sufficiently strong to form bound states (hence, it cannot be treated perturbatively), makes the above problem even more formidable. To proceed, we restrict our consideration to a simplified model that still has some important features of real systems. The major simplification stems from neglecting the particle kinetic energy. This does not make the model meaningless as long as we consider only thermodynamic equilibrium and not kinetic processes, which lead to the equilibrium. The system considered resembles lattice-gas models. A quantum counterpart of the problem will be accounted for by the Fermi statistics of electrons and holes. We shall demonstrate that even such a simple model has a surprisingly rich behavior characterized by several crossover regimes. We shall show that the presence of disorder may influence strongly the ionization equilibrium condition for *e*-*h* pairs.

II. MODEL HAMILTONIAN

We take the following model Hamiltonian:

$$H=\sum_{\boldsymbol{l};\,a=e,h} V_a(\boldsymbol{l})_a n_a(\boldsymbol{l})-E_0\sum_{\boldsymbol{l}}\, n_e(\boldsymbol{l})n_h(\boldsymbol{l}). \tag{1}$$

Here $n_a(\boldsymbol{l})$ takes values 0 or 1 and is the number of electrons ($a=e$) or holes ($a=h$) occupying the site $\boldsymbol{l}$ of an effective lattice; $E_0>0$ is the binding energy for an electron-hole pair occupying the same lattice site. The local nature of the electron-hole coupling implies that the effective lattice constant should be identified with the exciton size of the underlying quantum-mechanical problem, this size is proportional to an effective Bohr radius of the medium. Our consideration is restricted to temperatures T small compared to E_0, which is a necessary condition for the formation of coupled *e*-*h* states. This allows us to exclude from the system Hilbert space also the states that correspond to the presence of several electrons or holes within the same exciton-size box in the real space (their interaction would be comparable with the effective Rydberg energy $\sim E_0$). Within the model Eq. (1) this restriction is effectively described by the Fermi statistics of electrons and holes occupying the effective lattice sites. The quantities $V_a(\boldsymbol{l})$ are effective random potentials at the space region that corresponds to the effective lattice site $\boldsymbol{l}$ (it is implied that the original physical potentials are either relatively smooth on the microscopic scale or they are averaged over the exciton-size boxes). Assuming the absence of any correlation between the random potentials at different effective lattice sites we arrive at the model that allows an analytical treatment.

We do not take clearly into account the electron-hole annihilation. Instead, we assume that the system is in a contact with electron and hole reservoirs so that the electron-hole system in the sample is described by the grand-canonical ensemble with chemical potentials μ_e and μ_h, respectively. This corresponds to a quasi-equilibrium stationary regime at equal pump and annihilation rates of *e*-*h* pairs. Our aim is to calculate an averaged density n_2 of the bound electron-hole pairs in the limit of small averaged (dimensionless) densities

$$n_a=\left\langle\left\langle \frac{1}{\mathcal{N}}\sum_{\boldsymbol{l}}\, n_a(\boldsymbol{l})\right\rangle\right\rangle \ll 1, \quad a=e,h \tag{2}$$

of electrons and holes. We consider an electron and a hole as a bound pair if they occupy the same lattice site, so that

$$n_2=\left\langle\left\langle \frac{1}{\mathcal{N}}\sum_{\boldsymbol{l}}\, n_e(\boldsymbol{l})n_h(\boldsymbol{l})\right\rangle\right\rangle. \tag{3}$$

Of course, even in the absence of the electron-hole coupling, there is a finite probability to find ''accidentally bound'' states, however their density will be given by the product $n_e n_h$, which is considerably smaller than both n_e and n_h. In Eqs. (2) and (3) $\mathcal{N}$ is the total number of the lattice sites and the averaging is performed both with respect to the thermal distribution and realizations of the disorder:

$$\langle\langle A\rangle\rangle=\int \{dV_e\}\{dV_h\}P(\{V_e,V_h\}) \times\frac{\mathrm{Tr}\{A\exp[-\beta(H-\mu_e N_e-\mu_h N_h)]\}}{\Omega(V_e,V_h)}, \tag{4}$$

where $\beta=1/T$, N_a is the total number of carriers of type a and

$$\Omega(V_e,V_h)=\mathrm{Tr}\{\exp[-\beta(H-\mu_e N_e-\mu_h N_h)]\}. \tag{5}$$

To simplify the following expressions we consider the case of a neutral system with equal electron and hole densities $n_e=n_h\equiv n$ and coinciding distribution functions $P_{\boldsymbol{l}}^e(V)=P_{\boldsymbol{l}}^h(V)\equiv P_{\boldsymbol{l}}(V)$ for the local-random potentials $V_e(\boldsymbol{l})$ and $V_h(\boldsymbol{l})$, with no correlations between them. Under these symmetric conditions we have

$$\mu_e=\mu_h\equiv\mu \tag{6}$$

(the last equality should be applied only to final expressions).

III. BASIC EXPRESSIONS

Due to the local nature of the Hamiltonian Eq. (1) and the random potentials $V_e(\boldsymbol{l})$ and $V_h(\boldsymbol{l})$, the expression for the

averaged thermodynamic potential $\Omega=\langle\Omega(V_e,V_h)\rangle$ factorizes and we obtain

$$\Omega=-\mathcal{N}T\langle\ln[1+e^{-\beta(V_e-\mu_e)}+e^{-\beta(V_h-\mu_h)}+\epsilon e^{-\beta(V_e+V_h-\mu_e-\mu_h)}]\rangle, \tag{7}$$

where

$$\epsilon=\exp(\beta E_0). \tag{8}$$

The remaining averaging in Eq. (1) is performed over the random potentials V_e and V_h. The quantities of interest Eqs. (2) and (3) are given by

$$n=-\frac{1}{\mathcal{N}}\left[\frac{\partial\Omega}{\partial\mu_e}\right]_{\mu_e=\mu_h=\mu}=\left\langle\frac{y\exp(-\beta V_e)+\epsilon y^2\exp[-\beta(V_e+V_h)]}{z(V_e,V_h)}\right\rangle \tag{9}$$

and

$$n_2=-\frac{1}{\mathcal{N}}\left[\frac{\partial\Omega}{\partial E_0}\right]_{\mu_e=\mu_h=\mu}=\left\langle\frac{\epsilon y^2\exp[-\beta(V_e+V_h)]}{z(V_e,V_h)}\right\rangle. \tag{10}$$

Here, we introduced

$$y=\exp(\beta\mu) \tag{11}$$

and

$$z(V_e,V_h)=1+y\exp(-\beta V_e)+y\exp(-\beta V_h)+\epsilon y^2\exp[-\beta(V_e+V_h)]. \tag{12}$$

We shall study a dependence of the system state on the carrier density n so that the value of the chemical potential μ should be found from Eq. (9) and put into Eq. (10). Equations (9) and (10) describe completely the ionization equilibrium between bound and unbound e-h pairs for the model Eq. (1).

IV. ANALYSIS OF THE BASIC EQUATIONS IN PARTICULAR CASES

For further comparison, we describe first two simplest particular cases: (a) the case of an ordered system ($V_e=V_h\equiv 0$) and (b) the case of noninteracting ($E_0=0$) particles in a disordered potential.

A. An ordered system ($V_e=V_h\equiv 0$)

From Eqs. (9) we find the quantity $y=\exp(\beta\mu)$ and put it into Eq. (10). As a result we obtain the following exact connection between the total density of electrons (holes) and the density of bound pairs:

$$n_2=\frac{2\epsilon n^2}{1+2n(\epsilon-1)+\sqrt{(1-2n)^2+4\epsilon n(1-n)}}. \tag{13}$$

We are interested in the case of a low-carrier density $n\ll 1$. Consider some limiting situations. For temperatures T comparable to or exceeding the binding energy E_0, i.e., when the parameter ϵ Eq. (8) is of the order of unity, we find from Eq. (13): $n_2\approx n^2\ll n$, which just corresponds to the density of ''accidentally'' formed electron-hole pairs in the noninteracting system; the interaction plays a negligible role in this trivial ''high-temperature'' limit. We are mostly interested in the opposite, low-temperature case, when $\epsilon\gg 1$. The inequality Eq. (23) is only necessary but still not sufficient condition for an efficient electron-hole coupling. Indeed, in the limit of an ultralow-carrier density $n\ll 1/\epsilon$ we have $n_2\approx\epsilon n^2$. Though the number of bound e-h pairs (excitons) in this case exceeds considerably the number of sites accidentally occupied simultaneously by an electron and a hole ($\epsilon n^2\gg n^2$), the fraction of bound pairs is still relatively small ($n_2/n=\epsilon n\ll 1$).

In the opposite limit $n\gg 1/\epsilon$ the exciton density $n_2\approx n$ coincides with the total density of electrons (holes), which means that most of the carriers are in the bound state. The crossover between the two regimes may be determined by a condition

$$n=1/\epsilon, \tag{14}$$

which separates the two density regimes $n\ll 1/\epsilon$ (unbound pairs) and $n\gg 1/\epsilon$ (bound pairs) at a given temperature obeying $\epsilon\gg 1$. On the other hand, Eq. (14) determines a crossover temperature T_0 for a given density of carriers: $T_0=E_0/\ln(1/n)$. Note that for the considered low-density case $n\ll 1$ this temperature is considerably smaller than E_0. Equation (13) is a particular form of a general ''ionization equilibrium'' condition[10] applied to the model Eq. (1).

B. Noninteracting ($E_0=0$) particles in a disordered potential

Obviously, there are no e-h pairs in this case except those that are associated formally with lattice sites occupied (statistically independently) by an electron and a hole. This is easily seen from Eq. (10), which factorizes into the product $n_2=n_e n_h=n^2$. Equation (9) reduces in this case to the Fermi-distribution function averaged over the disorder

$$n=\int\frac{1}{\exp[\beta(V-\mu)+1]}P(V)\,dV. \tag{15}$$

At this point, we should specify the distribution function $P(V)$ for the site disorder. We are interested in the case when the magnitude V_0 of the typical variation of the disorder potential is considerably greater than the temperature: $\beta V_0\gg 1$. We shall count energies from the minimal value of the disorder potential taken as the origin of the energy scale. The inequality $\beta V_0\gg 1$ means that only the low-energy range of the distribution $P(V)$ contributes to Eq. (15) and the concrete form of the distribution $P(V)$ at $\beta V\gg 1$ is not important. Here we consider the case of a uniformly distributed disorder potential with

$$P(V)=\frac{1}{V_0}\theta(V)\,\theta(V_0-V), \tag{16}$$

where $\theta(x)$ is the step function. Using $\beta V_0\gg 1$ and Eq. (16) we obtain for Eq. (15):

$$n=\frac{1}{\beta V_0}\ln[1+\exp(\beta\mu)], \tag{17}$$

or, respectively, $\mu=(1/\beta)\ln[\exp(\beta V_0 n)-1]$. The density value $n=n_c$,

$$n_c=\frac{\ln 2}{\beta V_0}, \tag{18}$$

separates nondegenerated ($n<n_c$, $\mu<0$) and degenerated ($n>n_c$, $\mu>0$) Fermi gases. This value describes a relative fraction of ''available'' sites, i.e., those with site energies not exceeding the temperature T.

This fulfills our consideration of the limiting physical situations. In the next section, we consider the case of the simultaneous presence of the electron-hole interaction and the disorder.

V. ELECTRON-HOLE COUPLING IN THE PRESENCE OF A DISORDER

To perform averaging in Eqs. (9) and (10) over the random-potential distribution Eq. (16) for electrons and holes it is convenient to introduce new integration variables $x_1=\exp(-\beta V_e)$ and $x_2=\exp(-\beta V_h)$. We obtain

$$n=\frac{y}{(\beta V_0)^2}\int\int_{x_0}^{1}\frac{[1+\epsilon y x_2]\,dx_1 dx_2}{x_2[1+y(x_1+x_2)+\epsilon y^2 x_1 x_2]} \tag{19}$$

and

$$n_2=\frac{\epsilon y^2}{(\beta V_0)^2}\int\int_{x_0}^{1}\frac{dx_1 dx_2}{1+y(x_1+x_2)+\epsilon y^2 x_1 x_2}, \tag{20}$$

where $x_0=\exp(-\beta V_0)\ll 1$. In Eq. (20) we may straightforwardly take the limit $x_0\to 0$, which corresponds to neglecting exponentially small contributions from the high-energy part of the disorder distribution Eq. (16). However, one should be more careful with the part that corresponds to the first term in the nominator of Eq. (19). This part contains a singularity at small x_2 so that the corresponding contribution should be extracted before taking the above limit $x_0\to 0$. As a result, we find the following expressions for the density of carriers

$$n=\frac{\ln(1+y)}{\beta V_0}+\frac{y(\epsilon-1)}{(\beta V_0)^2}\int_0^1\frac{dx}{(1+yx)(1+\epsilon yx)}\times\{\ln[1+y+xy(1+\epsilon y)]-\ln[1+yx]\} \tag{21}$$

and for the density of bound pairs:

$$n_2=\frac{y\epsilon}{(\beta V_0)^2}\int_0^1\frac{dx}{1+\epsilon yx}\ln\left[\frac{1+y+xy(1+\epsilon y)}{1+yx}\right]. \tag{22}$$

These equations determine completely the ionization equilibrium between bound and free carriers for the model Eq. (1) in the presence of both interaction and disorder. The only assumption, which has been made in the derivation of Eqs. (21) and (22) is the inequality $x_0=\exp(-\beta V_0)\ll 1$. In the limiting case of vanishing interaction [$\epsilon=\exp(\beta E_0)\to 1$] the system (21) and (22) reduces to Eqs. (17) and $n_2=n^2$, respectively. Our task is to study the solution to Eqs. (21) and (22) for the range of parameters given by

$$n\ll 1,\quad 1\ll\epsilon,\quad 1\ll\beta V_0. \tag{23}$$

VI. ANALYTICAL CONSIDERATION

For an analytical treatment of Eqs. (21) and (22), we shall assume the case of a strong disorder $E_0\ll V_0$ and reduce our consideration to the case of a nondegenerate system of carriers when $y\ll 1$. It may be shown that for the considered range of parameters the system (21) and (22) takes a simplified form

$$n=\frac{y}{\beta V_0}+\frac{1}{(\beta V_0)^2}F(\epsilon y^2), \tag{24}$$

$$n_2=\frac{1}{(\beta V_0)^2}F(\epsilon y^2), \tag{25}$$

where

$$F(x)=\int_0^x\frac{\ln(1+\xi)}{\xi}\,d\xi; \tag{26}$$

$F(x)\approx x$ at $x\ll 1$; and $F(x)\approx(1/2)\ln^2(x)$ at $x\gg 1$.

Note that the second term on the rhs of Eq. (24) coincides with the density n_2 of bound pairs Eq. (25), therefore, the first term $y/(\beta V_0)$ should be interpreted as the density $n_1=n-n_2$ of unbound carriers. Our task is to investigate various regimes described by Eqs. (24) and (25). We are interested in the dependence of n_2 on n. Thus, we should find a solution $y=y(n)$ to Eq. (24) for a given n and put this value into Eq. (25). Because Eq. (24) does not allow an analytical solution in a general form, we have to restrict ourselves to a qualitative analysis of limiting situations. Below we shall consider consecutively different cases; the results of this somewhat long consideration are summarized at the end of this section.

(1) First, we suppose, that $\epsilon y^2\ll 1$, so that $y\ll 1/\sqrt{\epsilon}$ [the opposite case $\epsilon y^2\gg 1$ will be studied in (2) below]. With the use of the expansion $F(\epsilon y^2)\approx\epsilon y^2$, Eq. (24) reduces to a quadratic equation for y with a solution:

$$y=\beta V_0\frac{\sqrt{1+4\epsilon n}-1}{2\epsilon}, \tag{27}$$

which leads to the following expression for n_2 Eq. (25):

$$n_2=\frac{1}{4\epsilon}[\sqrt{1+4\epsilon n}-1]^2. \tag{28}$$

We should provide the compatibility of Eq. (27) with the above assumption $y\ll 1/\sqrt{\epsilon}$. We shall do it in two opposite limiting situations, $\epsilon n\ll 1$ [item (1A)] and $\epsilon n\gg 1$ [item (1B)].

(1A) At $\epsilon n\ll 1$ we have $y\approx\beta V_0 n$, and $n_2\approx\epsilon n^2$, so that $n_2/n\approx\epsilon n\ll 1$. This case ($n\ll 1/\epsilon$) corresponds to a system of unbound carriers. The condition $y\ll 1/\sqrt{\epsilon}$ leads to an additional restriction for $n\ll 1/(\sqrt{\epsilon}\beta V_0)$, thus we require $n\ll\min\{1/\epsilon,\ 1/(\sqrt{\epsilon}\beta V_0)\}$. Comparison of the two terms in the curly brackets shows that $1/\epsilon\ll 1/(\sqrt{\epsilon}\beta V_0)$ at $T\ll T_c$, and on the contrary, $1/\epsilon\gg 1/(\sqrt{\epsilon}\beta V_0)$ at $T\gg T_c$, where

$$T_c\equiv\frac{E_0}{2\ln(V_0/E_0)}. \tag{29}$$

The appearance of a new characteristic crossover temperature T_c Eq. (29) is a peculiar feature of the considered disordered system.

Thus, we have the following range for carrier densities which corresponds to the considered case (1A) of *unbound carriers*: $n \ll 1/\epsilon$ at $T \ll T_c$; and $n \ll 1/(\sqrt{\epsilon}\beta V_0)$ at $T_c \ll T$.

(1B) At $\epsilon n \gg 1$ we have $y \approx \beta V_0 \sqrt{n/\epsilon}$, and $n_2 \approx n$. This case corresponds to a system of bound pairs. The condition $y \ll 1/\sqrt{\epsilon}$ leads to the restriction $n \ll 1/(\beta V_0)^2$, so that together with the inequality $\epsilon n \gg 1$ we have: $1/\epsilon \ll n \ll 1/(\beta V_0)^2$. To be compatible, the latter double inequality requires $1/\epsilon \ll 1/(\beta V_0)^2$, which leads to $T \ll T_c$. Hence, we have the following range for carrier densities which corresponds to the considered case (1B) of *bound pairs*: $T \ll T_c$; $1/\epsilon \ll n \ll 1/(\beta V_0)^2$.

(2) Suppose now that $\epsilon y^2 \gg 1$, so that Eq. (24) looks like

$$n = \frac{y}{\beta V_0} + \frac{\ln^2(\epsilon y^2)}{2(\beta V_0)^2} = n_1 + n_2. \tag{30}$$

The solution for y should obey

$$1/\sqrt{\epsilon} \ll y \ll 1. \tag{31}$$

To solve the transcendental Eq. (30), below we consider the following limiting situations: $n_2 \ll n_1$ [item (2A)] and $n_2 \gg n_1$ [item (2B)].

(2A) Assume $n_2 \ll n_1$, hence $n_1 \approx n$ (the regime of unbound carriers). This means that $y = \beta V_0 n$ and the above range for y corresponds to the following interval for n:

$$1/(\sqrt{\epsilon}\beta V_0) \ll n \ll 1/(\beta V_0). \tag{32}$$

Besides this restriction, the solution for y should also obey the above assumption $n_2 \ll n_1 \approx n$, which means

$$(1/2)\ln^2[\epsilon(\beta V_0 n)^2] \ll n(\beta V_0)^2. \tag{33}$$

Further analysis splits into two variants, depending on the temperature range ($T_c \ll T$ or $T \ll T_c$).

(i) $T_c \ll T$, which means that $\epsilon \ll (\beta V_0)^2$. In this case we have an estimate for the logarithm argument: $1 \ll \epsilon(\beta V_0 n)^2 \ll (\beta V_0)^4 n^2$, i.e., the lhs of Eq. (33) is greater than 1 but less than $2\ln[n(\beta V_0)^2]$. Therefore, Eq. (33) is equivalent to an inequality $1 \ll n(\beta V_0)^2$, i.e., $1/(\beta V_0)^2 \ll n$. Together with Eq. (32) this leads to an inequality: $\max\{1/(\sqrt{\epsilon}\beta V_0), 1/(\beta V_0)^2\} \ll n \ll 1/(\beta V_0)$. At $T_c \ll T$ the first term in the curly brackets prevails, so that we find the density range for the realization of the *unbound* carrier regime: $T_c \ll T$, $1/(\sqrt{\epsilon}\beta V_0) \ll n \ll 1/(\beta V_0)$.

(ii) $T \ll T_c$, which means that $(\beta V_0)^2 \ll \epsilon$. In this case, Eq. (33) may be represented as

$$\{\ln[\sqrt{\epsilon}/(\beta V_0 n)] + \ln[n(\beta V_0)^2]\}^2 \ll n(\beta V_0)^2, \tag{34}$$

where the argument of the first logarithm in the curly brackets is large. Therefore, we may neglect the second term $\ln[n(\beta V_0)^2]$ in the curly brackets in Eq. (34) as compared to a greater quantity $n(\beta V_0)^2$. Hence, we arrive at an inequality $2\{\ln[\sqrt{\epsilon}/(\beta V_0)]\}^2/(\beta V_0)^2 \ll n$, which must be fulfilled in addition to Eq. (32). Note that for the considered range $T \ll T_c$ the quantity on the lhs of the new inequality is greater than the quantity $1/(\sqrt{\epsilon}\beta V_0)$ on the lhs of Eq. (32), so that the resulting inequality looks like

$$\frac{2\ln^2[\sqrt{\epsilon}/(\beta V_0)]}{(\beta V_0)^2} \ll n \ll 1/(\beta V_0). \tag{35}$$

In the considered limit $[T \ll T_c \to (\beta V_0)^2 \ll \epsilon]$, the left term in Eq. (35) may be approximated as $2\{\ln[\sqrt{\epsilon}/(\beta V_0)]\}^2/(\beta V_0)^2 \ll n \approx 2[\ln(\sqrt{\epsilon})]^2/(\beta V_0)^2 + O[(\ln(\beta V_0)/(\beta V_0)^2] \approx E_0^2/(2V_0^2)$. The compatibility of two inequalities in Eq. (35) reads as $E_0^2/V_0^2 \ll 1/(\beta V_0)$, which is valid under condition $T_m \ll T$, where

$$T_m = \frac{E_0^2}{2V_0} \tag{36}$$

is a new characteristic temperature of the system. Evidently, $T_m \ll T_c$. Thus, an additional range where the carriers are mostly *unbound*, is determined by the following conditions: $T_m \ll T \ll T_c$, $E_0^2/(2V_0^2) \ll n \ll 1/(\beta V_0)$.

(2B) We continue to analyze the case $\epsilon y^2 \gg 1$ but here we shall study the limit $n_1 \ll n_2 \approx n$ [which is opposite to one considered in (2A)]. In this regime of bound pairs ($n_2 \approx n$) we have from Eq. (30): $2n(\beta V_0)^2 = \ln^2(\epsilon y^2)$, so that

$$y = \frac{\exp\sqrt{n(\beta V_0)^2/2}}{\sqrt{\epsilon}}. \tag{37}$$

This quantity must obey the restriction Eq. (31), as well as the assumed inequality $n_1 = y/(\beta V_0) \ll n_2 \approx n$, i.e., $y \ll n\beta V_0$. As follows immediately from Eqs. (31) and (37), values of n are restricted to the interval

$$1/(\beta V_0)^2 \ll n \ll \epsilon/(\beta V_0)^2, \tag{38}$$

while the inequality $y \ll n\beta V_0$ may be represented in the form: $\sqrt{n}\beta V_0 \ll \ln[\epsilon(\beta V_0 n)^2]$. Similar to what has been done in the previous section, we represent the rhs of the last inequality as $\ln[\sqrt{\epsilon}/(\beta V_0)] + \ln[n(\beta V_0)^2]$ and neglect the second term as compared to a much greater value $\sqrt{n}\beta V_0$ on the lhs of the inequality. The latter takes the form: $\sqrt{n}\beta V_0 \ll \ln[\sqrt{\epsilon}/(\beta V_0)]$, which means, in particular, that $1 \gg \epsilon/(\beta V_0)$, i.e., $T \ll T_c$. Thus, we arrive at the following inequality:

$$n \ll \frac{\ln^2[\sqrt{\epsilon}/(\beta V_0)]}{(\beta V_0)^2} \approx \frac{E_0^2}{2V_0^2}, \tag{39}$$

where the approximation is in neglecting a small ratio $\sim \ln^2(\beta V_0)/(\beta V_0)^2$. It is easily seen that Eq. (39) imposes a stronger restriction than the right inequality in Eq. (38), therefore, we obtain the following conditions for the realization of the considered case (2B) of the *bound pairs* regime: $T \ll T_c$, $1/(\beta V_0)^2 \ll n \ll E_0^2/(2V_0^2)$.

Summary of the analytical analysis

Combining the results of the above analytical considerations, we obtain the following qualitatively different scenarios shown schematically in Fig. 2.

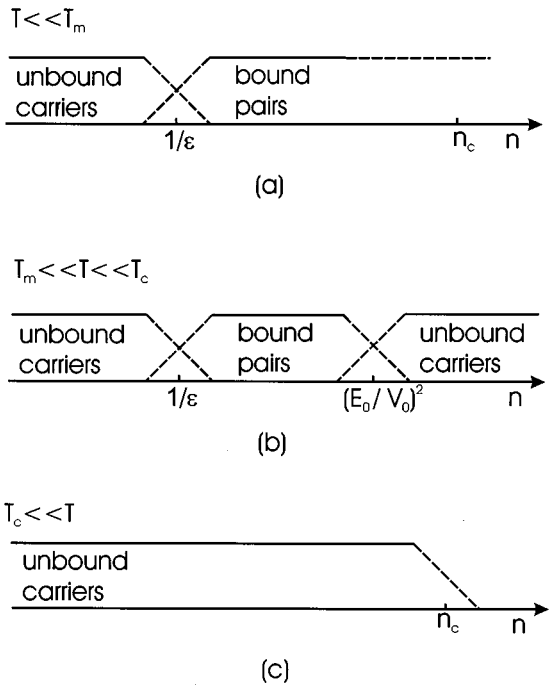

FIG. 2. Phases of the electron-hole system in the presence of disorder.

At very small temperatures $T \ll T_m$, the situation is similar to one considered in the Sec. IV A for an ordered system: the crossover between the regimes of free carriers and bound pairs is governed by the relation $\epsilon n \sim 1$, Fig. 2(a). The presence of the disorder in this case results only in the restriction of the temperature interval ($T < T_m$).

At intermediate temperatures $T_m \ll T \ll T_c$, we find that the regime of bound pairs exists only within an intermediate range of carrier densities $[1/\epsilon \ll n \ll (E_0/V_0)^2]$, while outside this density range the carriers are mostly unbound, Fig. 2(b).

At relatively high temperatures $T_c \ll T$ ($\ll 1/E_0$), the regime of bound pairs does not exist at all (if the density still obeys the requirement $n \ll 1$), Fig. 2(c).

We see, that the presence of disorder strongly influences the ionization equilibrium between bound *e-h* pairs and unbound carriers in the considered model. An explanation and discussion of the obtained results will be done in the next section.

VII. INTERPRETATION OF THE RESULTS

Despite the simplicity of the considered lattice-gas model Eq. (1), the system possesses a quite rich behavior, which is characterized by several regimes with corresponding crossover temperatures. Our present task is to give a physical explanation to the obtained results. For this aim we introduce several classes of lattice sites.

A class ''c'' of sites where the site potential for an electron (V_e) *or* for a hole (V_h) does not exceed temperature T. For the distribution Eq. (16) the density $2n_c$ of these sites is estimated according to $n_c = T/V_0$ [compare with Eq. (18)].

A more narrow class ''b'' of sites where the site potentials for an electron (V_e) *and* for a hole (V_h) do not exceed temperature T. The density n_b of these sites is $n_b = n_c^2 = (T/V_0)^2 \ll n_c$.

A class ''a'' of sites where the $V_e + V_h - E_0 < 0$. The density n_a of these sites is estimated as $n_a = (1/2)(E_0/V_0)^2$.

For the considered temperature range ($T \ll E_0 \ll V_0$) the density n_a of a sites is considerably greater than n_b. However the relation between n_a and n_c depends on the temperature—as is easily seen, at $T \ll T_m (T \gg T_m)$ $n_c \ll n_a$ ($n_c \gg n_b$). This explains the physical origin of the crossover temperature T_m (36). Consider subsequently the temperature intervals corresponding to Fig. 2.

$T \ll T_m (\ll T_c)$, Fig. 2(a). The range of ultralow carrier densities $n \ll n_b$ corresponds to the situation where the carriers are distributed mostly over the b sites (of the lowest energy). As the potential variation on b sites is of the order of T, i.e., small compared to the exciton binding energy E_0, this case corresponds to the limit of an ordered system and therefore the crossover between the regimes of free carriers and of bound pairs is governed by the relation $n \sim 1/\epsilon$ [see Fig. 2(a)]. With an increasing carrier density n, all the b sites become filled with bound pairs and a and c classes enter the game. As occupied with *e-h* pairs a sites are energetically more preferable than c sites, and $n_a \gg n_c$, most of the carriers fill a sites forming bound pairs. This lasts until all the a sites are filled with *e-h* pairs. The presence of a relatively low density of c sites does not influence the distribution of carriers. This qualitative scenario explains completely the phase states at Fig. 2(a).

$T_m \ll T \ll T_c$, Fig. 2(b). The range of ultralow-carrier densities $n \ll n_b = n_c^2$ does not differ from that considered in the previous item. However, at higher densities we deal with quite a different situation due to the fact that now $n_a \ll n_c$. This means that when a sites are occupied with bound pairs, a further increase of the carrier density results in filling single-particle c sites. As $n_a \ll n_c$, the density range $n_a \ll n \ll n_c$ corresponds to the situation when an overwhelming majority of carriers sit at c sites; thus, the regime of unbound carriers ($n_2 \ll n$) is recovered, Fig. 2(b).

($T_m \ll$) $T_c \ll T$ ($\ll E_0$), Fig. 2(c). In this temperature range $n_b \ll 1/\epsilon$, hence the low-density regime $n \leqslant n_b$ corresponds to unbound carriers. This regime will take place also for intermediate densities $n_b \ll n \ll n_a$. To prove this statement, we make the opposite assumption, i.e., that this density range corresponds to the regime of bound pairs, and come to a contradiction. Compare a statistical weight $W_p \sim \epsilon$ of a pair, which occupies one of the low-lying a sites, and a statistical weight $W_s \sim (n_s/n)^2$ of two charges on sites that form a shell of energetic width $\Delta E = E_0 \sqrt{n/n_a}$, the density n_s of these sites is $n_s = \Delta E / V_0$ [the above estimate for ΔE follows from an expression for the fraction of a sites occupied with pairs of density n: $n/n_a = (\Delta E)^2/E_0^2$]. The assumption $W_p/W_s \gg 1$ is equivalent to $1/\epsilon \ll n$. In the considered density range $n < < n_a$ this would imply that $1/\epsilon \ll n_a = E_0^2/(2V_0^2)$, which contradicts the considered inequality $T_c \ll T$. Therefore, at $T_c \ll T$ the intermediate density range $n_b \ll n \ll n_a$ also corresponds to the regime of unbound carriers. And finally, for densities $n_a \ll n$, when the density n_a of sites occupied with pairs is small compared to the total density n of carriers, the regime corresponds, evidently, to unbound carriers. Thus, the temperature interval $T_c \ll T$ corresponds to unbound carriers for all (low) densities, at which the system of carriers remains nondegenerate; Fig. 2(c).

Here we have presented a simple physical explanation for the scenarios and crossover parameters found in the qualitative analysis of the equations of state, which has been carried out in the previous section.

VIII. CONCLUSION

We have presented an effective lattice-gas model of electron-hole coupling in disordered semiconductor structures. This model is based on the assumption of a thermal (quasi) equilibrium in the electron-hole gases, therefore, it is mainly applied to systems of spatially separated electrons and holes (or indirect excitons) where electron-hole annihilation processes are strongly suppressed and the exciton lifetime is much longer than in bulk samples. Despite its simplicity, the model turns out to be quite rich. In particular, it possesses several crossover regimes characterized by the corresponding temperatures, such as T_m (36) and T_c (36). This is a peculiar feature of the considered disordered system.

It has been shown that the disorder promotes dissociation of bound electron-hole pairs (excitons) and may decrease considerably the range of existence of exciton gas. This finding might be relevant for interpreting recent experiments on indirect excitons in thin $GaAs/Al_xGa_{1-x}As$ quantum wells. As is known[8] effects of disorder are quite important in these systems; in particular, there is a drastic reduction of the exciton diffusion coefficient. An additional warning, which stems from the above analysis is that even the presence of exciton phase itself in strongly disordered nanostructures should be a subject of an examination.

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[1] See, e.g., *Bose-Einstein Condensation*, edited by A. Griffin, D.W. Snoke, and S. Stringari (Cambridge University Press, Cambridge, England, 1995).

[2] Yu.E. Lozovik and V.I. Yudson, Pis'ma Zh. Éksp. Teor. Fiz. **22**, 556 (1975) [JETP Lett. **22**, 274 (1975)]; Zh. Éksp. Theor. Fiz. **71**, 738 (1976) [Sov. Phys. JETP **44**, 389 (1976)].

[3] X.M. Chen and J.J. Quinn, Phys. Rev. Lett. **67**, 895 (1993); H.C. Tso, P. Vasilopoulos, and F.M. Peeters, *ibid.* **70**, 2146 (1993); L. Świerkowski, J. Szymanski, and Z.W. Gortel, *ibid.* **74**, 3245 (1995); X. Zhu, P.B. Littlewood, M.S. Hybertsen, and T.M. Rice, *ibid.* **74**, 1633 (1995); M. Alatalo, M.A. Salmi, P. Pietiläinen, and T. Chakraborty, Phys. Rev. B **52**, 7845 (1995).

[4] L. Liu, L. Świerkowski, D. Neilson, and J. Szymanski, Phys. Rev. B **53**, 7923 (1996).

[5] G. Vignale and A.H. MacDonald, Phys. Rev. Lett. **76**, 2786 (1996).

[6] V.I. Yudson, Phys. Rev. Lett. **77**, 1564 (1996).

[7] T. Fukuzawa, E.E. Mendez, and J.M. Hong, Phys. Rev. Lett. **64**, 3066 (1990); J.E. Golub, K. Kash, J.P. Harbison, and L.T. Florez, Phys. Rev. B **41**, 8564 (1990); J.A. Kash, M. Zachau, E.E. Mendez, J.M. Hong, and T. Fukuzawa, Phys. Rev. Lett. **66**, 2247 (1991); U. Sivan, P.M. Solomon, and H. Shtrikman, *ibid.* **68**, 1196 (1992); B.E. Kane, J.P. Eisenstein, W. Wegscheider, L.N. Pfeiffer, and K.W. West, Appl. Phys. Lett. **65**, 3266 (1994); J-P. Cheng, J. Kono, B.D. McCombe, I. Lo, W.C. Mitchel, and C.E. Stutz, Phys. Rev. Lett. **74**, 450 (1995).

[8] L.V. Butov, A. Zrenner, G. Abstreiter, G. Böhm, and G. Weimann, Phys. Rev. Lett. **73**, 304 (1994).

[9] I.V. Lerner and Yu.E. Lozovik, Solid State Commun. **23**, 453 (1977); Zh. Éksp. Theor. Fiz. **80**, 1488 (1981) [Sov. Phys. JETP **53**, 763 (1981)]; Y. Kuramoto and C. Horie, Solid State Commun. **25**, 713 (1978).

[10] L.D. Landau and E.M. Lifshitz, *Statistical Physics*, 3rd ed., edited by E.M. Lifshitz and L.P. Pitaevskii (Pergamon Press, New York, 1980), P. I.