Universality in the crossover between edge-channel and bulk transport in the quantum Hall regime

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We present a theoretical approach for the integer quantum Hall effect, which is able to describe the inter-plateau transitions as well as the transition to the Hall insulator. We find two regimes (metalliclike and insulatorlike) of the top Landau level, in which the dissipative bulk current appears in different directions. The regimes are separated by a temperature-invariant point. [S0163-1829(98)01447-7]

Even more than 15 years after the discovery of the integer quantum Hall effect (IQHE) in two-dimensional electronic systems,[1] the nature of the transitions between adjacent QH plateaus is still a controversial question. Already years ago Kivelson, Lee, and Zhang[2] developed a global-phase diagram, which maps out insulator and QH liquid phases onto a magnetic field-disorder plane. However, a quantitative modeling of the complete transport regime of the IQHE has not been given so far to our knowledge. While the quantized values of the Hall resistance are well described by the edge channel (EC) picture,[3] it is widely believed that the EC picture is insufficient to describe also the transport regime between the IQHE plateaus. The ongoing studies of the inter-plateau transitions are particularly stimulated by recent results of the work on the Hall insulator (HI).[4–6] Using QHE samples with not too low disorder, the HI regime is entered directly from the $\nu=1$ IQHE regime without observing the fractional QHE. An analysis of the transport ranging from the HI to the adjacent HQ liquid regime suggests the existence of a close relation between the transport behavior in the two regimes:[6] By defining a critical filling factor ν_c it is possible to distinguish two regimes, that are coupled by the relation $\rho_{xx}(\Delta\nu)=1/\rho_{xx}(-\Delta\nu)$, where $\Delta\nu$ is the filling factor relative to ν_c. Another important experimental fact is the existence of a critical longitudinal resistivity ρ_{xx}^{c}, which appears at the transition point from the QH liquid to the HI regime.[4] This critical point ν_c is indicated by the crossing of the temperature dependent ρ_{xx} traces and the value of ρ_{xx}^{c} was found to be close to h/e^2. Using a tensor-based analysis of the experimental data, Shahar *et al.*[7] have been able to extract the contribution of the top Landau level (LL) (referred to as ρ_{xx}^{top}) to the total ρ_{xx} in the transition regime between the first and second QH plateau. They found that ρ_{xx}^{top} shows the same behavior as ρ_{xx}^{ins} in the HI regime, namely, a monotonous increase with increasing magnetic field without any peaklike behavior. It was possible to collapse all temperature-dependent traces onto each other by plotting ρ_{xx}^{ins} as well as ρ_{xx}^{top} with respect to $(\nu-\nu_c)T^{-\kappa}$ using the same $\kappa=0.45$. Another experimental fact is that ρ_{xy} remains quantized on the $\nu=1$ plateau also in the HI regime below the critical filling factor, while ρ_{xx}^{ins} already steeply rises. Furthermore, Shahar *et al.* demonstrated that the conductivity components in the HI regime ($\sigma_{xx}^{\mathrm{ins}}$ and $\sigma_{xy}^{\mathrm{ins}}$) as well as the extracted components for the top LL ($\sigma_{xx}^{\mathrm{top}}$ and $\sigma_{xy}^{\mathrm{top}}$) for the $1\rightarrow 2$ plateau transition fulfill a semicircle relation ($\sigma_{xx}^2+\sigma_{xy}^2\propto\sigma_{xy}$).[8]

In this paper, we show that by a modification of the Landauer-Büttiker formalism,[3] which is based on an alternative formulation of backscattering, it is possible to model the full IQHE behavior, i.e., the Hall plateaus as well as the transport regime between them. Several of the above referenced experimental facts can be modeled without assuming a particular function for the dependence of the backscattering on the filling of the Landau levels. Introducing an exponential function for this dependence, further details of the experimental findings are reproduced.

Several attempts for modeling a four-terminal experiment with a discrete backscattering barrier or a disordered region between ideal conductors have been made already.[3,9,10] As an example, for the case of a discrete backscattering barrier in a four-terminal arrangement Büttiker[3,9] obtains $R_{xx}=(h/e^2)[R/(NT)]$, where N is the number of channels, R and T are the reflection and transmission coefficients of the barrier. The Landauer-Büttiker formalism is completely general, where the transmitting channels are not necessarily edge channels. However, for the explanation of the quantized Hall resistance values Büttiker introduces the EC picture. We also use this picture[11] as an input to our model in which we use an alternative representation of backscattering. Details of our approach can be obtained from Ref. 12. The essence of Ref. 12 is that the coupling of the states at opposite edges via backscattering leads to a coupling between R_{xx} and R_{xy} according to $R_{xx}=PR_{xy}$, where P is a backscattering parameter.[13] For the case of a *single* LL, which is represented by a single pair of EC's, we substitute R_{xy} by h/e^2:

$$R_{xx}=P\frac{h}{e^2}. \tag{1}$$

Equation (1) agrees perfectly with Büttiker's above result: The factor R/T can be identified with P. Since we have $N=1$ and $T=1-R$, a variation of R between 0 and 1 corresponds to a variation of P between 0 and ∞, which is the appropriate range for our model.

For a standard QH system, as e.g., in $Al_xGa_{1-x}As/GaAs$, backscattering appears only in the top LL in the regime between plateaus, while the transport in the lower LL's remains dissipationless. For a transport model in the EC picture one

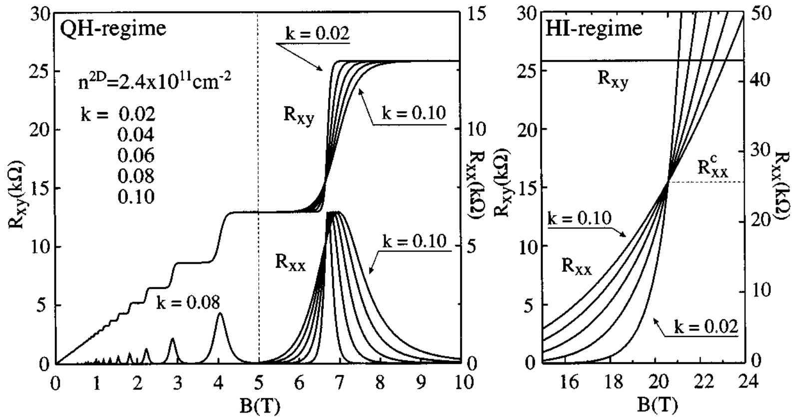

FIG. 1. R_{xx} and R_{xy} calculated according to Eqs. (4a) and (4b) for a sheet carrier density of $n^{2D}=2.4\times10^{11}$ cm^{-2} and different factors k in the exponent of $P(\Delta\nu)$. The range below $B=5$ T shows just the traces for $k=0.08$, the range above $B=5$ T shows the traces for all different k values as given in the figure. The HI regime is shown separately with a different R_{xx} scale on the right.

has therefore to combine one pair of EC's with nonzero backscattering ($P>0$) and a set of EC pairs without backscattering ($P=0$). R_{xx} and R_{xy} of the complete system must finally result from the current distribution between both EC systems.[14] For treating these parallel systems we use the components of the conductance tensor (G_{xx} and G_{xy}), which can be obtained from the components of the resistance tensor (R_{xx} and R_{xy}) by the well-known relation $G_{xx,yx}=R_{xx,xy}(R_{xx}^2+R_{xy}^2)^{-1}$. The use of this equation means that we restrict our analysis to the case of a symmetric behavior where $R_{yy}=R_{xx}$. In comparison to classical transport this corresponds to the case of a quadratically shaped conductor. Consequently the equations are formally identical with the equations for the resistivities ρ_{xx},ρ_{xy} and conductivities σ_{xx},σ_{xy}. By use of Eq. (1) we get for the top LL,

$$G_{xx}^{\mathrm{top}}=\frac{e^2}{h}\,\frac{P}{1+P^2}. \tag{2}$$

Due to the absence of backscattering in the lower LL's we have $G_{xx}^{\mathrm{low}}=0$ and therefore the total G_{xx} is given by G_{xx}^{top}. In an analogous way we calculate the Hall components

$$G_{xy}^{\mathrm{top}}=\frac{e^2}{h}\,\frac{1}{1+P^2}, \tag{3a}$$

$$G_{xy}^{\mathrm{low}}=\frac{e^2}{h}\,\overline{\nu}, \tag{3b}$$

where $\overline{\nu}$ is the number of filled LL's below the top LL. The total Hall conductance G_{xy} is given by the sum of Eqs. (3a) and (3b). Now, using $R_{xx,xy}=G_{xx,yx}(G_{xx}^2+G_{xy}^2)^{-1}$ we obtain

$$R_{xx}=\frac{h}{e^2}\,\frac{P}{(\overline{\nu}+1)^2+(\overline{\nu}P)^2}, \tag{4a}$$

$$R_{xy}=\frac{h}{e^2}\left[\overline{\nu}+\frac{1}{1+P^2}\right]\left[\overline{\nu}^2+\frac{2\overline{\nu}+1}{1+P^2}\right]^{-1}, \tag{4b}$$

where the backscattering parameter P depends on the partial filling ν^{top} of the top LL. Even without knowing yet the function $P(\nu^{\mathrm{top}})$, one can directly see that Eqs. (2) and Eq. (3a) fulfill the semicircle relation $G_{xx}^2+G_{xy}^2\propto G_{xy}$, which is valid also for the complete system. It was experimentally found to be valid for the top LL as well as for the HI regime.[7]

It is widely accepted that in the plateau transition regions backscattering in the top LL enables bulk conduction if the Fermi level (E_F) is near the center of the broadened LL. In the bulk a transition to an insulating state occurs if E_F moves out of the center on either side.[15] However, in a model which is based on the EC picture, a symmetry in E_F with respect to the center of the top LL cannot exist: By starting with E_F above a completely filled top LL, EC's are formed and the transport can be described by the EC picture without backscattering. With E_F approaching the center of the LL, dissipation because of backscattering becomes possible and finally with E_F moving below the center of the top LL the associated pair of EC's disappears. We will show that despite this asymmetric behavior of the top LL, the overall behavior of the total system can be still symmetric. Considering Eq. (2) one can see that G_{xx} is proportional to P for $P\ll1$, while it changes to a reciprocal dependence on P for $P\gg1$. For a symmetric G_{xx} we have to look for a suitable monotonous function $P(\nu^{\mathrm{top}})$, which is able to produce such a symmetric behavior around ν_c. To get perfect symmetry, the form of Eq. (2) requires a function that fulfills the relation $P(\Delta\nu)=1/P(-\Delta\nu)$ with $\Delta\nu=\nu^{\mathrm{top}}-\nu_c$. It is easily seen, that the only function which is also in agreement with the experimental observation[6] is of the form[16]

$$P(\Delta\nu)=\exp(-\Delta\nu/k) \tag{5}$$

with k being a constant but possibly temperature-dependent factor. Since the maximum of G_{xx} is identified with the center of the top LL, ν_c corresponds to half filling. From Fig. 1 one can see that the calculation based on Eqs. (4a), (4b), and (5) reproduces very well the typical traces known from the experimental curves at different temperatures.

Already without needing the particular function of Eq. (5) we get a number of important results. We can consider two regimes that are divided by the point at which $P(\Delta\nu)=1$: The regime $0<P<1$ corresponds to E_F above the center of the top LL, while $P>1$ corresponds to E_F below the center of the top LL. Figures 2(a) and 2(b) show schematically the situation in the two regimes: While E_F moves towards the

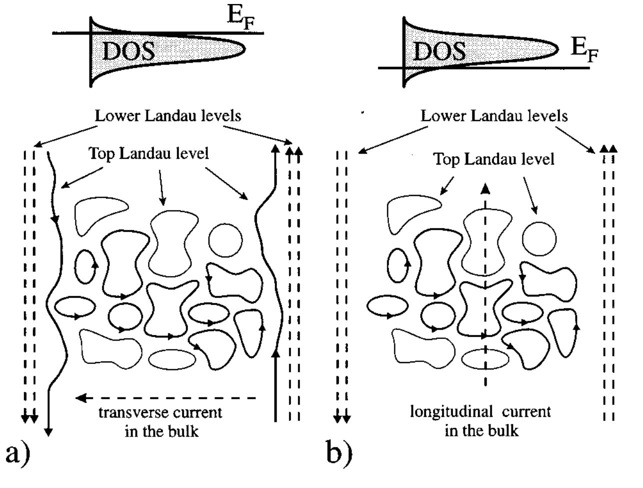

FIG. 2. (a) Edge channel conduction in the top LL in the presence of localized magnetic bound states. The transport across the loops appears as a transverse current, which acts as a backscattering process. (b) Conduction in the top LL in the presence of localized magnetic bound states but in absence of an associated EC. In contrast to the situation sketched in (a), the transport across the loops appears now as a longitudinal current. The EC's of the lower LL's are indicated by the dashed arrows. The relative position of the Fermi level with respect to the LL is indicated at the top of the figure.

center of the broadened top LL [Fig. 2(a)], localized magnetic bound states are created in the bulk region in addition to the associated pair of EC's. Therefore, some transport across those loops by tunneling becomes possible, which finally enables backscattering. According to Eq. (1), R_{xx}^{top} is directly proportional to the backscattering rate in this regime. For describing this type of transport in the bulk region, basically a network model such as, e.g., that one of Chalker and Coddington[17] would be suitable. A situation with E_F below the LL center is schematically shown in Fig. 2(b) with one major difference to Fig. 2(a), namely, that the associated EC pair is not present, while the transport mechanism in the bulk itself may remain the same. Consequently the transport in the bulk does no longer act as a coupling between opposite edges, but may contribute now via a current in the longitudinal direction instead. Characterizing the dissipative transport through the bulk by a conductivity σ_{bulk}, we get basically $R_{xx}^{\mathrm{top}} \propto \sigma_{\mathrm{bulk}}$ for E_F above the LL center and $R_{xx}^{\mathrm{top}} \propto \sigma_{\mathrm{bulk}}^{-1}$ for E_F below the LL center. Consequently, any influence of an eventually existing temperature dependence of σ_{bulk} on the longitudinal transport properties must appear with opposite sign in the two regimes. This implies that there must be a crossover of the two regimes, where the temperature dependence of R_{xx} is canceled. In this way our model indicates correctly the existence of metalliclike and insulatorlike regimes. One can also interpret the two regimes as two different phases of the top LL with perpendicular directions of the dissipative bulk current. This is a striking agreement with Ruzin and Feng,[8] who also found that for a correct description of the transport behavior the bulk current directions in both phases must be perpendicular to each other. It is easily found that the critical point in the crossover regime occurs at $P=1$. According to Eq. (1) this means that at the critical point R_{xx}^{top} approaches the quantized value h/e^2. $P=1$ also means that for the transport in a single LL $G_{xx}=G_{xy}=0.5e^2/h$, in agreement with Ref. 18.

In Ref. 7 also the R_{xx} peak between the first and second plateau has been analyzed. It has been found that the maximum value is $h/4e^2$, while R_{xx}^c at the critical point appears is $h/5e^2$. In our model the critical point appears at $P=1$, for which we get a value of $R_{xx}^c = h/5e^2$, in agreement with Ref. 7. Considering the maximum of Eq. (4a) for $\overline{\nu}=1$, we find $P=2$, which leads to $R_{xx}^{\max} = h/4e^2$, also in agreement with Ref. 7.

Using the particular function $P(\Delta\nu)$ according to Eq. (5), we can go a step further: With the help of Eq. (1) we obtain $R_{xx}^{\mathrm{top}} = (h/e^2)\exp(-\Delta\nu/k)$, which is a monotonous function and covers both regimes $P>1$ and $P<1$. Now, we can also consider the principal behavior of σ_{bulk} in the tails of the LL ($P \gg 1$ and $P \ll 1$) by using $\sigma_{\mathrm{bulk}} \propto R_{xx}^{\mathrm{top}}$ for $\Delta\nu>0$ and $\sigma_{\mathrm{bulk}} \propto 1/R_{xx}^{\mathrm{top}}$ for $\Delta\nu<0$. As generally expected for pure bulk transport, we obtain a symmetric function around the LL center $\sigma_{\mathrm{bulk}}(\Delta\nu) \propto \exp(-|\Delta\nu|/k)$. Thus, it is demonstrated, that our model provides the correct framework to include also dissipative bulk transport.

The experimental evidence for the nonsymmetric transport behavior of R_{xx}^{top} comes with E_F in the lowest LL ($\overline{\nu}=0$, see Eq. (4a)]. There R_{xx} is identical to R_{xx}^{top} and increases monotonically with decreasing filling factor. This is exactly the regime of the HI, which has been experimentally very well investigated already: R_{xx}^{ins} has been indeed found to be monotonously increasing without any peak behavior and R_{xy} stays at the quantized value h/e^2,[7] in agreement with Eq. (4b) for $\overline{\nu}=0$. Therefore, we can interpret the behavior in the HI regime to be a direct consequence of the asymmetric transport behavior of a single LL. Since in our model the transition to the HI as well as the interplateau transitions are described by the same function $P(\Delta\nu)$, the experimentally observed equivalent behavior of R_{xx}^{top} and R_{xx}^{ins} (Ref. 7) is an inherent property of our model.

The fact, that the temperature dependence disappears at a certain point, suggests that the temperature T enters only the factor k in the exponent of Eq. (5). Moreover, $\Delta\nu=0$ in Eq. (5) means that $P=1$ and therefore $R_{xx}^c = h/e^2$ [Eq. (4a) for $\overline{\nu}=0$], in agreement with Ref. 4. This is also evident from Fig. 1, where the traces cross each other at $R_{xx}^c = h/e^2$ (at $B=20$ T).

A widely used basis for the discussion of experimental data is the plot of the ρ_{xx} peak width ΔB as a function of temperature. In this context we analyze the width of the G_{xx} peak, which is described in Eq. (2): $G_{xx} \propto 1/(P+1/P)$ is symmetric in P with respect to $P=1$ and the maximum appears at $P=1$. On the basis of this symmetry we choose a point on each side of the G_{xx} maximum. The associated values of the backscattering function are $P_1=W$ and $P_2=1/W$, respectively, with W being a constant, except unity. We can write $P_1=\exp(\Delta\nu_w/k)$ and $P_2=\exp(-\Delta\nu_w/k)$ and obtain $P_1/P_2 = W^2 = \exp(2\Delta\nu_w/k)$, where $2\Delta\nu_w$ can be identified as the width of the G_{xx} peak on the filling-factor scale. This results in $\ln(W^2) = 2\Delta\nu_w/k = \mathrm{const}$, which means that the temperature dependence of $2\Delta\nu_w(T)$ and $k(T)$ must be the same, regardless of any particular temperature dependence $k(T)$. The fact that all experimentally obtained traces of R_{xx}^{ins} and R_{xx}^{top} of Ref. 7 collapse onto a single trace, if plotted with respect to $(\nu-\nu_c)T^{-\kappa}$, suggest that the argument of the exponential function should have the form $\alpha(\nu-\nu_c)T^{-\kappa}$ with α being a constant. However, as evident from above, also an alternative temperature dependence $k(T)=\alpha+\beta T$, which has been suggested recently by Shahar *et al.*,[19] can be used.

In summary, we have presented a model for the IQHE with a novel representation of backscattering, which is shown to be in agreement with the general form of the Landauer-Büttiker formalism. It successfully describes the transport regime also between IQHE plateaus. Even though we use the edge channel approach for the IQHE as an input for our model, the results are more general and the model provides the correct framework to include also dissipative bulk transport. Quite a number of well-known facts can be obtained without needing the particular function of backscattering versus Landau level filling $P(\Delta\nu)$: (i) the semicircle relation between σ_{xx} and σ_{xy} for the complete QH regime as well as for the Hall insulator (HI) regime, (ii) the critical value $\rho_{xx}^{c}=h/e^2$ in the HI regime, (iii) the value $\sigma_{xx}=\sigma_{xy}=0.5e^2/h$ at the critical point for a single Landau level, (iv) the maximum value $\rho_{xx}^{\mathrm{max}}=h/4e^2$ for the 1-2 transition, (v) the critical value $\rho_{xx}^{c}=rh/5e^2$ for the 1-2 transition. Using an exponential function for $P(\Delta\nu)$ we obtain further: (vi) the validity of the relation $\rho_{xx}(\Delta\nu)=1/\rho_{xx}(-\Delta\nu)$ between the HI and the adjacent QH-liquid regime, (vii) the equivalence of the temperature scaling of ρ_{xx} in the HI regime, of ρ_{xx} of the top LL and of the ρ_{xx}-peak width. (viii) regarding the temperature dependence [using any $k(T)$ monotonously increasing with T], the model indicates correctly the existence of metalliclike $(P<1)$ and insulator like $(P>1)$ regimes.

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[14] In what follows we present an analytical version of our model. The results are fully in agreement with those of a numerical model which does not use the tensor relations. In the numerical model the sample current is allowed to flow via the two parallel EC systems, which are connected at the metallic contacts only. The potential differences at the contacts are then obtained in an iterative way from current conservation considerations. The numerical version of our model is able to give correct results for nonlocal contact configurations as well (considered for publication elsewhere).
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