3*e* tunneling processes in a superconducting single-electron tunneling transistor

P. Hadley, E. Delvigne, E. H. Visscher, S. Lähteenmäki, and J. E. Mooij
Applied Physics and DIMES, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands
(Received 11 June 1998)

A current due to a tunneling event that involves three times the charge of an electron was observed in the current-voltage characteristics of a superconducting single-electron tunneling transistor. In this tunnel event, a Cooper pair tunnels through one tunnel barrier simultaneously with a quasiparticle that tunnels through a second tunnel barrier which is about 0.5 μm distant from the first tunnel barrier. This current was observed in a bias regime where current flow due to sequential quasiparticle tunneling is forbidden due to the Coulomb blockade. [S0163-1829(98)01347-2]

A superconducting single-electron tunneling (SET) transistor consists of a small superconducting island that is coupled to three leads, a gate and two output leads.[1] The two output leads are connected to the island by tunnel junctions and the gate is capacitively coupled to the island. The quantum nature of this device is manifested in the periodic modulation of the current that flows through the output leads as the charge on the gate is varied. One modulation period corresponds to adding one electron charge e to the island. By monitoring the current, one can make very sensitive measurements of the charge at the gate. The charge sensitivity of a SET transistor in the superconducting state is better than the charge sensitivity of a SET transistor in the normal state, which makes the superconducting SET transistor the most sensitive device now available for measuring charge.[2,3]

The characteristics of a superconducting SET transistor depend on the relative magnitudes of three energies: the charging energy E_C, the Josephson energy E_J, and the superconducting gap Δ.[4] The charging energy is the energy associated with charging the island with a single electron charge, $E_C=e^2/(2C_\Sigma)$. Here C_Σ is the total capacitance of the island. The Josephson energy is related to the junction critical current I_c, $E_J=\hbar I_c/2e$, and the superconducting gap can be seen as the addition energy that is required for a superconducting island to have an odd number of electrons rather than an even number of electrons.[5] In devices with large junctions, the Josephson energy is much larger than the charging energy, $E_J \gg E_C$, and a supercurrent is observed. As the junctions are made smaller, E_J decreases while E_C increases. When $E_J \approx E_C$, the supercurrent can be modulated by applying a voltage to the gate, while for $E_J \ll E_C$ the supercurrent is suppressed. If $\Delta > E_C > E_J$, parity effects are observed.[6] It is then possible to determine if the number of electrons on the island is an odd or even number. In the present experiment $\Delta \approx E_C \gg E_J$ and no supercurrent was observed.

The SET transistor studied consisted of two $Al/AlO_x/Al$ tunnel junctions that were fabricated by shadow evaporation. The two junction capacitances were $C_1=1.78\times10^{-16}$ F and $C_2=2.10\times10^{-16}$ F, the gate capacitance was $C_g=1.07\times10^{-18}$ F, the total resistance of the device was $R_1+R_2=1.7\times10^6$ Ω, the superconducting gap was $\Delta=203$ μeV, and the charging energy was $E_C=206$ μeV. Under normal operating conditions, the current that flows through a superconducting SET transistor is primarily due to the sequential tunneling of normal quasiparticles. However, at low bias voltages, the tunneling of individual quasiparticles is suppressed by a combination of the Coulomb blockade and the absence of states in the superconducting gap. At these low bias voltages, other transport mechanisms can be observed such as cotunneling,[7] the Josephson-quasiparticle cycle,[8,9] Andreev reflection,[10] the resonant tunneling of Cooper pairs,[11] and singularity matching.[12] Here we report the experimental observation of a current that flows due to the simultaneous tunneling of a Cooper pair and a quasiparticle. The Cooper pair and the quasiparticle simultaneously tunnel through two different tunnel barriers that are spaced about 0.5 μm from each other.

The thresholds for the various tunnel events that occur in a SET transistor can be determined by examining the electrostatic energy of the circuit. To calculate the change in electrostatic energy when an electron tunnels, one can treat the circuit as a network of capacitors. It is convenient to also treat the voltage sources as capacitors with very large capacitances. At the end of the calculation the limit of very large capacitance for the voltage sources is taken. Figure 1 shows the equivalent capacitor network for an asymmetrically biased SET transistor. The electrostatic energy of this network of capacitors is the sum of the electrostatic energies of the capacitors,

$$E=\frac{1}{2}C_1V^2+\frac{1}{2}C_2(V-V_b)^2+\frac{1}{2}C_g(V-V_g)^2+\frac{1}{2}C_bV_b^2+\frac{1}{2}C_bV_g^2. \quad (1)$$

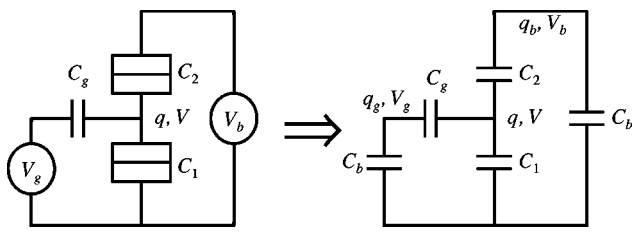

FIG. 1. The equivalent capacitor network used to calculate the electrostatic energy of a SET transistor.

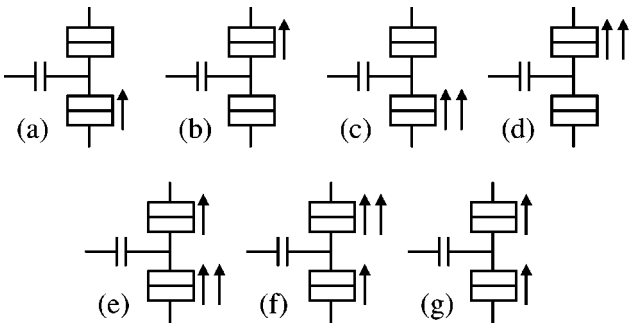

FIG. 2. Nine tunnel processes were observed in the experiment. Each arrow indicates that a charge of e has passed through that junction.

Taking the derivatives of the electrostatic energy with respect to the three voltages (V,V_b,V_g) yields a set of three coupled equations which can be written in the form $q_i = \partial E/\partial V_i = \Sigma_j C_{ij} V_j$. Here q_i are the charges on the islands and C_{ij} is the capacitance matrix. The electrostatic energy of the circuit can then be rewritten as $E = \frac{1}{2}\Sigma_{i,j} C_{ij}^{-1} q_i q_j$.[13] This form was used to calculate the change in electrostatic energy when charge tunneled. Figure 2 illustrates the tunnel events that were considered. Each arrow indicates that a charge of e has passed through that tunnel junction. In the limit $C_b \gg C_1, C_2, C_g$, the changes in the electrostatic energies are

$$\delta E = \frac{e}{C_\Sigma}\left[\frac{e}{2} - ne - q_0 - C_2 V_b - C_g V_g\right], \tag{2a}$$

$$\delta E = \frac{e}{C_\Sigma}\left[\frac{e}{2} + ne + q_0 - (C_1 + C_g) V_b + C_g V_g\right], \tag{2b}$$

$$\delta E = \frac{2e}{C_\Sigma}[e - ne - q_0 - C_2 V_b - C_g V_g], \tag{2c}$$

$$\delta E = \frac{2e}{C_\Sigma}[e + ne + q_0 - (C_1 + C_g) V_b + C_g V_g], \tag{2d}$$

$$\delta E = \frac{e}{C_\Sigma}\left[\frac{e}{2} - ne - q_0 - (C_1 + 2C_2 + C_g) V_b - C_g V_g\right], \tag{2e}$$

$$\delta E = \frac{e}{C_\Sigma}\left[\frac{e}{2} + ne + q_0 - (2C_1 + C_2 + 2C_g) V_b + C_g V_g\right]. \tag{2f}$$

Equations (2a)–(2f) correspond to the tunnel event illustrated in Figs. 2(a)–2(f). The changes in electrostatic energy can be used to construct a stability diagram for the superconducting SET transistor as shown in Fig. 3. Each line in Fig. 3 represents the threshold for a certain tunnel process. The position of the threshold is dependent on the number of electrons on the island n. This results in a periodic stability diagram with a periodicity e. The lines which are determined by the tunneling of charge only through junction 1 [Figs. 2(a) and 2(c)] have a slope of $-C_g/C_2$. The lines which are determined by the tunneling of charge only through junction 2 [Figs. 2(b) and 2(d)] have a slope of $C_g/(C_1 + C_g)$. The threshold determined by the tunneling of $3e$ of charge as

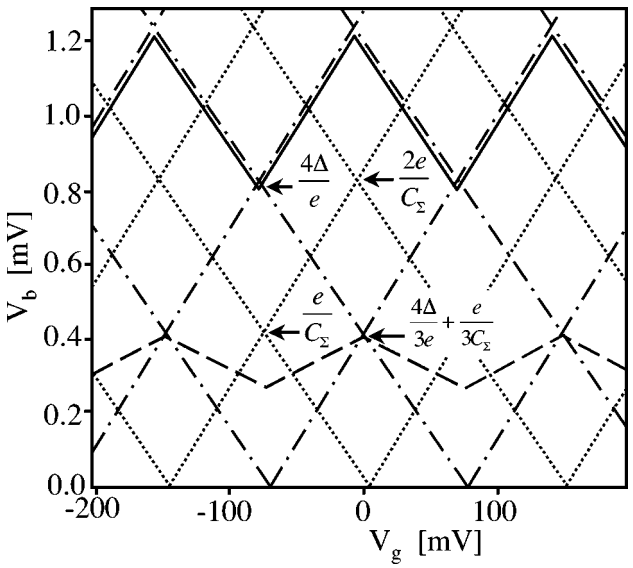

FIG. 3. The thresholds for various tunnel processes in a superconducting SET transistor as a function of gate voltage and bias voltage. The solid line is the threshold for sequential quasiparticle tunneling [Fig. 2(a) and Fig. 2(b), $\delta E = -2\Delta$], the dot-dash lines are the thresholds for singularity matching [Fig. 2(a) and Fig. 2(b), $\delta E = 0$], and coincide with the threshold for the Coulomb blockade in the normal state (gray line). The dotted lines are the resonant conditions for tunnel events involving Cooper pair tunneling [Fig. 2(c) and Fig. 2(d), $\delta E = 0$]. The dashed lines are the thresholds for the tunneling of $3e$ of charge [Fig. 2(e) and Fig. 2(f), $\delta E = -2\Delta$]. The experimental values were used to generate this figure. The program that was used to generate the figure is available at http://vortex.tn.tudelft.nl/research/set/stability/stability.html

shown in Fig. 2(e) has a slope of $-C_g/(C_1 + 2C_2 + C_g)$ and the slope of the threshold determined by the tunnel process shown in Fig. 2(f) is $C_g/(2C_1 + C_2 + 2C_g)$.

Figure 4(a) shows the measured current through the superconducting SET transistor as a function of the bias voltage and the gate voltage. The logarithm of the current was taken so that the high bias data and low bias data could be presented in the same figure. Figure 4(b) shows the derivative of the same data. The current is periodic in gate voltage with a periodicity of e/C_g. The inverted white triangles at the top of Fig. 4(a) form the threshold for sequential quasiparticle tunneling through the SET transistor. In this process, a single quasiparticle tunnels onto the island through one junction and then another quasiparticle tunnels out through the other junction. These tunnel processes are shown in Fig. 2(a) ($\delta E = -2\Delta$) and Fig. 2(b) ($\delta E = -2\Delta$). The minimum bias voltage for the threshold for sequential quasiparticle tunneling is $4\Delta/e$ and the maximum is $4\Delta/e + e/C_\Sigma$. Here C_Σ is the total capacitance, $C_\Sigma = C_1 + C_2 + C_g$. The change in electrostatic energy when a quasiparticle tunnels must be $\delta E = -2\Delta$ because there are no quasiparticle states within the superconducting gap.

Also clearly visible in Fig. 4(a) are intersecting ridges of current that are due to the Josephson-quasiparticle (JQP) cycle. These are the white X's centered at about 0.8 mV. This transport mechanism can occur when the bias voltages are such that a Cooper pair can be transported through one of the junctions without changing the total energy of the system. There are then two degenerate charge states which are

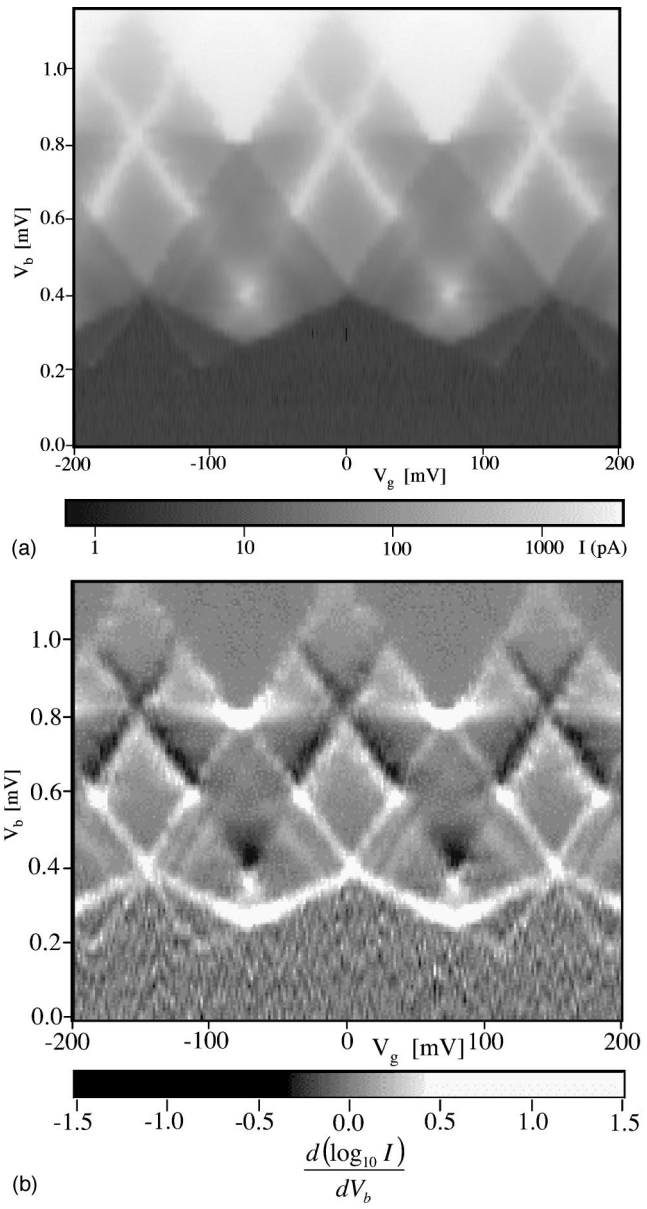

FIG. 4. (a) The logarithm of the current through a superconducting SET transistor is plotted as a function of the bias voltage and the gate voltage. (b) The same derivative of the same data shown in (a).

coupled by the Josephson energy E_J. This results in a mixing of the charge states and the probability of the Cooper pair being on either side of the junction oscillates with a frequency $E_J/\hbar = I_c/(2e)$. These oscillations produce no net current, however the oscillations can be interrupted by the tunneling of a quasiparticle through the other junction. The result of this interruption is that a Cooper pair is transported through one of the junctions while a quasiparticle is transported through the other junction. The charge of the island changes by e, and the mixing of the charge states ceases. If the bias voltage is greater than $2\Delta/e + e/C_\Sigma$, then a second quasiparticle can tunnel returning the system to its original charge state and the process can start over again. The JQP current ridges intersect at a bias voltage of $2e/C_\Sigma$.

There are also isolated current peaks located at a bias of 0.4 meV in Fig. 4. These peaks lie on the extensions of the JQP current ridges at a bias voltage of e/C_Σ.[14,15] Two sequential tunneling events are responsible for these current peaks that are similar to the first tunnel process in the JQP cycle described above. First Cooper pair tunneling is resonant across junction 1. When the tunneling of a quasiparticle through junction 2 interrupts the mixing of the charge states, a charge of $-2e$ is transported through junction 1 and a charge of $-e$ is transported through junction 2. This adjusts the potential of the island so that Cooper pair tunneling is resonant across junction 2. Then a quasiparticle can tunnel onto the island through junction 1 while a Cooper pair is transported off the island through junction 2. This returns the system to its original charge state and the process repeats.

The horizontal line at 4Δ in Fig. 4 is due to the rather abrupt onset of cotunneling of quasiparticles at a bias voltage of 4Δ. This cotunneling is illustrated in Fig. 2(g). Cotunneling of quasiparticles for bias voltages less than 4Δ is suppressed by the lack of quasiparticle states in the superconducting gap.[16]

Now we focus on the sawtooth threshold for current that lies just below 0.4 mV in Fig. 4. This threshold is e periodic and the lines that form the threshold have a slope that is one third of the slope of the threshold for sequential quasiparticle tunneling or the JQP cycle. The tunnel process responsible for this threshold is one where a Cooper pair and a quasiparticle tunnel simultaneously. This sort of cotunneling event involving a Cooper pair and a quasiparticle was first described by Maassen van den Brink *et al.*[17] First the charge on the island decreases by $-e$ via the tunnel event shown in Fig. 2(e) with $\delta E = -2\Delta$. Then the island returns to its initial charge state via the tunnel event in Fig. 2(f) with $\delta E = -2\Delta$. The minimum bias voltage for this threshold is $4\Delta/(3e)$ and the maximum bias voltage for this threshold for this process is $4\Delta/(3e) + e/(3C_\Sigma)$. A similar simultaneous $3e$ tunneling threshold should also occur for SET transistors in the normal state [Fig. 2(e) and Fig. 2(f), $\delta E = 0$]. However in that case three particles would have to tunnel simultaneously so the rate would be much lower.

The tunneling of $3e$ of charge also forms part of a sequence of tunnel events that is responsible for the current observed in the diamond-shaped regions that extend from a bias voltage of about 0.4 mV to 1.2 mV. In this region, first $3e$ of charge tunnels as in Fig. 2(e) [or Fig. 2(f)] with $\delta E = -2\Delta$. Then the charge of the island returns to its initial state by the tunneling of a quasiparticle as in Fig. 2(b) [or Fig. 2(a)] with $\delta E = -2\Delta$.

At bias voltages between 0.2 mV and 0.4 mV a small current that is $2e$ periodic is observed. This current arises from the sequential tunneling of a quasiparticle and the tunneling of $3e$ of charge as described above. If the initial state of the island is odd, then a quasiparticle can tunnel on or off the island in the tunnel processes illustrated in Fig. 2(a) or Fig. 2(b) with $\delta E = 0$.[18] In this tunnel process, the quasiparticle that tunnels pairs with the odd quasiparticle on the island. The island then returns to its initial charge state via a Cooper pair-quasiparticle cotunneling event [Fig. 2(e) or Fig. 2(f), $\delta E = -2\Delta$]. A similar process cannot occur if the initial state of the island is even since the quasiparticle that tunnels from the lead has no partner to condense with to form a Cooper pair. Consequently, this current is $2e$ periodic.

In summary, the thresholds for a number of distinct charge-transport mechanisms were observed in the current-voltage characteristics of a superconducting SET transistor. These cycles involve the sequential tunneling of quasiparticles, the sequential tunneling of Cooper pairs and quasiparticles (JQP cycles), cotunneling of quasiparticles, and the sequential cotunneling of Cooper pairs and quasiparticles with the tunneling of quasiparticles. Of particular interest are the currents that arise from cycles which include cotunneling of a Cooper pair and a quasiparticle. In this tunnel process, a charge of $3e$ tunnels and the Cooper pair and quasiparticle are transported simultaneously through two different tunnel barriers. Cotunneling of a Cooper pair and a quasiparticle also plays a role in a sequence of tunnel events that leads to a $2e$ periodic current at low bias voltages.

We are indebted to Sarah Pohlen, Leonid Glazman, Yuli Nazarov, Gerd Schön, and Arkadi Odintsov for their enlightening comments on this work. We also thank Caspar van der Wal for assistance with the measurements. Support from Esprit project 22953, CHARGE, is gratefully acknowledged.

[1] K. K. Likharev, IEEE Trans. Magn. **23**, 1142 (1987).
[2] A. N. Korotkov, Appl. Phys. Lett. **69**, 2593 (1996).
[3] R. J. Schoelkopf, P. Wahlgren, A. A. Koshevnikov, P. Delsing, and D. E. Prober, Science **280**, 5367 (1998).
[4] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996).
[5] D. V. Averin and Yu. V. Nazarov, Phys. Rev. Lett. **69**, 1993 (1992).
[6] M. T. Tuominen, J. M. Hergenrother, T. S. Tighe, and M. Tinkham, Phys. Rev. Lett. **69**, 1997 (1992).
[7] D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, City, 1991).
[8] T. A. Fulton, P. L. Gammel, D. J. Bishop, L. N. Dunkleberger, and G. J. Dolan, Phys. Rev. Lett. **63**, 1307 (1989).
[9] A. J. Manninen, Yu. A. Pashkin, A. N. Korotkov, and J. P. Pekola, Europhys. Lett. **39**, 305 (1997).
[10] R. J. Fitzgerald, S. L. Pohlen, and M. Tinkham, Phys. Rev. B **57**, 11 073 (1998).
[11] D. B. Haviland, Y. Harada, P. Delsing, C. D. Chen, and T. Claeson, Phys. Rev. Lett. **73**, 1541 (1994).
[12] Y. Nakamura, A. N. Korotkov, C. D. Chen, and J. S. Tsai, Phys. Rev. B **56**, 5116 (1997).
[13] J. D. Jackson, *Classical Electrodynamics* (John Wiley and Sons, New York, 1975).
[14] A. Maassen van den Brink, A. A. Odintsov, P. A. Bobbert, and G. Schön, Z. Phys. B **85**, 459 (1991).
[15] Jens Siewert and Gerd Schön, Phys. Rev. B **54**, 7421 (1996).
[16] D. V. Averin, A. N. Korotkov, A. J. Manninen, and J. P. Pekola, Phys. Rev. Lett. **78**, 4821 (1997).
[17] Alec Maassen van den Brink, Gerd Schön, and L. J. Geerligs, Phys. Rev. Lett. **67**, 3030 (1991).
[18] G. Schön, J. Siewert, and A. D. Zaikin, Physica B **203**, 340 (1994).