

# Fluctuation-induced diamagnetism below the critical temperature in high- $T_c$ superconductors

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We have calculated the fluctuation-induced diamagnetism ( $\Delta\chi$ ) of the high- $T_c$  superconductors below the critical temperature  $T_c$  and in the weak-field region. The theoretical formulation estimates the relation between  $\Delta\chi$  and the reduced temperature. [S0163-1829(98)02242-5]

## I. INTRODUCTION

The thermodynamic fluctuations are important in high- $T_c$  superconductors (HTSC) because of their short coherence length. There are numerous studies both theoretical as well as experimental of the fluctuation-induced conductivity, magnetoconductivity, specific heat, and diamagnetism above the critical temperature.<sup>1,2</sup> The fluctuation in the diamagnetic susceptibility was first theoretically studied by Schmidt above the critical temperature.<sup>3</sup> According to this formulation the diamagnetic susceptibility above  $T_c$  caused by the thermal fluctuation is expressed as follows:

$$\chi_D \propto (T - T_c)^{-1}. \quad (1)$$

Above the critical temperature, the fluctuation in diamagnetism is calculated by Schmid starting from the Ginzburg-Landau free-energy functional.<sup>4</sup> According to this formulation the fluctuation in diamagnetism is expressed as follows:

$$\chi = - \frac{\pi T \xi_{\parallel}^2(0)}{3d\phi_0\epsilon}. \quad (2)$$

Here  $\epsilon$  is the reduced temperature defined as

$$\epsilon = \frac{T - T_c}{T_c}$$

and  $T_c$  is the critical temperature where a HTSC starts to show negative susceptibility during the cooling.  $d$  and  $\xi_{\parallel}(0)$  are the interlayer separation and the  $ab$ -plane coherence length at absolute zero, respectively, and  $\phi_0$  is the flux quanta. Koshelev has estimated the magnetization as a function of the applied magnetic field and reduced temperature of high- $T_c$  superconductors above the critical temperature using the Ginzburg-Landau free-energy functional.<sup>5</sup> The magnetization is estimated both for the high and low magnetic fields. Baraduc *et al.* have reported the fluctuation in magnetization in the high- $T_c$  superconductors above the critical temperature with the help of the Lawrence-Doniach free-energy functional.<sup>6</sup> Klemm has described fluctuation in diamagnetism in multiperiodic layered superconductors.<sup>7</sup> Lee, Klemm, and Johnston have reported the fluctuation in diamagnetism of biperiodic superconductors above  $T_c$ . The result shows that fluctuation is observed from around  $2T_c$  in the weak-field regime.<sup>8</sup> The magnetization measurement of Bi-2212 up to 5 T below  $T_c$  by Kes *et al.* has revealed that quasi-two-dimensional fluctuations are responsible for the field-induced suppression of phase transition.<sup>9</sup> Tesanovic *et al.* have studied the thermodynamic quantities in HTSC in the critical

region by taking into account the high magnetic field.<sup>10</sup> Ramallo, Torron, and Vidal have studied the fluctuation-induced diamagnetism above  $T_c$  in biperiodic HTSC systems.<sup>11</sup> Dorin *et al.* have studied fluctuation conductivity of layered superconductors in a magnetic field parallel to the  $c$  direction. The contributions from the fluctuation in the single quasiparticle density of states (DOS) and the Maki-Thompson contribution are compared to the Aslamazov-Larkin contribution in terms of an effective quasiparticle nearest-neighbor interlayer hopping energy.<sup>12</sup>

Skocpol and Tinkham have discussed the fluctuation below the critical temperature.<sup>1</sup> Bulaevskii, Ginzburg, and Sobyenin have suggested that the fluctuation can be studied below the critical temperature in the layered superconductors.<sup>13</sup> Later Bulaevskii, Ledvij, and Kogan have reported the fluctuation of vortices in layered superconductors.<sup>14</sup> The entropy contribution to the free energy is not too small in a broad temperature region below the critical temperature and they have calculated magnetization for a magnetic field  $B \gg B_{cr}$  below the critical temperature. According to this formulation magnetization is field and temperature dependent.<sup>14</sup> Varlamov and Livanov have investigated the effect of superconducting fluctuations on the electronic part of the thermoelectric force and thermal conductivity at a temperature above the critical temperature.<sup>15</sup> They have reported that thermoelectric force possesses a maxima at the critical temperature and then rapidly decreases to zero in the superconducting state. In a recent paper Houssa *et al.* have described the fluctuation contribution in thermal conductivity below the critical temperature.<sup>16</sup> Below the critical temperature thermal conductivity is enhanced both from the electronic and phonon contribution. A dimensional crossover from three to two dimensions is observed for normalized fluctuation contribution below the critical temperature. The Ginzburg-Landau parameter is extracted in this paper from the crossover temperature and the critical temperature.<sup>16</sup> To our knowledge, there is no explicit study of fluctuation-induced diamagnetism below the critical temperature in the weak-field region in HTSC. We have estimated the fluctuation-induced diamagnetism  $\Delta\chi$  below the critical temperature as a function of the reduced temperature in the high- $T_c$  system.

## II. THEORETICAL FORMULATIONS

### A. Magnetic moment of high- $T_c$ superconductors below $T_c$

The Lawrence-Doniach functional describes the fluctuation phenomena in layered high- $T_c$  superconductors with the

weak coupling between the CuO<sub>2</sub> planes. If a magnetic field is applied, the free-energy functional takes the form as<sup>6</sup>

$$F[\psi] = \sum_n \int d^2r \left[ a |\psi_n|^2 + \frac{b}{2} |\psi_n|^4 + \frac{\hbar^2}{4m_{\parallel}} \left| \left( \nabla_{\parallel} - \frac{2ie}{\hbar c} A_{\parallel} \right) \right|^2 + t |\psi_{n+1} - \psi_n|^2 \right], \quad (3)$$

where  $\psi_n$  is the order parameter of the  $n$ th superconducting layer and  $a = \alpha(T - T_c)/T_c = \alpha\epsilon$ .

As the magnetic field is applied to a superconductor, the order parameter will be perturbed. The thermal energy causes another perturbation in  $\psi_n$ . So the free-energy functional will be perturbed by  $\Delta F_{fl}$ , which is termed as the fluctuation contribution to the free energy. This is written as<sup>7</sup>

$$\Delta F_{fl}(h, T) = - \frac{Vk_B T h}{4\pi^2 \xi_{\parallel}^2(0) d} \int_{-\pi}^{+\pi} dz \sum_{n=0}^{\infty} \ln \frac{\pi k_B T}{\alpha[(2n+1)h + r(1 - \cos z) + \epsilon]}, \quad (4)$$

with  $h = B/H'_{c2}$  and the magnetic field is applied along the  $z$  direction or the  $c$  axis. The parameter  $r$  is given by<sup>6</sup>

$$r = 2 \frac{\xi_c^2(0)}{d^2},$$

where  $\xi_c(0)$  is the coherence length along the  $c$  direction.

Baraduc *et al.* have calculated the magnetic moment using the fluctuation in the free energy below and above the critical temperature as a function of the magnetic field and reduced temperature.<sup>6</sup>

The magnetic moment below the critical temperature is given as follows:<sup>6</sup>

$$M(-|\epsilon|) = \Delta M + 2M(\epsilon=0) - M(|\epsilon|). \quad (5)$$

### B. Calculation of $\Delta M$ and its variation with the reduced temperature

$\Delta M$  is expressed as

$$\Delta M = - \frac{Vk_B T}{2\pi \xi_{\parallel}^2(0) d H'_{c2}} \frac{\partial}{\partial h} h \sum_{n=0}^{\infty} \ln \frac{\phi(P)^2}{\phi(P+|\theta|)\phi(P-|\theta|)}, \quad (6)$$

where

$$\phi(N) = N + \sqrt{N^2 - \rho^2}, \quad (7)$$

with  $N = n + \epsilon/2h + \rho$  and  $\rho = r/2h$ . The  $n$  has the cutoff with a value  $n_0 \approx 1/h$ , because the Ginzburg-Landau approximation is valid only for this choice.<sup>4</sup>

We have derived  $\Delta M$  presented in Appendix A that can be expressed as

$$\Delta M = - \frac{Vk_B T}{\phi_0 d} \left[ \sum_{n=0}^{n_0} \ln \frac{\phi(P)^2}{\phi(P+|\theta|)\phi(P-|\theta|)} + h S_T \right], \quad (8)$$

with

$$S_T = \sum_{n=0}^{n_0} T_1 + T_2 + T_3, \quad (9)$$

where  $T_1$ ,  $T_2$ , and  $T_3$  can be expressed as follows:

$$T_1 = - \frac{r}{h^2 [P + (P^2 - \rho^2)^{1/2}]} \left[ 1 + P(P^2 - \rho^2)^{-1/2} \right], \quad (10)$$

$$T_2 = - \frac{1 + \{(P + |\theta|)^2 - \rho^2\}^{-1/2} (P + |\theta|)}{(P + |\theta|) + \{(P + |\theta|)^2 - \rho^2\}^{1/2}} \left( \frac{r + \epsilon}{2h^2} \right), \quad (11)$$

$$T_3 = - \frac{1 + \{(P - |\theta|)^2 - \rho^2\}^{-1/2} (P - |\theta|)}{(P - |\theta|) + \{(P - |\theta|)^2 - \rho^2\}^{1/2}} \left( \frac{r - \epsilon}{2h^2} \right). \quad (12)$$

In expressions for  $T_1$ ,  $T_2$ , and  $T_3$ , we get

$$P + |\theta| = n + \frac{1}{2} + \frac{r + \epsilon}{2h} \quad (13)$$

and

$$P - |\theta| = n + \frac{1}{2} + \frac{r - \epsilon}{2h}. \quad (14)$$

In the limit  $r \gg \epsilon$ ,  $P$  becomes equal to  $P + |\theta|$  and  $P - |\theta|$ , which can be expressed as follows:

$$P = P \pm |\theta| = n + \frac{1}{2} + \frac{r}{2h}. \quad (15)$$

Therefore, in the above limit,  $\Delta M$  becomes zero and is independent of  $\epsilon$  otherwise so that

$$\frac{d\Delta M}{d\epsilon} = 0. \quad (16)$$

### C. Calculation of $M(|\epsilon|)$

The third part of the magnetic moment deduced by Koshchev below the critical temperature  $M(|\epsilon|)$  is written as<sup>4</sup>

$$M(B, \epsilon) = - \frac{T}{d\phi_0} m_{sc}(b), \quad (17)$$

where

$$m_{sc}(b) = \frac{d}{db} \left[ b \sum_{n=0}^{n_0} \int_{-1/2}^{+1/2} dx \ln \frac{1 + b[2(n+x) + 1]}{1 + b(2n+1)} \right], \quad (18)$$

with  $b = h/\epsilon$  and  $\phi_0 = 2\pi \xi_{\parallel}^2(0) H'_{c2}$ . After the integration followed by the differentiation in Eq. (18), given in Appendix B, we get  $m_{sc}(b)$  as given below,

$$m_{sc}(b) = \sum_{n=0}^{n_0} \left[ (n+1) \ln(1+2nb+2b) - n \ln(1+2nb) - \ln(1+2nb+b) - b \frac{2n+1}{1+(2n+1)b} \right]. \quad (19)$$

So, the term for magnetization  $M(|\epsilon|)$  is obtained in the form

$$M(|\epsilon|) = -\frac{T}{d\phi_0} \sum_{n=0}^{n_0} \left[ (n+1) \ln(1+2nb+2b) - n \ln(1+2nb) - \ln(1+2nb+b) - b \frac{2n+1}{1+(2n+1)b} \right]. \quad (20)$$

#### D. Calculation of $M(\epsilon=0)$

Now to calculate  $M(\epsilon=0)$ , we reform  $m_{sc}(b)$  as (see Appendix C) follows:

$$m_{sc}(\epsilon) = \sum_{n=0}^{n_0} \left[ (n+1) \{ \epsilon + 2(n+1)h \} - n \ln(\epsilon + 2nh) - \ln \{ \epsilon + (2n+1)h \} - \frac{(2n+1)h}{\epsilon + (2n+1)h} \right]. \quad (21)$$

So, from Eq. (21), we get

$$m_{sc}(\epsilon=0) = \sum_{n=0}^{n_0} \left[ (n+1) \ln(2n+1)h - n \ln 2nh - \ln(2n+1)h - 1 \right]. \quad (22)$$

On simplification, Eq. (22) is transformed to

$$m_{sc}(\epsilon=0) = \sum_{n=0}^{n_0} \left[ n \ln \frac{n+1}{n} + \ln \frac{n+1}{n+\frac{1}{2}} - 1 \right]. \quad (23)$$

This is same as that for  $m_{sc}(b \gg 1)$  and is equal to  $-0.346$ . At  $\epsilon=0$ ,  $T=T_c$ , so that  $M(\epsilon=0)$  can be written as follows:

$$M(\epsilon=0) = \frac{0.346T_c}{d\phi_0}. \quad (24)$$

Therefore, we have

$$\frac{dM(\epsilon=0)}{d\epsilon} = 0. \quad (25)$$

#### E. The diamagnetic susceptibility $\chi$ and $d\chi/d\epsilon$

We know that the susceptibility  $\chi$  is obtained by the relation

$$\chi = \frac{M(-|\epsilon|)}{B}. \quad (26)$$

So, the variation of  $\chi$  with the reduced temperature is given by the relation

$$\frac{d\chi}{d\epsilon} = \frac{1}{B} \frac{dM(-|\epsilon|)}{d\epsilon}. \quad (27)$$

It follows from Eq. (3),

$$\frac{dM(-|\epsilon|)}{d\epsilon} = \frac{d\Delta M}{d\epsilon} + 2 \frac{dM(\epsilon=0)}{d\epsilon} - \frac{dM(|\epsilon|)}{d\epsilon}. \quad (28)$$

Using Eqs. (16), (25), and (28) we get

$$\frac{dM(-|\epsilon|)}{d\epsilon} = -\frac{dM(|\epsilon|)}{d\epsilon}. \quad (29)$$

We know that  $b = B/H'_{c2}\epsilon$ . Therefore it is found that

$$d\epsilon = -\frac{B}{H'_{c2}b^2} db. \quad (30)$$

Therefore, the variation of  $\chi$  with the reduced temperature is found from Eqs. (27), (29), and (30) as follows:

$$\frac{d\chi}{d\epsilon} = -\frac{1}{H'_{c2}\epsilon^2} \frac{dM(|\epsilon|)}{db}. \quad (31)$$

#### F. Calculation of $dM(|\epsilon|)/db$

Differentiating Eq. (20), as shown in Appendix D, we get

$$\frac{dM(|\epsilon|)}{db} = -\frac{T_c(1+\epsilon)}{d\phi_0} \sum_{n=0}^{n_0} \left[ \frac{2(n+1)^2}{1+2nb+2b} - \frac{4n}{1+2nb} + \frac{(2n+1)(2+2nb+b)}{(1+2nb+b)^2} \right]. \quad (32)$$

#### G. Calculation of diamagnetic fluctuation below the critical temperature $T_c$

If the applied magnetic field is very small or zero, then using  $b \approx 0$  we have

$$\frac{dM(|\epsilon|)}{db} = -\frac{2T_c(1+\epsilon)}{d\phi_0} \sum_{n=0}^{n_0} (n^2 + 2n + 2). \quad (33)$$

Using Eqs. (31) and (33) we get

$$\frac{d\chi}{d\epsilon} = \frac{4\pi T_c \xi_{||}^2(0)}{d\phi_0^2} \frac{1+\epsilon}{\epsilon^2} \Omega(n), \quad (34)$$

with

$$\Omega(n) = \sum_{n=0}^{n_0} (n^2 + 2n + 2). \quad (35)$$

Integrating Eq. (34) we have the susceptibility  $\chi$  below the critical temperature  $T_c$  represented by

$$\chi = \frac{4\pi T_c \xi_{||}^2(0)}{d\phi_0^2} \Omega(n) \int \left( \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \right) d\epsilon. \quad (36)$$

The above expression for  $\chi$  can be rewritten as

$$\chi = \eta \left[ \ln \epsilon - \frac{1}{\epsilon} \right] + C, \quad (37)$$

with

$$\eta = \frac{4\pi T_c \xi_{\parallel}^2(0)}{d\phi_0^2} \Omega(n). \quad (38)$$

The diamagnetic susceptibility saturates with a value  $\chi_0$  at some lower temperature  $T < T_c$ , where  $|\epsilon| \approx 1$ . With this boundary condition we can write the diamagnetic susceptibility as follows.

$$\chi = \eta \left[ \ln \epsilon - \frac{1}{\epsilon} \right] + \chi_0 + \eta. \quad (39)$$

At the critical temperature the superconductors should show perfect diamagnetism with the susceptibility  $\chi_0$ . But the perfect diamagnetism is found at a temperature lower than  $T_c$ . The difference  $\chi_0 - \chi$  is the fluctuation in diamagnetism ( $\Delta\chi$ ) caused by the thermal fluctuations. The temperature dependence of  $\Delta\chi$  is obtained from the above equation as expressed below,

$$\Delta\chi = \chi_0 - \chi = -\eta \left[ \ln \epsilon - \frac{1}{\epsilon} + 1 \right]. \quad (40)$$

Now we have  $\ln \epsilon \approx \epsilon - 1$  because it is known that  $2 \geq \epsilon \geq 0$ . Using this we can write  $\Delta\chi$  as follows:

$$\Delta\chi = \chi_0 - \chi = -\eta \left[ \epsilon - \frac{1}{\epsilon} \right]. \quad (41)$$

Since  $\epsilon^2 \ll 1$  for the discussed temperature region, the fluctuation-induced diamagnetism is given by the following equation:

$$\Delta\chi = -\frac{\eta}{\epsilon} = -\frac{4\pi T_c \xi_{\parallel}^2(0)}{d\phi_0^2} \Omega(n) \epsilon^{-1}. \quad (42)$$

### III. SUMMARY

The fluctuation-induced diamagnetism of HTSC is estimated as a function of the reduced temperature below the critical temperature. Below the critical temperature,  $\Delta\chi$  varies inversely proportional to the reduced temperature  $\epsilon$ . Also, the critical temperature and the number of superconducting layers influence the absolute magnitude of  $\Delta\chi$ . There may be a dimensional crossover below the critical temperature obtained between two regimes with different  $\epsilon$  as is reported in the case of thermal conductivity.<sup>16</sup> Depending upon the nature of the pair-breaking (weak or strong) Maki-Thompson contribution below  $T_c$  may be an important study. Though the contribution from the density of states is larger above the crossover temperature above  $T_c$ , the contribution to  $\Delta\chi$  may be affected to a certain extent in weak magnetic field below the critical temperature. These two contributions are controlled by the applied magnetic field. In addition, the  $ab$ -plane coherence length is an important factor to alter the fluctuation effect in diamagnetism in HTSC. This is unlike the fluctuation above the critical temperature.

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### APPENDIX A

We have the expression for  $\Delta M$  as follows:

$$\Delta M = \text{const} \times \frac{\partial}{\partial h} h \sum_{n=0}^{n_0} \frac{\phi(P)^2}{\phi(P+|\theta|)\phi(P-|\theta|)}. \quad (A1)$$

We get from the above equation,

$$\begin{aligned} & \frac{\partial}{\partial h} \sum_{n=0}^{n_0} \frac{\phi(P)^2}{\phi(P+|\theta|)\phi(P-|\theta|)} \\ &= \frac{\partial}{\partial h} \sum_{n=0}^{n_0} [2 \ln \phi(P) - \ln \phi(P+|\theta|) - \ln \phi(P-|\theta|)] \\ &= \sum_{n=0}^{n_0} \frac{2\phi'(P)}{\phi(P)} \frac{\partial P}{\partial h} - \frac{\phi'(P+|\theta|)}{\phi(P+|\theta|)} \frac{\partial(P+|\theta|)}{\partial h} \\ & \quad - \frac{\phi'(P-|\theta|)}{\phi(P-|\theta|)} \frac{\partial(P-|\theta|)}{\partial h}. \end{aligned} \quad (A2)$$

Now,  $T_1$  can be expressed as

$$T_1 = \frac{2\phi'(P)}{\phi(P)} \frac{\partial P}{\partial h} = \frac{2\phi'(P)}{\phi(P)} \frac{\partial}{\partial h} [n + \rho + \frac{1}{2}],$$

so that

$$T_1 = \frac{2}{P + \sqrt{P^2 - \rho^2}} \phi'(P) \frac{\partial}{\partial h} \left( \frac{r}{2h} \right).$$

Again putting  $\phi'(P)$  in the above expression we can rewrite  $T_1$  as follows:

$$T_1 = \frac{r}{P + \sqrt{P^2 - \rho^2}} \left( -\frac{1}{h^2} \right) [1 + P(P^2 - \rho^2)^{-1/2}]. \quad (A3)$$

The second term  $T_2$  can be evaluated as follows:

$$T_2 = \frac{\phi'(P+|\theta|)}{\phi(P+|\theta|)} \frac{\partial}{\partial h} (P+|\theta|). \quad (A4)$$

Finally, after the differentiation we have

$$T_2 = \frac{1 + \{(P+|\theta|)^2 - \rho^2\}^{-1/2} (P+|\theta|)}{P+|\theta| + \sqrt{(P+|\theta|)^2 - \rho^2}} \left( -\frac{r+\epsilon}{2h^2} \right). \quad (A5)$$

Similarly, the third term  $T_3$  can be expressed as follows:

$$T_3 = \frac{1 + \{(P-|\theta|)^2 - \rho^2\}^{-1/2} (P-|\theta|)}{P-|\theta| + \sqrt{(P+|\theta|)^2 - \rho^2}} \left( -\frac{r+\epsilon}{2h^2} \right). \quad (A6)$$

## APPENDIX B

$$m_{sc} = \frac{d}{db} \left[ b \sum_{n=0}^{n_0} I_1 \right], \quad (\text{B1})$$

with

$$I_1 = \int_{-1/2}^{+1/2} \ln \frac{1+b[2(n+x)+1]}{1+b(2n+1)} dx. \quad (\text{B2})$$

Let us assume  $1+b[2(n+x)+1]=y$  so that  $I_1$  can be written as

$$I_1 = \frac{1}{2b} \int_{1+2nb}^{1+b(2n+2)} \ln \frac{y}{1+b(2n+1)} dy. \quad (\text{B3})$$

On integration of the above we have  $I_1$  as given below,

$$I_1 = \frac{1}{2b} [(1+2nb+2b)\ln(1+2nb+2b) - 2b \ln(1+2nb) - (1+2nb)\ln(1+2nb) - 2b \ln(1+2nb+b)]. \quad (\text{B4})$$

Now we can rewrite  $m_{sc}(b)$  as follows:

$$m_{sc}(b) = \frac{d}{db} \left[ b \sum_{n=0}^{n_0} \frac{1}{2b} \{ (1+2nb+2b) \times \ln(1+2nb+2b) - 2b - (1+2nb) \times \ln(1+2nb) - 2b \ln(1+2nb+b) \} \right]. \quad (\text{B5})$$

After the differentiation we have

$$m_{sc}(b) = \sum_{n=0}^{n_0} \left[ (n+1)\ln(1+2nb+2b) - n \ln(1+2nb) - \ln(1+2nb+b) - b \frac{2n+1}{1+(2n+1)b} \right]. \quad (\text{B6})$$

## APPENDIX C

The  $b$  is defined with the help of the reduced temperature as follows:

$$b = \frac{h}{\epsilon}. \quad (\text{C1})$$

Putting  $b$  in the expression for  $m_{sc}(b)$  we get after a simplification,

$$m_{sc}(b) = \sum_{n=0}^{n_0} \left[ (n+1)\ln\{\epsilon+2(n+1)h\} - n \ln\{\epsilon+2nh\} - \ln\{\epsilon+(2n+1)h\} - \frac{(2n+1)h}{\epsilon+(2n+1)h} \right]. \quad (\text{C2})$$

Now putting  $\epsilon=0$  in the above expression we rewrite  $m_{sc}(\epsilon=0)$  as follows:

$$m_{sc}(\epsilon=0) = \sum_{n=0}^{n_0} [(n+1)\ln\{2h(n+1)\} - n \ln 2nh - \ln\{(2n+1)h\} - 1]. \quad (\text{C3})$$

After rearrangement of the above equation we get

$$m_{sc}(\epsilon=0) = \sum_{n=0}^{n_0} \left[ n \ln \frac{n+1}{n} + \ln \frac{n+1}{n+\frac{1}{2}} - 1 \right]. \quad (\text{C4})$$

## APPENDIX D

Differentiating both sides of Eq. (18) with respect to  $b$  we obtain

$$\frac{dM(|\epsilon|)}{db} = -\frac{T}{d\phi_0} \sum_{n=0}^{n_0} \left[ \frac{2(n+1)^2}{1+2nb+2b} - 2 \frac{2n}{1+2nb} - \frac{2n+1}{1+2nb+b} - \frac{2n+1}{1+(2n+1)b} + b \frac{(2n+1)^2}{(1+2nb+b)^2} \right]. \quad (\text{D1})$$

After simplification we get

$$\frac{dM(|\epsilon|)}{db} = -\frac{T}{d\phi_0} \sum_{n=0}^{n_0} \left[ \frac{2(n+1)^2}{1+2nb+2b} - \frac{4n}{1+2nb} + \frac{(2n+1)(2+2nb+b)}{(1+2nb+b)^2} \right]. \quad (\text{D2})$$

Putting  $T=T_c(1+\epsilon)$  in the above equation we get

$$\frac{dM(|\epsilon|)}{db} = -\frac{T_c(1+\epsilon)}{d\phi_0} \sum_{n=0}^{n_0} \left[ \frac{2(n+1)^2}{1+2nb+2b} - \frac{4n}{1+2nb} + \frac{(2n+1)(2+2nb+b)}{(1+2nb+b)^2} \right]. \quad (\text{D3})$$

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