Coherent versus incoherent *c*-axis Josephson tunneling between layered superconductors

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We calculate $I_c(T)$ and R_n for both coherent and incoherent electron tunneling across a *c*-axis break junction between two $\nu=s,d_{x^2-y^2}$-wave layered superconducting half spaces, each with *c*-axis bandwidth $2J$. Coherent quasiparticle tunneling only occurs for voltages $V<2J/e$, leading to difficulties in measuring R_n for underdoped samples. The coherent part of $I_c(0)$ is independent of $\Delta_\nu(0)$ for $J/\Delta_\nu(0)\ll 1$, and can be large. Our results are discussed with regard to recent experiments. [S0163-1829(98)01745-7]

It is presently possible to prepare high quality *c*-axis Josephson junctions of $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BSCCO).[1–3] The standard measure of Josephson junction quality is the product I_cR_n of the critical current times the normal resistance at the temperature $T=0$, for which Ambegaokar-Baratoff (AB) gave $I_cR_n=\pi\Delta(0)/2e$, where $\Delta(T)$ is the superconducting order parameter (OP) amplitude.[4] Real Josephson junctions almost never exceed this value. Early thin film atomic layer-by-layer (ALL) molecular beam epitaxy (MBE) preparations of trilayer junctions of BSCCO separated by a thin layer of Dy-doped BSCCO were found to have I_cR_n values consistently about 0.5 mV, with I_c and R_n varying greatly.[1] Recently, they prepared a single Josephson junction within a single unit cell of underdoped BSCCO, and reported $I_cR_n\approx 5$–10 mV.[1] Also, a blunt point contact tip pressed onto a BSCCO surface often resulted in an apparent *c*-axis break junction.[2] Overdoped junctions typically had $I_cR_n\approx 2.4$ mV, well below the weak coupling $[\Delta(0)=1.76T_c]$ AB result of 15–20 mV for T_c values of 62–83 K, where T_c is the transition temperature. However, two underdoped break junction samples had $I_cR_n\approx 15$–25 mV, apparently in *excess* of the weak coupling AB result. Furthermore, exceedingly clean *c*-axis break junctions were prepared by cleavage and subsequent refusion of BSCCO, with or without a twist about the *c* axis.[3] We expect $I_c(T)R_n$ data to be available shortly.[1–3]

We consider tunneling across an untwisted *c*-axis break (or single intrinsic Josephson) junction which is much less conductive than the bulk, intrinsic junctions between neighboring CuO_2 layer pairs. For underdoped samples, standard measurements of R_n at voltages $V>2\Delta(0)/e$ can be unreliable, since they do not fully measure the coherent processes that can dominate at $V=0$. Hence, the large values of I_cR_n reported for underdoped samples could be questionable.[1,2]

We assume a *c*-axis break junction between two untwisted half spaces of cross-sectional area A, each consisting of $N\gg 1$ identical clean superconducting layers separated a distance s apart. We label the upper (u) and lower (ℓ) half spaces by $\mu=u,\ell$, and index the layers in each half space by $j=1,\ldots,N$, with $j=1$ being the layer in each half space adjacent to the break junction. We allow $\psi_{\mu,j,\sigma}(\mathbf{k})[\psi^\dagger_{\mu,j,\sigma}(\mathbf{k})]$ to annihilate [create] a quasiparticle with spin $\sigma=\pm 1$ and two-dimensional (2D) wave vector $\mathbf{k}$ on the jth layer within the μth layered half space. Within each layer in the μth half space, the quasiparticles propagate with energy dispersions $\xi_{\mu 0}(\mathbf{k})=\epsilon_{\mu 0}(\mathbf{k})-E_F$ relative to the Fermi energy E_F [for free particles, $\epsilon_{\mu 0}(\mathbf{k})=\mathbf{k}^2/(2m_\mu)$], and interact with intralayer BCS-like pairing interaction $\lambda_\mu(\mathbf{k},\mathbf{k}')=\lambda_{\mu\nu 0}\varphi_\nu(\phi_{\mathbf{k}})\varphi_\nu(\phi_{\mathbf{k}'})$, where $\nu=s,d$, $\varphi_s(\phi_{\mathbf{k}})=1$ and $\varphi_d(\phi_{\mathbf{k}})=\sqrt{2}\cos(2\phi_{\mathbf{k}})$ are the eigenfunctions for s and $d_{x^2-y^2}$-wave intralayer pairing, respectively. We only consider here the purely *s*-wave and *d*-wave cases $\lambda_{\mu s0}\neq\lambda_{\mu' d0}=0$ and $\lambda_{\mu d0}\neq\lambda_{\mu' s0}=0$. Between neighboring layers in the μth half space, the quasiparticles tunnel with matrix element $J_\mu/2$.[5] The *c*-axis resistivity $\rho_c(T)$ above T_c suggests the limits $J_\mu/T_c\gg 1$ and $J_\mu/T_c\ll 1$ apply to overdoped (metallic) and underdoped (poorly metallic) materials, respectively.[6]

In addition, we take the single particle tunneling Hamiltonian $H_{\mathcal{T}}$ across the break junction to be

$$H_{\mathcal{T}}=\frac{1}{A^2}\sum_{\mathbf{k},\mathbf{k}',\sigma}\mathcal{T}_{\mathbf{k},\mathbf{k}'}\psi^\dagger_{u,1,\sigma}(\mathbf{k})\psi_{\ell,1,\sigma}(\mathbf{k}')+\text{H.c.},\qquad(1)$$

which transfers a quasiparticle from the $j=1$ layer in the ℓ half space to the $j=1$ layer in the u half space, and vice versa; $\mathcal{T}_{\mathbf{k},\mathbf{k}'}=\mathcal{T}^*_{\mathbf{k}',\mathbf{k}}$. We set $\hbar=c=k_B=1$.

For generality, we assume both coherent and incoherent break junction tunneling. The *spatially constant coherent* tunneling preserves the intralayer wave vectors, $\mathbf{k}=\mathbf{k}'$, allowing for both *s*- and *d*-wave Josephson tunneling. However, pure *spatially random incoherent* tunneling assumes $\mathbf{k}$ and $\mathbf{k}'$ are *independent* of each other,[4] which allows no *d*-wave incoherent Josephson tunneling. Hence, to allow for a finite (albeit extremely small) amount of *d*-wave incoherent Josephson tunneling, we assume to second order in $\mathcal{T}_{\mathbf{k},\mathbf{k}'}$,[7,8]

$$\langle\mathcal{T}_{\mathbf{k},\mathbf{k}'}\mathcal{T}_{\mathbf{k}',\mathbf{k}''}\rangle=A\,\delta_{\mathbf{k},\mathbf{k}''}[|\mathcal{T}_0|^2A\,\delta_{\mathbf{k},\mathbf{k}'}+f_{\text{inc}}(\mathbf{k}-\mathbf{k}')],\qquad(2)$$

where

$$f_{\rm inc}(\mathbf{k}-\mathbf{k}')=\frac{1}{2\pi N_{\rm 2D}(0)}\left[\frac{1}{\tau_{\perp s}}+\frac{2\cos[2(\phi_{\mathbf{k}}-\phi_{\mathbf{k}'})]}{\tau_{\perp d}}\right], \tag{3}$$

$1/\tau_{\perp d}\ll 1/\tau_{\perp s}$, and $N_{\rm 2D}(0)=[N_{2Du}(0)N_{2D\ell}(0)]^{1/2}$ is the geometric mean 2D density of states; for free particles, $N_{\rm 2D}(0)=\overline{m}/(2\pi)$, where $\overline{m}=(m_u m_\ell)^{1/2}$. In Eq. (2), $\langle\cdots\rangle$ denotes a 2D spatial average.

For intralayer pairing in a half space with one conducting layer per unit cell, the regular and anomalous temperature Green's functions $G_{\mu,j}(\mathbf{k},\omega)$ and $F_{\mu,j}(\mathbf{k},\omega)$, where ω represents the Matsubara frequencies, for propagation within layer j in the μth half space, explicitly depend upon j.[9,10] However, the OP Δ_μ is independent of j.[9] Nevertheless, $\Delta_\mu(\phi_{\mathbf{k}})\equiv\Delta_{\mu\nu}(T)\varphi_\nu(\phi_{\mathbf{k}})$ implicitly depends upon $\nu=s,d$.[11]

For purely s-wave incoherent tunneling between 3D superconductors, I_cR_n is independent of the properties of the junction.[4] However, in our model these quantities must be evaluated separately. R_n is found from the quasiparticle current I_{qp} to leading order in $\mathcal{T}_{\mathbf{k},\mathbf{k}'}$,[12]

$$I_{qp}=\frac{4e}{A^2\pi}\sum_{\mathbf{k},\mathbf{k}'}\langle|\mathcal{T}_{\mathbf{k},\mathbf{k}'}|^2\rangle\int_{-\infty}^{\infty}d\epsilon[f(\epsilon_u)-f(\epsilon_\ell)]$$
$$\times{\rm Im}[G_{u,1}(\mathbf{k},-i\epsilon_u)]{\rm Im}[G_{\ell,1}(\mathbf{k}',-i\epsilon_\ell)], \tag{4}$$

where $f(x)$ is the Fermi function, $\epsilon_u=\epsilon$, $\epsilon_\ell=\epsilon+eV$, and the $G_{\mu,1}(\mathbf{k},-i\epsilon_\mu)$ are obtained from $G_{\mu,1}(\mathbf{k},\omega)$ by the analytic continuations $\omega\rightarrow -i\epsilon_\mu$. Since the tunneling takes place between the $j=1$ in the two half spaces, the only relevant wave vectors are 2D. Hence, we set $\Sigma_{\mathbf{k}}\rightarrow AN_{\rm 2D}(0)\int_{-\infty}^{\infty}d\xi_{\mu 0}\int_0^{2\pi}d\phi_{\mathbf{k}}/(2\pi)$.

We consider separately the coherent and incoherent processes, and separately the $G_{\mu,1}(\mathbf{k},\omega)$ as evaluated exactly for the layered half spaces, and as *approximated* using the bulk layered states. In the bulk-space treatment, we assume $G_{\mu,1}(\mathbf{k},\omega)\approx G_{\mu,b}(\mathbf{k},\omega)=\int_0^\pi(dz/\pi)/[i\omega-\xi_{\mu 0}-J_\mu\cos z]$, or $G_{\mu,b}(\mathbf{k},\omega)=1/R_\mu(i\omega)$, where $R_\mu(z)\equiv[(z-\xi_{\mu 0})^2-J_\mu^2]^{1/2}$ depends upon $\mathbf{k}$ only through $\xi_{\mu 0}(\mathbf{k})$.[5] However, when one properly takes account of the surface at the weak break junction,[9] $G_{\mu,1}=\int_0^\pi(2dz/\pi)\sin^2 z/[i\omega-\xi_{\mu 0}-J_\mu\cos z]$, or $G_{\mu,1}(\mathbf{k},\omega)=\Xi_\mu(i\omega)$, where

$$\Xi_\mu(z)=2/[z-\xi_{\mu 0}+R_\mu(z)]. \tag{5}$$

Using either the bulk or half-space states and the identity $\int_{-\infty}^{\infty}d\epsilon[f(\epsilon)-f(\epsilon+eV)]=eV$, the incoherent quasiparticle c-axis break junction tunneling current is Ohmic,

$$I_{qp}^{\rm inc}=V/R_n^{\rm inc}=2e^2VN_{\rm 2D}(0)/\tau_{\perp s}. \tag{6}$$

For the coherent quasiparticle c-axis break junction tunneling current, we only consider tunneling between identical materials with $J_\ell=J_u=J$, etc., and obtain

$$I_{qp}^{\rm coh}(V)=V/R_n^{\rm coh}(V)=64C/(3\pi)\gamma\Theta(1-\gamma^2)Q(\gamma), \tag{7}$$

where $C=2e|\mathcal{T}_0|^2N_{\rm 2D}(0)$, $Q(\gamma)=(1+|\gamma|)[(1+\gamma^2)E(k)-2|\gamma|K(k)]$, $k=(1-|\gamma|)/(1+|\gamma|)$, $\gamma=eV/(2J)$, $K(z)$, $E(z)$ are standard complete elliptic integrals, and $\Theta(z)$ is the Heaviside step function. In the bulk state approximation,

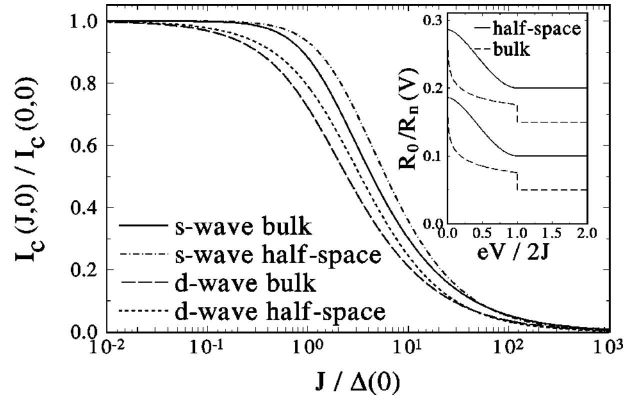

FIG. 1. $I_{c,\nu}^{\rm coh}(J,0)/I_{c,\nu}^{\rm coh}(0,0)$ and $I_{c,\nu,b}^{\rm coh}(J,0)/I_{c,\nu,b}^{\rm coh}(J,0)$ are plotted versus $\log_{10}[J/\Delta_\nu(0)]$ for $\nu=s,d$. Inset: Plot of $R_0/R_n(V)$, where $R_0=J/[8\pi^2e^2N_{\rm 2D}(0)|\mathcal{T}_0|^2]$, and its bulk space approximation versus $eV/2J$. For clarity, the curves calculated with the half-space (bulk) states have $J/[4\pi^2|\mathcal{T}_0|^2\tau_{\perp s}]$ values of 0.1 and 0.2 (0.05 and 0.15), respectively.

$Q(\gamma)$ is replaced by $(3/8)K(k)/(1+|\gamma|)$, which leads to a spurious, non-Ohmic $\ln V$ dependence of $1/R_n^{\rm coh}$ as $V\rightarrow 0$.

Although $I_{qp}^{\rm coh}(V)$ is Ohmic for $V\rightarrow 0$, it is non-Ohmic for finite V, and *vanishes* for $|eV|\geq 2J$. Thus, for $|eV|\geq 2J$, the only quasiparticle tunneling process allowed is *incoherent*. This result arises mainly from the geometry: the half spaces are *layered*, each with bandwidth $2J\ll E_F$ along the c axis. For bulk (3D) systems with large bandwidths $W\sim E_F$ along the tunneling direction, such limitations upon $I_{qp}^{\rm coh}$ are irrelevant. In addition, this limitation is only exact for very clean layers with intralayer scattering rate $1/\tau_{||}\ll J$, as assumed here. For $1/\tau_{||}\gg J$, $I_{qp}^{\rm coh}(V)$ is a Lorentzian in eV with a width of $1/\tau_{||}$.[7]

Thus, the quasiparticle current consists of two parts, $I_{qp}(V)=I_{qp}^{\rm inc}+I_{qp}^{\rm coh}(V)=V/R_n(V)$. In the inset of Fig. 1, we have plotted $R_0/R_n(V)$, where $R_0=J/[8\pi^2e^2N_{\rm 2D}(0)|\mathcal{T}_0|^2]$, as a function of $eV/2J$, for fixed values of $J/|\mathcal{T}_0|^2\tau_{\perp s}$. Note that one requires the break junction conductance to be much less than the conductance across neighboring layers in each half space. This implies that both $1/\tau_{\perp s}$ and $|\mathcal{T}_0|^2/J$ are small with respect to $J^2\tau_{||}$.[7] However, this does not restrict the relative magnitudes of $I_{qp}^{\rm coh}$ and $I_{qp}^{\rm inc}$.

In the superconducting state, I_c for Josephson tunneling across the break junction between arbitrary layered half spaces is given to lowest order in $\mathcal{T}_{\mathbf{k},\mathbf{k}'}$ by[7]

$$I_c(T)=\frac{4eT}{A^2}\sum_{\omega,\mathbf{k},\mathbf{k}'}\langle|\mathcal{T}_{\mathbf{k},\mathbf{k}'}|^2\rangle F_{u,1}(\mathbf{k},\omega)F_{\ell,1}^\dagger(\mathbf{k}',\omega), \tag{8}$$

where quite generally $F_{\mu,1}=-\Delta_\mu(\phi_{\mathbf{k}}){\rm Im}\,\Gamma_\mu/D_\mu$, $\Gamma_\mu=[\exp(ik_+s)-\exp(ik_-s)]/(iJ_\mu)$ and $D_\mu\equiv[|\Delta_\mu|^2+\omega^2]^{1/2}$.[9] The quantities $\exp(ik_\pm s)$ are obtained from the equation $J_\mu\cos(k_\pm s)=-\xi_{\mu 0}(\mathbf{k})\pm iD_\mu$,[9] which leads to $\Gamma_\mu=\Xi_\mu(iD_\mu)$, where $\Xi_\mu(z)$ is given by Eq. (5). In the bulk state approximation for $F_{\mu,1}$, Γ_μ is replaced by $R_\mu^{-1}(iD_\mu)$. Note that as $J_\mu\rightarrow 0$, the half space and bulk expressions both reduce to the familiar 2D form.

The incoherent part of the break junction I_c between arbitrary layered superconductors with $\nu=s,d$ is

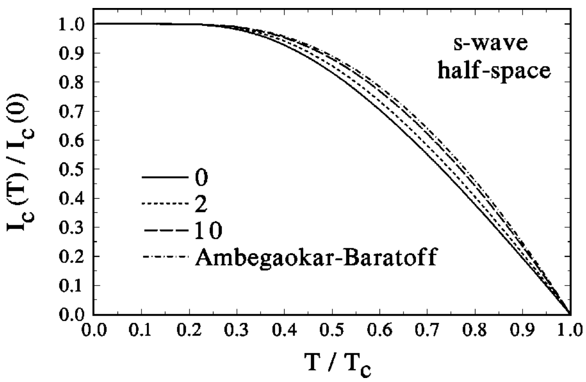

FIG. 2. Plots of $I^{\rm coh}_{c,s}(J,T)/I^{\rm coh}_{c,s}(J,0)$ for $J/\Delta_s(0)=0,2,10$ and of $I^{\rm inc}_{c,s}(T)/I^{\rm inc}_{c,s}(0)$ (AB) versus T/T_c, for tunneling between identical s-wave half-space superconductors.

$$I^{\rm inc}_{c,\nu}(T)=\frac{2eN_{\rm 2D}(0)T\pi}{\tau_{\perp\nu}}\sum_{\omega}\prod_{\mu=u}^{\ell}\int_0^{2\pi}\frac{d\phi_{\bf k}\Delta_\mu\varphi_\nu}{2\pi D_\mu}. \quad (9)$$

Note that $\Delta_\mu(\phi_{\bf k})$ implicitly depends upon ν. Equation (9) is obtained using either the bulk or half-space states. For s-wave pairing, $I^{\rm inc}_{c,s}(T)R^{\rm inc}_n$ equals the AB result.[4] For $\Delta_u=\Delta_\ell=\Delta$, $I^{\rm inc}_{c,\nu}(0)=\pi eN_{\rm 2D}(0)\Delta_\nu(0)A_\nu/\tau_{\perp\nu}$, where $A_s=1$, and $A_d=0.865$, so that $I^{\rm inc}_{c,d}(0)/I^{\rm inc}_{c,s}(0)=0.744\tau_{\perp s}/\tau_{\perp d}\ll 1$. $I^{\rm inc}_{c,s}(T)/I^{\rm inc}_{c,s}(0)$ and $I^{\rm inc}_{c,d}(T)/I^{\rm inc}_{c,d}(0)$ are plotted in Figs. 2 and 3, respectively.

For coherent c-axis break junction Josephson tunneling between identical materials, we drop the subscripts μ, noting that $\Delta(\phi_{\bf k})=\Delta_\nu(T)\varphi_\nu(\phi_{\bf k})$, etc. Writing Γ in F_1 in integral form as above Eq. (5), and performing two integrals analytically, we have

$$I^{\rm coh}_{c,\nu}(J,T)=\frac{4CT}{\pi}\sum_{\omega}\int_0^{2\pi}d\phi_{\bf k}|\Delta|^2\int_0^{\pi}\frac{dzX(z)}{D^3W(z)}, \quad (10)$$

where $W(z)=[1+J^2\sin^2 z/D^2]^{1/2}$, $X(z)=\sin^4 z-\sin^2 z/[1+W(z)]$, and $C=2e|\mathcal{T}_0|^2N_{\rm 2D}(0)$. Using the bulk states, $I^{\rm coh}_{c,\nu,b}(J,T)$ is obtained from Eq. (10) by replacing $X(z)$ by $\sin^2 z/8$. In the limit $T\rightarrow 0$, we set $\alpha(\phi_{\bf k})=|J/\Delta_\nu(0)\varphi_\nu(\phi_{\bf k})|$, $\beta(\phi_{\bf k})=\frac{1}{2}(1+[1+\alpha^2]^{1/2})$, and $I^{\rm coh}_{c,\nu}(J,0)=CY_\nu[J/\Delta_\nu(0)]$ reduces to

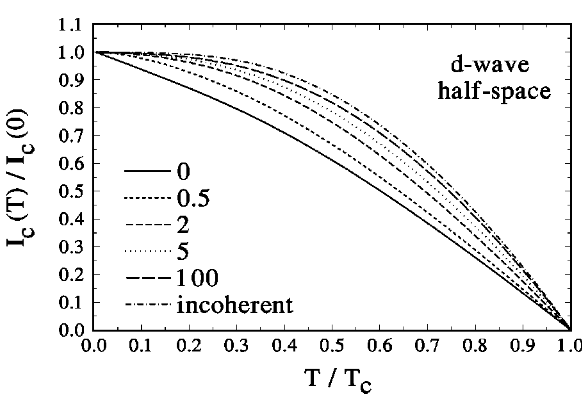

FIG. 3. Plots of $I^{\rm coh}_{c,d}(J,T)/I^{\rm coh}_{c,d}(J,0)$ for $J/\Delta_d(0)=0,0.5,2,5,100$ and of $I^{\rm inc}_{c,d}(T)/I^{\rm inc}_{c,d}(0)$ versus T/T_c, for tunneling between identical d-wave half-space superconductors.

$$I^{\rm coh}_{c,\nu}(J,0)=\frac{4C}{\pi}\int_0^{2\pi}d\phi_{\bf k}\left(\frac{1+8\beta}{24\beta^2}-\frac{\ln\beta}{\alpha^2}\right). \quad (11)$$

For the bulk states, $I^{\rm coh}_{c,\nu,b}(J,0)$ is obtained from $I^{\rm coh}_{c,\nu}(J,0)$ by replacing the integrand by $(8\alpha)^{-1}\sinh^{-1}\alpha$. In Fig. 1, we plotted $I^{\rm coh}_{c,\nu}(J,0)/I^{\rm coh}_{c,\nu}(0,0)$ and $I^{\rm coh}_{c,\nu,b}(J,0)/I^{\rm coh}_{c,\nu,b}(0,0)$ versus $\log_{10}[J/\Delta_\nu(0)]$ for s- or d-wave OP's. The most surprising point is that for small $J/\Delta_\nu(0)$, $Y_\nu(0)=1$, so that $I^{\rm coh}_{c,\nu}(J,0)\rightarrow C$, *independent* of $\Delta_\nu(0)$.[13]

From Fig. 1, $I^{\rm coh}_{c,d}(J,0)$ is slightly more sensitive to J than is $I^{\rm coh}_{c,s}(J,0)$. Also, for $J/\Delta_\nu(0)\sim 1$, the s- and d-wave bulk curves closely approximate the respective half-space curves obtained by reducing $J/\Delta_\nu(0)$ by the constant factor $1/\sqrt{2}$. For $J/\Delta_\nu(0)\gg 1$, however, the bulk and half-space curves are distinctly different. Whereas the correct $I^{\rm coh}_{c,\nu}=CY_\nu[J/\Delta_\nu(0)]\rightarrow 16C\Delta_\nu(0)B_\nu/(3J)$, where $B_s=1$, and $B_d=2\sqrt{2}/\pi$, spuriously $I^{\rm coh}_{c,\nu,b}(J,0)\rightarrow CB_\nu[\Delta_\nu(0)/J]\ln[2J/D_\nu\Delta_\nu(0)]$, where $D_s=1$, $D_d=0.5203$.

It is interesting to compare the coherent and incoherent results for identical half spaces. At $T=0$, the $V=0$ $I^{\rm coh}_{qp}(0)/I^{\rm inc}_{qp}\propto|\mathcal{T}_0|^2\tau_{\perp s}/J$, whereas for $T>T_c$, J/T_c distinguishes overdoped from underdoped behavior.[6] Since $\Delta_\nu(0)\approx T_c$, for $J/T_c\ll 1$, $I^{\rm coh}_{c,\nu}(J,0)/I^{\rm inc}_{c,\nu}(0)\propto|\mathcal{T}_0|^2\tau_{\perp\nu}/T_c$. For $J/T_c\gg 1$, $I^{\rm coh}_{c,\nu}(J,0)$ and $I^{\rm inc}_{c,\nu}(0)$ both $\propto\Delta_\nu(0)$, but $I^{\rm coh}_{c,\nu}(J,0)/I^{\rm inc}_{c,\nu}(0)\propto|\mathcal{T}_0|^2\tau_{\perp\nu}/J$. These results lead to the curious conclusions that for $J/T_c\ll 1$, the underdoped normal state tunneling is incoherent, the $T=0$ quasiparticle break junction tunneling could be either coherent or incoherent, but d-wave break junction pair tunneling would be mainly coherent. On the other hand, for $J/T_c\gg 1$, the overdoped normal state half-space tunneling would be coherent, but the $T=0$ quasiparticle break junction tunneling and the s-wave break junction pair tunneling could be incoherent.

In the limits $J_\mu\rightarrow 0$, one can evaluate the $J=0$, $T=0$ limit of the coherent part of $I_c(T)$ from Eq. (8) as a function of $r=\Delta_u/\Delta_\ell$, obtaining $I^{\rm coh}_c(0,0)=2Cr\ln(r)/(r^2-1)$, where C is given following Eq. (7). In the limit $r\rightarrow 1$, $I^{\rm coh}_c(0,0)\rightarrow C$. For either two s-wave or two d-wave superconductors, r is independent of $\phi_{\bf k}$.

In Fig. 2, we plotted $I^{\rm coh}_{c,s}(J,T)/I^{\rm coh}_{c,s}(J,0)$, as a function of T/T_c, for tunneling between two layered s-wave superconductors. Typical curves with $J/\Delta_s(0)=0,2,10$ are shown, along with the AB curve, Eq. (9). For $J/\Delta_s(0)=100$, the curve is almost identical to the AB curve. Using the bulk states does not change these curves very much, except for large $J/\Delta_s(0)$. Clearly, $I_{c,s}(J,T)/I_{c,s}(J,0)$ is rather indistinguishable from that of AB, independent of the microscopic details.

In Fig. 3, we plotted $I^{\rm coh}_{c,d}(J,T)/I^{\rm coh}_{c,d}(J,0)$, as a function of T/T_c. Typical curves with $J/\Delta(0)=0,0.5,2,10$ are shown along with the d-wave analog of AB, $I^{\rm inc}_{c,d}(T)/I^{\rm inc}_{c,d}(0)$. Note that the magnitude of $I^{\rm inc}_{c,d}(T)$ is very small, due to the factor of $1/\tau_{\perp d}$. Unlike the s-wave curves in Fig. 2, the $I^{\rm coh}_{c,d}(J,T)/I^{\rm coh}_{c,d}(J,0)$ curves with small values of $J/\Delta_d(0)$ are distinctly linear at low T, and are thus distinguishable from the AB curve in Fig. 2. However, for $J/\Delta_d(0)\gg 1$, $I^{\rm coh}_{c,d}(J,T)/I^{\rm coh}_{c,d}(J,0)$ and $I^{\rm inc}_{c,d}(T)/I^{\rm inc}_{c,d}(0)$ are nearly indistinguishable from the AB curve.

Summarizing our results, we have for $\nu=s,d$,

$$I_{c,\nu}(J,0)R_n(0)=\frac{\pi[Z_\nu\Delta_\nu(0)+3J\eta Y_\nu/16]}{2e[1+\eta]},\qquad(12)$$

where $Z_\nu=A_\nu\tau_{\perp s}/\tau_{\perp\nu}$, $\eta=32|\mathcal{T}_0|^2\tau_{\perp s}/(3\pi J)$, and $Y_\nu=I^{\rm coh}_{c,\nu}(J,0)/C$. Since I_c is measured at $V=0$, one requires $R_n(0)$. For overdoped samples, $J/T_c\gg1$, and $I_{c,\nu}(J,0)R_n(0)$ reduces to $\pi\Delta_\nu(0)[Z_\nu+\eta B_\nu]/\{2e[1+\eta]\}$. For both $\nu=s,d$, this is proportional to $\Delta_\nu(0)$, and nearly independent of the break junction properties for *s*-wave superconductors. For *d*-wave superconductors, one requires a substantial $I^{\rm coh}_{c,d}$ in order to obtain a non-negligible I_cR_n. However, for underdoped samples, $J/T_c\ll1$, the situation is far more complicated. First, one cannot measure $R_n(0)$ in the usual way, since the coherent contribution, which can be large at $V=0$, essentially vanishes for $eV/2\Delta_\nu(0)>1$. Second, $I_{c,\nu}(J,0)$ is dominated by coherent tunneling and independent of $\Delta_\nu(0)$ for $|\mathcal{T}_0|^2\gg\pi A_\nu\Delta_\nu(0)/2\tau_{\perp\nu}$, which is especially likely for *d*-wave superconductors.

In conclusion, we found that for *c*-axis break junction tunneling between two layered superconductors, a crossover from incoherent quasiparticle to coherent pair tunneling can occur. This greatly complicates the determination of $R_n(0)$, the coherent part of which cannot be seen from measurements with $eV/2\Delta_\nu(0)>1$, unless $J\gg\Delta_\nu(0)$, which corresponds to overdoped samples. For underdoped samples, I_cR_n values tend to be overestimated. The approximate bulk electronic states lead to correct incoherent, but incorrect coherent, tunneling results. Incoherent *d*-wave pair tunneling leads only to very small I_cR_n values. For coherent pair tunneling, both *s*-wave and *d*-wave pair tunneling are large in magnitude for small $J/\Delta_\nu(0)$, and cross over to the AB form for large $J/\Delta_\nu(0)$. The T dependence of coherent *d*-wave tunneling is distinctly different from that for AB for small $J/\Delta_\nu(0)$.Thus, accurate measurements of the T dependence and magnitude of I_cR_n in such break junctions could give important information regarding the questions of the order parameter symmetry and of the coherence of the pair tunneling.

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