Collective excitations in an asymmetrically spin-polarized quantum well

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We present a phenomenological picture for the many-body excitations of a two-dimensional electron gas in a quantum well spin polarized by a dc magnetic field at an angle θ with the axis of the well. In the framework of the Landau theory of charged Fermi liquids, we determine the frequencies of collective modes by solving a transport equation for quasiparticles in the local electromagnetic field associated with the charge- and spin-density fluctuations. In the long-wavelength limit, analytic solutions for $\omega(\vec{q})$ are obtained as functions of the degree of spin polarization and of the angle θ. [S0163-1829(98)06144-X]

I. INTRODUCTION

Under the application of a dc magnetic field $\vec{B}$ the spin degeneracy of the electronic levels of an electron gas in a quantum well—n electrons per unit area in the $\hat{x}$-$\hat{z}$ plane—is lifted and an equilibrium state characterized by a spin population imbalance results. The difference between the number of spins parallel or antiparallel to the direction of $\vec{B}$ is described by the spin polarization $\zeta=(n_\uparrow-n_\downarrow)/(n_\uparrow+n_\downarrow)$ a continuous function of B, that can take on any value between -1 and 1.

A weak electromagnetic perturbation—an electric field $\vec{E}(\vec{q},\omega)$ and a magnetic induction $\vec{b}(\vec{q},\omega)$ of arbitrary orientation—can excite collective spin- and charge-density fluctuations at those values of the frequency ω that are the poles of the response functions. At resonance, the excitations propagate undamped in the system, sustained solely by the Coulomb interaction among electrons. The departure of the excitation frequencies from the single-particle transition is a measure of the many-body interaction. The inelastic light-scattering spectra of quantum wells reveal such effects with remarkable accuracy.[1]

The character of the collective modes excited in a quantum well spin polarized along the direction of its symmetry axis (considered $\hat{z}$ for simplicity), as reflected in spectroscopic measurements,[2] is the consequence of two main causes. The initial imbalance in the number of spins breaks the symmetry of the density-dependent Coulomb interaction and determines the collapse of the poles of the dielectric function and of the induced magnetization along $\hat{z}$. The resonant density oscillations of electrons with spin parallel to $\hat{z}$ have a spin-symmetric and a spin-antisymmetric component. The coupling between the former, a charge-density wave, and the latter, a spin-density wave (in which the direction of the spin is parallel to $\hat{z}$) is a function of ζ. The spin direction, however, fluctuates, under the application of b_x and b_y, and spin-flip processes can occur. They generate an induced magnetization perpendicular on $\hat{z}$ and corresponding linear independent spin waves in which the spin direction is in the $\hat{x}$-$\hat{y}$ plane. This phenomenological description[3] is supported by numerous microscopic models, which employ a large range of approximations for the many-body interaction, from Hartree-Fock[4] to local spin-density functional theory.[5]

When a dc magnetic field is applied at an angle θ to the $\hat{z}$ axis, the electron gas is spin polarized along $\hat{u}=(0,\sin\theta,\cos\theta)$ and acquires a cyclotron motion driven by the transverse component $B_y=B\sin\theta$. (We assume the quantum well to be infinitely thin, such that the cyclotron motion is constrained to the $\hat{x}$-$\hat{z}$ plane.) The many-body excitations induced in the well are now generated by fluctuations in the density of spins parallel to $\hat{u}$ and spin-flip processes about $\hat{u}$, projected on the usual system of axes. This is equivalent to a rotation of angle θ in the spin space through a nondiagonal matrix. The result is expected to be a linear combination of charge and spin waves, whose coefficients are bound to be very sensitive to the spin dependent part of the electron-electron interaction. Furthermore, cyclotron excitations, driven by $B_y=B\sin\theta$, will be mixed in.

Following the traditional analogy between the two-dimensional (2D) electron gas and a Fermi liquid,[6] we adopt the phenomenological Landau-Silin theory of the electron liquid as our background. In this framework we solve a transport equation for quasiparticles, entities of charge $-e$, effective mass m^*, and gyromagnetic factor γ^*, moving in the self-consistent local electromagnetic field associated with the charge fluctuations. (m^* and γ^* are the renormalized values of the band effective mass and gyromagnetic factor by considering the quasiparticle interaction.[7]) Such a semiclassical approach, which neglects the Landau quantization of the electron orbits, is valid when the cyclotron frequency $\hbar\omega_c^*=\hbar eB\sin\theta/m^*c$ is much smaller than the Zeeman splitting of the electron levels in the magnetic field $2\gamma^*B$. We solve the transport equation and obtain analytic solutions for the excitation frequencies in the limit of two simplifying conditions-long wavelength and small angle—in terms of the phenomenological parameters of the Landau theory, considered functions of ζ.

II. QUASIPARTICLE DYNAMICS

In the Landau-Silin theory of Fermi liquids, the elementary excitations of a 2D electron gas are quasiparticles of momentum $\vec{k}$ and spin $\vec{\sigma}$, described by the deviation $\delta n^0_{k\sigma}$ from thermal equilibrium. The thermal equilibrium distribution function arises from the noninteracting ground state (which consists of two Fermi discs of radii $k_{F\sigma}=\sqrt{4\pi n_\sigma}$) by adiabatically turning on the electron-electron interaction.

In a spin-polarized quantum well, the spin-operator eigenvectors correspond to spin projection 1/2 or $-1/2$ along the direction $\vec{B}=B(0,\sin\theta,\cos\theta)$, with associated quasiparticle distributions $\delta n^0_{k\uparrow}$ and $\delta n^0_{k\downarrow}$, respectively. When summed over $\vec{k}$, the difference $\delta n^0_{k\uparrow}-\delta n^0_{k\downarrow}$ is equal to $n\zeta$. In the usual spin-space basis, formed by the eigenvectors of $\{\vec{S}^2,S_z\}$, a quasiparticle of spin $\vec{\sigma}$ has an equilibrium distribution $\delta n_{k\sigma}$ given by

$$\delta n_{k\sigma}=\frac{\delta n^0_{k\uparrow}+\delta n^0_{k\downarrow}}{2}+(\vec{\sigma}\cdot\hat{u})\frac{\delta n^0_{k\uparrow}-\delta n^0_{k\downarrow}}{2}. \tag{1}$$

The interaction with an electromagnetic field of wave vector $\vec{q}$ and frequency ω creates new quasiparticles and changes δn_σ to $\delta\tilde{n}_{k\sigma}(\vec{q},\omega)$. (Henceforth, the dependence on $\vec{q}$ and ω is implicitly understood.) Employing the usual Pauli spin matrices, the new distribution function can be written as[8]

$$\delta\tilde{n}_{k\sigma}=\frac{\delta\tilde{n}_{k\uparrow}+\delta\tilde{n}_{k\downarrow}}{2}+\sigma_z\frac{\delta\tilde{n}_{k\uparrow}-\delta\tilde{n}_{k\downarrow}}{2}+\frac{\sigma^+\delta\tilde{n}^-_k}{4}+\frac{\sigma^-\delta\tilde{n}^+_k}{4}. \tag{2}$$

The total particle-density fluctuation $\delta\tilde{n}_k$ is identified as $\mathrm{Tr}(\delta\tilde{n}_{k\sigma})$, whereas the magnetization along the $\hat{z}$ axis $(\delta\tilde{n}_{k\uparrow}-\delta\tilde{n}_{k\downarrow})$ is just $\mathrm{Tr}(\sigma_z\delta\tilde{n}_{k\sigma})$. These are density fluctuations of electrons whose spin is parallel to the $\hat{z}$ axis. The transverse magnetization is $\delta\tilde{n}^\pm_k=\mathrm{Tr}(\sigma^\pm\delta\tilde{n}_{k\sigma})$ and results from spin-flip processes driven by $b^\pm=b_x\pm ib_y$.

Two quasiparticles $(\vec{k}\sigma)$ and $(\vec{k}'\sigma')$ interact through a momentum and spin-symmetric function:

$$\Phi_{k\sigma;k'\sigma'}=\phi_{k;k'}+(\vec{\sigma}\cdot\vec{\sigma}')\psi_{k,k'}, \tag{3}$$

which in a translationally invariant system depends only on the magnitude of the relative momentum between particles $|\vec{k}-\vec{k}'|$. The quasiparticle energy is a functional of the distribution function of the entire system of quasiparticles:

$$\epsilon_{k\sigma}=\epsilon^0_{k\sigma}+\mathrm{Tr}_{\sigma'}\sum_{k'}\Phi_{k\sigma;k'\sigma'}\delta\tilde{n}_{k'\sigma'}, \tag{4}$$

with $\epsilon^0_k=\hbar^2k^2/2m-\gamma^*\vec{\sigma}(\vec{B}+\vec{b})$, the bare quasiparticle energy in the local magnetic field (the effective band mass m is involved). The interaction $\delta\mathcal{E}_{k\sigma}$ is obtained when $\delta\tilde{n}_{k\sigma}$, Eq. (2), and $\Phi_{k\sigma;k'\sigma'}$, Eq. (3), are substituted into Eq. (4).

$$\delta\mathcal{E}_{k\sigma}=\sum_{k'}[(\phi_{kk'}+\sigma_z\psi_{kk'})\delta\tilde{n}_{k'\uparrow}+(\phi_{kk'}-\sigma_z\psi_{kk'})\delta\tilde{n}_{k'\downarrow}+\sigma^-\psi_{kk'}\delta\tilde{n}^+_{k'}/2+\sigma^+\psi_{kk'}\delta\tilde{n}^-_{k'}/2]. \tag{5}$$

In a semiclassical approximation, the dynamics of spin and charge fluctuations is governed by the solution of a transport equation. Quasiparticles with velocity $\vec{v}=\nabla_k\epsilon_{k\sigma}$ move in a local potential that consists of the external perturbation and the electromagnetic field associated self-consistently with the density fluctuations. In a collisionless regime, $\delta\tilde{n}_k$ satisfies[8]

$$i\hbar\frac{\partial\delta\tilde{n}_{k\sigma}}{\partial t}+\left(\hbar\omega^*_{c\sigma}\frac{\partial}{\partial\varphi}+\vec{v}_{k\sigma}\cdot\nabla_r\right)\left[\delta\tilde{n}_{k\sigma}+\left(-\frac{dn_{k\sigma}}{d\epsilon_{k\sigma}}\right)\delta\mathcal{E}_{k\sigma}\right]+e\vec{v}_{k\sigma}\cdot\vec{E}\left(\frac{dn_{k\sigma}}{d\epsilon_{k\sigma}}\right)+2i\gamma^*[\delta\tilde{n}_{k\sigma},\vec{B}]=0. \tag{6}$$

([. . . , . . .] is the quantum-mechanical commutator.)

In a linear response approximation, a solution $\delta\tilde{n}_{k\sigma}$ to Eq. (6) depends on the equilibrium distribution function $\delta n_{k\sigma}$ or in a more general way, through Eq. (2), on a superposition of $\delta n^0_{k\uparrow}$ and $\delta n^0_{k\downarrow}$. Since quasiparticles of spin σ are well defined only in the vicinity of a spin σ-Fermi surface, where $(-d\delta n^0_{k\sigma}/\epsilon_{k\sigma})$ behaves like a delta function,[7] $\delta\tilde{n}_{k\sigma}$ can be expressed in terms of two new momentum dependent functions $\nu_{\sigma\uparrow}(\vec{k})$ and $\nu_{\sigma\downarrow}(\vec{k})$ as a linear superposition of delta functions:

$$\delta\tilde{n}_{k\sigma}=\nu_{\sigma\uparrow}(\vec{k})\left(-\frac{d\delta n^0_{k\uparrow}}{d\epsilon_{k\uparrow}}\right)+\nu_{\sigma\downarrow}(\vec{k})\left(-\frac{d\delta n^0_{k\downarrow}}{d\epsilon_{k\downarrow}}\right). \tag{7}$$

$\nu_{\sigma\uparrow}$ and $\nu_{\sigma\downarrow}$ are, of course, 2×2 matrices in the spin space. The role of the second index $\uparrow$ or $\downarrow$, respectively, is to specify the Fermi surface in the proximity of which the quasiparticles are located. We impose that $\nu_{\sigma\uparrow}(\vec{k})$ and $\nu_{\sigma\downarrow}(\vec{k})$ have the same formal expression as Eq. (2), to include all fluctuations generated by variations in the quasiparticle density at both spin Fermi surfaces,

$$\nu_{\sigma\uparrow}=(\nu_{\uparrow\uparrow}+\nu_{\downarrow\uparrow})/2+\sigma_z(\nu_{\uparrow\uparrow}-\nu_{\downarrow\uparrow})/2+(\sigma^+\nu^+_\uparrow+\sigma^-\nu^-_\uparrow)/4, \tag{8}$$

$$\nu_{\sigma\downarrow}=(\nu_{\uparrow\downarrow}+\nu_{\downarrow\downarrow})/2+\sigma_z(\nu_{\uparrow\downarrow}-\nu_{\downarrow\downarrow})/2+(\sigma^+\nu^+_\downarrow+\sigma^-\nu^-_\downarrow)/4. \tag{9}$$

The delta functions require all momenta be equated to the corresponding $k_{F\sigma}$, and consequently, the new variable becomes the angle φ made by $\vec{k}$ with the $\hat{z}$ axis. The equations in $\vec{k}$ are coupled by the interaction terms $\delta\mathcal{E}_\sigma$. It is preferable to solve for the Fourier components of $\nu_{\sigma\uparrow}$ and $\nu_{\sigma\downarrow}$, which are considered periodic functions of φ. The Fourier component, indexed by an integer l is defined in the usual way,

$\nu_{l\sigma\sigma'}(k_{F\sigma},k_{F\sigma'}) = (1/2\pi)\int_0^{2\pi} d\varphi\, e^{-il\varphi}\nu_{\sigma\sigma'}(k_{F\sigma},k_{F\sigma'},\varphi)$, and is a parametric function of the two Fermi momenta $k_{F\sigma}$ and $k_{F\sigma'}$ of the interacting quasiparticles.

For a sinusoidal variation of the electromagnetic perturbation $\sim e^{i(\omega t - qx)}$, the matrix $\nu_{l\sigma\uparrow}$ is a solution of

$$-i\omega\nu_{l\sigma\uparrow}+il\omega^*_{\sigma}\mathcal{E}_{l\uparrow}-\frac{qv_{F\uparrow}}{2}[\mathcal{E}_{(l-1)\sigma\uparrow}-\mathcal{E}_{(l+1)\sigma\uparrow}]$$
$$=\frac{ev_{F\uparrow}}{2}[(E_z+iE_x)\delta_{l,1}+(E_z-iE_x)\delta_{l,-1}]. \quad (10)$$

$\nu_{l\sigma\downarrow}$ satisfies the complex conjugate of Eq. (10), in which the direction of all spins has been changed. The Fourier transform of the quasiparticle interaction energy $\mathcal{E}_{\sigma\uparrow}$ can be readily calculated from Eq. (3), once the coefficients of the interaction function, normalized by a constant density of states at the corresponding Fermi surface $N(0)=m^*_{\sigma}/2\pi\hbar^2$ are introduced. Because of the spin rotation from $\hat{u}$ to $\hat{z}$, which makes each quasiparticle state of spin σ (along $\hat{z}$) a linear combination of quasiparticle states with spin ↑ or ↓ (along $\hat{u}$), four sets of coefficients are necessary:

$$\alpha_{l\sigma}=\frac{m^*_{\sigma}}{2\pi\hbar^2}\int_0^{2\pi}d\varphi\, e^{-il\varphi}(\phi+\psi)_{k_{F\sigma};k'_{F\sigma}}(\varphi), \quad (11)$$

$$\mu_{l\sigma}=\frac{m^*_{\sigma}}{2\pi\hbar^2}\int_0^{2\pi}d\varphi\, e^{-il\varphi}(\phi-\psi)_{k_{F\sigma};k'_{F\sigma}}(\varphi), \quad (12)$$

$$\lambda_{l\sigma}=\frac{m^*_{\sigma}}{2\pi\hbar^2}\int_0^{2\pi}d\varphi\, e^{-il\varphi}(\phi+\psi)_{k_{F\sigma};k'_{F\bar{\sigma}}}(\varphi), \quad (13)$$

$$\beta_{l\sigma}=\frac{m^*_{\sigma}}{2\pi\hbar^2}\int_0^{2\pi}d\varphi\, e^{-il\varphi}(\phi-\psi)_{k_{F\sigma};k'_{F\bar{\sigma}}}(\varphi). \quad (14)$$

When estimated at the same Fermi surface, the interaction $\Phi_{kk'}$, generates a spin-symmetric coefficient $\alpha_{l\sigma}$, and a spin-antisymmetric one $\mu_{l\sigma}$. When the two interacting quasiparticles are at different Fermi surfaces, the coefficients become, respectively, $\lambda_{l\sigma}$ and $\beta_{l\sigma}$. Their dependence on the Fermi momenta $k_{F\sigma}=\sqrt{2\pi n(1-\sigma_z\zeta)}$ is relevant for their variation with ζ. It is straightforward to write now $\mathcal{E}_{l\sigma\uparrow}$:

$$\mathcal{E}_{l\sigma\uparrow}=\Big\{[(\alpha_{l\uparrow}+\mu_{l\uparrow})+\sigma_z(\alpha_{l\uparrow}-\mu_{l\uparrow})]\frac{\nu_{l\uparrow\uparrow}}{2}+[(\alpha_{l\uparrow}+\mu_{l\uparrow})$$
$$-\sigma_z(\alpha_{l\uparrow}-\mu_{l\uparrow})]\frac{\nu_{l\downarrow\uparrow}}{2}+[(\lambda_{l\downarrow}+\beta_{l\downarrow})+\sigma_z(\lambda_{l\downarrow}$$
$$-\beta_{l\downarrow})]\frac{\nu_{l\uparrow\downarrow}}{2}+[(\lambda_{l\downarrow}+\beta_{l\downarrow})-\sigma_z(\lambda_{l\downarrow}-\beta_{l\downarrow})]\frac{\nu_{l\downarrow\downarrow}}{2}$$
$$+\sigma^-(\alpha_{l\uparrow}-\mu_{l\uparrow})\frac{\nu^+_{l\uparrow}}{4}+\sigma^+(\alpha_{l\uparrow}-\mu_{l\uparrow})\frac{\nu^-_{l\uparrow}}{4}$$
$$+\sigma^-(\lambda_{l\downarrow}-\beta_{l\downarrow})\frac{\nu^+_{l\downarrow}}{4}+\sigma^+(\lambda_{l\downarrow}-\beta_{l\downarrow})\frac{\nu^-_{l\downarrow}}{4}\Big\}$$
$$\times(1+\vec{\sigma}\cdot\hat{u})/2. \quad (15)$$

For certain values of wave vector and frequency, the local electromagnetic field, created by the charge and spin fluctuations themselves, sustains the oscillations even after the perturbation has been removed. The charged quasiparticle flow is equivalent to an electric current $\vec{j}$ equal to the sum of all bare electrons momenta weighed by the deviation from equilibrium of the quasiparticle distribution function:

$$\vec{j}=-e\,\mathrm{Tr}_{\sigma}\sum_k \frac{\hbar\vec{k}}{m^*_{\sigma}}\delta\tilde{n}_{k\sigma}. \quad (16)$$

The self-consistent electric field, which drives the drift motion of the electrons in Eq. (6), is related to the electric current through Maxwell's equations, which in our geometry, lead to[9]

$$i\omega\left(-\frac{\epsilon_0}{2\pi q}E_x,0,\frac{qc^2}{2\pi\omega^2}E_z\right)=\vec{j}. \quad (17)$$

The local contribution of the magnetic field associated with the spin-density fluctuations is negligible by comparison with the electric field, and we set $\vec{b}=0$.

Equations (10), (17), and (16) form a self-consistent set, which can be solved for $\nu_{l\sigma\sigma'}$. This infinite homogeneous system admits a nontrivial solution only for those values of $\omega(\vec{q})$ that resolve the secular equation obtained by cancelling its determinant. The diagonal components correspond to charge and longitudinal spin oscillations—propagation along $\hat{x}$ and spin direction parallel to $\hat{z}$. The off diagonal terms give the frequency for the spin waves that propagate along $\hat{x}$, with the spin direction in the $\hat{x}$-$\hat{y}$ plane. Solving for the Fourier transform of the fluctuations decouples the equations of the system in the momentum space. The interaction term $\mathcal{E}_{l\sigma\sigma'}$, however, preserves the linear superposition of the charge and spin-density waves in terms dependent on the angle θ. The coefficients of this coupling are generated by a rotation in the spin space from the direction $\hat{u}$ of the applied dc magnetic field to $\hat{z}$, the direction along which the response of the system is studied.

The fundamental determinant of the system is a 16×16 block that contains the same order l of the density fluctuations $\nu_{\sigma\sigma'}$. The strength of the coupling between the longitudinal spin-density wave and the transverse spin waves is proportional to $\sin\theta$, whereas the charge density and the longitudinal spin-density waves are coupled through linear combinations of $\sin^2\theta/2$ and $\cos^2\theta/2$.

In two particular cases, for $\theta=0$ and $\theta=90°$, a solution to the secular equation of the collective excitations can be obtained without difficulty.[8,10] We investigate the case of a small angle θ when the secular equation still admits an analytic solution that bears the distinguishable character of the coupling between the charge and spin waves.

III. EXCITATION FREQUENCIES

An important simplification occurs in the long wavelength limit, when at the Fermi surface $qv_{k\sigma}\ll\omega^*_{c\sigma}$. In the lowest-order approximation in $qv_{F\sigma}/\omega^*_{c\sigma}$, the plasma waves, which are collective modes driven by the local electric field ($|l|<2$), and the cyclotron harmonics, which are determined by the transverse dc magnetic field $B_y=B\sin\theta$, alone ($|l|\geqslant 2$), are linear independent.

The plasma excitations that occur in the presence of a dc magnetic field, usually called magnetoplasmons, are resonant density fluctuations of quasiparticles whose spin is parallel to $\hat{z}$. The high-frequency excitation ω_{CDW} is a superposition of two magnetoplasmons. If $\omega_{p\sigma}^2=2\pi n_\sigma e^2 q/m^*$ is the plasma frequency of a 2D noninteracting electron gas of spin σ in a dielectric medium of permitivity ϵ_s, $\omega_{\rm CDW}$ is simply

$$\omega_{CDW}^2(q)=\overline{\omega_\uparrow^2}+\overline{\omega_\downarrow^2}, \tag{18}$$

with $\overline{\omega_\sigma}$ the plasma frequency for an electron gas of spin σ modified by the quasiparticle interaction:

$$\overline{\omega_\sigma^2}=\omega_{p\sigma}^2\left\{1+\alpha_{1\sigma}+\beta_{1\sigma}\sqrt{n_{\bar\sigma}/n_\sigma}-\frac{\theta^2}{4}\left[(2\alpha_{1\sigma}-\beta_{1\sigma}-\mu_{1\sigma})+\sqrt{n_{\bar\sigma}/n_\sigma}(2\beta_{1\sigma}-\alpha_{1\sigma}-\lambda_{1\sigma})\right]\right\}+\frac{q^2}{2}v_{F\uparrow}^2\left[(1+\alpha_{1\uparrow})(1+\alpha_{0\uparrow})+\left(\frac{m_\uparrow^*}{m_\downarrow^*}\right)\sqrt{\frac{n_\downarrow}{n_\uparrow}}\beta_{1\uparrow}\beta_{0\downarrow}\right]. \tag{19}$$

The variation of $\overline{\omega_\sigma^2}$ from $\omega_{p\sigma}^2$ has two sources. The interaction of quasiparticles of spin parallel to $\hat{z}$ is described by $\alpha_{1\sigma}+\beta_{1\sigma}\sqrt{n_{\bar\sigma}/n_\sigma}$, where $\alpha_{1\sigma}$ refers to the same-spin interaction, a parametric function of $k_{F\sigma}$, while β_σ is for the opposite-spin interaction, dependent on both $k_{F\sigma}$ and $k_{F\bar\sigma}$. The coupling with the spin-flip processes along $\hat{u}$ gives the term proportional to θ^2, driven entirely by the spin dependent part of the interaction $\psi_{kk'}$, whose Fourier coefficients are linearly combined in $(2\alpha_{1\sigma}-\beta_{1\sigma}-\mu_{1\sigma})$.

The low-frequency collective excitation is a spin-density wave:

$$\omega_{SDW}^2(q)=\frac{\theta^2}{4}\left\{\omega_{p\uparrow}^2\left[(\alpha_{1\uparrow}-\mu_{1\uparrow})-\sqrt{\frac{n_\downarrow}{n_\uparrow}}(\lambda_{1\uparrow}-\beta_{1\uparrow})\right]+\omega_{p\downarrow}^2\left[(\alpha_{1\downarrow}-\mu_{1\downarrow})-\sqrt{\frac{n_\uparrow}{n_\downarrow}}(\lambda_{1\downarrow}-\beta_{1\downarrow})\right]\right\}+q^2v_{F\uparrow}v_{F\downarrow}[(1+\alpha_{1\uparrow})(1+\alpha_{1\downarrow})-\beta_{1\uparrow}\beta_{1\downarrow}]\times\frac{\left\{\left[\sqrt{\frac{m_\uparrow^*}{m_\downarrow^*}}(1+\alpha_{0\uparrow})-\sqrt{\frac{m_\downarrow^*}{m_\uparrow^*}}\beta_{0\uparrow}\right]+\left[\sqrt{\frac{m_\downarrow^*}{m_\uparrow^*}}(1+\alpha_{0\downarrow})-\sqrt{\frac{m_\uparrow^*}{m_\downarrow^*}}\beta_{0\downarrow}\right]\right\}}{\left\{\left[\sqrt{\frac{n_\uparrow}{n_\downarrow}\frac{m_\downarrow^*}{m_\uparrow^*}}(1+\alpha_{1\uparrow})+\sqrt{\frac{n_\downarrow}{n_\uparrow}}\beta_{1\uparrow}\right)+\left[\sqrt{\frac{n_\downarrow}{n_\uparrow}\frac{m_\uparrow^*}{m_\downarrow^*}}(1+\alpha_{1\downarrow})+\sqrt{\frac{n_\uparrow}{n_\downarrow}}\beta_{1\downarrow}\right]\right\}}. \tag{20}$$

This mode is a superposition between a spin-symmetric plasma oscillation, driven by the spin-dependent part of the interaction, the first term, and a longitudinal spin wave, proportional to q^2. The origin of the first term is the spin-flip processes along the direction of the dc magnetic field $\hat{u}$, which generate contributions to the magnetization along the z axis. The spin excitations in the second term are driven by the $l=0$ and $l=1$ Fourier components of the spin-antisymmetric part of the quasiparticle interaction, weighted by the ratio of the spin populations. The excitation frequencies of the two magnetoplasmons have a quadratic dependence on ζ, since they are associated with fluctuations in the particle density, invariant under the change in direction of $\vec{B}$.

The spin waves for $|l|<2$ propagate along $\hat{x}$, with the electron spin in the $\hat{x}$-$\hat{y}$ plane. They correspond to the poles of the transverse magnetization, induced by up-down and down-up spin flips. The excitations start at the Zeeman spin-splitting energy, corrected by the quasiparticle interaction, plus a term proportional to θ^2, that describes the coupling with the high frequency plasmonic mode $\omega_{\rm CDW}$:

$$\omega_{\downarrow\to\uparrow}=-2\gamma^*B(1+\alpha_{1\downarrow}-\mu_\downarrow)+\frac{\theta^2}{4}\omega_{p\downarrow}^2\times[(\alpha_\downarrow-\mu_\downarrow)-\sqrt{n_\downarrow/n_\uparrow}(\lambda_\downarrow-\beta_\downarrow)]/\omega_{\rm CDW},$$

$$\omega_{\uparrow\to\downarrow}=2\gamma^*B(1+\alpha_{1\uparrow}-\mu_\uparrow)+\frac{\theta^2}{4}\omega_{p\uparrow}^2\times[(\alpha_\uparrow-\mu_\uparrow)-\sqrt{n_\uparrow/n_\downarrow}(\lambda_\downarrow-\beta_\downarrow)]/\omega_{\rm CDW}. \tag{21}$$

In first order in θ, the fundamental absorption $l=1$ occurs at $2\gamma^*B(1+\alpha_{1\sigma}-\mu_{1\sigma})=2\gamma B$, because of the renormalization of the gyromagnetic factor on account of the quasiparticle interaction.[3] The term proportional to θ^2 is mixed in through the spin dependent part of the interaction, and reflects the coupling between the spin-flip processes about the $\hat{z}$ axis with the density fluctuations along the direction of the initial polarization. The dependence with ζ is linear, as a consequence of the initial spin imbalance in the system, preserved in a spin-density wave that is a spin-antisymmetric property of the system.

In addition to the modes described above for $|l|>2$, in the system propagate coupled cyclotron harmonics associated with the electron motion in the static magnetic field $B_y = B\sin\theta$. These excitations begin at

$$\omega^*_{c\pm} = \frac{\theta}{2}\Big[\omega^*_\sigma \alpha_{l\sigma} + \omega^*_{\bar\sigma}\alpha_{l\bar\sigma} \pm \sqrt{4\omega^*_\sigma \omega^*_{\bar\sigma}\beta_{l\sigma}\beta_{l\bar\sigma} + (\omega^*_\sigma \alpha_{l\sigma} - \omega^*_{\bar\sigma}\alpha_{l\bar\sigma})^2}\Big], \quad (22)$$

with $\omega^*_\sigma = eB/m^*_\sigma c$. The linear θ dependence is generated entirely by the component of the magnetic field perpendicular on the layer $B_y = B\sin\theta$. Equation (22) regains previous obtained results for the motion of the 2D spin-polarized electron gas in a magnetic field.[3]

The spin waves for $|l|\geqslant 2$ are excited at

$$\omega_{l\uparrow\to\downarrow} = (l\omega^*_\uparrow \theta + 2\gamma^* B)(1+\alpha_{l\uparrow} - \mu_{l\uparrow}),$$
$$\omega_{l\downarrow\to\uparrow} = (l\omega^*_\downarrow \theta - 2\gamma^* B)(1+\alpha_{l\downarrow} - \mu_{l\downarrow}). \quad (23)$$

The effect of the spin polarization on these values is determined by the change in the effective mass, as well as by the change in the Fermi surface parameters for the up-spin electrons.

IV. SUMMARY

We have demonstrated that the collective excitations of a spin-polarized quantum well can be treated within the Landau-Silin theory of charged Fermi liquids. When the dc magnetic field is oriented at a small angle θ to the layer, the plasmonic excitations and the spin waves are coupled through terms that depend exclusively on the spin part of the quasiparticle interaction function. The coupling is obtained in terms of the Fourier coefficients of $\psi_{kk'}$, parametric functions of the Fermi momenta $k_{F\sigma}$. By comparison with experimental data, these results can be used to determine the phenomenological parameters used in the Landau theory. Such results can serve as guidance for microscopic models of the spin-spin interaction.

[1] L. B. Liao, D. Heiman, P. F. Hopkins, and A. C. Gossard, Phys. Rev. B **49**, 16 825 (1994).
[2] A. Pinczuk *et al.*, Phys. Rev. Lett. **68**, 3623 (1992).
[3] D. C. Marinescu and J. J. Quinn, Phys. Rev. B (to be published).
[4] J. C. Ryan, Phys. Rev. B **43**, 4499 (1990).
[5] I. K. Marcomos and S. Das Sarma, Phys. Rev. B **45**, 13 396 (1992).
[6] T. Ando, A. B. Fowler, and B. Stern, Rev. Mod. Phys. **54**, 457 (1982).
[7] D. Pines and P. Nozieres, *The Theory of Quantum Liquids* (Benjamin, New York, 1966).
[8] S. C. Ying and J. J. Quinn, Phys. Rev. **180**, 193 (1969).
[9] K. W. Chiu and J. J. Quinn, Phys. Rev. B **9**, 4727 (1974).
[10] T. K. Lee and J. J. Quinn, Phys. Rev. B **11**, 2144 (1975).