Theory of fluctuations around a nonequilibrium state maintained by interband optical and driving electric field in semiconductors

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Theory of fluctuations around a nonequilibrium state slowly varying in space and time in two-band semiconductors is developed. The derivation of kinetic equations for fluctuations within the framework of the Keldysh formalism is given. The system is supposed to be under the action of a classic optical and external driving electric field, which can displace the system substantially from equilibrium with a thermal bath, while the carriers can interact also with phonons, with one another via the Coulomb potential, and with the thermal photons causing interband recombination and generation. Matrix of correlation functions for Langevin random forces is obtained. It is shown that in the nonequilibrium state there is an extra correlation contribution of Coulomb and phonon scattering to the correlation functions of the Langevin forces. [S0163-1829(98)02939-7]

I. INTRODUCTION.

Investigation of fluctuations around a nonequilibrium stationary or slowly varying state is important in many areas of physics. The equations for the time-displaced as well as one-time correlation functions were obtained by Lax.[1–3] It has been assumed that the basic stochastic process is a one-particle transition probability and hence, the explicit results for the case of one-particle collisions have been obtained.

Fluctuations around a nonequilibrium stationary state taking into account the two-particle collisions were considered theoretically by Gantsevich, Gurevich, Katilius[4,5] and Kogan and Shul'man[6] in the framework of nondegenerate Boltzmann statistics. Generalizations to the degenerate Fermi statistics were obtained in Ref. 7. The many-particle dynamical-screening effect on the collisions between charged particles was included in the fluctuation theory in Refs. 7 and 8. The generation-recombination effects in the case of a single band and trapping centers were taken into account in Ref. 9 (Boltzmann statistics).

Although Lax's method historically precedes the Gantsevich *et al.* approach (method of moments) and has been extensively employed by many authors in noise phenomena investigations,[10,11] we will closely follow the latter. In our opinion, it is reasonable to use the Gantsevich *et al.* method not only because it gives deep insight into fluctuation phenomena but also because the main equations of this approach can be readily obtained in the framework of the Keldysh formalism,[12] which is a great convenience in calculations while Lax's method is less formal and requires some art and intuition.

Although Lax's formulation includes multilevel atomic transitions the results turn out to be far from being easily applied in the investigation of noise phenomena in semiconductor amplifiers and lasers. That is why, to our mind, some authors prefer generalization of results of the theory of fluctuations near the equilibrium state (in particular, fluctuation-dissipation theorem or Callen-Welton relation) in their study of intensity and phase noise in semiconductor optical amplifiers and lasers (see, e.g., Refs. 10, 11, and 13). On the other hand, Gantsevich *et al.*'s usual concern was a single band case.

Recently a substantial number of works have been devoted to derivation of quantum kinetic equations describing nonequilibrium optical properties of semiconductors. To mention only a few, let us note[14–17] where such equations were derived by different approaches in the form of effective Bloch equations for the density matrix in the case of two-band semiconductors. While quantum kinetic theory describing the nonequilibrium state in semiconductors can be regarded as more or less developed, to our knowledge, a theory of fluctuations around such a nonequilibrium state is still not available. In this paper we intend to fill this gap. One of the problems in quantum electronics attracting the most attention is investigation of noise in various semiconductor optical amplifiers and generators. For instance, the noise power generated by spontaneous emission in optical amplifiers requires a correlation function of random Langevin forces that enters the right-hand side of the quantum kinetic equation for a nondiagonal in the band indices density matrix component.[10,11,13] The same correlation function determines the spectral linewidth of a semiconductor injection laser.[11,18]

Thus our purpose is to give an adequate description of fluctuation phenomena for the case of two-band semiconductors near a nonequilibrium state slowly varying in space and time. We present a first-principles derivation of the equations governing evolution of fluctuations. The essential difference between the present treatment and that of Refs. 10 and 11 consists in the following main aspects: first, our treatment naturally leads to the so-called extra correlation terms in the correlation functions, which are not δ correlated in $\mathbf{k}$ space and arise due to the simultaneous appearance of two electron states $\mathbf{k}$ and $\mathbf{k}'$ (and the disappearance of two other states) as a result of many-body interactions while the approach of Refs. 10 and 11 has not taken into account these extra correlation sources.

The other main difference is due to a new dynamical variable—nondiagonal over band indices—density matrix component involved also in the scattering processes again in contrast to Refs. 10 and 11 where the equation for the nondiagonal component has been introduced phenomenologically.

The last difference exists due to our investigation of the nonequilibrium state slowly varying in space. Therefore, the present approach being more general than the previous ones includes a theory of fluctuations in a single-band case (which enables one to describe current noise) as well as optical band-to-band transitions on the equal footing.

In Sec. II we introduce a correlation function matrix and obtain an equation of motion for this matrix. We consider quasiclassical fluctuations and discuss underlying approximations. The three subsections are devoted to interband recombination-generation processes, electron-phonon scattering, and electron-electron scattering. For all the interactions explicit expressions for a relaxation operator are derived that take into account the particle-particle scattering as well as the scattering on polarization. Section III gives a derivation of the differential equation, which determines an initial condition (one-time matrix of correlation functions) of the equation of motion derived in Sec. II for two-time correlation functions. In the first subsection we give expressions for a source of fluctuations that enter the right-hand side of the equation for one-time correlation functions. Here we use the concept of Langevin random forces and the last subsection is devoted to a derivation of correlation functions for these forces. The last section contains concluding remarks.

II. EQUATION FOR MATRIX OF CORRELATION FUNCTIONS

Fluctuations are described by the time-displaced two-particle correlation function in the mixed or Wigner representation

$$g^{iklm}_{\alpha\beta\gamma\delta}(t+\tau,\mathbf{kr};t,\mathbf{k}'\mathbf{r}')=\sum_{\mathbf{qq}'} e^{-i\mathbf{qr}-i\mathbf{q}'\mathbf{r}'}g^{iklm}_{\alpha\beta\gamma\delta} \times(t+\tau,\mathbf{k}_-;t\mathbf{k}'_-;t\mathbf{k}'_+;t+\tau,\mathbf{k}_+), \tag{1}$$

where $\alpha\beta\gamma\delta$ are band indices (c for the conduction band and v for valence band) and $iklm$ are indices in the Keldysh space (in our notation $i,k,l,m=1,2$). Here shorthand notation $\mathbf{k}_\pm$ stands for

$$\mathbf{k}_+=\mathbf{k}+\frac{\mathbf{q}}{2};\quad \mathbf{k}_-=\mathbf{k}-\frac{\mathbf{q}}{2}.$$

We closely follow Ref. 7 and define $g^{iklm}_{\alpha\beta\gamma\delta}$ as

$$g^{iklm}_{\alpha\beta\gamma\delta}(t+\tau+\delta',\mathbf{k}_1;t+\delta,\mathbf{k}_2;t,\mathbf{k}_3;t+\tau,\mathbf{k}_4) =\langle Ta_{\alpha\mathbf{k}_1}(t+\tau+\delta')_i a_{\beta\mathbf{k}_2}(t+\delta)_k a^\dagger_{\gamma\mathbf{k}_3}(t)_l a^\dagger_{\delta\mathbf{k}_4}(t+\tau)_m\rangle, \quad \delta,\delta'\to 0, \tag{2}$$

where $\langle\ \rangle$ denotes taking the statistical average, T denotes ordering over the Keldysh contour, $a_{\alpha\mathbf{k}}$ and $a^\dagger_{\alpha\mathbf{k}}$ are the Heisenberg annihilation and creation operators for an electron in the band α with a definite wave vector $\mathbf{k}$.

In this section we derive the equation of motion for the correlation function $g^{iklm}_{\alpha\beta\gamma\delta}(t,\tau)_{\mathbf{kk}'}$ regarding it as a function of two independent time variables t and τ. It is easily seen that at the fixed Keldysh indices lk for any im components the following identity holds:

$$g^{iklm}_{\alpha\beta\gamma\delta}(t+\tau,t)_{\mathbf{kk}'}=g^{kl}_{\alpha\beta\gamma\delta}(t+\tau,t)_{\mathbf{kk}'},\quad \tau>0. \tag{3}$$

This correlation function obeys an integral equation of the ladder type analogous to the pair Green's function (GF), which can be represented diagrammatically in the usual way (see, e.g., Ref. 7), i.e., we represent it through the Keldysh Green's functions G^{ij} and the kernel K. As is easily seen from the diagrammatic representation one can write the equation for $g^{iklm}_{\alpha\beta\gamma\delta}$ in the right-hand or the left-hand form. The equation in the right-hand form analytically reads

$$\begin{aligned}
&g^{iklm}_{\alpha\beta\gamma\delta}(t+\tau+\delta',\mathbf{k}_-;t+\delta,\mathbf{k}'_-;t\mathbf{k}'_+;t+\tau,\mathbf{k}_+)\\
&\quad=G^{il}_{\alpha\gamma}(t+\tau+\delta',\mathbf{k}_-;t\mathbf{k}'_+)G^{km}_{\beta\delta}(t+\delta,\mathbf{k}'_-;t+\tau,\mathbf{k}_+)\\
&\quad\quad-G^{im}_{\alpha\delta}(t+\tau+\delta',\mathbf{k}_-;t+\tau,\mathbf{k}_+)G^{kl}_{\beta\gamma}(t+\delta,\mathbf{k}'_-;t\mathbf{k}'_+)\\
&\quad\quad+\sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4}\int dt_1dt_2dt_3dt_4 G^{ii'}_{\alpha\alpha'}\\
&\quad\quad\times(t+\tau+\delta',\mathbf{k}_-;t_1\mathbf{k}_1)G^{m'm}_{\delta'\delta}(t_4\mathbf{k}_4;t+\tau,\mathbf{k}_+)\\
&\quad\quad\times K^{i'k'l'm'}_{\alpha'\beta'\gamma'\delta'}(t_1\mathbf{k}_1,t_2\mathbf{k}_2,t_3\mathbf{k}_3,t_4\mathbf{k}_4)g^{l'klk'}_{\gamma'\beta\gamma\beta'}\\
&\quad\quad\times(t_3\mathbf{k}_3;t+\delta,\mathbf{k}'_-;t\mathbf{k}'_+;t_2\mathbf{k}_2).
\end{aligned} \tag{4}$$

Here and henceforth we employ the Einstein summation convention.

First of all we identify the group of diagrams constituting a product of two unlinked Green's functions, taking into account interaction to all orders. Since this group has nothing to do with correlation, we exclude this group from our definition of the correlation function, i.e., we consider equations for

$$g^{iklm}_{\alpha\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{kk}'}\equiv g^{iklm}_{\alpha\beta\gamma\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{kk}'} +G^{im}_{\alpha\delta\mathbf{k}}(t+\tau,\mathbf{r})G^{kl}_{\beta\gamma\mathbf{k}'}(t,\mathbf{r}'), \tag{5}$$

which can be rewritten in the form (this will become apparent later)

$$g^{iklm}_{\alpha\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{kk}'}=\langle[\hat{n}_{\alpha\delta\mathbf{k}}(t+\tau,\mathbf{r})-n_{\alpha\delta\mathbf{k}}(t+\tau,\mathbf{r})] \times[\hat{n}_{\beta\gamma\mathbf{k}'}(t,\mathbf{r}')-n_{\beta\gamma\mathbf{k}'}(t,\mathbf{r}')]\rangle. \tag{6}$$

In the following we usually omit the redundant Keldysh kl and band $\beta\gamma$ indices if there is no likelihood of confusion.

For nonstationary problems it is convenient to convert our integral equations into differential ones with respect to the time displacement τ. Taking the time derivative of Eq. (4), and making use of the right-hand and left-hand (or conjugate) Dyson equations for the GF

$$iG^{ij}_{\alpha\beta}(\mathbf{k}t,\mathbf{k}'t')=\langle Ta_{\alpha\mathbf{k}}(t)_i a^\dagger_{\beta\mathbf{k}'}(t')_j\rangle$$

in the differential form

$$i\partial_t G^{ij}_{\alpha\beta}(t\mathbf{p},t'\mathbf{p}') = \delta_{\alpha\beta}\delta_{ij}(-1)^{i+1}\delta(t-t')\delta_{\mathbf{pp}'}+\varepsilon^{ii'}_{\alpha\alpha'}(\mathbf{p})G^{i'j}_{\alpha'\beta}(t\mathbf{p},t'\mathbf{p}') +(-1)^{i+1}\sum_{\mathbf{k}}\int dt_1\Sigma^{ii'}_{\alpha\alpha'}(t\mathbf{p},\mathbf{k}t_1) \times G^{i'j}_{\alpha'\beta}(t_1\mathbf{k},t'\mathbf{p}') \tag{7}$$

and an analogous equation for $\partial_{t'}G^{ij}_{\alpha\beta}(t\mathbf{p},t'\mathbf{p}')$, where we have introduced the energy matrix

$$\varepsilon^{ij}_{\alpha\beta}(\mathbf{k})=\delta_{\alpha\beta}\delta_{ij}\varepsilon_{\alpha}(\mathbf{k});\quad \alpha=\{c,v\}, \tag{8}$$

we obtain

$$\begin{aligned}\partial_\tau g^{im}_{\alpha\delta}&(t+\tau+\delta',\mathbf{k}_-;t+\delta,\mathbf{k}'_-;t\mathbf{k}'_+;t+\tau,\mathbf{k}_+)\\ &=i[\varepsilon_\delta(\mathbf{k}_+)-\varepsilon_\alpha(\mathbf{k}_-)]g^{im}_{\alpha\delta}(t+\tau+\delta',\mathbf{k}_-;t+\delta,\mathbf{k}'_-;t\mathbf{k}'_+;t+\tau,\mathbf{k}_+)\\ &\quad+i\sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}\int dt_1dt_2dt_3[(-1)^iG^{m'm}_{\delta'\delta}(t_3\mathbf{k}_3;t+\tau,\mathbf{k}_+)K^{ik'l'm'}_{\alpha\beta'\gamma'\delta'}\\ &\quad\times(t+\tau+\delta',\mathbf{k}_-,t_1\mathbf{k}_1,t_2\mathbf{k}_2,t_3\mathbf{k}_3)g^{l'k'}_{\gamma'\beta'}(t_2\mathbf{k}_2;t+\delta,\mathbf{k}'_-;t,\mathbf{k}'_+;t_1\mathbf{k}_1)-(-1)^mG^{ii'}_{\alpha\alpha'}\\ &\quad\times(t+\tau+\delta',\mathbf{k}_-;t_1\mathbf{k}_1)K^{i'k'l'm}_{\alpha'\beta'\gamma'\delta}(t_1\mathbf{k}_1,t_2\mathbf{k}_2,t_3\mathbf{k}_3,t+\tau\mathbf{k}_+)\\ &\quad\times g^{l'k'}_{\gamma'\beta'}(t_3\mathbf{k}_3;t+\delta,\mathbf{k}'_-;t,\mathbf{k}'_+;t_2\mathbf{k}_2)]+\sum_{\mathbf{p}}\int dt'[i(-1)^i\Sigma^{ir}_{\alpha\mu}(t+\tau+\delta',\mathbf{k}_-;t'\mathbf{p})\\ &\quad\times g^{rm}_{\mu\delta}(t'\mathbf{p};t+\delta,\mathbf{k}'_-;t\mathbf{k}'_+;t+\tau,\mathbf{k}_+)-i(-1)^mg^{ir}_{\alpha\mu}\\ &\quad\times(t+\tau+\delta',\mathbf{k}_-;t+\delta,\mathbf{k}'_-;t\mathbf{k}'_+;t'\mathbf{p})\Sigma^{rm}_{\mu\delta}(t'\mathbf{p};t+\tau,\mathbf{k}_+)].\end{aligned} \tag{9}$$

Note that we have included the external driving scalar potential $\varphi(\mathbf{r})$ into the definition of the self-energy (SE) $-i\Sigma^{ij}_{\alpha\beta}$ as well as the external classical optical field causing interband transitions.

Equation (9) is formally exact, as is Eq. (4). The reason for the form of this equation is that the Feynman rules in the Keldysh space for calculating the self-energy Σ and kernel K exist. Since the physical quantities such as densities and particle currents (including interband currents) should be expressed in terms of the equal-time, one-particle density matrix, Eq. (9) is useless until it is closed.

Let us note that in the course of evaluation of integrals we encounter the correlation functions with more complicated dependence on the time variable than $g^{im}_{\alpha\delta}(t+\tau,\mathbf{r};t,\mathbf{r}')_{\mathbf{kk}'}$; they involve the annihilation and creation operators at different instants. The latter can be expressed through the former by the simple relation, they differ from the former only by the exponential factor, corresponding to the free motion:

$$g^{im}_{\alpha\delta}(t_1\mathbf{p},t+\delta\mathbf{k},t\mathbf{k},t_2\mathbf{p}) =\begin{cases} g^{im}_{\alpha\delta}(t_2,t)_{\mathbf{pk}}e^{-i\varepsilon_{\alpha\mathbf{p}}(t_1-t_2)}, & \delta\to 0 \quad (10)\\ g^{im}_{\alpha\delta}(t_1,t)_{\mathbf{pk}}e^{i\varepsilon_{\delta\mathbf{p}}(t_2-t_1)}, & \delta\to 0. \quad (11)\end{cases}$$

Later we will need also formulas relating the GF to the density matrix. We make here an assumption similar to that we have just made. Since one-time Green's functions are

$$G^{12}_{\alpha\beta\mathbf{p}}(\mathbf{r}t,t)=in_{\alpha\beta\mathbf{p}}(t,\mathbf{r});\quad G^{21}_{\alpha\beta\mathbf{p}}(\mathbf{r}t,t)=G^{12}_{\alpha\beta\mathbf{p}}(\mathbf{r}t,t)-i\delta_{\alpha\beta}, \tag{12}$$

where we have used the relation between G^{12} and G^{21} at the coincident instants and introduced the density matrix in the mixed representation by

$$n_{\alpha\beta\mathbf{p}}(t,\mathbf{r})=\int d\mathbf{x}e^{-i\mathbf{px}}\left\langle\psi^+_\beta\left(t,\mathbf{r}-\frac{\mathbf{x}}{2}\right)\psi_\alpha\left(t,\mathbf{r}+\frac{\mathbf{x}}{2}\right)\right\rangle;$$

$$n_{cc\mathbf{p}}(t,\mathbf{r})\equiv n_{c\mathbf{p}}(t,\mathbf{r}),\quad n_{cv\mathbf{p}}(t,\mathbf{r})\equiv p_{\mathbf{p}}(t,\mathbf{r}),$$
$$n_{vc\mathbf{p}}(t,\mathbf{r})\equiv p^*_{\mathbf{p}}(t,\mathbf{r}),n_{vv\mathbf{p}}(t,\mathbf{r})\equiv n_{v\mathbf{p}}(t,\mathbf{r}), \tag{13}$$

we have

$$G^{12}_{\alpha\beta\mathbf{p}}(\mathbf{r}t,t')=in_{\alpha\beta\mathbf{p}}(t,\mathbf{r})e^{i\varepsilon_{\beta\mathbf{k}}(t'-t)}, \tag{14}$$

$$G^{12}_{\alpha\beta\mathbf{p}}(\mathbf{r}t,t')=in_{\alpha\beta\mathbf{p}}(t',\mathbf{r})e^{i\varepsilon_{\alpha\mathbf{k}}(t'-t)}. \tag{15}$$

The notations we have just introduced are commonly used: $n_{\alpha\mathbf{p}}$ stands for the distribution function within the $\alpha=c,v$ bands while $p_{\mathbf{k}}$ describes a mixed electron-hole state and is closely related to the macroscopic polarization.

We will consider quasiclassical fluctuations; i.e., in what follows we will transform Eq. (9) to the Wigner representation and use the gradient approximation. Spatial $1/q$ and time $1/\omega$ scales of quasiclassical fluctuations have to be large as compared with the de Broglie wavelength and the time of formation of the quantum state $1/\varepsilon_{\mathbf{p}}$ having momentum $\mathbf{p}$ and kinetic energy $\varepsilon_{\mathbf{p}}$:

$$\frac{q}{p}\ll 1,\quad \frac{\omega}{\varepsilon_{\mathbf{p}}}\ll 1. \tag{16}$$

Needless to say, in the course of derivation of kinetic equations one has to impose constraints similar to Eq. (16); e.g., see Ref. 7.

Performing a Taylor expansion one observes that the convolution in the mixed representation

$$\sum_{\mathbf{q}} e^{-i\mathbf{qr}} \sum_{\mathbf{p}} \Sigma(\mathbf{k}_-,\mathbf{k}+\mathbf{p}) g(\mathbf{k}+\mathbf{p},\mathbf{k}_+) \tag{17}$$

is given by

$$\sum_{\mathbf{q}} e^{-i\mathbf{qr}} \sum_{\mathbf{p}} \Sigma(\mathbf{k}_-,\mathbf{k}+\mathbf{p}) g(\mathbf{k}+\mathbf{p},\mathbf{k}_+) = \exp\left[-\frac{i}{2}(\partial_{\mathbf{k}}^{\Sigma}\partial_{\mathbf{r}}^{g} - \partial_{\mathbf{k}}^{g}\partial_{\mathbf{r}}^{\Sigma})\right]\Sigma(\mathbf{k},\mathbf{r})g(\mathbf{k},\mathbf{r}). \tag{18}$$

Expanding the exponential in Eq. (18) and keeping the first two terms one can get the gradient approximation, it is easily seen that the second term is merely the Poisson brackets, finally

$$\sum_{\mathbf{q}} e^{-i\mathbf{qr}} \sum_{\mathbf{p}} \Sigma(\mathbf{k}_-,\mathbf{k}+\mathbf{p}) g(\mathbf{k}+\mathbf{p},\mathbf{k}_+) = \Sigma(\mathbf{k},\mathbf{r})g(\mathbf{k},\mathbf{r}) + \frac{i}{2}[\Sigma,g]_P, \tag{19}$$

where

$$[\Sigma,g]_P = (\partial_{\mathbf{r}}\Sigma\,\partial_{\mathbf{k}}g - \partial_{\mathbf{k}}\Sigma\,\partial_{\mathbf{r}}g).$$

Making use of Eq. (19) we take a Fourier transform of Eq. (9) over the ''center-of-mass'' coordinate $\mathbf{r}$ and get

$$\begin{aligned}
\partial_\tau g^{im}_{\alpha\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{kk}'} ={}& i[\varepsilon_{\delta\mathbf{k}}-\varepsilon_{\alpha\mathbf{k}}]g^{im}_{\alpha\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{kk}'} - \frac{1}{2}\frac{\partial(\varepsilon_{\alpha\mathbf{k}}+\varepsilon_{\delta\mathbf{k}})}{\partial\mathbf{k}}\frac{\partial g^{im}_{\alpha\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{kk}'}}{\partial\mathbf{r}} - e\frac{\partial\varphi}{\partial\mathbf{r}}\frac{\partial g^{im}_{\alpha\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{kk}'}}{\partial\mathbf{k}}\\
&-\frac{e}{m\omega_0}(\mathbf{E}_0 e^{-i\omega_0(t+\tau)}-\text{c.c.})[\mathbf{p}_{\alpha\bar{\alpha}}(\mathbf{k})g^{im}_{\bar{\alpha}\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{kk}'} - \mathbf{p}_{\bar{\delta}\delta}(\mathbf{k})g^{im}_{\alpha\bar{\delta}}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{kk}'}]\\
&+\frac{1}{2}\int dt'\{-(-1)^i[\Sigma^{ir}_{\alpha\mu\mathbf{k}}(t+\tau+\delta',t'),g^{rm}_{\mu\delta}(t'\mathbf{r};t+\delta,\mathbf{r}';t,\mathbf{r}';t+\tau,\mathbf{r})_{\mathbf{kk}'}]_P\\
&+(-1)^m[g^{ir}_{\alpha\mu}(t+\tau+\delta',\mathbf{r};t+\delta,\mathbf{r}';t\mathbf{r}';t',\mathbf{r})_{\mathbf{kk}'},\Sigma^{rm}_{\mu\delta\mathbf{k}}(t',t+\tau)]_P\}\\
&+\sum_{\mathbf{pqs}} e^{-i\mathbf{qr}}U_{\mathbf{p}}\Big[G^{im}_{\alpha\delta}(t+\tau+\delta',\mathbf{k}_-;t+\tau,\mathbf{k}_+-\mathbf{p})g^{mm}_{\mu\mu}(t+\tau+\delta',\mathbf{s}-\mathbf{p};t+\tau+\delta',\mathbf{s})\\
&-G^{im}_{\alpha\delta}(t+\tau+\delta',\mathbf{k}_-+\mathbf{p};t+\tau,\mathbf{k}_+)g^{ii}_{\mu\mu}\left(t+\tau+\delta',\mathbf{s}+\frac{\mathbf{p}}{2};t+\tau+\delta',\mathbf{s}-\frac{\mathbf{p}}{2}\right)\Big]\\
&+i\int dt'[(-1)^i\Sigma^{ir}_{\alpha\mu\mathbf{k}}(t+\tau+\delta',t')g^{rm}_{\mu\delta}(t'\mathbf{r};t+\delta,\mathbf{r}';t,\mathbf{r}';t+\tau,\mathbf{r})_{\mathbf{kk}'} - (-1)^m g^{ir}_{\alpha\mu}\\
&\times(t+\tau+\delta',\mathbf{r};t+\delta,\mathbf{r}';t\mathbf{r}';t',\mathbf{r})_{\mathbf{kk}'}\Sigma^{rm}_{\mu\delta\mathbf{k}}(t',t+\tau)] + i\sum_{\mathbf{p}}\int dt_1dt_2dt_3[(-1)^i G^{m'm}_{\delta'\delta\mathbf{k}}\\
&\times(t_3;t+\tau)K^{ik'l'm'}_{\alpha\beta'\gamma'\delta'}(t+\tau+\delta',\mathbf{k};t_2\mathbf{p},t_3\mathbf{p},t_1\mathbf{k}) - (-1)^m G^{ii'}_{\alpha\alpha'\mathbf{k}}(t+\tau+\delta';t_1)K^{i'k'l'm}_{\alpha'\beta'\gamma'\delta}\\
&\times(t_1\mathbf{k},t_2\mathbf{p},t_3\mathbf{p},t+\tau\mathbf{k})]g^{l'k'}_{\gamma'\beta'}(t_3\mathbf{p};t+\delta,\mathbf{k}';t,\mathbf{k}';t_2\mathbf{p}).
\end{aligned} \tag{20}$$

Here we have taken into account that the expressions for the SE due to the instantaneous driving potential φ and the optical field with interband frequency ω_0

$$\mathbf{E} = \mathbf{E}_0 e^{-i(\omega_0 t - \mathbf{k}_0\mathbf{r})} + \text{c.c.} \tag{21}$$

are

$$-i\Sigma^{ij}_{\alpha\beta}(t\mathbf{p};t'\mathbf{p}') = i(-1)^i\delta_{\alpha\beta}\delta_{ij}\delta(t-t')(-e\varphi_{\mathbf{p}-\mathbf{p}'}), \tag{22}$$

$$\begin{aligned}
-i\Sigma^{ij}_{\alpha\beta}(t\mathbf{p};t'\mathbf{p}') ={}& -i(-1)^{i+1}\delta_{ij}\delta_{\alpha\bar{\beta}}\delta(t-t')\frac{e}{m\omega_0}\\
&\times[-i\langle\alpha\mathbf{p}|e^{i\mathbf{k}_0\mathbf{r}}\hat{\mathbf{p}}|\beta\mathbf{p}'\rangle\mathbf{E}_0 e^{-i\omega_0 t}\\
&+i\langle\alpha\mathbf{p}|e^{-i\mathbf{k}_0\mathbf{r}}\hat{\mathbf{p}}|\beta\mathbf{p}'\rangle\mathbf{E}_0^* e^{i\omega_0 t}],
\end{aligned} \tag{23}$$

where the matrix element of momentum is expressible through the dipole moment $\mathbf{d}_{\alpha\beta\mathbf{p}}$:

$$-\frac{e}{m}\langle\alpha\mathbf{p}|e^{\pm i\mathbf{k}_0\mathbf{r}}\hat{\mathbf{p}}|\beta\mathbf{p}'\rangle = i\omega_{\alpha\beta\mathbf{p}}\mathbf{d}_{\alpha\beta\mathbf{p}}\delta_{\mathbf{p}\pm\mathbf{k}_0;\mathbf{p}'};\ \omega_{\alpha\beta\mathbf{p}} = \varepsilon_{\alpha\mathbf{p}} - \varepsilon_{\beta\mathbf{p}} \tag{24}$$

and the field amplitude can be written in terms of the spatial density of photons $\mathcal{N}_{\omega_0}$ with the frequency ω_0, $|\mathbf{E}_0| \propto (\hbar\omega_0\mathcal{N}_{\omega_0})^{1/2}$. Here and henceforth the bar over band indices means that $\overline{c}=v$ and $\overline{v}=c$.

The first term on the right-hand side of Eq. (20) describes the free oscillation of the interband polarization providing $\alpha\neq\delta$, the second term takes into account intraband motion, the third and fourth terms describe the interaction with the driving force and with the interband optical field. The terms constituting the Poisson brackets describe the self-consistent field produced by the inhomogeneity in the electron state and the renormalization of the velocity of intraband motion due to interaction, which reflects the fact that electrons between collisions do not behave as noninteracting particles. The next term describes the self-consistent field of fluctuations (SCF),

$$U(\mathbf{r})=\sum_{\mathbf{q}} U_{\mathbf{q}}e^{i\mathbf{qr}};\quad U_{\mathbf{q}}=\frac{4\pi e^2}{\varepsilon q^2} \tag{25}$$

is the Coulomb potential, ε being the dielectric constant. The corresponding expression for K is

$$\begin{aligned}K^{iklm}_{\alpha\beta\gamma\delta}&(t_1\mathbf{k}_1,t_2\mathbf{k}_2,t_3\mathbf{k}_3,t_4\mathbf{k}_4)\\ &=-i(-1)^i\delta_{\mathbf{k}_1+\mathbf{k}_2,\mathbf{k}_3+\mathbf{k}_4}\delta_{ik}\delta_{im}\delta_{il}\delta(t_1-t_2)\\ &\quad\times[\delta_{\alpha\gamma}\delta_{\beta\delta}\delta(t_1-t_3)\delta(t_2-t_4)U_{\mathbf{k}_3-\mathbf{k}_1}\\ &\quad-\delta_{\alpha\delta}\delta_{\beta\gamma}\delta(t_1-t_4)\delta(t_2-t_3)U_{\mathbf{k}_4-\mathbf{k}_1}],\end{aligned} \tag{26}$$

where the first term in the brackets accounts for the exchange contribution while the second describes direct Coulomb interaction.

Our primary concern is a contribution from the direct interaction. We can easily take the Fourier transform and in the gradient approximation the term with the Coulomb potential to the first order originated from K is given by (we put $i=1$ and $m=2$)

$$\begin{aligned}-\hat{B}\hat{g}_{\mathrm{scf}}=&\frac{\partial}{\partial\mathbf{k}}n_{\alpha\delta\mathbf{k}}\frac{\partial}{\partial\mathbf{r}}\int d\mathbf{r}_1U(\mathbf{r}-\mathbf{r}_1)\\ &\times\sum_{\mathbf{p}} g_{\mu\mu}(t+\tau+\delta',\mathbf{r}_1;t\mathbf{r}')_{\mathbf{pk}'}.\end{aligned} \tag{27}$$

This expression clearly demonstrates the influence of the self-consistent fluctuation field. Here and henceforth $n_{\alpha\delta\mathbf{k}}$ will stand for $n_{\alpha\delta\mathbf{k}}(t+\tau,\mathbf{r})$, it will cause no confusion since whenever we encounter the density matrix component with the wave vector $\mathbf{k}(\mathbf{k}')$ it is obvious that the time and space arguments are $t+\tau,\mathbf{r}(t,\mathbf{r}')$.

Let us calculate now the exchange contribution. The first term in Eq. (26) after substitution into Eq. (20) and evaluation yields the following expression:

$$\begin{aligned}-\hat{B}\hat{g}_{\mathrm{ex}}=i\sum_{\mathbf{q}} &U_{\mathbf{q}}\{g_{\alpha\mu}(t,\tau)_{\mathbf{k}+\mathbf{q},\mathbf{k}'}n_{\mu\delta\mathbf{k}}\\ &-n_{\alpha\mu\mathbf{k}}g_{\mu\delta}(t,\tau)_{\mathbf{k}+\mathbf{q},\mathbf{k}'}\}.\end{aligned} \tag{28}$$

Notice that in the single band case this contribution vanishes and the next exchange term in the gradient approximation can be neglected.

The contribution of the self-consistent field due to exchange of a phonon is to be neglected since we have taken into consideration a more efficient and stronger contribution, which stems from the Coulomb potential. In what follows we will also neglect the contribution to the self-consistent electron field produced by the exchange of a phonon since again we will take into account the stronger contribution arising from the Coulomb potential. Evaluating the analytical expression for the SE,

$$\begin{aligned}-i\Sigma^{ij}_{\alpha\beta}(t\mathbf{p}_-,t'\mathbf{p}_+)=&i(-1)^i\delta_{ij}\delta_{\alpha\beta}\delta(t-t')U_{\mathbf{q}}\\ &\times\sum_{\mu\mathbf{k}}[-iG^{ii}_{\mu\mu}(t+0(-1)^i,\mathbf{k}_-,t\mathbf{k}_+)],\end{aligned} \tag{29}$$

and using it when evaluating the term with the Poisson brackets in Eq. (20) we get for the self-consistent electron (SCE) field

$$\begin{aligned}-\hat{B}\hat{g}_{\mathrm{SCE}}=&\frac{\partial}{\partial\mathbf{k}}g_{\alpha\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{kk}'}\frac{\partial}{\partial\mathbf{r}}\\ &\times\int d\mathbf{r}_1U(\mathbf{r}-\mathbf{r}_1)\sum_{\mu\mathbf{p}} n_{\mu\mathbf{p}}(\mathbf{r}_1).\end{aligned} \tag{30}$$

To take into account the charge of lattice ions one should subtract from $\Sigma_{\mu\mathbf{p}}n_{\mu\mathbf{p}}$ the total number of carriers. The SCE field vanishes in the case of spatially homogeneous distribution of carriers.

The self-consistent electron field can be incorporated into the effective scalar potential if one replaces φ in Eq. (20) by φ^{eff} according to

$$\varphi^{\mathrm{eff}}=\varphi-\frac{1}{e}\int d\mathbf{r}_1U(\mathbf{r}-\mathbf{r}_1)\sum_{\mu\mathbf{p}} n_{\mu\mathbf{p}}(\mathbf{r}_1). \tag{31}$$

The last two terms in Eq. (20) take into account the scattering between states; they are out and in terms, respectively.

To complete this section let us write the expression for the term that gives the exchange contribution to the energy

$$\begin{aligned}-i\Sigma^{ij}_{\alpha\beta}(t\mathbf{p}_-,t'\mathbf{p}_+)=&(-1)^i\delta_{ij}\delta(t-t')\sum_{\mathbf{s}} iU_{\mathbf{s}}iG^{ii}_{\alpha\beta}\\ &\times(t\mathbf{p}_-+\mathbf{s};t'\mathbf{p}_++\mathbf{s})\end{aligned} \tag{32}$$

or in the mixed representation

$$\begin{aligned}-i\Sigma^{ij}_{\alpha\beta\mathbf{p}}(t\mathbf{r},t'\mathbf{r})=&(-1)^i\delta_{ij}\delta(t-t')\\ &\times\sum_{\mathbf{s}} iU_{\mathbf{s}}iG^{ii}_{\alpha\beta\mathbf{p}+\mathbf{s}}(t\mathbf{r};t'\mathbf{r}).\end{aligned} \tag{33}$$

Inserting this expression into the seventh term in Eq.(20) we get the following term in the right-hand side of Eq. (20):

$$\begin{aligned}i\sum_{\mathbf{s}} U_{\mathbf{s}}[&(-i)G^{ii}_{\alpha\mu\mathbf{k}+\mathbf{s}}(t+\tau+0)g^{im}_{\mu\delta}(t,\tau,\mathbf{r},\mathbf{r}')_{\mathbf{kk}'}\\ &-g^{im}_{\alpha\mu}(t,\tau,\mathbf{r},\mathbf{r}')_{\mathbf{kk}'}(-i)G^{mm}_{\mu\delta\mathbf{k}+\mathbf{s}}(t+\tau-0)].\end{aligned} \tag{34}$$

Using Eq. (12) and for definiteness putting the Keldysh indices $i=1$ and $m=2$ the last expression can be reduced to

$$-\hat{B}\hat{g}_{\mathrm{enr}}=i\sum_{\mathbf{s}} U_{\mathbf{s}}[n_{\alpha\mu\mathbf{k}+\mathbf{s}}g_{\mu\delta}(t,\tau)_{\mathbf{kk}'}-g_{\alpha\mu}(t,\tau)_{\mathbf{kk}'}n_{\mu\delta\mathbf{k}+\mathbf{s}}]. \tag{35}$$

The last expression can be incorporated into the definition of effective energy matrix by

$$\varepsilon^{\mathrm{eff}}_{\alpha\beta\mathbf{k}}=\varepsilon_{\alpha\mathbf{k}}\delta_{\alpha\beta}-\sum_{\mathbf{s}} U_s n_{\alpha\beta\mathbf{k}-\mathbf{q}} \tag{36}$$

and the first term in Eq. (20) can then be cast into the form

$$i[g_{\alpha\mu\mathbf{kk}'}\varepsilon^{\mathrm{eff}}_{\mu\delta\mathbf{k}}-\varepsilon^{\mathrm{eff}}_{\alpha\mu\mathbf{k}}g_{\mu\delta\mathbf{kk}'}]. \tag{37}$$

We notice that a sum of exchange operators in Eqs. (28) and (35) has a particle conserving property, i.e.,

$$\sum_{\mathbf{k}} [(B^{\alpha\beta}_{\mathbf{k}}g_{\mathbf{kk}'})_{\mathrm{ex}}+(B^{\alpha\beta}_{\mathbf{k}}g_{\mathbf{kk}'})_{\mathrm{enr}}]=0. \tag{38}$$

A. Interaction with photons. Interband generation and recombination

Let us return to Eq. (20) and consider the interaction with photons causing band-to-band transitions (interband recombination-generation processes). The electron-photon interaction Hamiltonian has the form

$$H_{\mathrm{el}\text{-}phot}=\sum_{\alpha\mathbf{k};\mathbf{q}\neq 0}(C^*_{\alpha\mathbf{q}}b^\dagger_{\alpha\mathbf{q}}a^\dagger_{v\mathbf{k}}a_{c\mathbf{k}+\mathbf{q}}+C_{\alpha\mathbf{q}}b_{\alpha\mathbf{q}}a^\dagger_{c\mathbf{k}}a_{v\mathbf{k}-\mathbf{q}}). \tag{39}$$

The photon one-time Green's function $\hat{D}$ is related to the number of photons $\mathcal{N}_{\alpha\mathbf{q}}$ by the expression

$$i\hat{D}^{12}_{\alpha\mathbf{q}}(t)=\mathcal{N}_{\alpha\mathbf{q}}(t);$$

$$i\hat{D}^{21}_{\alpha\mathbf{q}}(t)=1+i\hat{D}^{12}_{\alpha\mathbf{q}}(t), \tag{40}$$

which, in its turn, can be related to the two-time function by

$$i\hat{D}^{12}_{\alpha\mathbf{q}}(t,t')=\mathcal{N}_{\alpha\mathbf{q}}(t)e^{-i\omega_{\alpha\mathbf{q}}(t-t')}. \tag{41}$$

The electron-photon interaction brings about an additional matrix over the band indices $\delta_{\alpha\bar{\beta}}$ for the electron-photon vertex and by virtue of Eq. (39) gives for the line representing the exchange of photons between the valence- and conduction-band electrons

$$D^{ij}_{\alpha\mathbf{q}}(t,t')=\hat{D}^{ij}_{\alpha\mathbf{q}}(t,t')+\hat{D}^{ji}_{\alpha,-\mathbf{q}}(t',t). \tag{42}$$

For the term that describes the lowest-order generation (gain) term and is obtained from the kernel K we have

$$\begin{aligned}K^{iklm}_{\alpha\beta\gamma\delta}&(t_1\mathbf{k}_1,t_2\mathbf{k}_2,t_3\mathbf{k}_2,t_4\mathbf{k}_1)\\&=\sum_{\nu} i(-1)^{i+k}\delta_{il}\delta_{mk}\delta_{\alpha\bar{\beta}}\delta_{\gamma\bar{\alpha}}\delta_{\delta\bar{\beta}}\delta(t_1-t_3)\\&\quad\times\delta(t_2-t_4)D^{ik}_{\nu\mathbf{k}_1-\mathbf{k}_2}(t_1,t_2).\end{aligned} \tag{43}$$

Let us also specify the analytical expression for the SE, which describes the lowest-order recombination (loss) term

$$-i\Sigma^{ij}_{\alpha\beta\mathbf{k}}(t,t')=-(-1)^{i+j}\sum_{\nu\mathbf{q}} iG^{ij}_{\overline{\alpha\beta}\mathbf{k}-\mathbf{q}}(t,t')iD^{ij}_{\nu\mathbf{q}}(t,t'). \tag{44}$$

Let us begin with the gain term. Inserting Eq. (43) into the last term in Eq. (20) and performing integration over t_2 and t_3 we get

$$\begin{aligned}\sum_{\nu\mathbf{p}}\int dt'&[(-1)^{i'}G^{ii'}_{\alpha\delta\mathbf{k}}(t+\tau+\delta',t')D^{i'm}_{\nu\mathbf{k}-\mathbf{p}}\\&\times(t',t+\tau)g^{i'm}_{\overline{\delta\delta}}(t',t+\delta,t,t+\tau)_{\mathbf{pk}'}-(-1)^{m'}G^{m'm}_{\alpha\delta\mathbf{k}}\\&\times(t',t+\tau)D^{im'}_{\nu\mathbf{k}-\mathbf{p}}(t+\tau+\delta',t')g^{im'}_{\overline{\alpha\alpha}}\\&\times(t+\tau+\delta',t+\delta,t,t')_{\mathbf{pk}'}].\end{aligned} \tag{45}$$

In accordance with causality principle here we can replace the upper integration limit by the instant $t+\tau$ although the integration over t' formally runs from $-\infty$ to $+\infty$. One can easily verify that the future contribution vanishes due to the GF, SE, and kernel properties.

It is now straightforward, by employing Eq. (10) and Eq. (14) to reduce the two-time correlation function and the GF in the first term in the brackets to the one-time functions and Eq. (11) together with Eq. (15) in the second term, to obtain

$$\begin{aligned}\pi\sum_{\nu\mathbf{p}}&[(-1)^{i'}G^{ii'}_{\alpha\delta\mathbf{k}}(t+\tau+0)g^{i'm}_{\overline{\delta\delta}}(t,\tau)_{\mathbf{pk}'}\\&\times\{\hat{D}^{i'm}_{\nu\mathbf{k}-\mathbf{p}}(t+\tau-0)\delta_-(\varepsilon_{\delta\mathbf{k}}-\varepsilon_{\bar{\delta}\mathbf{p}}-\omega_{\nu\mathbf{k}-\mathbf{p}})+\hat{D}^{mi'}_{\nu\mathbf{p}-\mathbf{k}}\\&\times(t+\tau-0)\delta_-(\varepsilon_{\delta\mathbf{k}}-\varepsilon_{\bar{\delta}\mathbf{p}}+\omega_{\nu\mathbf{p}-\mathbf{k}})\}-(-1)^{m'}G^{m'm}_{\alpha\delta\mathbf{k}}\\&\times(t+\tau-0)g^{im'}_{\overline{\alpha\alpha}}(t,\tau)_{\mathbf{pk}'}\{\hat{D}^{im'}_{\nu\mathbf{k}-\mathbf{p}}(t+\tau+0)\delta_+\\&\times(\varepsilon_{\alpha\mathbf{k}}-\varepsilon_{\bar{\alpha}\mathbf{p}}-\omega_{\nu\mathbf{k}-\mathbf{p}})+\hat{D}^{im'}_{\nu\mathbf{p}-\mathbf{k}}(t+\tau+0)\delta_+\\&\times(\varepsilon_{\alpha\mathbf{k}}-\varepsilon_{\bar{\alpha}\mathbf{p}}+\omega_{\nu\mathbf{p}-\mathbf{k}})\}].\end{aligned} \tag{46}$$

Here we have introduced $\delta_-(x)$ and $\delta_+(x)$ by the identities

$$\begin{aligned}\int_{-\infty}^{0}dte^{itx}&=\frac{-i}{x-i0}=\pi\delta(x)-\mathcal{P}\frac{i}{x}\\&=\pi\delta_-(x);\delta_+(x)=\delta_-(-x),\end{aligned} \tag{47}$$

where $\mathcal{P}$ is the Cauchy principle part. Now using Eqs. (12) and (40) which express the electron and photon GF in terms of $n_{\alpha\beta}$ and $\mathcal{N}_{\mathbf{q}}$ we have

$$-\hat{B}\hat{g}_{\rm gen}=\pi\sum_{\nu\mathbf{q}}|C_{\nu\mathbf{q}}|^2\{[n_{\alpha\delta\mathbf{k}}+\mathcal{N}_{\nu\mathbf{q}}\delta_{\alpha\delta}]\times[g_{\overline{\delta\delta}}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}\delta_-(\varepsilon_{\delta\mathbf{k}}-\varepsilon_{\bar{\delta}\mathbf{k-q}}-\omega_{\nu\mathbf{q}})+g_{\overline{\alpha\alpha}}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}\delta_+(\varepsilon_{\alpha\mathbf{k}}-\varepsilon_{\bar{\alpha}\mathbf{k-q}}-\omega_{\nu\mathbf{q}})]+[\delta_{\alpha\delta}-n_{\alpha\delta\mathbf{k}}+\mathcal{N}_{\nu\mathbf{q}}\delta_{\alpha\delta}][g_{\overline{\delta\delta}}(t,\tau)_{\mathbf{k+q},\mathbf{k}'}\delta_-\times(\varepsilon_{\delta\mathbf{k}}-\varepsilon_{\bar{\delta}\mathbf{k+q}}+\omega_{\nu\mathbf{q}})+g_{\overline{\alpha\alpha}}(t,\tau)_{\mathbf{k+q},\mathbf{k}'}\delta_+\times(\varepsilon_{\alpha\mathbf{k}}-\varepsilon_{\bar{\alpha}\mathbf{k+q}}+\omega_{\nu\mathbf{q}})]\}. \tag{48}$$

Performing summation over band indices and retaining only the resonant terms we obtain the following nonzero operators:

$$-B^{cc}g_{\rm gen}=2\pi\sum_{\nu\mathbf{q}}|C_{\nu\mathbf{q}}|^2\delta(\varepsilon_{c\mathbf{k}}-\varepsilon_{v\mathbf{k-q}}-\omega_{\nu\mathbf{q}})\times[n_{c\mathbf{k}}+\mathcal{N}_{\nu\mathbf{q}}]g_{vv}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}, \tag{49}$$

$$-B^{cv}g_{\rm gen}=-\pi p_{\mathbf{k}}\sum_{\nu\mathbf{q}}|C_{\nu\mathbf{q}}|^2\{g_{cc}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}\delta_-(\varepsilon_{v\mathbf{k}}-\varepsilon_{c\mathbf{k-q}}+\omega_{\nu\mathbf{q}})-g_{vv}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}\delta_+(\varepsilon_{c\mathbf{k}}-\varepsilon_{v\mathbf{k-q}}-\omega_{\nu\mathbf{q}})\}. \tag{50}$$

Here we first encounter processes that do not conserve energy; this is explicitly displayed by δ_- and δ_+, which have imaginary parts. This is not surprising since the quantum kinetic equation for $p_{\mathbf{k}}$ includes terms that also do not have δ-function structure and describe both interband transition energy renormalization (imaginary parts of δ_- and δ_+) and relaxation (real parts),

$$-B^{vc}g_{\rm gen}=-\pi p^*_{\mathbf{k}}\sum_{\nu\mathbf{q}}|C_{\nu\mathbf{q}}|^2\{g_{cc}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}\delta_+(\varepsilon_{v\mathbf{k}}-\varepsilon_{c\mathbf{k-q}}+\omega_{\nu\mathbf{q}})-g_{vv}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}\delta_-(\varepsilon_{c\mathbf{k}}-\varepsilon_{v\mathbf{k-q}}-\omega_{\nu\mathbf{q}})\}, \tag{51}$$

$$-B^{vv}g_{\rm gen}=2\pi\sum_{\nu\mathbf{q}}|C_{\nu\mathbf{q}}|^2\delta(\varepsilon_{v\mathbf{k}}-\varepsilon_{c\mathbf{k+q}}+\omega_{\nu\mathbf{q}})[1-n_{v\mathbf{k}}+\mathcal{N}_{\nu\mathbf{q}}]g_{cc}(t,\tau)_{\mathbf{k+q},\mathbf{k}'}. \tag{52}$$

We should note here that $B^{\alpha\beta}_{\mathbf{k}}$ are still operators since in the gain (or in–) terms they includes summation over $\mathbf{q}$ of expressions like $(\cdots)g_{\mathbf{k-q}}$ describing how fluctuations with $\mathbf{k-q}$ vanish and fluctuations with the wave vector $\mathbf{k}$ appear.

As for the loss (recombination) term, substituting the expression for the self-energy Eq. (44) into the next to the last term in Eq. (20) and by the similar calculation as above we obtain

$$-\hat{B}\hat{g}_{\rm rec}=-\pi\sum_{\nu\mu\mathbf{q}}|C_{\nu\mathbf{q}}|^2\{g_{\mu\delta}(t,\tau)_{\mathbf{k},\mathbf{k}'}([\delta_{\overline{\alpha\mu}}-n_{\overline{\alpha\mu}\mathbf{k-q}}+\mathcal{N}_{\nu\mathbf{q}}\delta_{\overline{\alpha\mu}}]\delta_-(\varepsilon_{\bar{\mu}\mathbf{k-q}}-\varepsilon_{\mu\mathbf{k}}+\omega_{\nu\mathbf{q}})+[n_{\overline{\alpha\mu}\mathbf{k+q}}+\mathcal{N}_{\nu\mathbf{q}}\delta_{\overline{\alpha\mu}}]\delta_-(\varepsilon_{\bar{\mu}\mathbf{k+q}}-\varepsilon_{\mu\mathbf{k}}-\omega_{\nu\mathbf{q}}))+g_{\alpha\mu}(t,\tau)_{\mathbf{k},\mathbf{k}'}([\delta_{\overline{\mu\delta}}-n_{\overline{\mu\delta}\mathbf{k-q}}+\mathcal{N}_{\nu\mathbf{q}}\delta_{\overline{\mu\delta}}]\delta_+(\varepsilon_{\bar{\mu}\mathbf{k-q}}-\varepsilon_{\mu\mathbf{k}}+\omega_{\nu\mathbf{q}})+[n_{\overline{\mu\delta}\mathbf{k+q}}+\mathcal{N}_{\nu\mathbf{q}}\delta_{\overline{\mu\delta}}]\delta_+(\varepsilon_{\bar{\mu}\mathbf{k+q}}-\varepsilon_{\mu\mathbf{k}}-\omega_{\nu\mathbf{q}}))\}, \tag{53}$$

or after summation over band indices

$$-B^{cc}_{\alpha\beta}g^{\rm rec}_{\alpha\beta}=-2\pi g_{cc}(t,\tau)_{\mathbf{k},\mathbf{k}'}\sum_{\nu\mathbf{q}}|C_{\nu\mathbf{q}}|^2[1-n_{v\mathbf{k-q}}+\mathcal{N}_{\nu\mathbf{q}}]\delta(\varepsilon_{v\mathbf{k-q}}-\varepsilon_{c\mathbf{k}}+\omega_{\nu\mathbf{q}}), \tag{54}$$

$$-B^{cv}g^{\rm rec}=-\pi g_{cv}(t,\tau)_{\mathbf{k},\mathbf{k}'}\sum_{\nu\mathbf{q}}|C_{\nu\mathbf{q}}|^2\{[1-n_{v\mathbf{k-q}}+\mathcal{N}_{\nu\mathbf{q}}]\delta_-(\varepsilon_{v\mathbf{k-q}}-\varepsilon_{c\mathbf{k}}+\omega_{\nu\mathbf{q}})+[n_{c\mathbf{k+q}}+\mathcal{N}_{\nu\mathbf{q}}]\delta_+(\varepsilon_{c\mathbf{k+q}}-\varepsilon_{v\mathbf{k}}-\omega_{\nu\mathbf{q}})\}, \tag{55}$$

$$-B^{vc}g^{\rm rec}=-\pi g_{vc}(t,\tau)_{\mathbf{k},\mathbf{k}'}\sum_{\nu\mathbf{q}}|C_{\nu\mathbf{q}}|^2\{[1-n_{v\mathbf{k-q}}+\mathcal{N}_{\nu\mathbf{q}}]\delta_+(\varepsilon_{v\mathbf{k-q}}-\varepsilon_{c\mathbf{k}}+\omega_{\nu\mathbf{q}})\}+\{[n_{c\mathbf{k+q}}+\mathcal{N}_{\nu\mathbf{q}}]\delta_-(\varepsilon_{c\mathbf{k+q}}-\varepsilon_{v\mathbf{k}}-\omega_{\nu\mathbf{q}})\}, \tag{56}$$

$$-B^{vv}g^{\rm rec}=-2\pi g_{vv}(t,\tau)_{\mathbf{k},\mathbf{k}'}\sum_{\nu\mathbf{q}}|C_{\nu\mathbf{q}}|^2[n_{c\mathbf{k+q}}+\mathcal{N}_{\nu\mathbf{q}}]\delta(\varepsilon_{c\mathbf{k+q}}-\varepsilon_{v\mathbf{k}}-\omega_{\nu\mathbf{q}}), \tag{57}$$

B. Interaction with phonons

This subsection is devoted to calculation of the last two terms in Eq. (20) for the case when the carriers scatter on collective excitations of the lattice. The Hamiltonian for the electron-phonon interaction is

$$H_{\text{el-}phon}=\sum_{\alpha\mathbf{k},\mathbf{q}\neq 0}(c_{\mathbf{q}}b_{\mathbf{q}}a^{\dagger}_{\alpha\mathbf{k}}a_{\alpha\mathbf{k-q}}+c^*_{\mathbf{q}}b^{\dagger}_{\mathbf{q}}a^{\dagger}_{\alpha\mathbf{k}}a_{\alpha\mathbf{k+q}}), \tag{58}$$

where $b^{\dagger}_{\mathbf{q}}$ and $b_{\mathbf{q}}$ are the creation and annihilation operators for phonons with dispersion $\omega_{\mathbf{q}}$. It is easily seen that the electron-phonon interaction Hamiltonian (58) closely resembles that for electron-photon interaction (39) except that for now the interaction vertex does not compromise the factor $\sigma^x_{\alpha\beta}$ since the interaction (58) does not involve interband transitions. As soon as such a resemblance has been established one can process the above written expressions (48), (53) and simplify them to describe the electron-phonon interaction. Now the photon GF is replaced by the phonon GF and we have for the in-scattering term

$$-\hat{B}\hat{g}_{\rm in}=\pi\sum_{\mu\mathbf{q}}|c_{\mathbf{q}}|^2(\{[n_{\alpha\mu\mathbf{k}}+\mathcal{N}_{\mathbf{q}}\delta_{\alpha\mu}]g_{\mu\delta}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}\delta_-\times(\varepsilon_{\mu\mathbf{k}}-\varepsilon_{\mu\mathbf{k-q}}-\omega_{\mathbf{q}})+[n_{\mu\delta\mathbf{k}}+\mathcal{N}_{\mathbf{q}}\delta_{\mu\delta}]\times g_{\alpha\mu}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}\delta_+(\varepsilon_{\mu\mathbf{k}}-\varepsilon_{\mu\mathbf{k-q}}-\omega_{\mathbf{q}})\}+\{[\delta_{\alpha\mu}-n_{\alpha\mu\mathbf{k}}+\mathcal{N}_{\mathbf{q}}\delta_{\alpha\mu}]g_{\mu\delta}(t,\tau)_{\mathbf{k+q},\mathbf{k}'}\delta_-\times(\varepsilon_{\mu\mathbf{k}}-\varepsilon_{\mu\mathbf{k+q}}+\omega_{\mathbf{q}})+[\delta_{\mu\delta}-n_{\mu\delta\mathbf{k}}+\mathcal{N}_{\mathbf{q}}\delta_{\mu\delta}]\times g_{\alpha\mu}(t,\tau)_{\mathbf{k+q},\mathbf{k}'}\delta_+(\varepsilon_{\mu\mathbf{k}}-\varepsilon_{\mu\mathbf{k+q}}+\omega_{\mathbf{q}})\}). \qquad (59)$$

Performing summation over band indices we have, e.g., for $B^{cc}g^{\rm in}$,

$$-B^{cc}g^{\rm in}=2\pi\sum_{\mathbf{q}}|c_{\mathbf{q}}|^2g_{cc}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}\{[n_{c\mathbf{k}}+\mathcal{N}_{\mathbf{q}}]\delta(\varepsilon_{c\mathbf{k}}-\varepsilon_{c\mathbf{k-q}}-\omega_{\mathbf{q}})+[1-n_{c\mathbf{k}}+\mathcal{N}_{\mathbf{q}}]\delta(\varepsilon_{c\mathbf{k}}-\varepsilon_{c\mathbf{k-q}}+\omega_{\mathbf{q}})\}+\pi\sum_{\mathbf{q}}|c_{\mathbf{q}}|^2\{p_{\mathbf{k}}g_{vc}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}[\delta_-(\varepsilon_{v\mathbf{k}}-\varepsilon_{v\mathbf{k-q}}-\omega_{\mathbf{q}})-\delta_-(\varepsilon_{v\mathbf{k}}-\varepsilon_{v\mathbf{k-q}}+\omega_{\mathbf{q}})]+p^*_{\mathbf{k}}g_{cv}(t,\tau)_{\mathbf{k-q},\mathbf{k}'}[\delta_+(\varepsilon_{v\mathbf{k}}-\varepsilon_{v\mathbf{k-q}}-\omega_{\mathbf{q}})-\delta_+(\varepsilon_{v\mathbf{k}}-\varepsilon_{v\mathbf{k-q}}+\omega_{\mathbf{q}})]\}. \qquad (60)$$

The remaining operators $B^{\alpha\delta}$ can be detailed in the same way and can be easily obtained from Eq. (59).

Treating the out-scattering problem in the same way, with the SE being given by an expression similar again to the one obtained for photons, we observe that the result can be obtained by merely removing overbars from the band indices in Eq. (53):

$$-\hat{B}\hat{g}_{\rm out}=-\pi\sum_{\mu\mathbf{q}}|c_{\mathbf{q}}|^2(g_{\mu\delta}(t,\tau)_{\mathbf{k},\mathbf{k}'}\{[\delta_{\alpha\mu}-n_{\alpha\mu\mathbf{k-q}}+\mathcal{N}_{\mathbf{q}}\delta_{\alpha\mu}]\delta_-(\varepsilon_{\mu\mathbf{k-q}}-\varepsilon_{\mu\mathbf{k}}+\omega_{\mathbf{q}})+[n_{\alpha\mu\mathbf{k+q}}+\mathcal{N}_{\mathbf{q}}\delta_{\alpha\mu}]\delta_-(\varepsilon_{\mu\mathbf{k+q}}-\varepsilon_{\mu\mathbf{k}}-\omega_{\mathbf{q}})\}+g_{\alpha\mu}(t,\tau)_{\mathbf{k},\mathbf{k}'}\{[\delta_{\mu\delta}-n_{\mu\delta\mathbf{k-q}}+\mathcal{N}_{\mathbf{q}}\delta_{\mu\delta}]\delta_+(\varepsilon_{\mu\mathbf{k-q}}-\varepsilon_{\mu\mathbf{k}}+\omega_{\mathbf{q}})+[n_{\mu\delta\mathbf{k+q}}+\mathcal{N}_{\mathbf{q}}\delta_{\mu\delta}]\delta_+(\varepsilon_{\mu\mathbf{k+q}}-\varepsilon_{\mu\mathbf{k}}-\omega_{\mathbf{q}})\}). \qquad (61)$$

Again performing summation over dummy indices one can easily obtain operators $B^{\alpha\delta}$, for instance, for B^{cc} we have

$$-B^{cc}g^{\rm out}=-2\pi g_{cc}(t,\tau)_{\mathbf{k},\mathbf{k}'}\sum_{\mathbf{q}}|c_{\mathbf{q}}|^2\{[1-n_{c\mathbf{k-q}}+\mathcal{N}_{\mathbf{q}}]\delta(\varepsilon_{c\mathbf{k-q}}-\varepsilon_{c\mathbf{k}}+\omega_{\mathbf{q}})+[n_{c\mathbf{k+q}}+\mathcal{N}_{\mathbf{q}}]\delta(\varepsilon_{c\mathbf{k+q}}-\varepsilon_{c\mathbf{k}}-\omega_{\mathbf{q}})\}-\pi\sum_{\mathbf{q}}|c_{\mathbf{q}}|^2\{g_{vc}(t,\tau)_{\mathbf{k},\mathbf{k}'}p_{\mathbf{k-q}}[\delta_-(\varepsilon_{v\mathbf{k-q}}-\varepsilon_{v\mathbf{k}}-\omega_{\mathbf{q}})-\delta_-(\varepsilon_{v\mathbf{k-q}}-\varepsilon_{v\mathbf{k}}+\omega_{\mathbf{q}})]+g_{cv}(t,\tau)_{\mathbf{k},\mathbf{k}'}p^*_{\mathbf{k-q}}[\delta_+(\varepsilon_{v\mathbf{k-q}}-\varepsilon_{v\mathbf{k}}-\omega_{\mathbf{q}})-\delta_+(\varepsilon_{v\mathbf{k-q}}-\varepsilon_{v\mathbf{k}}+\omega_{\mathbf{q}})]\}. \qquad (62)$$

One can verify that the operators have the particle number conserving property, indeed, summing Eqs. (59) and (61) over $\mathbf{k}$ and changing integration variable $\mathbf{k}\to\mathbf{k}+\mathbf{q}$ in (59) and adding (59) and (61) we get

$$\sum_{\mathbf{k}}[(B^{\alpha\delta}_{\mathbf{k}}g_{\mathbf{kk}'})_{\rm in}+(B^{\alpha\delta}_{\mathbf{k}}g_{\mathbf{kk}'})_{\rm out}]=0. \qquad (63)$$

C. Electron-electron scattering

Although within the framework of Keldysh's GF formalism taking into account the screening does not present any difficulties (see, e.g., Refs. 7 and 16) for the sake of brevity in this section we will restrict ourselves to the unscreened Coulomb interaction. We will take screening into account only in one of our final specific expression for correlation function of Langevin random forces (see Sec. III B).

The second-order terms for the SE describe collisions between carriers and the exchange contribution to the scattering processes (this contribution is smaller than the contribution of the direct Coulomb scattering due to phase space constraints). We evaluate here only direct scattering and the corresponding expression for the SE reads

$$-i\Sigma^{ij}_{\alpha\beta\mathbf{k}}(t,t')=-i(-1)^{i+j}\sum_{\mathbf{pq}}U^2_{\mathbf{q}}G^{ij}_{\alpha\beta\mathbf{k-q}}(t,t')\times G^{ij}_{\mu\nu\mathbf{p+q}}(t,t')G^{ji}_{\nu\mu\mathbf{p}}(t',t). \qquad (64)$$

Substituting this expression into Eq. (20) and utilizing Eqs. (10), (11), (14), (15), (12) we have for the out-scattering term in the right-hand side of Eq. (20)

$$-\hat{B}\hat{g}_{a,\mathrm{el}}=-\pi\sum_{\mathbf{pq}} U_{\mathbf{q}}^2\{g_{\lambda\delta}(t,\tau)_{\mathbf{kk}'}\delta_-(\Delta\varepsilon_{\lambda\mu})[(n_{\alpha\lambda\mathbf{k}-\mathbf{q}}-\delta_{\alpha\lambda})(n_{\nu\mu\mathbf{p}+\mathbf{q}}-\delta_{\nu\mu})n_{\mu\nu\mathbf{p}}-n_{\alpha\lambda\mathbf{k}-\mathbf{q}}n_{\nu\mu\mathbf{p}+\mathbf{q}}(n_{\mu\nu\mathbf{p}}-\delta_{\mu\nu})]$$
$$+g_{\alpha\lambda}(t,\tau)_{\mathbf{kk}'}\delta_+(\Delta\varepsilon_{\lambda\nu})[(n_{\lambda\delta\mathbf{k}-\mathbf{q}}-\delta_{\lambda\delta})(n_{\nu\mu\mathbf{p}+\mathbf{q}}-\delta_{\nu\mu})n_{\mu\nu\mathbf{p}}-n_{\lambda\delta\mathbf{k}-\mathbf{q}}n_{\nu\mu\mathbf{p}+\mathbf{q}}(n_{\mu\nu\mathbf{p}}-\delta_{\mu\nu})]\}. \quad (65)$$

We have introduced shorthand notation for the energy difference

$$\Delta\varepsilon_{\alpha\beta}=\varepsilon_{\alpha\mathbf{k}-\mathbf{q}}+\varepsilon_{\beta\mathbf{p}+\mathbf{q}}-\varepsilon_{\beta\mathbf{p}}-\varepsilon_{\alpha\mathbf{k}},$$

$$-B^{cc}g_{a,\mathrm{el}}=-2\pi g_{cc}(t,\tau)_{\mathbf{kk}'}\sum_{\mathbf{pq}} U_{\mathbf{q}}^2\{\delta(\Delta\varepsilon_{cc})[(n_{c\mathbf{k}-\mathbf{q}}-1)(n_{c\mathbf{p}+\mathbf{q}}-1)n_{c\mathbf{p}}-n_{c\mathbf{k}-\mathbf{q}}n_{c\mathbf{p}+\mathbf{q}}(n_{c\mathbf{p}}-1)]+\delta(\Delta\varepsilon_{cv})((n_{c\mathbf{k}-\mathbf{q}}-1)$$
$$\times(n_{v\mathbf{p}+\mathbf{q}}-1)n_{v\mathbf{p}}-n_{c\mathbf{k}-\mathbf{q}}n_{v\mathbf{p}+\mathbf{q}}(n_{v\mathbf{p}}-1))\}+\pi\sum_{\mathbf{pq}} U_{\mathbf{q}}^2 g_{cc}(t,\tau)_{\mathbf{kk}'}\{p_{\mathbf{p}+\mathbf{q}}p_{\mathbf{p}}^*[\delta_-(\Delta\varepsilon_{cv})+\delta_+(\Delta\varepsilon_{cc})]+\text{c.c.}\}$$
$$-\pi\sum_{\mathbf{pq}} U_{\mathbf{q}}^2 g_{cv}(t,\tau)_{\mathbf{kk}'}p_{\mathbf{k}-\mathbf{q}}^*\{\delta_+(\Delta\varepsilon_{vc})[n_{c\mathbf{p}+\mathbf{q}}-n_{c\mathbf{p}}]+\delta_+(\Delta\varepsilon_{vv})[n_{v\mathbf{p}+\mathbf{q}}-n_{v\mathbf{p}}]\}$$
$$-\pi\sum_{\mathbf{pq}} U_{\mathbf{q}}^2 g_{vc}(t,\tau)_{\mathbf{kk}'}p_{\mathbf{k}-\mathbf{q}}\{\delta_-(\Delta\varepsilon_{vc})[n_{c\mathbf{p}+\mathbf{q}}-n_{c\mathbf{p}}]+\delta_-(\Delta\varepsilon_{vv})[n_{v\mathbf{p}+\mathbf{q}}-n_{v\mathbf{p}}]\}. \quad (66)$$

Here the first term is conventional except that it includes also and polarization $p_{\mathbf{k}}$ are created by the same external perturbation, in this case $|p_{\mathbf{k}}|^2$ and $n_{c\mathbf{k}}$ (or $1-n_{v\mathbf{k}}$) turn out to be of the same (second) order in the perturbation.

Now let us consider the part of Eq. (20) involving the kernel. For the kernel we have three terms [we denote them by (b),(c),(d)], which bring about operators $B^{\alpha\beta}g_{b,\mathrm{el}}$, $B^{\alpha\beta}g_{c,\mathrm{el}}$ and $B^{\alpha\beta}g_{d,\mathrm{el}}$. Explicit expressions for these operators are given in the Appendix A.

Again the sum of operators describing the electron-electron scattering has a particle conserving property, that is easily seen after simple transformations under the sum over $\mathbf{k}$ of the sum of corresponding operators (a,b,c,d). For instance, the replacement $\mathbf{k}\to\mathbf{k}+\mathbf{q}$ in the sum of Eq. (A4) (c) over $\mathbf{k}$ clearly demonstrates that this sum coincides with the sum of Eq. (A2) (b) over $\mathbf{k}$ except for the sign. Hence, we encounter the following relation:

$$\sum_{\mathbf{k}}[(B_{\mathbf{k}}^{\alpha\delta}g_{\mathbf{kk}'})_{a,\mathrm{el}}+(B_{\mathbf{k}}^{\alpha\delta}g_{\mathbf{kk}'})_{b,\mathrm{el}}+(B_{\mathbf{k}}^{\alpha\delta}g_{\mathbf{kk}'})_{c,\mathrm{el}}+(B_{\mathbf{k}}^{\alpha\delta}g_{\mathbf{kk}'})_{d,\mathrm{el}}]=0. \quad (67)$$

Collecting all operators $\hat{B}_j$ and denoting the sum of them by $\hat{B}$ and also incorporating the first four terms in the right-hand side of Eq. (20) into the definition of the operator $\hat{B}$ we have

$$[\delta_{\alpha\alpha'}\delta_{\delta\delta'}\partial_\tau+B_{\alpha'\delta'\mathbf{k}}^{\alpha\delta}(t+\tau,\mathbf{r})]g_{\alpha'\beta\gamma\delta'}(t+\tau\mathbf{r},t\mathbf{r}')_{\mathbf{kk}'}=0, \quad \tau>0. \quad (68)$$

The operator B describes relaxation of small deviations to a steady distribution and thus is called a relaxation operator. As is demonstrated by this formula there is no difference between a fluctuation deviation evolution and an evolution of deviation caused by some external perturbation. As has been noted by Gantsevich *et al.* in their comprehensive review article[4] Eq. (68) is nothing but an application of the famous Onsager's hypothesis to the nonequilibrium fluctuations.

Once the formula (68) is established the problem is reduced to the seeking of the one-time two-particle correlation function $g_{\alpha\beta\gamma\delta}(t\mathbf{r},t\mathbf{r}')_{\mathbf{kk}'}$, which stands as an initial condition to our differential equation for the time-displaced correlation function $g_{\alpha\beta\gamma\delta}(t+\tau\mathbf{r},t\mathbf{r}')_{\mathbf{kk}'}$. First, we observe that Eq. (4) at $\tau=0$ includes two terms: the first term corresponds to the unlinked diagrams (cf. Ref. 7)

$$G_{\alpha\gamma}^{11}(t+\delta',\mathbf{k}_-;t\mathbf{k}'_+)G_{\beta\delta}^{12}(t+\delta,\mathbf{k}'_-;t,\mathbf{k}_+)$$

and its Fourier transform results in

$$\delta_{\mathbf{rr}'}\delta_{\mathbf{kk}'}n_{\beta\delta\mathbf{k}}(t\mathbf{r})[\delta_{\alpha\gamma}-n_{\alpha\gamma\mathbf{k}}(t\mathbf{r})], \quad (69)$$

the third (the second has been included into our definition of correlation function) corresponds to the group of linked diagrams. Adopting for the latter at $\tau=0$ the notation $G_{\alpha\beta\gamma\delta}(t;\mathbf{rr}')_{\mathbf{kk}'}$ we write

$$g_{\alpha\beta\gamma\delta}(t\mathbf{r},t\mathbf{r}')_{\mathbf{kk}'}=\delta_{\mathbf{rr}'}\delta_{\mathbf{kk}'}n_{\beta\delta\mathbf{k}}(t\mathbf{r})[\delta_{\alpha\gamma}-n_{\alpha\gamma\mathbf{k}}(t\mathbf{r})]+G_{\alpha\beta\gamma\delta}(t;\mathbf{rr}')_{\mathbf{kk}'}. \quad (70)$$

As for the initial condition, in its turn, it has to be obtained from an equation of motion for G, we are going to derive such an equation in the next section.

III. EQUATION FOR ONE-TIME CORRELATION FUNCTION

In the same way as we have derived equation for the time-displaced correlation function for the one-time correlation function $G_{\alpha\beta\gamma\delta}(t;\mathbf{rr}')_{\mathbf{kk}'}$ we obtain the differential equation with respect to t. Since we have already established the equation with respect to τ (or with respect to the first time variable $t+\tau$), we note that we need derivative with respect to the second time variable t. The left-hand form integral equation [counterpart of Eq. (4) written in the left-hand form] is suitable for such a differentiation and it can be easily established that the procedure leads to the same operator B which in this case acts on variables corresponding to the second time variable: band indices $\beta\gamma$ and electron wave vector $\mathbf{k}'$ and spatial position $\mathbf{r}'$. Thus, the equation of motion for the one-time correlation function reads

$$\begin{aligned}\partial_t G_{\alpha\beta\gamma\delta}(t;\mathbf{rr}')_{\mathbf{kk}'} = {} & L_{\alpha\beta\gamma\delta}(t;\mathbf{rr}')_{\mathbf{kk}'} - [\delta_{\beta\beta'}\delta_{\gamma\gamma'}B^{\alpha\delta}_{\alpha'\delta'\mathbf{k}}(t\mathbf{r}) \\ & + \delta_{\alpha\alpha'}\delta_{\delta\delta'}B^{\beta\gamma}_{\beta'\gamma'\mathbf{k}'}(t\mathbf{r}')] \\ & \times G_{\alpha'\beta'\gamma'\delta'}(t;\mathbf{rr}')_{\mathbf{kk}'} . \end{aligned} \tag{71}$$

Here we have introduced the source term $L_{\alpha\beta\gamma\delta}$, which describes a creation of correlation. Thus the influence of the collisions as well as self-consistent fields on the kinetics is twofold: the electron-electron, electron-photon, electron-phonon collisions enter Eq. (71) through the operator B, which describes relaxation processes; on the other hand the very existence of the extra source term $L_{\alpha\beta\gamma\delta}(t;\mathbf{rr}')_{\mathbf{kk}'}$ creating a correlation is due to the same collision events.[5]

The source term in our diagrammatic procedure appears in the following way: among the diagrams representing the correlation function we observe the group with four Green's functions linked by the kernel K. Ladder repetition of the group constitutes the one-time correlation function. The source term stems from

$$\begin{aligned} l_{\alpha\beta\gamma\delta} = \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} \int & dt_1 dt_2 dt_3 dt_4 G^{ii'}_{\alpha\alpha'}(t+\delta',\mathbf{k}_-;t_1\mathbf{k}_1) \\ & \times G^{m'm}_{\delta'\delta}(t_4\mathbf{k}_4;t,\mathbf{k}_+) K^{i'k'l'm'}_{\alpha'\beta'\gamma'\delta'} \\ & \times (t_1\mathbf{k}_1,t_2\mathbf{k}_2,t_3\mathbf{k}_3,t_4\mathbf{k}_4) G^{l'l}_{\gamma'\gamma}(t_3\mathbf{k}_3;t\mathbf{k}'_+) \\ & \times G^{kk'}_{\beta\beta'}(t+\delta\mathbf{k}'_-;t_2\mathbf{k}_2) \end{aligned} \tag{72}$$

after differentiation with respect to time t, which leads to the replacing one of the GF's by the corresponding δ function in accordance with Eq. (7). Since we have already written expressions for K we need no extra formulas. In the next subsection we are going to process each of them separately and obtain explicit expressions for all interactions we have considered so far and included in Eq. (68) and Eq. (71).

A. Extra correlation source

Let us begin with the Coulomb interaction, we take into account first direct Coulomb interaction and replace K in Eq. (72) according to Eq. (26). Calculations are lengthy but straightforward and we get the following extra source term

$$\begin{aligned} L_{\alpha\beta\gamma\delta}(t;\mathbf{rr}')_{\mathbf{kk}'} = {} & \frac{\partial U(\mathbf{r}-\mathbf{r}')}{\partial \mathbf{r}} \frac{\partial n_{\alpha\delta\mathbf{k}}}{\partial \mathbf{k}} (n_{\beta\gamma\mathbf{k}'} - n_{\beta\mu\mathbf{k}'} n_{\mu\gamma\mathbf{k}'}) \\ & + \frac{\partial U(\mathbf{r}-\mathbf{r}')}{\partial \mathbf{r}'} \frac{\partial n_{\beta\gamma\mathbf{k}'}}{\partial \mathbf{k}'} (n_{\alpha\delta\mathbf{k}} - n_{\alpha\mu\mathbf{k}} n_{\mu\delta\mathbf{k}}). \end{aligned} \tag{73}$$

In a single band case these terms are in exact agreement with the results of Ref. 7.

For the remaining extra source terms we get

$$L_{\alpha\beta\gamma\delta}(t;\mathbf{rr}')_{\mathbf{kk}'} = \delta_{\mathbf{rr}'} L_{\alpha\beta\gamma\delta}(t\mathbf{r})_{\mathbf{kk}'} , \tag{74}$$

where the factor $\delta_{\mathbf{rr}'}$ indicates that the scattering particles are at the same spatial point.

For exchange Coulomb interaction we get the following expression for the extra source term:

$$\begin{aligned} L_{\alpha\beta\gamma\delta}(t\mathbf{r})_{\mathbf{kk}'} = {} & -iU_{\mathbf{k}'-\mathbf{k}} \{ n_{\alpha\gamma\mathbf{k}'} [n_{\beta\mu\mathbf{k}'} - \delta_{\beta\mu}] n_{\mu\delta\mathbf{k}} \\ & - n_{\alpha\gamma\mathbf{k}} n_{\beta\mu\mathbf{k}'} [n_{\mu\delta\mathbf{k}} - \delta_{\mu\delta}] n_{\alpha\mu\mathbf{k}} n_{\mu\gamma\mathbf{k}'} \\ & \times [n_{\beta\delta\mathbf{k}} - n_{\beta\delta\mathbf{k}'}]\}, \end{aligned} \tag{75}$$

which has no analogy in the single band case.

Let us write some of the extra terms, describing generation-recombination processes (explicit expressions for these extra sources are given in Appendix B). First we let $\alpha=\delta=c$ and $\beta=\gamma=v$ in Eq. (B2), which corresponds to the extra term entering the right-hand side of the equation for one-time correlation function $\langle \delta\hat{n}_{c\mathbf{k}} \delta\hat{n}_{v\mathbf{k}'} \rangle$ and have

$$\begin{aligned} L_{cvvc}(t\mathbf{r})_{\mathbf{kk}'} = {} & 2\pi \sum_\nu |C_{\nu\mathbf{k}-\mathbf{k}'}|^2 \delta(\varepsilon_{c\mathbf{k}} - \varepsilon_{v\mathbf{k}'} - \omega_{\nu\mathbf{k}-\mathbf{k}'})(n_{c\mathbf{k}} \\ & - n_{v\mathbf{k}'})[(\mathcal{N}_{\nu\mathbf{k}-\mathbf{k}'}+1) n_{c\mathbf{k}}(n_{v\mathbf{k}'}-1) \\ & - \mathcal{N}_{\nu\mathbf{k}-\mathbf{k}'}(n_{c\mathbf{k}}-1) n_{v\mathbf{k}'}] \end{aligned} \tag{76}$$

This extra term describes how two electron states $\mathbf{k}$ and $\mathbf{k}'$ in different bands become occupied (and two others become unoccupied) simultaneously as a result of emission and absorption of photons with interband frequency $\omega_{\nu\mathbf{k}-\mathbf{k}'}$. This expression reveals the important property of the extra source, namely, the interband fluctuation source vanishes in the thermal equilibrium. Indeed, in this case the expression in the square brackets in Eq. (76) vanishes provided that the photons are described by the Bose distribution function $\mathcal{N}_{\nu\mathbf{k}-\mathbf{k}'}$ with $\omega_{\nu\mathbf{k}-\mathbf{k}'} = \varepsilon_{c\mathbf{k}} - \varepsilon_{v\mathbf{k}'}$. Physically this means that in this case the mean flow into the states $\mathbf{k}$ and $\mathbf{k}'$ in different bands is equal to the mean flow out of these states.

Let us now obtain the extra source for $\langle \delta\hat{p}_{\mathbf{k}} \delta\hat{p}^\dagger_{\mathbf{k}'} \rangle$, letting $\alpha=\gamma=c$ and $\beta=\delta=v$ in Eq. (B2) we get

$$\begin{aligned} L_{cvcv}(t\mathbf{r})_{\mathbf{kk}'} = {} & \pi \sum_\nu |C_{\nu\mathbf{k}-\mathbf{k}'}|^2 p^*_{\mathbf{k}'} p_{\mathbf{k}} \{ \delta_+(\varepsilon_{c\mathbf{k}} - \varepsilon_{v\mathbf{k}'} \\ & - \omega_{\nu\mathbf{k}-\mathbf{k}'})[1 - n_{v\mathbf{k}'} + \mathcal{N}_{\nu\mathbf{k}-\mathbf{k}'}] + \delta_-(\varepsilon_{c\mathbf{k}} \\ & - \varepsilon_{v\mathbf{k}'} - \omega'_{\nu\mathbf{k}-\mathbf{k}})[n_{c\mathbf{k}} + \mathcal{N}_{\nu\mathbf{k}-\mathbf{k}'}] + \delta_+(\varepsilon_{c\mathbf{k}'} \\ & - \varepsilon_{v\mathbf{k}} - \omega_{\nu\mathbf{k}'-\mathbf{k}})[n_{c\mathbf{k}'} + \mathcal{N}_{\nu\mathbf{k}'-\mathbf{k}}] + \delta_-(\varepsilon_{c\mathbf{k}'} \\ & - \varepsilon_{v\mathbf{k}} - \omega_{\nu\mathbf{k}'-\mathbf{k}})[1 - n_{v\mathbf{k}} + \mathcal{N}_{\nu\mathbf{k}'-\mathbf{k}}]\}. \end{aligned} \tag{77}$$

Actual calculation of the extra sources for the electron-phonon and electron-electron scattering mechanisms is presented in Appendix B. Since the extra correlation is expressed only through the kernel [see Eq. (72)] we again separate the electron-electron scattering terms as we did before (see Appendix A).

Let us write the same extra sources as above for the electron-electron scattering; for the sake of brevity here we restrict ourselves to the case when $p_{\mathbf{k}}\to 0$ and discard all terms that include $p_{\mathbf{k}}$. This approximation corresponds to the quasiequilibrium theory, where one regards $p_{\mathbf{k}}$ as the linear response to a weak perturbation $\mathbf{d}_{cv\mathbf{k}}E_0$ and neglects all second and higher terms of $p_{\mathbf{k}}$. Moreover, carriers within the bands are described by a nonequilibrium Fermi distribution $n^F_{\alpha\mathbf{k}}$ with chemical potentials μ_α to be self-consistently obtained from normalization condition $N_\alpha=\Sigma_{\mathbf{k}} n^F_{\alpha\mathbf{k}}$, and number of carriers within α band N_α is to be obtained from the summed over $\mathbf{k}$ equation for density matrix component $n_{\alpha\mathbf{k}}$. In the mentioned approximation we obtain as a contribution, which stems from the first term [see Eq. (B4) in Appendix B],

$$L_{cvvc}(t\mathbf{r})_{\mathbf{k}\mathbf{k}'}=2\pi(n_{c\mathbf{k}}+n_{v\mathbf{k}'}-1)\sum_{\mathbf{q}} U^2_{\mathbf{q}}\delta(\varepsilon_{c\mathbf{k}+\mathbf{q}}+\varepsilon_{v\mathbf{k}'-\mathbf{q}}-\varepsilon_{v\mathbf{k}'}-\varepsilon_{c\mathbf{k}})[n_{c\mathbf{k}}(n_{c\mathbf{k}+\mathbf{q}}-1)(n_{v\mathbf{k}'-\mathbf{q}}-1)n_{v\mathbf{k}'}-(n_{c\mathbf{k}}-1)n_{c\mathbf{k}+\mathbf{q}}n_{v\mathbf{k}'-\mathbf{q}}(n_{v\mathbf{k}'}-1)] \tag{78}$$

for L_{cvvc} and

$$L_{cvcv}(t\mathbf{r})_{\mathbf{k}\mathbf{k}'}=0. \tag{79}$$

Performing summation over band indices in Eq. (B5) and neglecting expressions involving $p_{\mathbf{k}}$ we get the following contribution from the second term:

$$L_{cvvc}(t\mathbf{r})_{\mathbf{k}\mathbf{k}'}=2\pi(n_{c\mathbf{k}}-n_{v\mathbf{k}'})\sum_{\mathbf{q}} U^2_{\mathbf{q}}\delta(\varepsilon_{v\mathbf{k}'-\mathbf{q}}+\varepsilon_{c\mathbf{k}}-\varepsilon_{v\mathbf{k}'}-\varepsilon_{c\mathbf{k}-\mathbf{q}})[n_{v\mathbf{k}'}(1-n_{c\mathbf{k}})n_{c\mathbf{k}-\mathbf{q}}(1-n_{v\mathbf{k}'-\mathbf{q}})-n_{c\mathbf{k}}(1-n_{v\mathbf{k}'})(1-n_{c\mathbf{k}-\mathbf{q}})n_{v\mathbf{k}'-\mathbf{q}}] \tag{80}$$

and for L_{cvcv} we again obtain zero. The last term [see Eq. (B6) in Appendix B] does not contribute in the accepted approximation to the source L_{cvvc}, while the rather lengthy explicit expression for L_{cvcv} can be easily obtained from Eq. (B6) in Appendix B.

The extra correlation created by collisions was called in Ref. 4 a kinetic correlation, since the quite important property of the extra source term L in the single-band case is that it vanishes in the thermal equilibrium state. It can be easily verified that in this case the in and out terms cancel each other in the expressions for L. This property of the extra terms remains valid whenever one deals with sources concerning fluctuations of occupation numbers, as follows from our expressions even for fluctuations of occupation numbers in different bands. We wish to note that the property does not hold for fluctuations of the nondiagonal in band indices density matrix component as is quite evident from Eq. (B6); this is not strange since the nondiagonal components describe a mixed electron-hole state.

Thus we have established quantum kinetic equations for fluctuations slowly varying in space and time, closely following the Gantsevich, Gurevich, and Katilius approach. The time-displaced correlation functions obey Eq. (68), the initial condition to the equation being determined by Eq. (71). As for the density matrix components that enter our formulas, they, in their turn, should be determined from the quantum kinetic equations they satisfy. Needless to say, they should contain all the interactions involved in the fluctuations kinetics and describe density matrix components slowly varying in space and time. We note that such equations can be restored from our fluctuations kinetic equations (68) since our equations are, as one can see, a linearized version of the quantum kinetic equations.

The structure of Eq. (68) suggests that it can be interpreted within a concept of fictitious Langevin random forces. Indeed, one can write the generalized Langevin equation for the fluctuations of density matrix by adding the matrix of random Langevin forces:

$$[\delta_{\alpha\alpha'}\delta_{\delta\delta'}\partial_t+B^{\alpha\delta}_{\alpha'\delta'\mathbf{k}}(t\mathbf{r})]\delta n_{\alpha'\delta'\mathbf{k}}(t\mathbf{r})=F_{\alpha\delta\mathbf{k}}(t\mathbf{r}). \tag{81}$$

This equation can be called *Bloch-Langevin equations* or linearized Bloch equation with fluctuations. The popularity of Langevin approach relies on the fact that within this approach one can deal with more simple and physically transparent linearized equations for the density matrix instead of a rather complicated method of moments' equations we have derived, provided, of course, that the correlation functions of the corresponding random forces are known.

In the next subsection we will give explicit expressions for the correlation functions of the random Langevin forces for the fluctuations near a nonequilibrium but stationary and spatially inhomogeneous case. Generally speaking, the Langevin forces are fictitious with rather little physical meaning, whereas their correlation functions enter expressions determining fluctuations of physical observables.

B. Matrix of Langevin random forces

For fluctuations from a stationary state our correlation function $g_{\alpha\beta\gamma\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{k}\mathbf{k}'}$ depends only on the time difference τ. We define the Fourier transform by

$$g_{\alpha\beta\gamma\delta}(\omega)_{\mathbf{k}\mathbf{k}'}=\int d\tau e^{i\omega\tau}g_{\alpha\beta\gamma\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{k}\mathbf{k}'}. \tag{82}$$

It is convenient to express the previous transform in terms of half Fourier transform $g^\dagger_\omega$

$$g_{\alpha\beta\gamma\delta}(\omega)_{\mathbf{k}\mathbf{k}'}=g^+_{\alpha\beta\gamma\delta}(\omega)_{\mathbf{k}\mathbf{k}'}+g^\dagger_{\alpha\beta\gamma\delta}(-\omega)_{\mathbf{k}\mathbf{k}'}, \tag{83}$$

where we introduced the half transforms by

$$g^\dagger_{\alpha\beta\gamma\delta}(\omega)_{\mathbf{k}\mathbf{k}'}=\int_0^{+\infty}d\tau e^{i\omega\tau}g_{\alpha\beta\gamma\delta}(t+\tau,\mathbf{r};t\mathbf{r}')_{\mathbf{k}\mathbf{k}'} \tag{84}$$

and

$$g^\dagger_{\alpha\beta\gamma\delta}(-\omega)_{\mathbf{k}\mathbf{k}'}=\int_0^{+\infty}d\tau e^{-i\omega\tau}g_{\alpha\beta\gamma\delta}(t,\mathbf{r};t+\tau,\mathbf{r}')_{\mathbf{k}\mathbf{k}'}. \tag{85}$$

The half Fourier transform $g^{\dagger}(\omega)$ can be obtained readily from Eq. (68) and $g^{\dagger}(-\omega)$ from the similar equation

$$[\delta_{\beta\beta'}\delta_{\gamma\gamma'}\partial_\tau + B^{\beta\gamma}_{\beta'\gamma'\mathbf{k}'}(\mathbf{r}')]g_{\alpha\beta'\gamma'\delta}(t\mathbf{r},t+\tau\mathbf{r}')_{\mathbf{kk}'}$$
$$=\delta(\tau)g_{\alpha\beta\gamma\delta}(\mathbf{r},\mathbf{r}')_{\mathbf{kk}'}. \tag{86}$$

We get

$$[-i\omega\delta_{\alpha\alpha'}\delta_{\delta\delta'} + B^{\alpha\delta}_{\alpha'\delta'\mathbf{k}}(\mathbf{r})]g^{\dagger}_{\alpha'\beta\gamma\delta'}(\omega)_{\mathbf{kk}'} = g_{\alpha\beta\gamma\delta}(\mathbf{rr}')_{\mathbf{kk}'}, \tag{87}$$

$$[i\omega\delta_{\beta\beta'}\delta_{\gamma\gamma'} + B^{\beta\gamma}_{\beta'\gamma'\mathbf{k}'}(\mathbf{r}')]g^{\dagger}_{\alpha\beta'\gamma'\delta}(-\omega)_{\mathbf{kk}'}$$
$$= g_{\alpha\beta\gamma\delta}(\mathbf{rr}')_{\mathbf{kk}'}. \tag{88}$$

On the other hand using Eq. (81) and the similar equation for $\delta n_{\beta'\gamma'\mathbf{k}'}$ and multiplying them and taking an average we have

$$[-i\omega\delta_{\alpha\alpha'}\delta_{\delta\delta'} + B^{\alpha\delta}_{\alpha'\delta'\mathbf{k}}(\mathbf{r})][i\omega\delta_{\beta\beta'}\delta_{\gamma\gamma'}$$
$$+B^{\beta\gamma}_{\beta'\gamma'\mathbf{k}'}(\mathbf{r}')]g_{\alpha'\beta'\gamma'\delta'}(\omega)_{\mathbf{kk}'}$$
$$=\langle F_{\alpha\delta\mathbf{k}}(\mathbf{r})F_{\beta\gamma\mathbf{k}'}(\mathbf{r}')\rangle_\omega. \tag{89}$$

Multiplying Eq. (87) by the operator that stands in Eq. (88) and vice versa and adding them we get the white spectral density of the random forces [we read Eq. (89) from the right to the left]

$$\langle F_{\alpha\delta\mathbf{k}}(\mathbf{r})F_{\beta\gamma\mathbf{k}'}(\mathbf{r}')\rangle_\omega$$
$$=B^{\alpha\delta}_{\alpha'\delta'\mathbf{k}}(\mathbf{r})\delta_{\mathbf{rr}'}\delta_{\mathbf{kk}'}n_{\beta\delta'\mathbf{k}}(\delta_{\alpha'\gamma}-n_{\alpha'\gamma\mathbf{k}})$$
$$+B^{\beta\gamma}_{\beta'\gamma'\mathbf{k}'}(\mathbf{r}')\delta_{\mathbf{rr}'}\delta_{\mathbf{kk}'}n_{\beta'\delta\mathbf{k}}(\delta_{\alpha\gamma'}-n_{\alpha\gamma'\mathbf{k}})$$
$$+L_{\alpha\beta\gamma\delta}(\mathbf{rr}')_{\mathbf{kk}'}, \tag{90}$$

where we have used Eqs. (70), (71). First of all, using the explicit expressions for the operator B and extra source L we notice that the self-consistent fluctuations' contribution to this correlation function vanishes as does the exchange contribution from operator B_{ex} in Eq. (28) and from the source L in Eq. (75).

It can be easily shown by giving band indices specific values and inserting corresponding relaxation operators and extra sources from the above that there are also nonzero cross-correlation functions of Langevin forces related to different bands $\langle F_{cc\mathbf{k}}F_{vv\mathbf{k}'}\rangle$. Although it is rather obvious we wish here to emphasize that if one includes, say, interband relaxation in the quantum kinetic equations for occupation numbers (or/and in the kinetic equation for concentration of carriers), to be self-consistent the above-mentioned cross-correlation functions have to be taken into account. Moreover, the very existence of such a correlation function is inevitable since it is closely related to the requirement of the charge neutrality. For instance, for spatially homogeneous fluctuations the following equality must be fulfilled

$$\sum_{\mathbf{k}'}\langle F_{cc\mathbf{k}}F_{vv\mathbf{k}'}\rangle_\omega = -\sum_{\mathbf{k}'}\langle F_{cc\mathbf{k}}F_{cc\mathbf{k}'}\rangle_\omega. \tag{91}$$

Let us calculate the sum over $\mathbf{k}$ of the left-hand side of this equation in the quasiequilibrium approximation: we assume that $|p_k|^2$ can be neglected as compared with $n_{i\mathbf{k}}(1-n_{i\mathbf{k}})$. From Eq. (90) and taking into account properties of relaxation operators and the explicit form of operators in Eqs. (49), (52),(54), (57), and (76) we get after some algebra

$$\sum_{\mathbf{kk}'}\langle F_{cc\mathbf{k}}F_{vv\mathbf{k}'}\rangle_\omega = -\delta_{\mathbf{rr}'}2\pi\sum_{\mathbf{kq}\nu}|C_{\nu\mathbf{q}}|^2\delta(\varepsilon_{c\mathbf{k}+\mathbf{q}}-\varepsilon_{v\mathbf{k}}-\omega_{\nu\mathbf{q}})$$
$$\times\{(1+\mathcal{N}_{\nu\mathbf{q}})n_{c\mathbf{k}+\mathbf{q}}(1-n_{v\mathbf{k}})$$
$$+\mathcal{N}_{\nu\mathbf{q}}(1-n_{c\mathbf{k}+\mathbf{q}})n_{v\mathbf{k}}\}. \tag{92}$$

Therefore the sum is expressed through the interband relaxation and generation operators that enter the right-hand side of the equation for a macroscopic variable, namely, carrier concentration. Here we get a sum of these operators while the difference of these operators enters the equation for carrier concentration, thus confirming the general concept that the correlation functions are expressed through the sum of corresponding gain and loss terms in the kinetic equation. Note the importance of the extra term in Eq. (76); it ensures the requirement of charge neutrality, Eq. (91).

Let us write explicit expressions in the same approximation as before for the sum of correlation functions

$$\delta_{\mathbf{rr}'}S_{\mathbf{kk}'}(\omega)=\langle F_{vc\mathbf{k}}F_{cv\mathbf{k}'}+F_{cv\mathbf{k}}F_{vc\mathbf{k}'}\rangle_\omega,$$

which is usually of importance in noise investigations (cf. Refs. 11,13). According to Eq. (90) and taking into account explicit expressions for the relaxation operators B we get the corresponding contributions of all interactions into the sum of correlation functions. The electron-phonon interaction will not be taken into account, the screening of the Coulomb potential in the electron-electron interaction will be taken properly into account in the relaxation operator but not in the extra term Eq. (B6) since the latter has no singularity $\mathbf{k}\neq\mathbf{k}'$. First of all let us write the generation-recombination contribution

$$S_{\mathbf{kk}'}(\omega)^{g-r}=\delta_{\mathbf{kk}'}W_{\mathbf{k}}2\pi\sum_{\nu\mathbf{q}}|C_{\nu\mathbf{q}}|^2\{(n_{c\mathbf{k}+\mathbf{q}}+\mathcal{N}_{\nu\mathbf{q}})\delta(\varepsilon_{c\mathbf{k}+\mathbf{q}}$$
$$-\varepsilon_{v\mathbf{k}}-\omega_{\nu\mathbf{q}})+(1-n_{v\mathbf{k}-\mathbf{q}}+\mathcal{N}_{\nu\mathbf{q}})$$
$$\times\delta(\varepsilon_{c\mathbf{k}}-\varepsilon_{v\mathbf{k}-\mathbf{q}}+\omega_{\nu\mathbf{q}})\}. \tag{93}$$

where $W_{\mathbf{k}}$ stands for

$$W_{\mathbf{k}}=n_{c\mathbf{k}}(1-n_{v\mathbf{k}})+n_{v\mathbf{k}}(1-n_{c\mathbf{k}}).$$

The generation-recombination contribution coincides with the results of Haug and Haken,[11] particularly if one replaces the damping coefficient $\gamma_{\mathbf{k}}$ for the nondiagonal density matrix component, phenomenologically introduced in Ref. 11

by our microscopical expression.

The external optical field yields the following nontrivial contribution:

$$S_{\mathbf{kk}'}(\omega)^{f}=\delta_{\mathbf{kk}'}2(n_{c\mathbf{k}}-n_{v\mathbf{k}})\{i(\mathbf{E}_0\mathbf{d}_{cv\mathbf{k}})^{*}p_{\mathbf{k}\omega_0}+\text{c.c.}\} \tag{94}$$

in contrast with the incorrectly obtained result of Refs. 10 and 11.

The electron-electron contribution includes three terms, two of them are due to the relaxation (out and in terms, correspondingly) and the last one represents the extra term. For the first term we have

$$S_{\mathbf{kk}'}(\omega)^{\text{el-}el,out}=\delta_{\mathbf{kk}'}iW_{\mathbf{k}}\sum_{\mathbf{q}}\left\{\left(\frac{1}{2}-n_{c\mathbf{k}-\mathbf{q}}\right)[U^{R}(\Delta\varepsilon_{c\mathbf{kk}-\mathbf{q}})-U^{A}(\Delta\varepsilon_{c\mathbf{kk}-\mathbf{q}})]-\frac{1}{2}U^{K}(\Delta\varepsilon_{c\mathbf{kk}-\mathbf{q}})+(c\rightarrow v)\right\}, \tag{95}$$

where $\Delta\varepsilon_{\alpha\mathbf{kk}'}=\varepsilon_{\alpha\mathbf{k}}-\varepsilon_{\alpha\mathbf{k}'}$, the retarded U^{R}, advanced U^{A} and Keldysh U^{K} Coulomb potentials in the most widely used random-phase approximation are

$$U^{R}(\omega)=U_{\mathbf{q}}\varepsilon_{\mathbf{q}(\omega)},\quad U^{A}(\omega)=U_{\mathbf{q}}\varepsilon^{*}_{\mathbf{q}(\omega)},\quad U^{K}(\omega)=\frac{U_{\mathbf{q}}^{2}}{|\varepsilon_{\mathbf{q}}(\omega)|^{2}}\Pi^{K}_{\mathbf{q}}(\omega),$$

$$\varepsilon_{\mathbf{q}}(\omega)=1-U_{\mathbf{q}}\Pi^{R}_{\mathbf{q}}(\omega),\quad \Pi^{R}_{\mathbf{q}}(\omega)=\sum_{\alpha\mathbf{p}}\frac{n_{\alpha\mathbf{k}-\mathbf{q}}-n_{\alpha\mathbf{k}}}{\omega-\Delta\varepsilon_{\alpha\mathbf{kk}-\mathbf{q}}+i0},$$

$$\Pi^{K}_{\mathbf{q}}(\omega)=-2i\pi\sum_{\alpha\beta\mathbf{p}}\delta(\omega-\Delta\varepsilon_{\beta\mathbf{kk}-\mathbf{q}})[n_{\alpha\beta\mathbf{p}}(n_{\beta\alpha\mathbf{p}-\mathbf{q}}-\delta_{\alpha\beta})+(n_{\alpha\beta\mathbf{p}}-\delta_{\alpha\beta})n_{\beta\alpha\mathbf{p}-\mathbf{q}}].$$

The second term can be written in the form

$$\begin{aligned}S_{\mathbf{kk}'}(\omega)^{\text{el-}el,in}={}&-iW_{\mathbf{k}}\left\{\left(\frac{1}{2}-n_{c\mathbf{k}'}\right)[U^{R}(\Delta\varepsilon_{c\mathbf{kk}'})-U_{\mathbf{k}-\mathbf{k}'}]-\left(\frac{1}{2}-n_{v\mathbf{k}'}\right)[U^{R}(-\Delta\varepsilon_{v\mathbf{kk}'})-U_{\mathbf{k}-\mathbf{k}'}]\right\}+iW_{\mathbf{k}'}\left\{\left(\frac{1}{2}-n_{c\mathbf{k}}\right)\right.\\&\left.\times[U^{R}(\Delta\varepsilon_{c\mathbf{kk}'})-U_{\mathbf{k}-\mathbf{k}'}]-\left(\frac{1}{2}-n_{v\mathbf{k}}\right)[U^{R}(-\Delta\varepsilon_{v\mathbf{kk}'})-U_{\mathbf{k}-\mathbf{k}'}]\right\}+\frac{1}{2}[W_{\mathbf{k}}+W_{\mathbf{k}'}]\int\frac{d\xi}{2\pi}U^{K}(\xi)\\&\times\left[\frac{1}{\xi-\Delta\varepsilon_{c\mathbf{kk}'}-i0}+\frac{1}{\xi+\Delta\varepsilon_{v\mathbf{kk}'}-i0}\right].\end{aligned} \tag{96}$$

The last term can be obtained from Eq. (B6) and is equal to

$$\begin{aligned}S_{\mathbf{kk}'}(\omega)^{\text{el-el,extra}}={}&2\pi U^{2}_{\mathbf{k}-\mathbf{k}'}(n_{v\mathbf{k}'}-n_{v\mathbf{k}})\sum_{\alpha\mathbf{p}}\delta_{-}(\varepsilon_{c\mathbf{k}'}-\varepsilon_{c\mathbf{k}}+\varepsilon_{\alpha\mathbf{p}-\mathbf{k}'}-\varepsilon_{\alpha\mathbf{p}-\mathbf{k}})[n_{c\mathbf{k}}(n_{c\mathbf{k}'}-1)n_{\alpha\mathbf{p}-\mathbf{k}}(n_{\alpha\mathbf{p}-\mathbf{k}'}-1)\\&-n_{c\mathbf{k}'}(n_{c\mathbf{k}}-1)n_{\alpha\mathbf{p}-\mathbf{k}'}(n_{\alpha\mathbf{p}-\mathbf{k}}-1)]+2\pi U^{2}_{\mathbf{k}-\mathbf{k}'}(n_{c\mathbf{k}'}-n_{c\mathbf{k}})\sum_{\alpha\mathbf{p}}\delta_{+}(\varepsilon_{v\mathbf{k}'}-\varepsilon_{v\mathbf{k}}+\varepsilon_{\alpha\mathbf{p}-\mathbf{k}'}-\varepsilon_{\alpha\mathbf{p}-\mathbf{k}})\\&\times[n_{v\mathbf{k}}(n_{v\mathbf{k}'}-1)n_{\alpha\mathbf{p}-\mathbf{k}}(n_{\alpha\mathbf{p}-\mathbf{k}'}-1)-n_{v\mathbf{k}'}(n_{v\mathbf{k}}-1)n_{\alpha\mathbf{p}-\mathbf{k}'}(n_{\alpha\mathbf{p}-\mathbf{k}}-1)].\end{aligned} \tag{97}$$

Here we wish to emphasize that this term describes extra correlation created by collisions in the correlation function of the nondiagonal (interband) Langevin forces F_{cv}, F_{vc} even though the Coulomb interaction cannot produce interband transitions. The real part of this extra term is nontrivial only in nonequilibrium situations and vanishes in the thermal intraband equilibrium. Therefore, our result does not, of course, violate the famous fluctuation-dissipation theorem, which is valid only in the thermal equilibrium.

IV. CONCLUSION

In the present paper a theory of fluctuations around a nonequilibrium state slowly varying in space and time maintained by interband optical and driving electric field in semiconductors is presented. The presentation is self-contained and is based on the moment method approach. We have used the Keldysh Green's-function formalism in the course of first-principles derivation of quantum kinetic equations for fluctuations in the case of two band semiconductors. The fluctuation kinetic equations include both equations for time displaced (two-time) correlation functions of occupation numbers in two bands and for correlation functions of the mixed interband state. These equations manifest themselves as a mathematical expression of the famous Onsager hypothesis, stating that a time evolution of fluctuation deviation from the physical point of view coincides with the evolution of a small deviation caused by some external perturbation. The explicit expressions for relaxation operators that enter the equation for the two-time correlation function are given and their properties are discussed. A one-time correlation function enters as an initial condition to the equation for the two-time correlation function. In its turn, the equation for the one-time correlation function, aside from the relaxation operators, includes extra source terms. Explicit microscopical expressions for the extra source terms for the interband generation-recombination processes, the electron-phonon, and electron-electron interactions are given. These two equations, i.e., the equation for the time displaced matrix of correlation functions (68) and the equation for one-time matrix of these correlation functions (71) exhaust the theory of fluctuations near the nonequilibrium state in a semiconductor.

The structure of the kinetic equation for the two-time correlation function brings about the interpretation of this equation within the notion of Langevin random forces. Thus, adding the random forces to the linearized kinetic equation for the density matrix we arrive at the Bloch-Langevin equation (81). The Langevin approach is very attractive due to its physical transparency and has been extensively used in many problems of physics. That is why we show how the correlation functions (closely related to the ''diffusion coefficients'' in Lax's terminology) of the Langevin forces can be obtained within our approach. Therefore, we give microscopical expressions (90) for the matrix of the Langevin random forces.

We demonstrated that even though the Coulomb interaction cannot cause band-to-band transitions it can in a nonequilibrium state contribute to the correlation functions of the Langevin forces. Since these correlation functions determine the phase and intensity noise in semiconductor lasers we show that the Coulomb scattering yields the nonequilibrium contribution to the linewidth and intensity of the laser radiation.

Note that through the relaxation operators the external driving electric field $\partial\varphi/\partial\mathbf{r}$ and interband optical field explicitly enter expressions for correlation functions of the Langevin forces. On the other hand, it is rather obvious that the fluctuation source, the very existence of which is due to random collision events, should not include external deterministic fields. It is easy to verify that this requirement can be satisfied if we eliminate the field terms making use of kinetic equations for the density matrix so that the final expressions for the source will contain only the density matrix components and the transition probabilities.

The general results concerning properties of the relaxation operators and correlation functions of the Langevin forces would prove to be useful as a checkup tool when one treats noise problems in a phenomenological way. Needless to say, in the absence of external optical field our results describe fluctuations in two bands (electron and hole gases) interacting through the self-consistent Coulomb field as well as due to recombination-generation processes.

Finally, let us note another field where our results can be applied: they can be employed in quantum electronics in order to evaluate the noise sources in diverse active semiconductor devices. In such investigations only the following correlation functions of the Langevin forces $\langle F_{cv\mathbf{k}}F_{vc\mathbf{k}'}\rangle$ and $\langle F_{c\mathbf{k}}F_{c\mathbf{k}'}\rangle$ are sufficient, usually they are incorporated into correlation functions of Langevin forces entering the right-hand side of the field equation and equation for the concentration of carriers. It is nearly obvious that, in general, in the course of solving the density matrix equation with corresponding Langevin forces the rest correlation functions also can be of importance.

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APPENDIX A: RELAXATION OPERATORS

To find operators describing electron-electron interaction we need analytical expressions for the kernel K; for the first (b) term we obtain

$$\begin{aligned}K^{iklm}_{\alpha\beta\gamma\delta}&(t_1\mathbf{k},t_2\mathbf{p},t_3\mathbf{p},t_4\mathbf{k})\\&=(-1)^{i+m}\delta_{ik}\delta_{ml}\delta(t_1-t_2)\delta(t_3-t_4)\\&\quad\times\sum_{\mathbf{q}}U^2_{\mathbf{q}}G^{im}_{\alpha\delta\mathbf{k}+\mathbf{q}}(t_1,t_3)G^{im}_{\beta\gamma\mathbf{p}-\mathbf{q}}(t_1,t_3).\end{aligned}\tag{A1}$$

Substitution of this expression into the last term of Eq. (20) and calculation gives

$$\begin{aligned}-\hat{B}\hat{g}_{b,el}=\pi\sum_{\mathbf{pq}}U^2_{\mathbf{q}}\{&\delta_+(\Delta\varepsilon_{\lambda\nu})[(n_{\alpha\lambda\mathbf{k}}-\delta_{\alpha\lambda})n_{\lambda\delta\mathbf{k}-\mathbf{q}}n_{\nu\mu\mathbf{p}+\mathbf{q}}\\&-n_{\alpha\lambda\mathbf{k}}(n_{\lambda\delta\mathbf{k}-\mathbf{q}}-\delta_{\lambda\delta})(n_{\nu\mu\mathbf{p}+\mathbf{q}}-\delta_{\nu\mu})]\\&+\delta_-(\Delta\varepsilon_{\lambda\mu})[n_{\alpha\lambda\mathbf{k}-\mathbf{q}}(n_{\lambda\delta\mathbf{k}}-\delta_{\lambda\delta})n_{\nu\mu\mathbf{p}+\mathbf{q}}\\&-n_{\lambda\delta\mathbf{k}}(n_{\alpha\lambda\mathbf{k}-\mathbf{q}}-\delta_{\alpha\lambda})(n_{\nu\mu\mathbf{p}+\mathbf{q}}\\&-\delta_{\nu\mu})]\}g_{\mu\nu}(t,\tau)_{\mathbf{pk}'}.\end{aligned}\tag{A2}$$

Now for the next (c) term we have

$$K^{iklm}_{\alpha\beta\gamma\delta}(t_1\mathbf{k},t_2\mathbf{p},t_3\mathbf{p},t_4\mathbf{k})$$
$$=(-1)^{i+m}\delta_{il}\delta_{mk}\delta(t_1-t_3)\delta(t_2-t_4)$$
$$\times\sum_{\mathbf{q}} U^2_{\mathbf{q}} G^{lm}_{\alpha\delta\mathbf{k-q}}(t_1,t_2)G^{ml}_{\beta\gamma\mathbf{p-q}}(t_2,t_1) \tag{A3}$$

and further calculations give

$$-\hat{B}\hat{g}_{c,\mathrm{el}}=\pi\sum_{\mathbf{pq}} U^2_{\mathbf{q}}\{\delta_+(\Delta\varepsilon_{\lambda\mu})[(n_{\alpha\lambda\mathbf{k}}-\delta_{\alpha\lambda})n_{\lambda\delta\mathbf{k-q}}(n_{\nu\mu\mathbf{p}}-\delta_{\nu\mu})-n_{\alpha\lambda\mathbf{k}}(n_{\lambda\delta\mathbf{k-q}}-\delta_{\lambda\delta})n_{\nu\mu\mathbf{p}}]+\delta_-(\Delta\varepsilon_{\lambda\nu})$$
$$\times[n_{\alpha\lambda\mathbf{k-q}}(n_{\nu\mu\mathbf{p}}-\delta_{\nu\mu})(n_{\lambda\delta\mathbf{k}}-\delta_{\lambda\delta})-(n_{\alpha\lambda\mathbf{k-q}}-\delta_{\alpha\lambda})n_{\nu\mu\mathbf{p}}n_{\lambda\delta\mathbf{k}}]\}g_{\mu\nu}(t,\tau)_{\mathbf{p+q,k'}}. \tag{A4}$$

For the last (d) term originated from K we get the following expression:

$$K^{iklm}_{\alpha\beta\gamma\delta}(t_1\mathbf{k},t_2\mathbf{p},t_3\mathbf{p},t_4\mathbf{k})$$
$$=(-1)^{i+k}\delta_{il}\delta_{mk}\delta(t_1-t_3)\delta(t_2-t_4)$$
$$\times\delta_{\alpha\gamma}\delta_{\beta\delta}U^2_{\mathbf{k-p}}\sum_{\mu\nu\mathbf{s}} G^{ik}_{\mu\nu\mathbf{s+k-p}}(t_1,t_2)G^{ki}_{\nu\mu\mathbf{s}}(t_2,t_1). \tag{A5}$$

Finally, substitution of this expression into Eq. (20) yields

$$-\hat{B}\hat{g}_{d,\mathrm{el}}=\pi\sum_{\mathbf{pq}} U^2_{\mathbf{q}}\{\delta_+(\Delta\varepsilon_{\lambda\mu})g_{\lambda\delta}(t,\tau)_{\mathbf{k-q,k'}}[(n_{\alpha\lambda\mathbf{k}}-\delta_{\alpha\lambda})n_{\mu\nu\mathbf{p+q}}(n_{\nu\mu\mathbf{p}}-\delta_{\nu\mu})-n_{\alpha\lambda\mathbf{k}}(n_{\mu\nu\mathbf{p+q}}-\delta_{\mu\nu})n_{\nu\mu\mathbf{p}}]$$
$$+\delta_-(\Delta\varepsilon_{\lambda\nu})g_{\alpha\lambda}(t,\tau)_{\mathbf{k-q,k'}}[n_{\mu\nu\mathbf{p+q}}(n_{\nu\mu\mathbf{p}}-\delta_{\nu\mu})\times(n_{\lambda\delta\mathbf{k}}-\delta_{\lambda\delta})-(n_{\mu\nu\mathbf{p+q}}-\delta_{\mu\nu})n_{\nu\mu\mathbf{p}}n_{\lambda\delta\mathbf{k}}]\}. \tag{A6}$$

APPENDIX B: EXTRA SOURCE TERMS

Inserting explicit expression (43) for K describing the electron-photon interaction of the lowest order we get for Eq. (72)

$$l^{iklm}_{\alpha\beta\gamma\delta}=(-1)^{i'+k'}\int dt_1dt_2G^{ii'}_{\alpha\mu\mathbf{k}}(t+\delta',t_1)G^{k'm}_{\mu\delta\mathbf{k}}(t_2;t)iD^{i'k'}_{\mathbf{k-k'}}(t_1,t_2)\times G^{i'l}_{\bar{\mu}\gamma\mathbf{k'}}(t_1,t)G^{kk'}_{\beta\bar{\mu}\mathbf{k'}}(t+\delta,t_2). \tag{B1}$$

Taking derivative with respect to t and calculating the l^{1112} component we have for the recombination-generation processes the following source term:

$$L_{\alpha\beta\gamma\delta}(t\mathbf{r})_{\mathbf{kk'}}=\pi\sum_{\nu}|C_{\nu\mathbf{k-k'}}|^2[(n_{\bar{\alpha}\gamma\mathbf{k'}}-\delta_{\bar{\alpha}\gamma})\{\delta_-(\varepsilon_{\bar{\alpha}\mathbf{k'}}-\varepsilon_{\alpha\mathbf{k}}+\omega_{\nu\mathbf{k-k'}})[\mathcal{N}_{\nu\mathbf{k-k'}}(n_{\alpha\delta\mathbf{k}}-\delta_{\alpha\delta})n_{\beta\bar{\alpha}\mathbf{k'}}-(\mathcal{N}_{\nu\mathbf{k-k'}}+1)n_{\alpha\delta\mathbf{k}}(n_{\beta\bar{\alpha}\mathbf{k'}}$$
$$-\delta_{\beta\bar{\alpha}})]+\delta_-(\varepsilon_{\bar{\alpha}\mathbf{k'}}-\varepsilon_{\alpha\mathbf{k}}-\omega_{\nu\mathbf{k'-k}})[(\mathcal{N}_{\nu\mathbf{k'-k}}+1)(n_{\alpha\delta\mathbf{k}}-\delta_{\alpha\delta})n_{\beta\bar{\alpha}\mathbf{k'}}-\mathcal{N}_{\nu\mathbf{k'-k}}n_{\alpha\delta\mathbf{k}}(n_{\beta\bar{\alpha}\mathbf{k'}}-\delta_{\beta\bar{\alpha}})]\}$$
$$+n_{\beta\bar{\delta}\mathbf{k'}}\{\delta_-(\varepsilon_{\delta\mathbf{k}}-\varepsilon_{\bar{\delta}\mathbf{k'}}-\omega_{\nu\mathbf{k-k'}})[\mathcal{N}_{\nu\mathbf{k-k'}}(n_{\alpha\delta\mathbf{k}}-\delta_{\alpha\delta})n_{\bar{\delta}\gamma\mathbf{k'}}-(\mathcal{N}_{\nu\mathbf{k-k'}}+1)n_{\alpha\delta\mathbf{k}}(n_{\bar{\delta}\gamma\mathbf{k'}}-\delta_{\bar{\delta}\gamma})]+\delta_-(\varepsilon_{\delta\mathbf{k}}-\varepsilon_{\bar{\delta}\mathbf{k'}}$$
$$+\omega_{\nu\mathbf{k'-k}})[(\mathcal{N}_{\nu\mathbf{k'-k}}+1)(n_{\alpha\delta\mathbf{k}}-\delta_{\alpha\delta})n_{\bar{\delta}\gamma\mathbf{k'}}-\mathcal{N}_{\nu\mathbf{k'-k}}n_{\alpha\delta\mathbf{k}}(n_{\bar{\delta}\gamma\mathbf{k'}}-\delta_{\bar{\delta}\gamma})]\}+(n_{\alpha\bar{\gamma}\mathbf{k}}-\delta_{\alpha\bar{\gamma}})\{\delta_-(\varepsilon_{\gamma\mathbf{k'}}-\varepsilon_{\bar{\gamma}\mathbf{k}}$$
$$+\omega_{\nu\mathbf{k-k'}})[(\mathcal{N}_{\nu\mathbf{k-k'}}+1)n_{\bar{\gamma}\delta\mathbf{k}}(n_{\beta\gamma\mathbf{k'}}-\delta_{\beta\gamma})-\mathcal{N}_{\nu\mathbf{k-k'}}(n_{\bar{\gamma}\delta\mathbf{k}}-\delta_{\bar{\gamma}\delta})n_{\beta\gamma\mathbf{k'}}]+\delta_-(\varepsilon_{\gamma\mathbf{k'}}-\varepsilon_{\bar{\gamma}\mathbf{k}}-\omega_{\nu\mathbf{k'-k}})$$
$$\times[\mathcal{N}_{\nu\mathbf{k'-k}}n_{\bar{\gamma}\delta\mathbf{k}}(n_{\beta\gamma\mathbf{k'}}-\delta_{\beta\gamma})-(\mathcal{N}_{\nu\mathbf{k'-k}}+1)(n_{\bar{\gamma}\delta\mathbf{k}}-\delta_{\bar{\gamma}\delta})n_{\beta\gamma\mathbf{k'}}]\}+n_{\bar{\beta}\delta\mathbf{k}}\{\delta_-(\varepsilon_{\bar{\beta}\mathbf{k}}-\varepsilon_{\beta\mathbf{k'}}-\omega_{\nu\mathbf{k-k'}})[(\mathcal{N}_{\nu\mathbf{k-k'}}$$
$$+1)n_{\alpha\bar{\beta}\mathbf{k}}(n_{\beta\gamma\mathbf{k'}}-\delta_{\beta\gamma})-\mathcal{N}_{\nu\mathbf{k-k'}}(n_{\alpha\bar{\beta}\mathbf{k}}-\delta_{\alpha\bar{\beta}})n_{\beta\gamma\mathbf{k'}}]+\delta_-(\varepsilon_{\bar{\beta}\mathbf{k}}-\varepsilon_{\beta\mathbf{k'}}+\omega_{\nu\mathbf{k'-k}})[\mathcal{N}_{\nu\mathbf{k'-k}}n_{\alpha\bar{\beta}\mathbf{k}}(n_{\beta\gamma\mathbf{k'}}-\delta_{\beta\gamma})$$
$$-(\mathcal{N}_{\nu\mathbf{k'-k}}+1)(n_{\alpha\bar{\beta}\mathbf{k}}-\delta_{\alpha\bar{\beta}})n_{\beta\gamma\mathbf{k'}}]\}]. \tag{B2}$$

For the terms that describe extra source for the electron-phonon scattering we have

$$L_{\alpha\beta\gamma\delta}(t\mathbf{r})_{\mathbf{kk'}}=\pi|c_{\mathbf{k-k'}}|^2\{(n_{\alpha\gamma\mathbf{k}}-n_{\alpha\gamma\mathbf{k'}})[\delta_-(\varepsilon_{\mu\mathbf{k'}}-\varepsilon_{\mu\mathbf{k}}+\omega_{\mathbf{k-k'}})[\mathcal{N}_{\mathbf{k-k'}}n_{\beta\mu\mathbf{k'}}(\delta_{\mu\delta}-n_{\mu\delta\mathbf{k}})-(\mathcal{N}_{\mathbf{k-k'}}+1)n_{\mu\delta\mathbf{k}}(\delta_{\beta\mu}-n_{\beta\mu\mathbf{k'}})]$$
$$+\delta_-(\varepsilon_{\mu\mathbf{k'}}-\varepsilon_{\mu\mathbf{k}}-\omega_{\mathbf{k'-k}})[(\mathcal{N}_{\mathbf{k'-k}}+1)n_{\beta\mu\mathbf{k'}}(\delta_{\mu\delta}-n_{\mu\delta\mathbf{k}})-\mathcal{N}_{\mathbf{k'-k}}n_{\mu\delta\mathbf{k}}(\delta_{\beta\mu}-n_{\beta\mu\mathbf{k'}})]]+(n_{\beta\delta\mathbf{k}}$$
$$-n_{\beta\delta\mathbf{k'}})[\delta_+(\varepsilon_{\mu\mathbf{k'}}-\varepsilon_{\mu\mathbf{k}}+\omega_{\mathbf{k-k'}})[\mathcal{N}_{\mathbf{k-k'}}n_{\mu\gamma\mathbf{k'}}(\delta_{\alpha\mu}-n_{\alpha\mu\mathbf{k}})-(\mathcal{N}_{\mathbf{k-k'}}+1)n_{\alpha\mu\mathbf{k}}(\delta_{\mu\gamma}-n_{\mu\gamma\mathbf{k'}})]+\delta_+(\varepsilon_{\mu\mathbf{k'}}-\varepsilon_{\mu\mathbf{k}}$$
$$-\omega_{\mathbf{k'-k}})[(\mathcal{N}_{\mathbf{k'-k}}+1)n_{\mu\gamma\mathbf{k'}}(\delta_{\alpha\mu}-n_{\alpha\mu\mathbf{k}})-\mathcal{N}_{\mathbf{k'-k}}n_{\alpha\mu\mathbf{k}}(\delta_{\mu\gamma}-n_{\mu\gamma\mathbf{k'}})]]\}. \tag{B3}$$

For the electron-electron scattering we treat corresponding terms separately, for the first term (see Appendix A) we get

$$
\begin{aligned}
L_{\alpha\beta\gamma\delta}(t\mathbf{r})_{\mathbf{kk}'} = {} & \pi\sum_{\mathbf{q}} U_{\mathbf{q}}^2\delta_-(\varepsilon_{\nu\mathbf{k}+\mathbf{q}}+\varepsilon_{\mu\mathbf{k}'-\mathbf{q}}-\varepsilon_{\mu\mathbf{k}'}-\varepsilon_{\nu\mathbf{k}})\{n_{\beta\lambda\mathbf{k}'}[n_{\nu\delta\mathbf{k}}(n_{\alpha\nu\mathbf{k}+\mathbf{q}}-\delta_{\alpha\nu})(n_{\lambda\mu\mathbf{k}'-\mathbf{q}}-\delta_{\lambda\mu})n_{\mu\gamma\mathbf{k}'}-(n_{\nu\delta\mathbf{k}} \\
& -\delta_{\nu\delta})n_{\alpha\nu\mathbf{k}+\mathbf{q}}n_{\lambda\mu\mathbf{k}'-\mathbf{q}}(n_{\mu\gamma\mathbf{k}'}-\delta_{\mu\gamma})]+(n_{\alpha\lambda\mathbf{k}}-\delta_{\alpha\lambda})[n_{\nu\delta\mathbf{k}}(n_{\lambda\nu\mathbf{k}+\mathbf{q}}-\delta_{\lambda\nu})(n_{\beta\mu\mathbf{k}'-\mathbf{q}}-\delta_{\beta\mu})n_{\mu\gamma\mathbf{k}'}-(n_{\nu\delta\mathbf{k}} \\
& -\delta_{\nu\delta})n_{\lambda\nu\mathbf{k}+\mathbf{q}}n_{\beta\mu\mathbf{k}'-\mathbf{q}}(n_{\mu\gamma\mathbf{k}'}-\delta_{\mu\gamma})]\}-\pi\sum_{\mathbf{q}} U_{\mathbf{q}}^2\delta_+(\varepsilon_{\nu\mathbf{k}+\mathbf{q}}+\varepsilon_{\mu\mathbf{k}'-\mathbf{q}}-\varepsilon_{\mu\mathbf{k}'}-\varepsilon_{\nu\mathbf{k}})\{n_{\lambda\delta\mathbf{k}}[(n_{\alpha\nu\mathbf{k}} \\
& -\delta_{\alpha\nu})n_{\nu\lambda\mathbf{k}+\mathbf{q}}n_{\mu\gamma\mathbf{k}'-\mathbf{q}}(n_{\beta\mu\mathbf{k}'}-\delta_{\beta\mu})-n_{\alpha\nu\mathbf{k}}(n_{\nu\lambda\mathbf{k}+\mathbf{q}}-\delta_{\nu\lambda})(n_{\mu\gamma\mathbf{k}'-\mathbf{q}}-\delta_{\mu\gamma})n_{\beta\mu\mathbf{k}'}]+(n_{\lambda\gamma\mathbf{k}'}-\delta_{\lambda\gamma})[(n_{\alpha\nu\mathbf{k}} \\
& -\delta_{\alpha\nu})n_{\nu\delta\mathbf{k}+\mathbf{q}}n_{\mu\lambda\mathbf{k}'-\mathbf{q}}(n_{\beta\mu\mathbf{k}'}-\delta_{\beta\mu})-n_{\alpha\nu\mathbf{k}}(n_{\nu\delta\mathbf{k}+\mathbf{q}}-\delta_{\nu\delta})(n_{\mu\lambda\mathbf{k}'-\mathbf{q}}-\delta_{\mu\lambda})n_{\beta\mu\mathbf{k}'}]\},
\end{aligned}
\tag{B4}
$$

which is the only one to keep in the nondegenerate and single band case for it would be the second order in occupation numbers while the rest at least the third. Treating the second term similarly we get the following expression for the source:

$$
\begin{aligned}
L_{\alpha\beta\gamma\delta}(t\mathbf{r})_{\mathbf{kk}'} = {} & \pi\sum_{\mathbf{q}} U_{\mathbf{q}}^2\delta_-(\varepsilon_{\mu\mathbf{k}'-\mathbf{q}}+\varepsilon_{\nu\mathbf{k}}-\varepsilon_{\mu\mathbf{k}'}-\varepsilon_{\nu\mathbf{k}-\mathbf{q}})\{n_{\mu\gamma\mathbf{k}'}(\delta_{\alpha\nu}-n_{\alpha\nu\mathbf{k}})[n_{\lambda\delta\mathbf{k}}n_{\nu\lambda\mathbf{k}-\mathbf{q}}(\delta_{\beta\mu}-n_{\beta\mu\mathbf{k}'-\mathbf{q}})-n_{\beta\lambda\mathbf{k}'}n_{\nu\delta\mathbf{k}-\mathbf{q}}(\delta_{\lambda\gamma} \\
& -n_{\lambda\gamma\mathbf{k}'-\mathbf{q}})]-n_{\alpha\nu\mathbf{k}}(\delta_{\mu\gamma}-n_{\mu\gamma\mathbf{k}'})[n_{\lambda\delta\mathbf{k}}(\delta_{\nu\lambda}-n_{\nu\lambda\mathbf{k}-\mathbf{q}})n_{\beta\mu\mathbf{k}'-\mathbf{q}}-n_{\beta\lambda\mathbf{k}'}(\delta_{\nu\delta}-n_{\nu\delta\mathbf{k}-\mathbf{q}})n_{\lambda\gamma\mathbf{k}'-\mathbf{q}}]\} \\
& -\pi\sum_{\mathbf{q}} U_{\mathbf{q}}^2\delta_+(\varepsilon_{\nu\mathbf{k}}+\varepsilon_{\mu\mathbf{k}'-\mathbf{q}}-\varepsilon_{\mu\mathbf{k}'}-\varepsilon_{\nu\mathbf{k}-\mathbf{q}})\{n_{\nu\delta\mathbf{k}}(\delta_{\beta\mu}-n_{\beta\mu\mathbf{k}'})[(\delta_{\lambda\gamma}-n_{\lambda\gamma\mathbf{k}'})(\delta_{\alpha\nu}-n_{\alpha\nu\mathbf{k}-\mathbf{q}})n_{\mu\lambda\mathbf{k}'-\mathbf{q}}-(\delta_{\alpha\lambda} \\
& -n_{\alpha\lambda\mathbf{k}})(\delta_{\lambda\nu}-n_{\lambda\nu\mathbf{k}-\mathbf{q}})n_{\mu\gamma\mathbf{k}'-\mathbf{q}}]-n_{\beta\mu\mathbf{k}'}(\delta_{\nu\delta}-n_{\nu\delta\mathbf{k}})[(\delta_{\lambda\gamma}-n_{\lambda\gamma\mathbf{k}'})n_{\alpha\nu\mathbf{k}-\mathbf{q}}(\delta_{\mu\lambda}-n_{\mu\lambda\mathbf{k}'-\mathbf{q}})-(\delta_{\alpha\lambda} \\
& -n_{\alpha\lambda\mathbf{k}})n_{\lambda\nu\mathbf{k}-\mathbf{q}}(\delta_{\mu\gamma}-n_{\mu\gamma\mathbf{k}'-\mathbf{q}})]\}.
\end{aligned}
\tag{B5}
$$

The remaining term (which corresponds to the diagram with the closed fermion loop) is equal to

$$
\begin{aligned}
L_{\alpha\beta\gamma\delta}(t\mathbf{r})_{\mathbf{kk}'} = {} & \pi U_{\mathbf{k}-\mathbf{k}'}^2\sum_{\mathbf{p}} \delta_-(\varepsilon_{\mu\mathbf{p}-\mathbf{k}'}+\varepsilon_{\nu\mathbf{k}'}-\varepsilon_{\nu\mathbf{k}}-\varepsilon_{\mu\mathbf{p}-\mathbf{k}})\{(n_{\alpha\gamma\mathbf{k}'}-n_{\alpha\gamma\mathbf{k}})[n_{\nu\delta\mathbf{k}}(n_{\lambda\mu\mathbf{p}-\mathbf{k}'}-\delta_{\lambda\mu})n_{\mu\lambda\mathbf{p}-\mathbf{k}}(n_{\beta\nu\mathbf{k}'}-\delta_{\beta\nu}) \\
& -(n_{\nu\delta\mathbf{k}}-\delta_{\nu\delta})n_{\lambda\mu\mathbf{p}-\mathbf{k}'}(n_{\mu\lambda\mathbf{p}-\mathbf{k}}-\delta_{\mu\lambda})n_{\beta\nu\mathbf{k}'}]\}-\pi U_{\mathbf{k}-\mathbf{k}'}^2\sum_{\mathbf{p}} \delta_+(\varepsilon_{\mu\mathbf{p}-\mathbf{k}'}+\varepsilon_{\nu\mathbf{k}'}-\varepsilon_{\nu\mathbf{k}}-\varepsilon_{\mu\mathbf{p}-\mathbf{k}})\{(n_{\beta\delta\mathbf{k}'}-n_{\beta\delta\mathbf{k}}) \\
& \times[(n_{\alpha\nu\mathbf{k}}-\delta_{\alpha\nu})n_{\mu\lambda\mathbf{p}-\mathbf{k}'}(n_{\lambda\mu\mathbf{p}-\mathbf{k}}-\delta_{\lambda\mu})n_{\nu\gamma\mathbf{k}'}-n_{\alpha\nu\mathbf{k}}(n_{\mu\lambda\mathbf{p}-\mathbf{k}'}-\delta_{\mu\lambda})n_{\lambda\mu\mathbf{p}-\mathbf{k}}(n_{\nu\gamma\mathbf{k}'}-\delta_{\nu\gamma})]\}.
\end{aligned}
\tag{B6}
$$

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