Quantum electrodynamic treatment of photon-assisted tunneling

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We present a quantum electrodynamic treatment of the photon-assisted tunneling sidebands that arise when an electron tunnels through a discrete electron state that is interacting with a coherent field. Virtual spontaneous emission of photons by the electron due to its interaction with the vacuum fluctuations of the electromagnetic field leads to an asymmetry in the sideband spectrum not present in the classical limit. For the case of strong coupling between the electron state and the electromagnetic field there can be significant differences between the quantum and classical results, even in the limit of high fields. [S0163-1829(98)02640-X]

INTRODUCTION

The application of time-dependent fields to tunnel junctions, whether through an ac voltage applied to the potential barrier itself or via laser illumination of the device, has been of interest since the advent of high-quality superconducting tunnel junctions (STJ's).[1,2] More recently, low-dimensional electron gases formed in semiconductor heterostructures have provided a range of resonant tunneling structures in which effects induced by an electromagnetic (EM) field may be investigated.[3–11]

Most theories for the effects of the applied field use a semiclassical model to describe the system. These theories ignore the quantum nature of the EM field and treat it as a classical potential energy, including it in the Hamiltonian for the electrons as a term $eV\cos\omega t$.[2] For a STJ or confined electron system the classical theory predicts the emergence of ''sidebands'' to the resonant level under the influence of the EM field. The sidebands form a symmetric spectrum of additional resonant tunneling channels through which the current can flow. This effect could form the basis for the design of fast optoelectronic switching devices, optical memory, or radiation detectors[3,5,10,12,13] if the device is biased such that it is not conducting in the dark, but illumination creates a sideband to the resonant level at the correct energy to allow a current to flow from emitter to collector. An example is a quantum dot biased in the Coulomb blockade regime.[8]

Classical extensions to the original theory have included a more realistic treatment of the field on the electron state[14] and increasing the complexity of the electronic system, for example, introducing coupling between neighboring confined electron gases.[12,15] The calculation has also been performed using a quantum electrodynamic (QED) formalism, but in the classical limit where the EM field operators are allowed to commute.[13,16] Using this method both sequential and coherent tunneling[17] have been treated, including the effects of the field on the emitter and collector regions, in addition to on the confined state itself.[12,13,16,17] The only fully QED results of which we are aware have been obtained for a thermal photon field.[18]

In this paper we present a QED calculation for the interacting system, in which the EM field is taken to be coherent, and both the electrons and photons are treated quantum mechanically. We compare our results with a semiclassical calculation and find that differences arise due to interaction of the electron with the vacuum fluctuations of the EM field. We emphasize that the classical field approximation is a valid one for present experiments, not because the field intensities are high, but because the coupling between the electrons and field is weak.

THE CURRENT

To calculate the current we use a sequential model for the tunneling event; the tunneling is taken to be a two-step process, from emitter to confined electron gas, and from confined electron gas to collector. The method assumes that the transmission coefficients across the individual barriers are small, such that the widths of the resonances are small compared with their separations and tunneling is a slow process.

In this model, the Hamiltonian in second quantized form for tunneling across a potential barrier is $H=H_L+H_R+H_{LR}$ where $H_{L(R)}$ is the Hamiltonian for the electrons on the left- (right-) hand side of the barrier, $H_{L(R)}=\Sigma_{L(R)}E_{L(R)}a^\dagger_{L(R)}a_{L(R)}$. The tunneling Hamiltonian[19] is given by

$$H_{LR}=\sum_{L,R}(T_{LR}a_L^\dagger a_R+T_{LR}^* a_R^\dagger a_L),$$

where the tunneling matrix element T_{LR} is calculated using the Bardeen transfer method.[20] The rate of change of the number of particles on the left-hand side is the commutator of the number operator with the Hamiltonian, $\dot{N}_L=[N_L,H]$. The current is then given by the expectation value $I_{LR}=-e\langle\dot{N}_L\rangle$ and is calculated from linear response theory to be[21]

$$I_{LR}=\left(\frac{2e}{\hbar}\right)\sum_{L,R}|T_{LR}|^2\int_{-\infty}^{\infty}d\varepsilon\,\chi_R(\varepsilon)\chi_L(\varepsilon+eV)\times[n_F(\varepsilon)-n_F(\varepsilon+eV)],$$

where V is the bias voltage and n_F is the Fermi occupation factor. This formula shows explicitly that the current depends on the product of the spectral functions $\chi_{L(R)}(\varepsilon)$ for

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the two sides of the barrier. The spectral function, equal to twice the imaginary part of the single electron retarded frequency Green's function $\chi_{L(R)}(\varepsilon)=2\,\mathrm{Im}[G^{\mathrm{ret}}_{L(R)}(\varepsilon)]$, is the probability that an electron in state $L(R)$ has energy ε. This derivation is for a single tunnel barrier. For a double-barrier system the current is calculated for emitter to well and separately for well to collector. Equating these two currents gives the full expression for the current from emitter to collector.[18]

THE INTERACTING ELECTRON-PHOTON SYSTEM

The interaction of the electron with the EM field is include in the Hamiltonian for the electrons in terms of A as $H_{\mathrm{el}}=(p-eA)^2/2m^*+V(r)$. In the QED description of the field the magnetic vector potential A is expressed in terms of $b_q^\dagger$ and b_q, the creation and annihilation operators for the photons of each mode q:[21]

$$A(r,t)=\sum_q\ (\hbar/2\varepsilon V\omega_q)^{1/2}\varepsilon_q(b_q e^{i\mathbf{q}\cdot\mathbf{r}-i\omega_q t}+b_q^\dagger e^{-i\mathbf{q}\cdot\mathbf{r}+i\omega_q t}).$$

Neglecting the quadratic term in A in the H_{el} as it merely shifts the zero of energy for the electrons, and normally by a negligible amount, H_{el} may be written as

$$H_{\mathrm{el}}=\sum_i\ \varepsilon_i a_i^\dagger a_i+\sum_{q,i,j} M_{q,ij}(b_q+b_q^\dagger)a_j^\dagger a_i,$$

where the first term refers to the electrons moving in the potential $V(r)$ in the absence of any coupling and the second term describes the interaction, the strength of which is determined by the matrix element $M_{q,ij}=-(e/m^*)(2\pi\hbar/V\omega_q\varepsilon)^{1/2}\langle i|e^{i\mathbf{q}\cdot\mathbf{r}}\hat{p}|j\rangle$. Physically the interaction term describes the scattering of an electron from state i to state j with the emission ($b_q^\dagger$) or absorption (b_q) of a photon of wave vector q. The full Hamiltonian H_{tot} for the interacting system is obtained by adding the Hamiltonian for the EM field $H_{\mathrm{EM}}=\Sigma_q\hbar\omega_q(b_q^\dagger b_q+1/2)$ to $\mathrm{H_{el}}$.[22]

The full Hamiltonian is a sum over the modes q, thus each mode may be treated independently. We make the additional simplification of considering only a single bound electron state, of energy E_0; thus the total Hamiltonian for each mode q also becomes diagonal in the electron terms:

$$H_{\mathrm{tot},q}=E_0 a^\dagger a+\hbar\omega(b^\dagger b+1/2)+M(b+b^\dagger)a^\dagger a.$$

This Hamiltonian is used to describe the electrons and photons in the well region only; there is assumed to be no coupling between electrons and photons in the emitter and collector for which the noninteracting form of the Green's and spectral functions are retained. This approximation is reasonable as the effect of the photons on a continuum of levels, such as would be found in the leads, is small; the structure in the spectral function is washed out by summing over the continuum of states. Therefore the current will be modified by the EM field mainly due to the change in the spectral function in the well, obtained from the single-electron retarded Green's function:

$$G(t,t')=-i\Theta(t-t')\langle Ta(t)a^\dagger(t')\rangle,$$

evaluated for the electron vacuum and the coherent state, which describes the EM field. The eigenstates of the bosonic part of the Hamiltonian for a single mode are the number states $|n\rangle$. However, a laser field is best described by the minimum uncertainty or single-mode coherent state $|\alpha\rangle=e^{-|\alpha|^2/2}\Sigma_n(\alpha^n/\sqrt{n!})|n\rangle$, a Poisson distribution of number states with mean value $\overline{n}=|\alpha|^2$, since the expectation value of the electric field in this state has the form of a classical field of amplitude $\sqrt{2\overline{n}\hbar\omega/\varepsilon V}$.[22]

In order to calculate the Green's function, we first perform a canonical transformation of the Hamiltonian of the interacting system to a new Hamiltonian $\overline{H}=e^S He^{-S}$, with $S=a^\dagger a\lambda(b^\dagger-b)$ and $\lambda=M/\hbar\omega$, giving the diagonal from $\overline{H}=a^\dagger a(E_0-\Delta)+\hbar\omega(b^\dagger b+1/2)$ with $\Delta=\lambda^2\hbar\omega$.[21] The Green's function is also transformed to $\overline{G}(t,t')=e^S G(t,t')e^{-S}$, factored into electron and photon parts and evaluated between the electron vacuum and the coherent photon state giving

$$\overline{G}(t,t')=-(i/\hbar)[1-n_F(E_0-\Delta)]\Phi(t,t')$$

with

$$\begin{aligned}\Phi(t,t')&=e^{-i(E_0-\Delta)(t-t')}e^{-\lambda^2(1-e^{-i\omega(t-t')})}\\&\quad\times e^{-\lambda\alpha(e^{i\omega t}-e^{i\omega t'})}e^{\lambda\alpha(e^{-i\omega t}-e^{-i\omega t'})}\\&=e^{-i(E_0-\Delta)(t-t')}e^{-\lambda^2(1-e^{-i\omega(t-t')})}\\&\quad\times\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}J_n(2\lambda\alpha)J_m(2\lambda\alpha)e^{in\omega t'}e^{-im\omega t},\end{aligned}$$

where $J_m(z)$ is the mth-order Bessel function. In the absence of a time-dependent field, this would depend only on the time difference $\tau=t-t'$. However, since the field removes the translational invariance with respect to time, this simplification cannot be made. Provided the frequency of the external field is larger than the inverse tunneling time we can, however, define a spectral function, by averaging the Green's function over one cycle of the EM field. Transforming to the Wigner coordinates $\tau=t-t'$, $T=(t+t')/2$ (Ref. 17) gives

$$\begin{aligned}\Phi\left(T+\frac{\tau}{2},T-\frac{\tau}{2}\right)&=-\frac{i}{\hbar}e^{-\lambda^2(1-e^{-i\omega\tau})}\\&\quad\times\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}J_n(2\lambda\alpha)J_m(2\lambda\alpha)\\&\quad\times e^{i\omega T(n-m)}e^{-i\omega\tau(n+m)/2}e^{-i(E_0-\Delta)\tau},\end{aligned}$$

yielding an average

$$\begin{aligned}\overline{\Phi}(\tau)&=\frac{1}{T_0}\int_0^{T_0}dT\,\Phi\left(T+\frac{\tau}{2},T-\frac{\tau}{2}\right)\\&=-\frac{i}{\hbar}e^{-\lambda^2}e^{-i(E_0-\Delta)\tau}\sum_{n=-\infty}^{\infty}e^{-in\omega\tau}\\&\quad\times\sum_{m=-\infty}^{n}J_m^2(2\lambda\alpha)\frac{\lambda^{2(n-m)}}{(n-m)!}.\end{aligned}$$

The spectral function is obtained by taking the Fourier transform of $\overline{G}(\tau)$, yielding

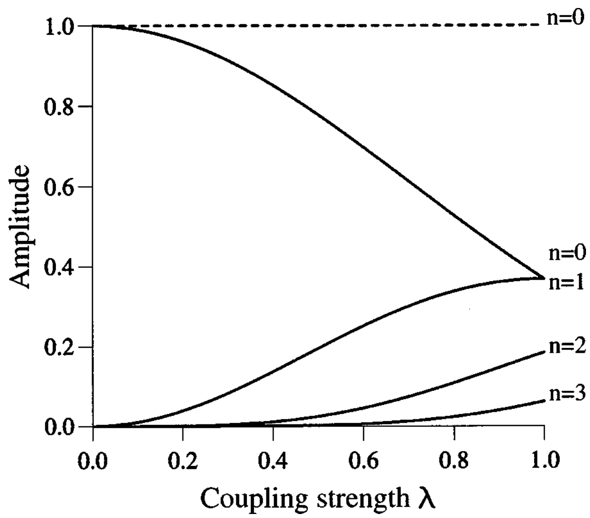

FIG. 1. The amplitude of the first four QED spectral lines at energies $E=E_0-\Delta+n\hbar\omega$ for $n=0,\ldots,3$ (see text) as a function of coupling strength λ in the absence of the EM field (full lines). The semiclassical result gives only one spectral line in the absence of a field (dashed line).

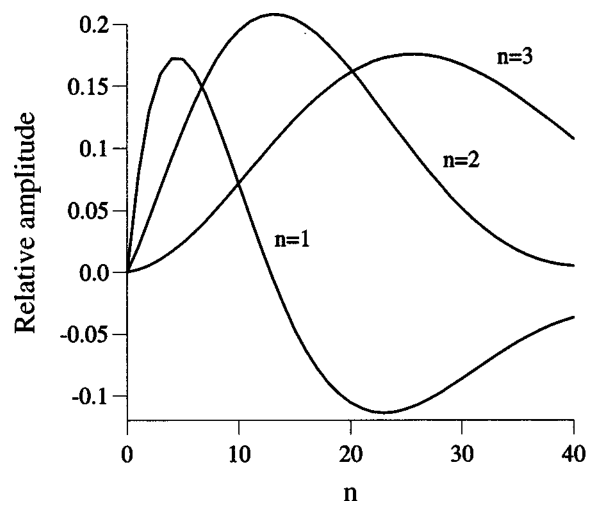

FIG. 2. The amplitude of the first three sidebands at energies $E=E_0-\Delta+n\hbar\omega$ for $n=1,\ldots,3$, relative to the zero field sideband amplitude (see Fig. 1), as a function of field intensity ($\overline{n}$) for a coupling strength λ of 0.4. As the field intensity increases, the sideband amplitude can become smaller than the vacuum background.

$$\chi(\varepsilon)=2\pi e^{-\lambda^2}\sum_{n=-\infty}^{n}\delta(\varepsilon-E_0-n\hbar\omega+\Delta)\times\sum_{m=-\infty}^{n}J_m^2(2\lambda\alpha)\frac{\lambda^{2(n-m)}}{(n-m)!}.$$

Under the influence of radiation, the single δ-function spectral function is shifted (by Δ) and splits into a series of peaks, corresponding to the spectrum of energies at which an electron can tunnel through the system.

DISCUSSION

The results of the semiclassical model[2] may be expressed in terms of the spectral function, $\chi_{\rm class}(\omega)=2\pi\Sigma_{n=-\infty}^{\infty}\delta(\varepsilon-E_0-n\hbar\omega)J_n^2(eV/\hbar\omega)$. As in the QED calculation, the semiclassical calculation finds that under laser illumination sidebands to the initial resonant level are formed; these levels are equally spaced and occur at integer multiples of a field quantum. In addition both models show that the sideband amplitudes oscillate with increasing field amplitude.

However, there are several important differences between the classical and quantum results: firstly the spectrum in the quantum case is displaced from the noninteracting spectrum (delta function) by Δ; this is a renormalization of the electron energy (electron self-energy) as a result of the interaction with the EM field. More importantly, the relative intensities of the quantum spectral lines are different from those obtained classically, and in particular the spectrum is asymmetric with respect to the $n=0$ line. In Fig. 1 it can be seen that this is also the case if the intensity α of the field is zero; here the spectral amplitudes are finite for sideband energies greater than the renormalized resonant energy whereas the semiclassical model gives only a single delta function at $E=E_0$. The sidebands still occur in the quantum case due to interaction of the confined particle with the vacuum fluctuations. Physically they correspond to the spontaneous emission by an electron of a photon to the field or, rather, a virtual spontaneous emission since the electron only emits a photon to the field while it is in the well as the tunneling process we consider is elastic. It should be noted that vacuum interaction occurs for all photon modes, not just for the single mode we are treating here. Summing over all the other zero-point modes gives a continuous spectral function for energies greater than the bound-state energy since all frequencies are included.

The asymmetry in the single field mode QED spectral function is retained as the field intensity increases, although the degree of asymmetry is reduced as higher n sidebands become stronger with increasing field strength α. Physically, the asymmetry arises at finite field strength since each of the sidebands induced by the field has a set of spontaneous emission sidebands above it. The calculation automatically includes the total probability associated with each spectra line; the nth spectral line contains contributions from all the spectral lines below it since an electron using the nth sideband can spontaneously emit photons and traverse the well using any sideband at lower energy. The difference between the quantum and classical results depends largely on the strength of the coupling, λ, and significant differences can be obtained for large couplings even at high field intensity. High-intensity EM fields (large number of photons) are frequently treated as classical, whereas our result shows that the high-intensity field limit is rather more subtle than this: it appears that the field may be treated as a classical entity only under rather specific conditions, namely, when the coupling between the electrons and the EM field is weak. However, present experimental investigations are limited to the weak coupling regime where it is appropriate to use the classical approximation.

The nonmonotonicity of the Bessel functions results in an oscillatory behavior of the spectral function amplitudes with increasing field strength. In the semiclassical model the amplitudes have real zeroes and tunneling can be completely suppressed at certain values of the field strength, even at the original resonance. However, the quantum spectral line intensity is finite for all values of the field strength since some zero-point spontaneous emission terms, of order λ^2, from the lower-energy lines always remain. Figure 2 illustrates that in the quantum model, for certain ranges of laser intensity, the amplitude of a spectral line can fall below the zero field-vacuum contribution at the energy, i.e., the laser has the effect of suppressing the tunneling contribution due to the vacuum interaction for that mode.

In summary, we have derived a time-averaged form for the QED spectral function for an irradiated confined electron gas, and found that it is different from the semiclassical result for all field intensities. The differences arise due to the interaction of the electrons with the vacuum fluctuations, and are only negligible in the limit of weak coupling between the electrons and the field. The field-induced sidebands to the main resonance oscillate in amplitude but, whereas the classical current peaks can be entirely suppressed by increasing the intensity of the field, the quantum sidebands cannot, but they can be reduced below the field-free vacuum amplitude.

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