Quantum Hall ferromagnet in a parabolic quantum wire

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The spin phase diagram of interacting electrons in a quantum wire under a high magnetic field is investigated by finite-system exact diagonalization. Various phases emerge as a function of interaction strength and aspect ratio. Stable spin-polarized states correspond to two-dimensional fractional quantum Hall states of filling fractions $1/q$, with q odd. Spin-singlet ground states for six electrons with filling fractions around $\frac{2}{5}-\frac{1}{2}$ are inspected in detail, and a real-space singlet pairing state peculiar to wire systems is found for a larger aspect ratio. [S0163-1829(98)06640-5]

I. INTRODUCTION

The fractional quantum Hall state (FQHS), which is realized by the many-body effect of a two-dimensional electron system (2DES) under a high magnetic field, has been extensively studied. Recently, a FQHS with an edge has been investigated and successfully described as a chiral Luttinger liquid.[1] Now, if two parallel edges are located very close together in the order of magnetic length, what can be expected? To simplify the problem, here I consider a system with only one kinetic subband occupied. When the edges are tightly squeezed, a usual Tomonaga-Luttinger liquid[2] is realized. In the opposite limit, the system is characterized by two independent edge channels. This paper addresses the intermediate regime between a wide 2DES and a squeezed one-dimensional interacting system under a high magnetic field.

Theoretical studies have been undertaken for wires with rectangularly confining potential, first by Chui,[3] who found a finite excitation gap where the electron filling fraction ν (defined by the average electron number per unit quantum flux) is $\frac{1}{2}$, which is related to charge-density-wave (CDW) states. Then Rezayi and Haldane[4] undertook an extensive study of the strongly squeezed state, whose excitation was found to be described as an interacting one-dimensional electron system with $1/r^2$ potentials. For a parabolic confining potential, Yoshioka, in his pioneering work,[5] demonstrated that there is a sequence of states which are close to the FQHS with filling fractions $\nu=\frac{2}{3}$, $\frac{1}{3}$, and $\frac{1}{5}$, depending on the strength of the interaction potential. More recently, an incompressible state has been reported at the filling fraction $\frac{1}{2}$.[6]

Almost all of these studies have focused on spin-polarized states. Numerical studies on 2DES's have asserted that some of the incompressible states are not fully spin polarized if the Zeeman energy is small.[7] There is experimental evidence of these states in the GaAs system,[8,9] whose g factor is much smaller than that of a free electron. Moreover at the edge of a system with several bulk Landau levels occupied, it is predicted that a strip of spin-polarized states will be formed if the edge-confining potential is smooth.[10] Hence the spin degree of freedom extends the variety of ground states (GS's) of this system. In this paper, the eigenvalue problem is solved exactly for up to eight electrons in a quantum wire in the FQHS regime, taking the spin degree of freedom into account. The main results are as follows. (1) States corresponding to a filling fraction $\nu=1/q$, where q is odd, are spin polarized. (2) High-spin states found near $\nu\sim\frac{1}{2}$ (Ref. 6) are not stable except for the four-electron system. (3) For the six-electron system, spin-singlet GS's are found for filling $\nu\sim\frac{2}{5}-\frac{1}{2}$.

The model and formulation are described in Sec. II. Section III presents results relating to GS's as a function of various parameters, and discussions follow. Finally, conclusions are presented in Sec. IV.

II. MODEL AND METHOD

Consider an interacting quasi-one-dimensional electron gas, subject to a magnetic field B in the z direction, strongly confined in the x-y plane and weakly confined by a parabolic potential $\frac{1}{2}m^*\omega_p^2 y^2$, where m^* is electron effective mass. Throughout this paper the discussion is limited to a regime of sufficiently high magnetic fields such that only the lowest kinetic-energy level is occupied, and it neglects the level mixing effects by the interactions. Zeeman energy is neglected in order to clarify intrinsic spin configurations. Following the parameterization of the system by Yoshioka,[5] the Hamiltonian is in the second quantized form:

$$H=\sum_{m\sigma} E_m a^\dagger_{m\sigma}a_{m\sigma}+\frac{V_0}{2}\sum_{mm_pm_q\sigma\sigma'} f_{m_p,m_q}a^\dagger_{m+m_q\sigma}a^\dagger_{m-m_p\sigma'} \times a_{m-m_p+m_q\sigma'}a_{m\sigma} \quad (1)$$

where $a^\dagger$ and a are creation and annihilation operators. The function f_{m_p,m_q} is the matrix element of the interaction potential constructed with single-particle bases in the Landau gauge, $\exp[i(m/\alpha)x]\chi_m(y)$, with $\chi_m(y)=(1/\pi^{1/4})e^{-(1/2)[y-(m/\alpha)]^2}$. The momentum quantum number m along the wire of length L is an integer (half integer), for the periodic (antiperiodic) boundary condition (BC), which also corresponds to the (y) displacement of the wave function relative to the wire center. The length scale is chosen to be the effective magnetic length $l_\Omega=(m^*\Omega/\hbar)^{-1/2}$, with $\Omega=\sqrt{\omega_p^2+\omega_c^2}$ and cyclotron frequency $\omega_c=eB/m^*$. The aspect ratio parameter of an N-electron system is defined as $\alpha=L/(2\pi l_\Omega)$. The normalized aspect ratio $\tilde{\alpha}=\alpha/N$ corresponds to the ratio of the electron average spacing along the

wire and the wire effective width. A system with larger $\tilde{\alpha}$ corresponds to a more squeezed wire. The energy is scaled by $\hbar\omega_p^2/(2\omega_c)$; hence the kinetic energy is $E_m=(m/\alpha)^2$, and V_0 is a parameter of interaction strength independent of α. Assuming $\omega_c \gg \omega_p$, the magnetic field only appears in parameters α and V_0, and does not affect the functional form of the potential energy. Care is needed for our results near $V_0=0$, which do not correspond to the states for a weak field ($\omega_c \ll \omega_p$). To investigate phases for a weak field, a new parameter, say, ω_c/ω_p, should be introduced. The results for this regime are reported elsewhere.[11]

In the absence of spin-orbit coupling, the many-particle wave function can be factored into a spatial part and a spin part. The quantum numbers M and S denote the total momentum and the total spin of a state, respectively. Since Zeeman energy is neglected, states with different total z components of spin S_z are energetically degenerate. The exact N-electron eigenstates $\Psi(N,M,S)$ are solved in a subspace of definite M and S in the following three steps: (a) enumerate all possible N-particle basis states $|NMS\gamma\rangle$, (b) calculate the interaction matrix $\langle NMS\gamma'|V|NMS\gamma\rangle$, and (c) diagonalize the matrix. For fixed M, there is an infinite set of γ's reflecting a continuous spectrum along the wire. To make numerical diagonalization possible, the largest $|m|$ is restricted to a finite value m_{max}. The criterion for selecting m_{max} is that the GS probability amplitude of the occupation of the state, $m=\pm m_{max}$ is sufficiently small. In particular, I choose $M=0$ subspace, which gives the absolute ground state (AGS) for a periodic (antiperiodic) BC and odd (even) N. The AGS is selected by comparing the GS for possible S, say, for odd N, $S=\frac{1}{2},\frac{3}{2},\ldots,N/2$ and for even N, $S=0,1,\ldots,N/2$.

The interaction potential is taken as being of Coulomb type $V(r)=e^2/(4\pi\epsilon r)$, with a truncated range of $L/2$ along the wire, whose matrix element

$$f_{m_p,m_q}=\int dy\ \exp\left\{-\frac{1}{2}\left[\left(\frac{m_q}{\alpha}\right)^2+\left(y-\frac{m_p}{\alpha}\right)^2\right]\right\} \times\int_{-\pi\alpha}^{\pi\alpha} dx\,\frac{\cos\dfrac{m_q}{\alpha}x}{A\sqrt{x^2+y^2}}, \tag{2}$$

with $A=(2\pi)^{3/2}\alpha$, is evaluated numerically. The interaction strength parameter V_0 is given in this case by $e^2/(4\pi\epsilon l_\Omega)2\omega_c/(\hbar\omega_p^2)$.

Since the Hamiltonian matrix is highly sparse, the Lanczos method is applied to obtain the lowest few eigenvalues and their eigenstates. The effective filling factor is defined as in a 2DES, $\nu_\sigma(y)=2\pi\rho_\sigma(y)$. The local electron density of spin σ, $\rho_\sigma(y)$, is given by $\Sigma_m|\chi_m(y)|^2 n_{m\sigma}/L$, where $n_{m\sigma}$ is the occupation of state with momentum m and spin σ averaged by the GS eigenstates. The pair correlation function $g_{s,s'}(\Delta x,y,y')$ is defined in a usual way, which corresponds to the joint probability of finding electrons with spin s' at $r'=(\Delta x,y')$ if one electron with spin s is put at $r=(0,y)$.

III. RESULTS

The phase diagram of the AGS for a six-electron system is shown in Fig. 1. The broadband regions have average

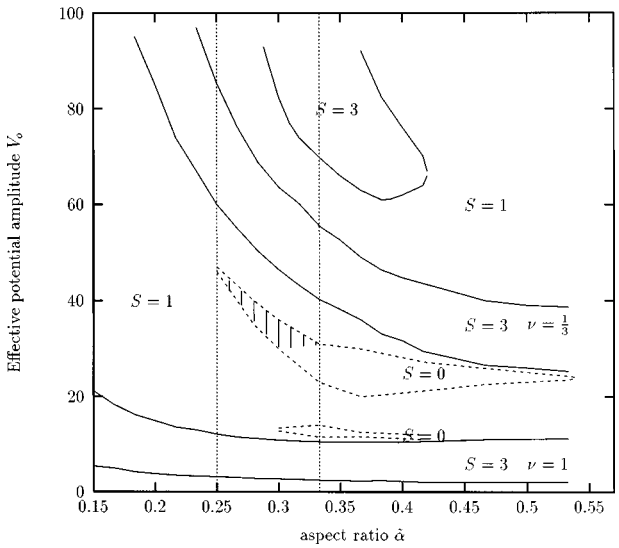

FIG. 1. Phase diagram of the AGS for a six-electron system.

fillings about $\nu=1$ and $\frac{1}{3}$ from the bottom to the top. The situation is very similar to that of a 2DES,[7] where the states with $1/q$ filling with odd q are spin polarized. There is little size dependence of the areas of these phases up to $N=8$. In the region of very small $\tilde{\alpha}$ these polarized states have periodic density modulation in the y direction, which has been investigated perturbatively[12] and numerically.[4] The typical excitation energy is 0.1–0.3, and the excited state has one smaller S than the GS.

Comparing the result of Coulomb potential and that of Haldane's pseudopotential[13] is particularly useful to assign the GS. The value of the nth pseudopotential U_n, defined in a circularly symmetric system, is given by momentum integration of the Fourier-transformed potential $V(q)$ multiplied by $\exp(-q^2)L_n(q^2)$, where L_n is the nth Laguerre polyno-

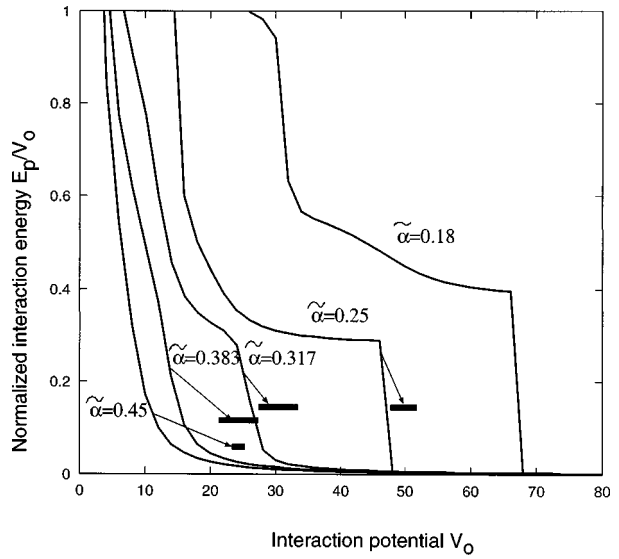

FIG. 2. Potential energy of a spin-singlet GS normalized by V_0 calculated with the HC potential as a function of V_0 for $N=6$ and $\tilde{\alpha}=0.18$, 0.25, 0.317, 0.383, and 0.45. Thick horizontal lines show the V_0 parameter region of the spin-singlet AGS calculated with the Coulomb potential for each $\tilde{\alpha}$.

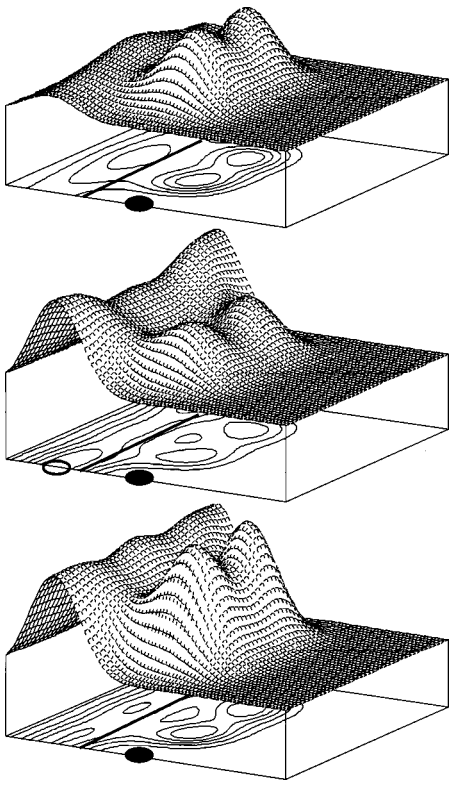

FIG. 3. Pair distribution functions for a spin-singlet GS at $\tilde{\alpha}=0.4$, $V_0=26$, and $N=6$. One electron is fixed at $r=(0,2)$, which is shown symbolically by black ovals. The center of the wire is shown by a thick solid line. The three figures, from top to bottom, are for spin parallel, spin staggered, and sum of spin parallel and staggered pairs.

mial. The matrix elements of these pseudopotentials are obtained analytically as $f^0_{m_p,m_q}, f^1_{m_p,m_q}, \ldots$. In the so-called hard-core (HC) potential ($f_n=0$ for $n>q-2$), Laughlin's $\nu=1/q$ state[14] is exact, and is a zero-eigenvalue state in a 2DES. While there remains a kinetic-energy term in a wire, the potential energy E_p is strongly suppressed in the broad regions by using the HC potential.[5] Therefore the spin-polarized states are assigned as Laughlin's $\nu=1/q$ state.

In spin-polarized subspace, there is an isolated incompressible state for about $\tilde{\alpha}\sim 0.23$ and $V_0\sim 40$. The electrons of this states are almost uniformly distributed with $\nu=\frac{1}{2}$, as reported in Ref. 6. For a zero Zeeman energy condition, this state is *not* an AGS except for the four-electron system, even in which case the excitation energy to $S=0$ and 1 states is very small (typically 20% of the gap for $\nu=1$ and $\frac{1}{3}$ AGS's).

For a four-electron system with an antiperiodic BC, there are only spin-singlet ($S=0$) AGS's and spin-polarized ($S=2$) AGS's. For a five-electron system with a periodic BC, the states of $S=\frac{1}{2}$ and $\frac{3}{2}$ are distributed between $\nu=1/q$ (q odd) stripes. In the following, I will concentrate on the details of the spin-singlet state for $N=6$. There are two regions of spin-singlet AGS's between the $\nu=1$ and $\frac{1}{3}$ spin-polarized stripes. Figure 2 shows the E_p of a spin-singlet GS normalized by V_0 as a function of V_0 calculated with the HC potential ($f^n=0$, $n>1$). There are clear transitions to the zero E_p state at $V_0=66$ for $\tilde{\alpha}=0.18$, or at $V_0=46$ for $\tilde{\alpha}=0.25$. In these zero E_p regions, the eigenstates

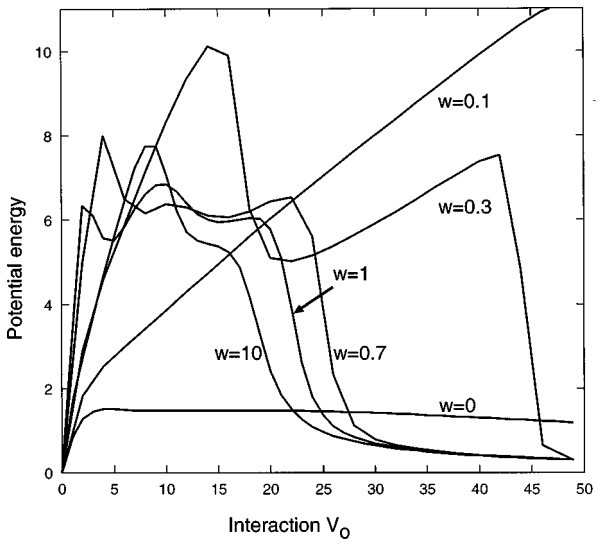

FIG. 4. Potential energy of a spin-singlet GS for $N=6$ as a function of V_0 with various pseudopotential parameters w (see the text for the definition).

calculated with the HC potential and that with the Coulomb potential overlap more than 90%, and their electron distribution is almost uniform and the average filling is about 0.4. For correlation with small distance, they show a r^6 dependence for parallel spin and r^4 for staggered spin pairs. This correlation is well described by the $\nu=\frac{2}{5}$ spin-singlet state

$$\Psi_{332}=\prod_{i,j}(z_i-z_j)^3\prod_{\alpha,\beta}(z_\alpha-z_\beta)^3\prod_{i,\alpha}(z_\alpha-z_i)^2 e^{-\Sigma_a|z_a|^2} \tag{3}$$

proposed by Halperin,[15] which is a zero eigenvalue exact GS of two-dimensional HC potential. The position with index i,j is for the spin-up state, the position with index α,β is for the spin-down state, and the position with index a is for both spin directions. The region where the spin singlet GS has a very large overlap with Ψ_{332} is shown by vertical lines in Fig. 1, which shows locations for a smaller $\tilde{\alpha}$.

What, then, are the other regions? As seen in Fig. 2, the potential energy of the spin-singlet GS calculated with the HC potential in these regions is apparently suppressed but is not completely zero. The pair distribution function for this region calculated with the Coulomb potential is shown in Fig. 3. Quite noticeable, and the largest difference from that of the $\nu=\frac{2}{5}$ state, is the large spin-singlet correlation in the lateral direction of the wire. This pair structure can be found in the same parameter range with $N=4$ or 5, as well as for the HC potential. There is also a finite probability that two electrons with staggered spin are located at the same position. The filling fraction of these states is close to $\nu=\frac{1}{2}$, although there are two small peaks in the lateral direction of the wire. This spin-singlet AGS disappears for $\tilde{\alpha}>0.55$, where no singlet pair can be formed in the lateral direction in such a squeezed wire.

There has been a spin singlet $\nu=\frac{1}{2}$ GS proposed by Haldane and Rezaji (HR) (Ref. 16) with a ''hollow core'' potential. While in spin-polarized states, the zeroth component of their pseudopotential U_0 is irrelevant because of the Pauli principle, a pair of staggered spins expel each other by

U_0 for spin-singlet states. HR showed that if U_0 is sufficiently small, the following function is an exact zero energy spin singlet eigenstate for $S=0$ and $\nu=\frac{1}{2}$,

$$\Psi_{HR}=\prod_{a,b}(z_a-z_b)^2\ \det|M|e^{-\Sigma_a|z_a|^2}, \tag{4}$$

where $M_{\alpha,i}=(z_\alpha-z_i)^{-2}$ is the $N/2\times N/2$ matrix. Following HR, Belkhir and co-workers tried to realize spin-singlet AGS's at $\nu=\frac{1}{2}$ for Coulomb or HC potentials. They succeeded for $N=4$ and 6 systems in a spherical geometry, but then failed for the $N=8$ system.[17] The pair-correlation functions for the HR state and the singlet state in the wire are different. According to the definition, the parallel spin pair correlation of HR states shows r^4, and the staggered spin correlation is constant. However, for the wire, the parallel spin correlation is only r^2.

To compare further with Ψ_{HR}, I used a pseudopotential defined as $f=wf^0+f^1$ to calculate spin-singlet states, and controlled the relative potential strength of the staggered spin by varying parameter w. E_p/V_0 as a function of V_0 for this pseudopotential is shown in Fig. 4. The state with $w=0$ is the one-dimensional CDW state, which is a periodic array of spin-singlet pairs with $\nu\sim 1$ at the center of the wire. As w increases, the CDW state become unstable, E_p increases, and the filling fraction decreases. For a fixed w, there is a critical potential strength V_0^{cr}, where E_p falls abruptly and corresponding filling fraction is about $\nu=1/2$. E_p does not change greatly if V_0 is increased further. This V_0^{cr} is a decreasing function of w, and this feature does not change even for $w\gg 1$. Therefore, the origin of the pairing for larger $\tilde{\alpha}$ is qualitatively *different* from the hollow core potential.

There is another spin-singlet state in the parameter region near $\nu=1$, whose filling factor is about $\nu=\frac{2}{3}$, and its region is much smaller than that of Laughlin's $\nu=\frac{2}{3}$ state.[5] The details of this state will be reported elsewhere.[11]

IV. CONCLUSIONS

I have investigated the phase diagram of interacting electrons in a quantum wire under a high magnetic field in a two-parameter space, namely, interaction strength and aspect ratio. I used the exact diagonalization method for a finite system, while neglecting the Zeeman energy. As in a 2DES, the odd-denominator filling state is a stable spin-polarized state. A low-spin GS at around a filling fraction of $\frac{2}{5}$, and small aspect ratio, has a large overlap with Halperin's spin-singlet GS. Another spin singlet GS was found at a filling fraction of about $\frac{1}{2}$ and for a larger aspect ratio, at which spin-singlet real-space pairing was realized.

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