Nonstationary state of superconductors: Application to nonequilibrium tunneling detectors

Yu. N. Ovchinnikov* and V. Z. Kresin
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
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The nonequilibrium state of a superconductor caused by an external source (e.g., x or γ rays) and its relaxation dynamics are studied. Microscopic theory allows one to evaluate the distribution functions and characteristic time scales for all cascade stages. The appearance of quasiparticles in the nonequilibrium state leads to an additional contribution to the tunneling current. The time dependence of the current flowing through a superconducting tunneling detector is evaluated. [S0163-1829(98)07441-4]

I. INTRODUCTION

This paper is concerned with nonequilibrium superconductivity. We focus on the situation when such a state is created in a superconductor by incoming radiation (e.g., by x or γ rays). The energy input leads to a transition into an excited state corresponding to a quasuparticle with energy $E_0 \gg \tilde{\Omega}$, where $\tilde{\Omega} \approx \Omega_D$ (Ω_D is the Debye energy, and Δ is the energy gap). As a result, the relaxation process will involve many collisions, leading to the appearance of many new quasiparticles. In other words, a decrease in the energy of the initial electronic excitation is accompanied by an increase in the number of quasiparticles. This relaxation process, called a cascade, is a nonstationary phenomenon. A microscopic description of this phenomenon is interesting for its own sake, but, in addition, is directly related to the problem of making sensitive superconducting detectors. The area of superconducting detectors has attracted a lot of interest (see, e.g., Refs. 1–4).

As is known, the cascade consists of three stages.[2,5,6] The first stage corresponds to the energy interval $E_1 < \varepsilon < E_0$, $E_1 \cong \tilde{\Omega}$, $\tilde{\Omega} \cong \Omega_D$. This high-energy region is dominated by electron-electron collisions, and the time scale for this stage is very short ($\approx 10^{-4}$ μs). During the second stage ($\Delta < \varepsilon < E_1$), the electron-phonon scattering plays an important role (see, e.g. Refs. 2 and 7). By the end of this stage, a noticeable number of quasiparticles are concentrated in the region near the edge, $\varepsilon \approx \Delta$; note that this region corresponds to the peak in the superconducting density of states. The final, third stage ($\varepsilon \approx \Delta$) is also dominated by the electron-phonon interaction. During this stage, the recombination process is very important.

The relaxation process has been described in several interesting papers (see, e.g., Refs. 5 and 8–10). Reference 5 contains a phenomenological model which has been used in many articles. A detailed microscopic treatment for the case close to equilibrium has been developed in Ref. 8; see also Ref. 10. In this paper we focus on the cascade which is a strong nonstationary process. It is essential also that this process creates also a nonequilibrium state of the phonon subsystem.

In this paper we consider an isotropic gapped superconductor; in addition, we assume $\tilde{\Omega} \gg \Delta$. The majority of conventional superconductors belong to this category. We discuss briefly the case of the high-T_c oxides in Sec. IV.

If the superconductor is a part of a tunnel junction, then deviation from equilibrium leads to the appearance of an additional tunnel current; this factor is the key to the operation of a tunneling detector.

As was mentioned above, the microscopic description of the relaxation process is directly related to the making of superconducting detectors. Indeed, an analysis of the detector, its parameters, and the time dependence of the tunneling current needs to be based on a theory of the cascade. One can study different types of detectors (see, e.g., Refs. 4 and 7), the relaxation process (cascade) is a key ingredient for all of them. In this paper we apply our theory to tunneling detectors.

The structure of the paper is as follows. The main equations are introduced in Sec. II. The different steps of the relaxation process (cascade) are described in Sec. III. Section IV contains a general discussion and a comparison with the experimental data.

II. NONSTATIONARY STATE: CASCADE

The most general description of the nonequilibrium state is provided by the time-dependent Green's functions method.[11,12] Based on this method, Larkin and one of the authors have shown[13,14] that the evolution of a nonstationary system is described by the equation

$$\frac{\partial f}{\partial t}\,\mathrm{Tr}(\hat{g}^R\hat{\tau}_z-\hat{\tau}_z\hat{g}^A)=-4I^{\mathrm{ph}}(f)-4I^{ee}(f)-4J^t(f). \qquad (1)$$

Here $\hat{g}^{R(A)}$ are the retarded and advanced Green's functions integrated over the energy ξ of the normal state. Such integrated Green's functions were introduced in Refs. 15 and 16 and have the following matrix form:

$$\hat{g}^{R(A)}=\pm[(\varepsilon\pm i\delta)^2-|\Delta|^2]^{-1/2}\hat{M}. \qquad (2)$$

The upper and lower signs correspond to $\hat{g}^R$ and $\hat{g}^A$, respectively, and

$$\hat{M}=\begin{pmatrix}\varepsilon & \Delta\\ -\Delta^* & -\varepsilon\end{pmatrix}. \qquad (2')$$

In Eq. (1), f is a scalar function directly related to the distribution function (see below and the Appendix), I^{ph} and I^{cc} describe the electron-phonon and electron-electron collisions, respectively, and $\hat{\tau}_z$ is the usual Pauli matrix. The last term in Eq. (1) is the tunneling term, which needs to be included if, in addition, there is also a tunneling channel present, e.g. The superconductor serves as a tunneling electrode (this is the case for tunneling detectors), Δ is the energy gap, and $\varepsilon=\sqrt{\xi^2+\Delta^2}$. We consider the case of the usual isotropic gapped superconductor without magnetic impurities. The case of high-T_c oxides will be discussed later. The electron-phonon collision integral can be written in the form:[13]

$$I^{\rm ph}=\frac{i\lambda}{4\pi}\int d\Omega_{p_1}\int\frac{d\varepsilon_1}{2\pi}\,\mathrm{Tr}(\hat{\delta}_{\bar p}(\varepsilon)\hat{\delta}_{\bar p_1}(\varepsilon_1)\{\hat{D}_{\bar p-\bar p_i}(\varepsilon_1-\varepsilon)$$
$$\times[f_{\bar p}(\varepsilon)-f_{\bar p_1}(\varepsilon_1)]+[D^R_{\bar p-\bar p_1}(\varepsilon_1-\varepsilon)$$
$$-D^A_{\bar p-\bar p_1}(\varepsilon_1-\varepsilon)][1-f_{\bar p}(\varepsilon)f_{\bar p_1}(\varepsilon_1)]\}). \qquad (3)$$

Here $\lambda=\pi\nu g^2/2$ is the electron-phonon coupling constant (ν is the density of states; g is the matrix element), $\delta_{\bar p}=(\hat{g}^R_{\bar p}-\hat{g}^A_{\bar p})/2$, and $D^{R(A)}$ is the retarded (advanced) phonon Green's function, so that

$$D^R_{\bar k}(\omega)=D^{A*}_{\bar k}(\omega)=\Omega^2(\bar{k})[\Omega^2(\bar{k})-(\omega+i\delta)^2]^{-1} \qquad (3')$$

and

$$\tilde{D}_{\bar p}(\omega)=(D^R_{\bar p}-D^A_{\bar p})(1+2N_{\rm ph}(|\omega|))\mathrm{sgn}\,\omega. \qquad (3'')$$

$N(\omega)$ is the phonon distribution function. In equilibrium, the scalar function f has the form $f=\tanh(\varepsilon/2T)$, as can be verified by a direct calculation. For a nonequilibrium state,

$$f=1-2n(\varepsilon), \qquad (4)$$

where $n(\varepsilon)$ is the quasiparticle distribution function. Since $D^R_{\bar p}(\omega)-D^A_{\bar p}(\omega)=-i\pi\Omega(\bar p)\cdot[\delta(\omega-\Omega(\bar p))-\delta(\omega+\Omega(\bar p))]$ and $|\bar p-\bar p_1|\cong 2p_{\hat F}\sin(\theta/2)$, where θ is the angle between $\bar p$ and $\bar p_1$, we obtain, from Eq. (3),

$$I_1^{\rm ph}(n(\varepsilon))=\lambda\int_{|\varepsilon_1|>\Delta}d\varepsilon_1\left(\frac{\varepsilon_1-\varepsilon}{\tilde\Omega}\right)\frac{\varepsilon\varepsilon_1-\Delta^2}{\chi(\varepsilon)\chi(\varepsilon_1)}$$
$$\times\{[1+2N(|\varepsilon_1-\varepsilon|)](n_{\varepsilon_1}-n_\varepsilon)$$
$$+(n_\varepsilon+n_{\varepsilon_1}-2n_\varepsilon n_{\varepsilon_1})\theta(\varepsilon_1-\varepsilon)$$
$$-(n_\varepsilon+n_{\varepsilon_1}-2n_\varepsilon n_{\varepsilon_1})\theta(\varepsilon-\varepsilon_1)\}, \qquad (5)$$

where $\tilde\Omega=sp_F$ (s is the sound velocity) and $\chi(\varepsilon)=[(\varepsilon+i\delta)^2-\Delta^2]^{1/2}$. Note that Eq. (5) is valid for any deviation from equilibrium. If we assume that $n=n_0+n_1$ with $n_1\ll n_0$, we recover an expression, linear in n_1, which can be obtained with the use of the Bogolubov transformation (see Refs. 17, 18, and also 19).

Incoming radiation (e.g., x rays) exites an electron into $E^{\rm exc}\equiv E_0$, so that $E_0\gg\Omega_D$, Δ. As was mentioned above, the consequent relaxation process (cascade) consists of three stages. At the end of the first stage, the electronic excitation energy decreases down to a value $E_1\cong\tilde\Omega$ (in the usual metals $\tilde\Omega\cong\Omega_D$). During this first fast stage ($\approx 10^{-14}$ s), electron-electron collisions play the dominant role. As a result, the number of excited quasiparticles increases (at the end of the stage, it is of the order of $E_0/\tilde\Omega$), while the average energy decreases down to the value $E_1\cong\tilde\Omega$.

During the second stage, electron-phonon collisions become dominant. Let us focus on this important stage.

III. RELAXATION PROCESS: MAIN EQUATIONS

A. Region $\Delta<\varepsilon<E_1$, $E_1\cong\tilde\Omega$

As was noted above, during this stage of the relaxation the term $I^{\rm ph}$ becomes dominant. Based on Eq. (5), we obtain the following system of equations for the distribution functions of quasiparticles $n(\varepsilon)$ and phonons $N(\varepsilon)$:

$$\frac{\partial n(\varepsilon)}{\partial t}=-\lambda\left[\left(\frac{\varepsilon^3}{3\tilde\Omega^2}\right)n(\varepsilon)+\int_\varepsilon^\infty d\varepsilon_1\left(\frac{\varepsilon_1-\varepsilon}{\tilde\Omega}\right)^2 n(\varepsilon_1)\right.$$
$$\left.+\int_0^\infty d\varepsilon_1\left(\frac{\varepsilon_1+\varepsilon}{\tilde\Omega}\right)^2 N(\varepsilon_1+\varepsilon)\right], \qquad (6)$$

$$\frac{\partial N(\varepsilon)}{\partial t}=\lambda\left(\frac{\varepsilon}{\tilde\Omega}\right)^2\left[\int_\varepsilon^\infty d\varepsilon_1 n(\varepsilon_1)\right.$$
$$\left.+\int_{-\infty}^{-\varepsilon}d\varepsilon_1[1-n(\varepsilon_1)]-N(\varepsilon)\varepsilon\right]. \qquad (6')$$

The first two terms on the right-hand side (RHS) of Eq. (6′) describe the increase in a number of phonons caused by the Cherenkov radiation of electrons and holes, correspondingly. The last term on the RHS of Eq. (6′) describes the process of pair creation.

Let us consider Eqs. (6) and (6′) in more detail. It is important that the characteristic time for these equations is (we put $h=1$)

$$\tau_{\rm ch}=\Delta^{-1}(\tilde\Omega/\Delta)^2. \qquad (6'')$$

This will be verified below. In a first approximation, one can neglect the derivative $\partial N(\varepsilon)/\partial t$. Indeed, $\partial N(\varepsilon)/\partial t\sim N/\tau_{\rm ch}\sim N\Delta^3/\Omega^2$, whereas the last term on the RHS of Eq. (6), $\lambda(\varepsilon/\Omega)^2N\varepsilon\sim N\lambda\varepsilon^3/\Omega^2\sim N\lambda\Omega\gg\partial N/\partial t$. The self-consistency of such a picture will be confirmed below. One can see directly from Eq. (6′) that this leads to the relation

$$N(\varepsilon)=2\varepsilon^{-1}\int_\varepsilon^\infty d\varepsilon_1 n(\varepsilon_1). \qquad (7)$$

We also took into the account the fact that $n(\varepsilon)=1-n(-\varepsilon)$; indeed, charge inbalance contributes to higher approximations only.

Inserting the expression (7) into Eq. (6), we arrive, after some manipulations, at the following equation for the quasiparticle distribution function $n(\varepsilon,t)$:

$$\frac{\partial n(\varepsilon,t)}{\partial t}=-\left(\frac{\lambda}{3}\right)\left(\frac{\varepsilon^3}{\Omega^2}\right)n(\varepsilon,t)+\left(\frac{2\lambda}{\Omega^2}\right)$$
$$\times\int_\varepsilon^\infty d\varepsilon_1\varepsilon_1(\varepsilon_1-\varepsilon)n(\varepsilon_1,t). \qquad (8)$$

Equation (8) is one of the main equations of our theory. It is significant that one can obtain an analytical solution of Eq. (8). Indeed, integrating over ε, one obtains

$$\frac{\partial}{\partial t}\int_0^\infty d\varepsilon n(\varepsilon,t)=\left(\frac{2\lambda}{3}\right)\tilde{\Omega}^{-2}\int_0^\infty d\varepsilon\ \varepsilon^3 n(\varepsilon,t). \qquad (8')$$

It is important to recognize that there is also an integral of the motion

$$E=\int_0^\infty n(\varepsilon,t)\varepsilon\ d\varepsilon=\text{const}. \qquad (9)$$

As was noted above, our goal is to solve Eq. (8). We are seeking a solution in the form of a steplike function:

$$n(\varepsilon,t)=n(t)\quad[0<\varepsilon<\gamma(t)];$$
$$n(\varepsilon,t)=0\quad[\varepsilon>\gamma(t)]. \qquad (10)$$

With the use of Eq. (10), we obtain

$$n(t)=\text{const}\times\gamma^{-2}(t). \qquad (11)$$

Based on Eqs. (8), (9), and (11) we find

$$\frac{\partial}{\partial t}\ \gamma^{-1}(t)=\left(\frac{\lambda}{6\tilde{\Omega}^2}\right)\gamma^2(t). \qquad (12)$$

The solution of Eq. (13) has the following form:

$$\gamma(t)=\tilde{\Omega}[1+(\lambda/2)\tilde{\Omega}t]^{-1/3}. \qquad (13)$$

Therefore, the solution of Eq. (8) is [see Eqs. (10), (11), and (13)]

$$n(\varepsilon,t)=\begin{cases} n_0[1+(\lambda/2)\tilde{\Omega}t]^{2/3}\tilde{\Omega}^{-}2 & [0<\varepsilon<\gamma(t)] \\ 0 & [\varepsilon>\gamma(t)]. \end{cases} \qquad (14)$$

Here $\gamma(t)$ is determined by Eq. (13) and $n_0=\text{const}$ (see below) Note that the characteristic time for the second state is, indeed, of the order of $\tau_{\text{ph}}=\Delta^{-1}(\tilde{\Omega}/\Delta)^2$. This estimate follows from Eq. (13); one should put $\gamma\cong\Delta$, since Δ corresponds to the end of this cascade stage.

The expression (14) allows us to evaluate the dependence $\tilde{n}(t)$, that is, the time dependence of the number of excitations present in the system during the second stage of the cascade. Indeed,

$$\tilde{n}\equiv\tilde{n}(t)=\int_0^\infty d\varepsilon\ n(\varepsilon,t). \qquad (15)$$

With the use of Eqs. (13), (14), and (15), we arrive at the following result:

$$\tilde{n}=\tilde{n}(0)[1+(\lambda/2)\tilde{\Omega}t]^{1/3}. \qquad (16)$$

Here $\tilde{n}(0)$ is the number of excitations at $t=0$ (the beginning of this stage). Therefore, the constant n_0 [see Eq. (14)] can be written as $n_0=\tilde{n}(0)\Omega^{-1}$.

Consider the important case when the superconductor forms a part of a tunnel junction. Then the relaxation process and the appearance of quasiparticles result in a tunneling current. The expression for this current follows directly from Eq. (16):

$$j(t)=j_1(1+\alpha t)^{1/3}. \qquad (17)$$

Here $\alpha=(\lambda/2)\tilde{\Omega}$; $j_1\equiv j_1(0)$ is the value of the current at the beginning of the present cascade stage. Thus the electron-phonon relaxation channel, which is dominant for $\Delta<\varepsilon<\Omega$, leads to a rise in the number of electronic excitations with time: $\propto(1+\alpha\tilde{\Omega}t)^{1/3}$.

By the end of the second stage of the relaxation process, the number of excitations has increased, while their average energy has come down to a value on the order of Δ.

B. Final stage of the cascade ($\varepsilon\approx\Delta$)

Let us turn to an analysis of the final stage of the relaxation process. As stated before, at the end of the second stage the electronic excitations have energy on the order of Δ. As a result, phonons emitted during the final stage of the cascade have energies $h\Omega\lesssim\Delta$, and so they are characterized by a relatively large mean free path l_{ph}^f.

The evolution of the distribution function is described by an equation which follows from Eq. (5):

$$\frac{\partial n(\varepsilon)}{\partial t}=-n(\varepsilon)\int_\Delta^\varepsilon d\varepsilon_1 W^-_{\varepsilon,\varepsilon_1}+\int_\varepsilon^\infty d\varepsilon_1 n(\varepsilon_1)W^-_{\varepsilon,\varepsilon_1}$$
$$-\int_{-\infty}^{-\Delta}d\varepsilon_1 n(\varepsilon)[1-n(\varepsilon_1)]W^+_{\varepsilon\varepsilon_1}$$
$$-\tilde{\gamma}n(\varepsilon)\theta(\varepsilon+eV-\Delta_2). \qquad (18)$$

Here

$$W^-_{\varepsilon\varepsilon_1}=\lambda\varepsilon^{-1}[(\varepsilon-\varepsilon_1)/\tilde{\Omega}]^2(\varepsilon\varepsilon_1-\Delta^2)(\varepsilon_1^2-\Delta^2)^{1/2},$$
$$W^+_{\varepsilon\varepsilon_1}=\lambda\varepsilon^{-1}[(\varepsilon+|\varepsilon_1|)/\tilde{\Omega}]^2(\varepsilon|\varepsilon_1|+\Delta^2)(\varepsilon_1^2-\Delta^2)^{1/2},$$
$$\tilde{\gamma}=(\gamma/D)(\varepsilon+eV)[(\varepsilon+eV)^2-\Delta_2^2]^{-1/2}, \qquad (18')$$

where D is the junction thickness.

The first two terms on the RHS of Eq. (18) describe the Cherenkov radiation of phonons; the third term corresponds to the recombination effect, that is, to electron-hole annihilation accompanied by the formation of a Cooper pair. The last term in Eq. (18) is the tunneling current (we again focus on the case when the superconductor is a part of a tunnel junction). The tunneling coefficient γ can be expressed in terms of the normal resistance R_n of the barrier: namely,

$$\gamma^{-1}=2e^2\nu SR_n, \qquad (19)$$

where ν is the density of states in the normal metal and S is the area of the junction. As is known, the superconducting density of states is peaked near Δ. If the voltage eV is such that $\Delta+eV$ is close to Δ_2, then one can simplify Eq. (18) (see the Appendix) and write

$$\partial\tilde{w}/\partial t=-8\lambda(\Delta/\tilde{\Omega})^2\tilde{w}^2-\tilde{\gamma}\tilde{w}. \qquad (20)$$

We have introduced the quantity $\tilde{w}=\int_\Delta^\infty d\varepsilon\ \varepsilon(\varepsilon^2-\Delta^2)^{-1/2}n(\varepsilon)$, which is proportional to the full number of excitations w: that is, $w=\nu V\tilde{w}$.

The first term in Eq. (20) describes the recombination phenomenon [see the discussion following Eq. (18)], and the second term corresponds to tunneling through the adjunct junction. The solution of Eq. (20) is

$$\tilde{w}(t)=\tilde{w}_m f(t), \tag{21}$$

where $f(t)=\Gamma[(\tilde{w}_m+\Gamma)\exp(\tilde{\gamma}t)-\tilde{w}_m]^{-1}$, $\Gamma=\tilde{\gamma}(\tilde{\Omega}/\Delta)^2/8\lambda$.

The tunneling current is proportional to the number of excitations. Consequently, we can write

$$j(t)=j_m f(t), \tag{22}$$

where j_m is the magnitude of the current at the beginning of the final stage and $f(t)$ is defined by Eq. (21). Therefore, the tunneling current varies with time as described by Eq. (22).

IV. DISCUSSION: TUNNELING DETECTORS

Equations (8), (14), (18), and (21) describe the dynamics of the relaxation phenomenon. The electron-phonon interaction plays a key role for energies $\varepsilon<\tilde{\Omega}$. This region can be separated into two stages (second and third stages of the cascade, see above).

(i) $\Delta<\varepsilon<\tilde{\Omega}$. This stage lasts up to t_{ch} $(0<t<t_{\mathrm{ch}})$, $t_{\mathrm{ch}}\cong\Delta^{-1}(\Omega/\Delta)^2$.

(ii) $\varepsilon\approx\Delta$, $t>t_{\mathrm{ch}}$.

As a result of many collisions, the relaxation process is accompanied by the creation of quasiparticle electronic excitations. The number of excitations depends on time, and this dependence is described by Eqs. (16) and (21).

A description of the nonstationary dynamics is, of course, interesting for its own sake. But it is also important for the production of a superconducting detector (e.g., an x-ray detector). Indeed, detector behavior is directly related to various features of the relaxation process (see above) and to the cascade dynamics.

In this paper we focus on tunneling detectors. Other types of detectors will be analyzed elsewhere. If the superconductor forms a part of a tunneling junction, then the incoming radiation is manifested in the appearance of an impulse of tunneling current [see Eqs. (17) and (22)]. The current increases during the time interval t_{ch} $(0<t<t_{\mathrm{ch}})$ [see Eq. (17)] and then decreases, in accordance with Eq. (22). If $\tilde{W}_m\ll\Gamma$, this decrease is exponential. Therefore, the time dependence of the tunneling current is

$$j(t)=j_1(1+\alpha t)^{1/3},\quad 0<t<t_{\mathrm{ch}},\quad t_{\mathrm{ch}}=\Delta^{-1}(\tilde{\Omega}/\Delta)^2,$$

$$j(t)=j_m\exp(-\tilde{\gamma}t);\quad t>t_{\mathrm{ch}}, \tag{23}$$

where $j_m\cong j_1(1+\alpha t_{\mathrm{ch}})^{1/3}$, $\alpha=(\lambda/2)\tilde{\Omega}$, and the current j_1 corresponds to the beginning of the second stage of the cascade; $\tilde{\gamma}t_{\mathrm{ch}}\ll 1$.

Experimentally, the dependence $j(t)$ has been studied in a number of papers (see, e.g., Refs. 20–23). The dependence (23) is in good agreement with the data. The current pulse, indeed, contains an initial sharp increase followed by exponential decay (see Fig. 1). Such a dependence has been observed in Ref. 20 for the Nb/Al-Al_2O_3-Al/Nb junction. A similar dependence was observed in Refs. 20–23. Based on Eq. (6″), one can estimate the rising time t_{ch}. With the use of values $\tilde{\Omega}=sp_F\approx5\times10^2$ K, $\Delta\approx0.3$ meV, we obtain $t_{\mathrm{ch}}\approx10^2$ ns, and this is in agreement with the data.[20–23] In addition, the decay time is described by the exponential dependence [cf. Eq. (23)]; according to Refs. 20 and 21, τ_{dec} greatly exceeds t_{ch}; $t_{\mathrm{dec}}\approx5-10\ \mu$s.

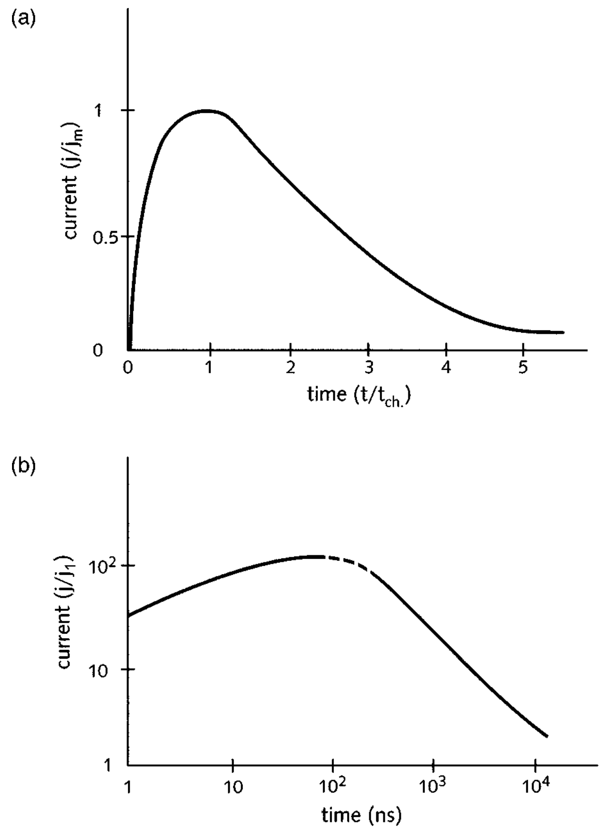

FIG. 1. Time dependence of the current: (a) general shape of the current pulse and (b) dependence $j(t)/j$, plotted on a logarithmic scale. The parameters used here are $\lambda=0.5$, $\tilde{\Omega}=5\times10^2$ K, $t_{\mathrm{ch}}=10^2$ ns, and $\gamma^{-1}=4\times10^2$ ns.

Note the important feature that it is helpful to use an asymmetric junction $(\Delta_2\neq\Delta_1)$ with an applied voltage eV close to the value $\Delta_2-\Delta_1$. In this case the tunneling current corresponds to the maximum of the superconducting density of states.

Note also that the detector should be used at a temperature $T\ll\Delta_1,\Delta_2$. Then one can neglect the contribution of thermal excitations $[n_{\mathrm{th}}\propto\exp(-\Delta_{1,2}/T)]$. Otherwise, they will also contribute to the tunneling current, interfering with the current due to the cascade. As a result, detectors based on ordinary low-temperature superconductors need to be used at very low temperatures. The use of the superconducting oxides such as Nd-Ce-Cu-O and Ba-Ca-Bi-O can be beneficial (see below). For example, a detector with Ba-Ca-Bi-P can be used at temperatures $T<20$ K, that is, above the liquid hydrogen temperature.

The total time dependence of j is presented in Fig. 1.

High-T_c oxides. The analysis described above was concerned with ordinary superconductors. They are characterized by a well-defined energy gap; moreover, the condition $\Omega\gg\Delta$ is also satisfied. Let us discuss now the special case of the high-T_c oxides. First of all, note that the above analysis is fully applicable to such oxides as Nd-Ce-Cu-O or Ba-Ca-Bi-O. Indeed, they have well-defined energy gaps; in addition, $\tilde{\Omega}\gg\Delta$ ($\Delta\cong6.5$ meV, $\tilde{\Omega}=40$ meV for the Nd-based cuprate and $\Delta=6.5$ meV, $\tilde{\Omega}=50$ meV for Ba-Ca-Bi-O). Note that according to Ref. 24, the La-Sr-Cu-O compound also has a sharp gap structure.

The situation is different for Y-Ba-Cu-O (YBCO) and other high-T_c cuprates. First of all, the position of the peak in the superconducting density of states is such that $\tilde{\Omega}\approx 2\Delta$. Therefore, the second and third stages of the cascade practically coincide. The inequality $\Omega \gg \Delta$ is needed during the second stage for a noticeable number of quasiparticles to collect at the edge $\varepsilon \approx \Delta$ during the second stage. This is important for detector efficiency. The absence of such a stage makes use of the aforementioned materials less efficient. In addition, YBCO and Bi- and Tl-based superconductors do not have a sharply defined gap spectrum. They display a gapless structure: that is, there are electronic states present down to $\varepsilon=0$. As a result, the recombination peak is not as sharp as for other superconductors. Note also that above we have assumed the absence of magnetic scattering. This is not the case for the aforementioned cuprates,[25,26] and the relaxation channels for these materials should be treated separately.

Therefore, the Nd-based cuprate and Ba-Ca-Bi-O are, probably, the best candidates for use in tunneling detectors.

The relaxation process in high-T_c oxides can be used in order to study the pairing mechanism. Indeed, the relaxation phenomenon has been used (see, e.g., Refs. 27–29) in order to generate phonons with $\Omega \approx 2\Delta$ during the recombination stage. The generation of phonons was caused by the electron-phonon coupling. Since $2\Delta \approx \tilde{\Omega}$ for the high-T_c cuprates, such as YBCO, Bi-based oxides, etc., one can conclude that the matrix elements for interaction with virtual $(\Omega \approx \tilde{\Omega})$ and real $(\Omega \approx 2\Delta)$ phonons are similar. Therefore, intensive generation of phonons by recombination would be a strong indication of an important phonon contribution to the pairing. This could be detected with the use of a second junction separated by a barrier, allowing the phonons to be transmitted. We will discuss this question in more detail elsewhere.

V. SUMMARY

In this paper we have described the nonequilibrium state caused by an external source (e.g., x rays) and the relaxation process (cascade) in isotropic gapped superconductors. The relaxation is accompanied by many collisions and by the generation of quasiparticles. As a result, we are faced with a nonstationary phenomenon and, consequently, with time-dependent dynamics. The cascade consists of several steps. The number of quasiparticles initially increases [during the time interval t_{ch} given by Eq. (6″)]; this is followed by an exponential decrease [Eqs. (21) and (23)]. We have obtained analytical expressions [Eqs. (14), (16), and (21)] describing the time dependence of the relaxation process.

A study of the relaxation process is important for the design of superconducting detectors. The current pulse in a tunneling detector is evaluated and is described by Eq. (23). This universal temperature dependence is in a good agreement with the data.[20–23]

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APPENDIX

(I) The nonstationary state of homogeneous superconductors is described by the time-dependent Green's function $\hat{g}^k$, which can be written in the following matrix form:

$$\hat{g}^k = \hat{g}^R \hat{f} - \hat{f} \hat{g}^A, \quad \hat{f} = f + \tau_z f_1. \tag{A1}$$

Here f and f_1 are scalar functions, τ_z is the Pauli matrix, and $\hat{g}^R$ and $\hat{g}^A$ are the retarded and advanced Green's functions.

The equations for $\hat{g}^k$ were obtained by Larkin and one of the authors[13,14] and have the following general form:

$$\begin{aligned}\frac{\partial f}{\partial t}\,\mathrm{Tr}(g^R\tau_z-\tau_z g^A)&-\frac{\partial f}{\partial\varepsilon}\,\mathrm{Tr}\left(\frac{\partial\hat{\Delta}}{\partial t}\right)(g^R-g^A)\\&-2if_1\mathrm{Tr}\,\hat{\Delta}\tau_z(g^R-g^A)\\&+\frac{\partial f_1}{\partial t}\,\mathrm{Tr}\,\hat{\Delta}\tau_z\,\frac{\partial(g^R+g^A)}{\partial\varepsilon}=-4I_1^{\mathrm{coll}}(f\ ),\end{aligned} \tag{A2}$$

$$\begin{aligned}\frac{\partial f_1}{\partial t}\,\mathrm{Tr}(g^R\tau_z-\tau_z g^A)&+\frac{\partial f}{\partial\varepsilon}\,\mathrm{Tr}\,\tau_z\,\frac{\partial\hat{\Delta}}{\partial t}\,(g^R+g^A)\\&+\frac{i}{2}\,\frac{\partial f}{\partial\varepsilon}\,\mathrm{Tr}\,\tau_z\,\frac{\partial^2\hat{\Delta}}{\partial t^2}\,\frac{\partial(g^R-g^A)}{\partial\varepsilon}-2if_1\mathrm{Tr}(g^R\hat{\Delta}+\hat{\Delta}g^A)\\&+\frac{\partial f_1}{\partial t}\,\mathrm{Tr}\,\hat{\Delta}\,\frac{\partial(g^R-g^A)}{\partial\varepsilon}=-4I_2(f_1).\end{aligned} \tag{A3}$$

Assume that the order parameter depends weakly on time. This corresponds to the condition that the number of particles in the nonequilibrium state be relatively small. In the absence of magnetic impurities one can use the expression (A2) and put $f_1=0$.[8] As a result, one obtains Eq. (1).

(II) Consider Eq. (18) and write $\varepsilon=\Delta+x$, $\varepsilon_1=\Delta+y$. We obtain

$$\begin{aligned}\frac{\partial n(x)}{\partial t}=&-\frac{0.86\lambda x^{7/2}}{\sqrt{\Delta}\Omega^2}\,n(x)+\frac{\lambda}{\sqrt{2}\Omega^2}\int_x^\infty dy\;n(y)(x+y)\\&\times(x-y)^2(\Delta y)^{-1/2}-\frac{8\lambda\Delta^{5/2}}{\sqrt{2}\Omega^2}\int_0^\infty\frac{dy\;n(x)n(y)}{\sqrt{y}}\\&-\gamma\,\frac{n(x)(\Delta+eV)}{d\sqrt{(\Delta+eV+x)^2-\Delta_2^2}}.\end{aligned} \tag{A4}$$

The density of states is peaked near Δ. As a result, for the quantity

$$\tilde{w}=\int_\Delta^\infty\frac{\sqrt{2}d\varepsilon}{\sqrt{\varepsilon^2-\Delta^2}}\,n(\varepsilon)\cong\int_0^\infty\frac{dx}{\sqrt{x\Delta}}\,n(x), \tag{A5}$$

which is proportional to the total number of excitations, we obtain Eq. (20).

*Permanent address: Landau Institute for Theoretical Physics, Russian Academy of Sciences, Moscow, Kosigyn Str. 2, 117334 Russia.

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