

# Conductance suppression in normal-metal–superconductor mesoscopic structures

R. Seviour and C. J. Lambert

*School of Physics and Chemistry, Lancaster University, Lancaster LA1 4YB, United Kingdom*

A. F. Volkov

*Institute of Radioengineering and Electronics of the Russian Academy of Sciences, Mokhovaya Street 11, Moscow 103907, Russia  
and School of Physics and Chemistry, Lancaster University, Lancaster LA1 4YB, United Kingdom*

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Using a scattering matrix approach and quasiclassical Green's function technique, we calculate the conductance of the superconductor–normal-metal (S/N) system (see Fig. 1). We establish that the difference between the superconducting and normal state conductance ( $\delta G = G_s - G_n$ ) is negative for large S/N interface resistances ( $R_{S/N}$ ) and changes sign with decreasing  $R_{S/N}$ . [S0163-1829(98)03742-4]

## I. INTRODUCTION

Recent studies of transport properties of mesoscopic, normal-metal–superconductor (N/S) structures (see Refs. 1, 2), have revealed a number of new physical phenomena. Examples include the measured subgap conductance of superconductor–insulator–normal-metal (SIN) junctions,<sup>3–6</sup> oscillations in the magnetoconductance of N/S systems with normal or superconducting loops,<sup>7–12</sup> and the nonmonotonic dependence of the conductance on temperature and voltage.<sup>10,13</sup> Although the majority of experimental results have been successfully explained, there are some which remain anomalous. In particular the increase in resistance of diffusive N/S systems in a certain temperature range below  $T_c$  (Refs. 14–17) has remained unexplained for a number of years. An early theoretical prediction<sup>18</sup> that superconductivity induced conductance suppression is a generic feature of N/S nanostructures was followed by quantitative theories of this effect in the ballistic and Anderson localized regions<sup>19</sup> as well as in resonant structures.<sup>20</sup> However a quantitative theory in the diffusive region has remained elusive. Several authors have suggested possible explanations of this puzzling phenomenon. In the simplest (Ref. 14) the resistance change  $\delta R_s = R_s - R_n$  is determined by a change in the interface resistance  $R_b$  which is larger in the superconducting state than in the normal state. Another possibility is presented in Refs. 21, 22 where a two-dimensional, multiprobe geometry was considered. The current  $I_a$  passes through two contacts on one side of the normal film contacting a superconductor and the voltage  $V_b$  is measured between two probes located on the opposite side of the normal film. The authors of Refs. 21, 22 showed that the quantity  $R_{ab} = V_b / I_a$  may exhibit an increase below  $T_{cs}$  compared with its normal state value. In this geometry the spatial distribution of the current is non-uniform. However in some experiments, the geometry is almost one dimensional with the current distribution across the width of the normal film almost uniform. Therefore this mechanism may not be responsible for all the experimental observations of enhanced resistance.

In this paper we suggest an alternative mechanism which determines the change in resistance  $\delta R_s$  (or the conductance  $\delta G_s \approx -\delta R_s / R_n^2$ ) of the structure shown in Fig. 1. We will

show that  $\delta R_s$  may be positive if the interface resistance  $R_b$  is large enough compared with the resistance of the metallic film in the normal state  $R_n$ . The variation  $\delta R_s$  is determined by two factors: a variation of the shunting interface resistance  $\delta R_b$ , leading to a positive change in resistance and a variation of the normal film resistance due to a condensate induced by the proximity effect. In view of these conflicting effects it is not obvious what sign  $\delta R_s$  will adopt for any given parameters of the system. In what follows we use two methods to study the change in resistance, namely an analytical quasiclassical technique and a numerical scattering approach. The scattering approach<sup>2</sup> complements the quasiclassical method, and enables us to probe areas of parameter space which lie outside the region of validity of the latter.

## II. QUASICLASSICAL THEORY

Consider the diffusive regime where the mean free path is shorter than any other characteristic length in the system (except the Fermi wavelength). Such a case is realized in most experiments performed on metallic films or on doped semiconductors. For diffusive S/N mesoscopic structures, equations for the quasiclassical Green's functions were derived many years ago and are presented (in the most convenient form suitable for the present analysis) in Larkin and Ovchinnikov's paper.<sup>23</sup> These equations must be supplemented by boundary conditions at the S/N interface derived by Zaitsev<sup>24</sup> (see also Refs. 25, 26) and have been used extensively for the theoretical study of transport properties of S/N mesoscopic structures.<sup>27–37</sup>

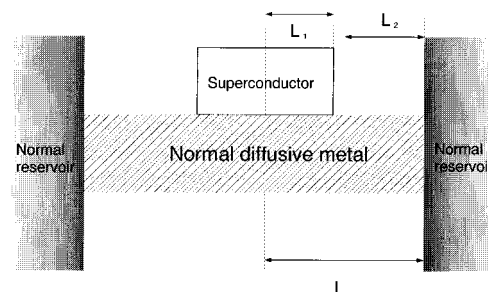


FIG. 1. The structure considered.

In this paper we shall assume that the proximity effect is weak, i.e., the amplitude of the condensate induced in the normal film is small. We will show that this is true for structures where the S/N interface resistance ( $R_b$ ), in the normal state, exceeds the resistance of the normal film. When this condition is satisfied, the condensate functions obey the linearized Usadel equation and the distribution function obeys the equation (see Refs. 29, 34),

$$L^2 \partial_x [(1-m) \partial_x f] = f g_b G_b \vartheta [x \in (S/N)], \quad (1)$$

where the function  $\vartheta(x)$  is equal to 1 in the S/N region and zero otherwise,  $m = \frac{1}{8} \text{Tr}(\hat{F}^R - \hat{F}^A)^2$  and  $\hat{F}^{R(A)}$  is the retarded (advanced) Green's function,  $g_b = \rho L^2 / R_{b\Box} d = g_{b1} (L/L_1)$ ,  $g_{b1}$  is the ratio of the normal film resistance to the S/N resistance,  $R_{b\Box}$  is the S/N interface resistance per unit area in the normal state, and  $\rho$  and  $d$  are the specific resistivity and the thickness of the normal film. The function  $G_b(x)$  determines the local normalized conductance of the S/N interface in the superconducting state,

$$G_b(x) = \nu_s \nu_n + \frac{1}{8} \text{Tr}(\hat{F}^R + \hat{F}^A)(\hat{F}_s^R + \hat{F}_s^A), \quad (2)$$

where the density of states (DOS) in the superconductor  $\nu_s = \text{Re}((\epsilon + i\Gamma) / \sqrt{(\epsilon + i\Gamma)^2 - \Delta^2})$  and  $\nu_n$  is the DOS in the normal film (for simplicity we assume  $\nu_n = 1$ , i.e., the S and N metals are regarded as identical apart from the critical temperature, we also assume that  $T_{cn} = 0$ ).  $\Gamma$  is the damping rate in the excitation spectrum of the superconductor. The first term in Eq. (2) describes the contribution of the quasiparticle current to the conductance (if  $\Gamma = 0$  it differs from zero only at energies  $|\epsilon| > \Delta$ ). The second term is due to Andreev reflection and describes a conversion of the low-energy quasiparticle current into the condensate current (if  $\Gamma = 0$ , the current is not zero for  $|\epsilon| < \Delta$ ). The condensate functions  $\hat{F}_s^{R(A)}$  in the superconductor are assumed undisturbed by the proximity effect [this is true provided that  $\Delta \gg \gamma_b = K_b^2 D$ , where  $K_b^2 = (R_{b\Box} d \sigma)^{-1}$ ], and they are equal to

$$\hat{F}_s^{R(A)} = i \hat{\tau}_y F_s^{R(A)}, \quad (3)$$

where  $F_s^{R(A)} = \Delta / \sqrt{(\epsilon + i\Gamma)^2 - \Delta^2}$ . Assuming that the right-hand side of Eq. (1) is a small perturbation we easily find a solution for

$$f = \begin{cases} \left\{ J_1 \left[ x + \int_0^x dx_1 (m + m_b) \right] \right\}, & 0 < x < L_1, \\ [J(x - L_1) + f(L_1)], & L_1 < x < L, \end{cases} \quad (4)$$

where  $J_1$  and  $J$  are the energy-dependent integration constants. The current  $I$  through the system is expressed in terms of  $J$  (see Refs. 33, 34),

$$I = (\sigma d / 2e) \int d\epsilon J(\epsilon). \quad (5)$$

The function  $m_b$  in Eq. (4) is given by

$$m_b = (g_b / L^2) \int_0^x dx_1 x_1 G_b(x_1). \quad (6)$$

In the reservoirs the distribution function has the equilibrium form

$$f(\epsilon, L) = F_v = [\tanh(\epsilon + eV)\beta - \tanh(\epsilon - eV)\beta] / 2, \quad (7)$$

where  $\beta = (2Tk_B)^{-1}$ . By matching the functions and their derivatives at  $x = L_1$  and using Eq. (7), we find for the ‘‘partial current’’  $J(\epsilon)$

$$J(\epsilon) = (F_v / L) [1 - \langle m \rangle + l_1 (m_{b1} - \langle m_b \rangle_1)]. \quad (8)$$

The angle brackets mean a spatial averaging over the regions  $(0, L)$  and  $(0, L_1)$ ;  $m_{b1} = m_b(L_1)$ ,  $l_1 = L_1 / L$ . With the aid of Eqs. (5) and (8), we find the normalized difference between the differential conductances of the system in the superconducting state and the normal state

$$\delta S = \frac{G_s - G_n}{G_n} = - \int_0^\infty d\epsilon \beta F'_v \{ \langle m \rangle + l_1 [\langle m_b - m_b^n \rangle_1 - (m_{b1} - m_{b1}^n)] \} / [1 + g_b l_1^3 / 3]. \quad (9)$$

Here  $G_{s,n} = (dI/dV)_{s,n}$  is the differential conductance below and above  $T_{cs}$ ;  $m_b^n$  is the function  $m_b(x)$  in the normal state:  $m_b^n = g_b x^2 / 2L^2$ ,  $F'_v = \partial F_v / \partial (eV\beta)$ .

Let us discuss the physical meaning of the different terms in Eq. (9). The first term gives a positive contribution to  $\delta S$  ( $\langle m \rangle$  is negative). This term arises from the renormalization of the normal film conductance caused by the induced condensate. This has been calculated in several papers,<sup>34–37</sup> where it was established that this term has a nonmonotonic voltage and temperature dependence decreasing to zero at  $[eV, T] = 0$  and  $[eV, T] \gg \epsilon_L$  ( $\epsilon_L = D/L^2$  is the Thouless energy). The second term in Eq. (9) determines the change in conductance due to different values of the S/N interface resistance in the normal and superconducting states. The contribution of this term to  $\delta S$  is negative because the S/N interface resistance in the superconducting state is larger than the normal state interface resistance (as long as the barrier transparency is not too high). Let us estimate the magnitudes of these terms. If  $g_b$  is small, the second term in Eq. (2) (the subgap conductance) is small compared with the first term. Therefore at low temperatures the contribution caused by the second term in Eq. (9) is related to a change in the DOS of the superconductor. This yields

$$\delta S_{\text{DOS}} \approx -g_b l_1^3 / 3. \quad (10)$$

As we shall see, the amplitude of the condensate functions  $\hat{F}^{R(A)}$  induced in the normal film by the proximity effect is of the order  $g_b l_1$ , i.e., of the order of the ratio of the normal film and the S/N interface resistances. The characteristic energy of decay of  $\hat{F}^{R(A)}(\epsilon)$  is the Thouless energy  $\epsilon_L$ . Thus the contribution to  $\delta S$  from the proximity effect [the first term in Eq. (9)] is (if  $\epsilon_L < T \ll \Delta$  and  $l_1 \ll 1$ ),

$$\delta S_{\text{pr}} \approx g_b^2 l_1^2. \quad (11)$$

Comparing Eqs. (10) and (11), we see that the sign of  $\delta S$  changes from negative to positive as  $g_b$  increases and  $\delta S_{\text{min}} \approx -l_1^4$  is reached when  $g_b \approx l_1$ .

In order to find  $\delta S$ , we need to calculate  $\hat{F}^{R(A)}$ . As noted above, in the limit of a weak proximity effect the function

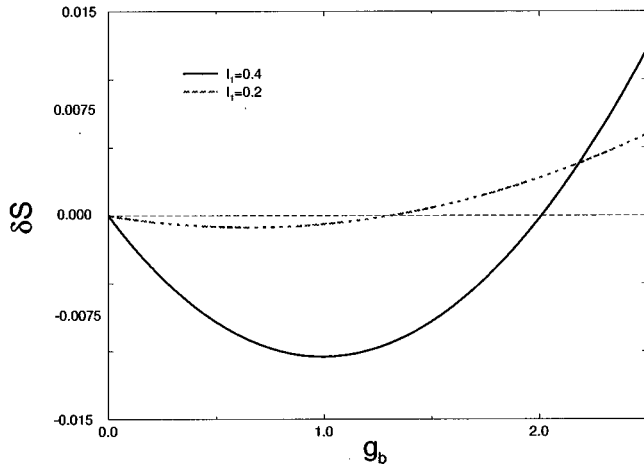


FIG. 2. The two curves show the dependence of  $\delta S$  on  $g_b$ , for the solid curve  $l_1=0.4$  and for the dashed curve  $l_1=0.2$ , where both curves have the parameters  $\Gamma=0.1(=\Gamma/\epsilon_L)$ ,  $\Delta=10(=\Delta/\epsilon_L)$ ,  $\gamma=0(=\gamma/\epsilon_L)$ ,  $\alpha=1.0[=\alpha/(2TK_B)]$ , and  $V=0(=V/\epsilon_L)$ .

$\hat{F}^{R(A)}$  obeys the linearized Usadel equation which may be presented in the form (see, for example, Refs. 33, 34),

$$\partial_{xx}\hat{F}^{R(A)} - (k^{R(A)})^2\hat{F}^{R(A)} = -(g_b/L^2)\hat{F}_s^{R(A)}\vartheta[x \in (S/N)], \quad (12)$$

where  $(k_b^{R(A)})^2 = (\mp 2i\epsilon + \gamma)/D$ ,  $\gamma$  is the depairing rate in the normal film. The solution to Eq. (12) satisfying the boundary conditions  $\hat{F}^{R(A)}(\pm L) = 0$  is the function

$$\hat{F} = (g_b/\theta^2)\hat{F}_s \begin{cases} [1 - c_2 \cosh(kx)], & 0 < x < L_1, \\ \{s_1 \sinh[k(L-x)]\}, & L_1 < x < L. \end{cases} \quad (13)$$

Here  $\theta = kL$ ,  $c_2 = \cosh(\theta_2)/\cosh(\theta)$ ,  $s_1 = \sinh(\theta_1)/\cosh(\theta)$ ,  $\theta_{1,2} = kL_{1,2}$ ,  $L_2 = L - L_1$ . For convenience we have dropped the indices  $R(A)$ . One can see from Eq. (13) that at characteristic energies  $\epsilon \approx \epsilon_L \ll \Delta$  the amplitude of  $\hat{F}^{R(A)}$  is of the order  $g_b l_1$ . It is worth noting that if  $l_1 \ll 1$  (this condition is satisfied in most experiments), then the magnitude  $g_b l_1$  corresponding to the actual  $g_b$  is of the order  $l_1^2$ , i.e., as  $l_1$  is small, the proximity effect is small.

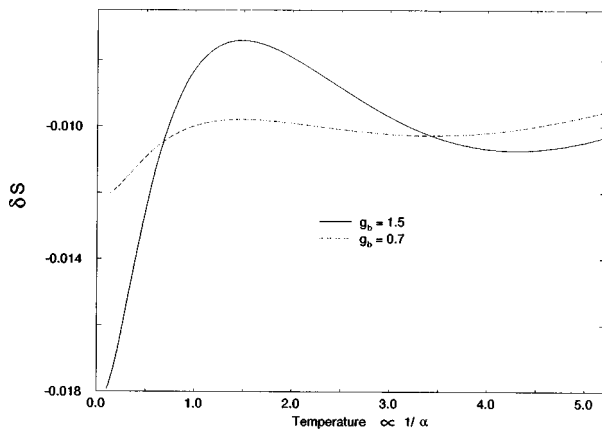


FIG. 3. The dependence of the conductance variation  $\delta S$  on temperature for  $g_b=1.5, 0.7$ ,  $\Gamma=0.1$ ,  $l_1=0.4$ ,  $\Delta=10$ ,  $\gamma=0$ , and  $V=0$ .

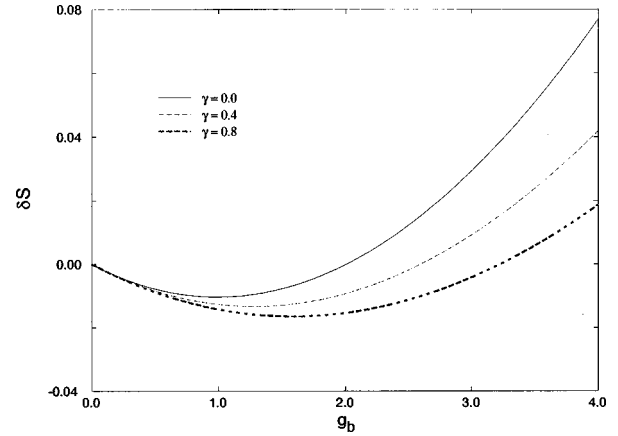


FIG. 4. Dependence of  $\delta S$  on  $g_b$ , for differing depairing rates  $\gamma=0, 0.4, 0.8$ , with  $\Gamma=0.1$ ,  $l_1=0.4$ ,  $\Delta=10$ , and  $V=0$ .

From Eq. (13), we can find the quantities  $\langle m \rangle$ ,  $m_{b1}$ , and  $\langle m_{b1} \rangle$ . Substituting them into Eq. (9), we obtain the variation of the normalized conductance  $\delta S$  as a function of temperature  $g_1$ ,  $l_1$ , etc. (see the Appendix). In the general case the expression for  $\delta S$  has a rather complicated form, but may be simplified drastically by taking the zero-bias, zero-temperature limit, in which case  $\delta S$  becomes

$$\delta S_0 = g_b l_1^3 \left[ g_b l_1 \left( 1 - l_1 \frac{4}{15} \right) - \frac{1}{3} \right]. \quad (14)$$

In this case the contribution due to the variation of the normal region conductance goes to zero [the term  $\langle m \rangle$  in Eq. (9)]. The variation  $\delta S$  is caused by a change in the S/N interface conductance  $G_b$ ; the first term in Eq. (2) gives a negative contribution, and the second term (i.e., the subgap conductance) gives a positive contribution. This expression also changes sign at  $g_b l_1 \approx 1/3$ . However, in this case the condensate amplitude is not small, and strictly speaking, Eq. (14) is not valid when  $g_b l_1$  is of order 1. Nevertheless, we show later using numerical calculations that this conclusion regarding the change in sign of  $\delta S$  remains valid in the zero-bias, zero-temperature limit. Figure 2 shows the dependence  $\delta S(g_b)$  for  $l_1=0.2$  and  $l_1=0.4$ . We see that in accordance with qualitative speculations given above,  $\delta S$  is negative at

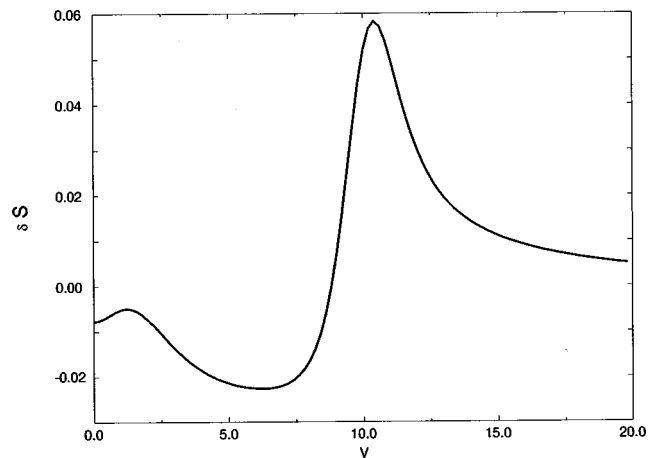


FIG. 5. Dependence of  $\delta S$  on  $V$  for  $\Gamma=0.1$ ,  $l_1=0.4$ ,  $\Delta=10$ ,  $\gamma=0$ ,  $g_b=1.5$ , and  $\alpha=1.0$ .

small  $g_b$ , reaching a minimum with increasing  $g_b$  and then changing sign. The magnitude of  $\delta S_{\min}$  decreases with decreasing  $l_1$  and depends on temperature in a complicated nonmonotonic way (see Fig. 3).

In Fig. 4 we show the dependence of  $\delta S$  on  $\gamma$  ( $\gamma$  may increase by applying an external magnetic field,  $\gamma \sim H^2$ ), the effect of a negative  $\delta S$  becomes more pronounced. This behavior is quite clear from a physical point of view. The applied magnetic field suppresses the proximity effect, but affects the DOS only weakly. Therefore the relative contribution  $\delta S_{\text{DOS}}$  increases with increasing magnetic field. A similar situation takes place in fluctuation paraconductivity in layered superconductors where an increase in the resistance due to superconducting fluctuations is enhanced by the magnetic field.<sup>38</sup>

In Fig. 5 we plot the voltage dependence of  $\delta S$ . One can see that two maxima exist in this dependence; one is close to zero bias and another one located at  $eV \approx \Delta$ . A similar voltage dependence of the phase coherent conductance has been

observed in the recent work of Ref. 39. We note that although the effect of the conductance decrease is small ( $\leq 5\%$ ), in dimensional units the conductance decrease  $\delta G$  may be much larger than the quantum conductance  $2e^2/h$  ( $\delta G \gg 2e^2/h$ ). Only in this limit can we use quasiclassical theory.

### III. NUMERICAL SIMULATIONS

In this section we use the scattering approach reviewed in Ref. 2 to determine  $\delta S$ , for a tight binding lattice with the geometry of Fig. 1. In the linear-response limit, at zero temperature, the conductance of a phase-coherent structure may be calculated from the fundamental current voltage relationship,<sup>40–44</sup>

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} v_1 - v \\ v_2 - v \end{pmatrix}, \quad (15)$$

where, at finite temperatures,

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \frac{2e^2}{h} \int_0^\infty dE \left( -\frac{\partial f(E)}{\partial E} \right) \begin{pmatrix} N_1^+(E) - R_0(E) + R_a(E) & T_a'(E) - T_0'(E) \\ T_a(E) - T_0(E) & N_2^+(E) - R_0'(E) + R_a'(E) \end{pmatrix}. \quad (16)$$

Equation (15) relates the current  $I_i$  from a normal reservoir  $i$  to the voltage differences  $(v_j - v)$ , where  $v = \mu/e$  ( $\mu$  is the chemical potential of the superconductor). The  $a_{ij}$ 's are linear combinations of normal ( $T_0, R_0$ ) and Andreev ( $T_a, R_a$ ) scattering coefficients. The primes on the coefficients refer to quasiparticles originating from the right-hand reservoir, while the coefficients without primes refer to particles from the left reservoir. Setting  $I_1 = I = -I_2$  and solving Eq. (15) the two probe conductance is (see Ref. 45),

$$G = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} + a_{12} + a_{21}}. \quad (17)$$

As noted in Ref. 45 in the presence of disorder, the various transmission and reflection coefficients can be computed by solving the Bogoliubov–de Gennes equation on a tight-binding lattice of sites, each labeled by an index  $i$  and possessing a particle (hole) degree of freedom  $\psi(i)$  [ $\psi(i)$  is the particle (hole) wave function]. In the presence of local  $s$ -wave pairing described by a superconducting order parameter  $\Delta_i$ , this takes the form

$$\begin{aligned} E \psi_i &= \epsilon_i \psi_i - \sum_{\delta} \tau (\psi_{i+\delta} + \psi_{i-\delta}) + \Delta_i \varphi_i, \\ E \varphi_i &= -\epsilon_i \varphi_i + \sum_{\delta} \tau (\varphi_{i+\delta} + \varphi_{i-\delta}) + \Delta_i^* \psi_i. \end{aligned} \quad (18)$$

In what follows, in the normal diffusive region, the on-site energy  $\epsilon_i$  is chosen to be a random number, uniformly distributed over the interval  $\epsilon_0 - 1$  to  $\epsilon_0 + 1$ , whereas in the clean  $N$  regions  $\epsilon_i = \epsilon_0$ . In the  $S$  region, the order parameter is set to a constant,  $\Delta_i = \Delta_0$ , while in all other regions,  $\Delta_i$

$= 0$ . The nearest neighbor hopping element  $\tau$  merely fixes the energy scale (i.e., the bandwidth), whereas  $\epsilon_0$  determines the band filling. In what follows we choose  $\tau = 1$ . By numerically solving for the scattering matrix of Eq. (18), exact results for the dc conductance can be obtained.<sup>22,41,44</sup> In the zero bias, zero temperature limit, Eq. (17) is greatly simplified and reduces to

$$G = T_0 + T_a + \frac{2(R_a R_a' - T_a T_a')}{R_a + R_a' + T_a + T_a'}. \quad (19)$$

For the structure shown in Fig. 1, with a superconductor of length  $2L_1$  and a barrier resistance  $R$ , evaluation of this expression yields results for  $\langle G_n \rangle$ ,  $\langle G_s \rangle$ , and  $\langle \delta G \rangle$  shown in Table I. In each case, the normal diffusive region is 40 sites wide and 64 sites long. The superconductor is of width 20 sites with  $\Delta_0 = 0.1$  ( $\Delta_0 = 0$ ) in the superconducting (normal) state. Results are obtained by averaging over 100 disorder realizations, yielding an estimated error in the mean values of approximately 0.04. The first row of the table shows results for  $L_1 = 30$ ,  $R = 2$  and demonstrates that a negative  $\delta G$

TABLE I. The numerical results for the structure shown in Fig. 1, for various lengths of superconductor ( $L_1$ ) with various barrier resistances ( $R$ ). The conductances shown are the ensemble averages.

$L_1$	$R$	$\langle G_n \rangle$	$\langle G_s \rangle$	$\langle \delta G \rangle$
30	2	3.70	3.40	-0.30
30	0.5	4.10	4.26	0.16
20	2	2.93	3.06	0.14
30	2	3.79	3.47	-0.33

can indeed occur, and comparison with the  $R=0.5$ , shows that lowering the interface resistance causes  $\delta G$  to change sign. As discussed previously, this result was expected although could not be proved using quasiclassical theory. Also, as discussed previously, when  $L_1$  is decreased (e.g., to  $L_1=20$  with  $R=2$ ) the table shows that  $\delta G$  changes sign and becomes positive. Finally, to examine the effect of a magnetic field, the fourth row of the table shows results with a magnetic field applied to the normal region (corresponding to 0.8 flux quanta through the whole structure). This demonstrates that the introduction of a magnetic field causes a negative  $\delta G$  to become more negative in agreement with the quasiclassical approach but in conflict with experimental evidence.

Finally we note that at a finite temperature ( $k_B T = \epsilon_L$ ), where the full integral of Eq. (16) needs to be evaluated, we find for the structure of row 1 in the table,  $\langle G_n \rangle = 3.68$ ,  $\langle G_s \rangle = 3.56$ ,  $\langle \delta G \rangle = -0.12$ , which confirms the prediction made using the quasiclassical approach, that the onset of superconductivity causes a drop in the conductance of the structure, even at finite temperatures.

#### IV. DISCUSSION

We have demonstrated that superconductivity-induced conductance suppression is an inherent property of the structure of Fig. 1. The suppression of the conductance at temperatures below  $T_{cs}$  is  $\leq 5\%$  and it is enhanced by the application of a magnetic field. In the experiment of Ref. 16

a stronger effect (10–20 %) is observed, which decreases when a rather weak magnetic field is applied. Meanwhile in the experiment of Ref. 17 a small increase of the resistance  $\Delta R$  was observed (less than 1%) and this effect of positive  $\Delta R$  (negative  $\Delta G$ ) was enhanced by applying a weak magnetic field. The results of the latter experiment are in qualitative agreement with our theoretical results. A quantitative comparison is difficult to carry out as the value of  $g_b$  which is crucial in the theory is unknown. If we accept that the conductances of the S/N interfaces (there were several superconducting strips on the  $N$  film) are comparable, i.e.,  $g_b \approx 1$  and  $l_1 \approx 1/2$ , then we obtain for  $\delta S \approx g_b l_1^3 / 3 \leq 5\%$ . As noted in Ref. 17 the effect of positive  $\Delta R$  and its magnetic field dependence is very sensitive to technological treatment. This suggests that the magnitude of this effect is dependent both on the geometry of the structure and on the fabrication method used. For the future, it would be of interest to confirm this experimentally by measuring the conductance of S/N structures of the type shown in Fig. 1 with different ratios of the normal channel and S/N interface resistances.

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#### APPENDIX

Using Eq. (12) we easily find the expression for the quantities in Eq. (8). We have

$$\langle m \rangle = -\frac{1}{2} [ |F_s|^2 \langle |b|^2 \rangle + \text{Re}(\langle b^2 \rangle F_s^2) ], \quad (\text{A1})$$

where  $F_s$  is defined in Eq. (3),

$$\langle |b|^2 \rangle = \frac{g_b^2}{|\theta|^4} \left\{ l_1 \left[ 1 + \frac{|c_2|^2}{2} \left( \frac{\sinh(2l_1\theta')}{2l_1\theta'} + \frac{\sinh(2l_1\theta'')}{2l_1\theta''} \right) - 2 \text{Re} \left( c_2 \frac{\sinh(l_1\theta)}{l_1\theta} \right) \right] + l_2 \frac{|s_1|^2}{2} \left( \frac{\sinh(2l_2\theta')}{2l_2\theta'} - \frac{\sinh(2l_2\theta'')}{2l_2\theta''} \right) \right\}, \quad (\text{A2})$$

$$\langle b^2 \rangle = \frac{g_b^2}{\theta^4} \left\{ l_1 \left[ 1 + \frac{c_2^2}{2} \left( \frac{\sinh(2l_1\theta)}{2l_1\theta} + 1 \right) - 2c_2 \frac{\sinh(l_1\theta)}{l_1\theta} \right] + l_2 \frac{s_1^2}{2} \left( \frac{\sinh(2l_2\theta)}{2l_2\theta} - 1 \right) \right\}, \quad (\text{A3})$$

$$m_{b1} - m_{b1}^n = g_b l_1^2 \left\{ \frac{\nu_s - 1}{2} + \frac{g_b}{2} \text{Re} \frac{A}{\theta^2} \left[ \frac{1}{2} - c_2 \left( \frac{\sinh(\theta l_1)}{\theta l_1} - \frac{\cosh(\theta l_1) - 1}{(\theta l_1)^2} \right) \right] \right\}, \quad (\text{A4})$$

$$\langle m_b \rangle_1 - \langle m_b^n \rangle_1 = g_b l_1^2 \left\{ \frac{\nu_s - 1}{6} + \frac{g_b}{2} \text{Re} \frac{A}{\theta^2} \left[ \frac{1}{6} - \frac{c_2}{(\theta l_1)^2} \left( \cosh(\theta l_1) + 1 - \frac{2 \sinh(\theta l_1)}{\theta l_1} \right) \right] \right\}, \quad (\text{A5})$$

$$c_2 = \cosh(\theta l_2) / \cosh(\theta), \quad s_1 = \sinh(\theta l_1) / \cosh(\theta), \quad \theta = kL, \quad A = |F_s|^2 - F_s^2, \quad \theta' = \text{Re}[\theta], \quad \theta'' = \text{Im}[\theta].$$

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