Zone-diagonal-dominated transport in high-T_c cuprates

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We present a Boltzmann equation analysis of the transport properties of a model of electrons with a lifetime that is short everywhere except near the Brillouin-zone diagonals. The assumed lifetime is directly implied by photoemission and c-axis transport data. We find quantitative agreement between calculations and ac and dc longitudinal and Hall resistivity, but the predicted longitudinal magnetoresistance disagrees with experiment. A possible microscopic origin of the anomalous lifetime is discussed. [S0163-1829(98)04038-7]

I. INTRODUCTION

Normal-state transport in high-T_c superconductors remains a controversial subject. There are two broad classes of theoretical approaches: the (generalized) Fermi liquid, in which the basic current carrying entities are electrons, and the non-Fermi liquid in which the basic entities are more exotic objects, e.g., spinons and holons,[1] phase fluctuations of a superconducting order parameter,[2,3] or fermions with definite charge conjugation symmetry.[4] In Fermi-liquid-based approaches one must invoke an anomalous scattering mechanism. Various models have been proposed;[5–7] a central concept is the ''hot spot,'' a small region on the Fermi surface [typically taken to be near the $(\pi,0)$ and $(0,\pi)$ points] where the electron lifetime is unusually short and has an anomalous temperature dependence. In these models the lifetime on most of the Fermi surface is much longer than in the hot spot and varies slowly with position on the Fermi surface. Hot spots arise, e.g., in models involving antiferromagnetic fluctuations strongly peaked at a particular momentum transfer. Models involving hot spots have had some successes, but have not led to complete and generally accepted descriptions of cuprate physics.[7,8]

In this paper we argue that the data are better described in terms of *cold spots*; small regions which we take to be near the zone diagonal and in which we assume the lifetime is much *longer* than elsewhere on the Fermi surface and varies rapidly as one moves along the Fermi surface away from the cold spot. A related model was recently put forward by Zheleznyak *et al.*[9] It differs in detail from ours (the ''hot'' regions, not the ''cold spots'' dominate the transport); the optical conductivity and magnetoresistance have not yet been considered. The idea bears an intriguing resemblance to the ''Fermi segments'' appearing in a phenomenological model of preformed pairs[10] and found in recent SU(2) gauge-theory calculations.[11] Also, in a paper devoted primarily to analysis of ''hot spot'' models, Stojkovic and Pines considered the σ_{xx} and weak field σ_{xy} that would arise in a cold-spot model of the type studied here.[12] They did not consider the frequency-dependent conductivity of the cold-spot model, but in a subsequent preprint[13] presented results for the magnetoresistance that are inconsistent with the ones presented here. We discuss the relationship of their work to ours in more detail in Secs. III and V.

We do not at present have a controlled calculation from a microscopic model that produces cold spots; we suggest however that they might arise from strong $d_{x^2-y^2}$ pairing fluctuations of the type proposed in Ref. 10, and we present a leading-order estimate along these lines in Sec. IV below. There are four phenomenological justifications. One is the apparently successful description of transport and most aspects of magnetotransport to be discussed at length below. A second justification comes from photoemission. Experiments[14,15] on optimally doped curprates show that for momenta parallel to (π,π) the electron spectral function exhibits a reasonably well defined quasiparticle peak, suggesting relatively weak scattering. However, for momenta near $(\pi,0)$ or $(0,\pi)$, there is no discernible peak (at $T>T_c$): the spectral function is very broad, suggesting relatively strong scattering. Thus photoemission implies a lifetime that is generically short but has a pronounced angular dependence. A third justification comes from c-axis transport. In optimally doped materials the observed ρ_c is large, and only weakly temperature dependent, while ρ_{ab} is small, and strongly temperature dependent. The anisotropy ρ_c/ρ_{ab} increases strongly as T is decreased. Note that although the temperature dependence of ρ_c depends strongly on the degree of doping and on which high-T_c compound is studied, in optimally and underdoped materials the anisotropy ρ_c/ρ_{ab} is always found to increase strongly as T is decreased. For example, in optimally doped $YBa_2Cu_3O_{7-\delta}$, $\rho_c(T)=a+bT$ with $a\approx 2500\ \mu\Omega$ cm and $b\sim 6\ \mu\Omega$ cm/K, whereas $\rho_{ab}=\lambda T$ with $\lambda\approx 0.6\ \mu\Omega$ cm/K and negligible intercept.[16] In all other optimally doped or underdoped materials ρ_c is constant or decreases as T is decreased, while ρ_{ab} remains T linear with negligible intercept.

Within a Fermi-liquid picture the observed strong temperature dependence of the resistivity anisotropy is explicable only if the two quantities are controlled by different parts of the Fermi surface. Band-theory calculations have shown that this may in fact occur in high-T_c materials: the calculated between-planes hopping $t_\perp(\mathbf{p})$ is strongly momentum dependent, being very small for momenta parallel to

(π,π) and being maximal for momenta parallel to $(\pi,0)$ for both two-plane[17,18] and one-plane[19] materials; for a review, see Ref. 20. In tetragonal materials the calculated $t_\perp$ is indistinguishable from 0 for momenta parallel to (π,π), while in $YBa_2Cu_3O_{7-\delta}$ (YBCO) the anisotropy due to the chains may lead to a zone-diagonal $t_\perp$ which, although still small compared to $t_\perp(0,\pi)$, is noticeably different from zero. In summary, the observation of very different temperature dependences of ρ_{ab} and ρ_c implies that ρ_{ab} must be controlled by carriers with momentum nearly parallel to (π,π) and that these carriers must have a much longer lifetime than do carriers on other parts of the Fermi surface. [It is tempting to relate the difference in $t_\perp$ to the observed difference in $\rho_c(T)$ between YBCO and other materials, but we emphasize that these differences are minor and do not affect our arguments.]

A fourth argument suggesting the ''cold spot'' picture concerns the anomalously small vortex viscosity measured by Parks *et al.*[21] or inferred from resistivity data.[22] This, it has been argued, can only be explained by a strongly anisotropic scattering rate.

The remainder of this paper is organized as follows. In Sec. II we present the model and derive an approximate solution. In Sec. III we compare the solution to data and present new experimental tests. In Sec. IV we discuss a possible microscopic origin. Section V is a conclusion.

II. MODEL

To motivate the model, we consider the c-axis conductivity σ_c. In very anisotropic systems such as high-T_c superconductors an adequate theoretical expression is

$$\sigma_c(T)=\int \frac{d^2p}{(2\pi)^2} t_\perp(p)^2 G_R(p,\omega)G_A(p,\omega)\frac{\partial f}{\partial\omega}. \quad (1)$$

Here $t_\perp(p)$ is the interplane hopping, $f(\omega)$ is the Fermi function, and $G_{R,A}$ are the retarded and advanced in-plane Green's functions evaluated at the chemical potential, which in our conventions is $\omega=0$. Band-structure calculations[17,20,18,19] yield

$$t_\perp(p)=\frac{t_0}{4}[\cos(p_x)-\cos(p_y)]^2$$

with $t_0\approx 0.15$ eV for bilayer BSCCO. As mentioned in the introduction, $t_\perp$ may have a small constant term in $YBa_2Cu_3O_{7-\delta}$, but this will not modify our conclusions in any important way so we neglect it here. This form for $t_\perp$ was used by Xiang and Wheatley to explain the difference in temperature dependence between the c axis and ab-plane superfluid stiffness.[23]

In a Fermi liquid,

$$G_{R,A}(p,\omega)=Z/(\omega-\epsilon_p\pm i\Gamma_p)+G_{inc}. \quad (2)$$

Here $1>Z>0$ is the quasiparticle renormalization factor, ϵ_p is the (renormalized) Fermi velocity, and Γ_p is a scattering rate. In a conventional Fermi liquid at low T, $\Gamma_p=(2\tau_{imp})^{-1}+\omega^2+T^2$ with τ_{imp} an impurity scattering time. Evaluating Eq. (1) using Eq. (2) and neglecting the incoherent part yields

$$\sigma_c(T)=\int (dp)t_\perp(p)^2N(p)Z^2(p)/2\Gamma_p. \quad (3)$$

Here (dp) denotes an integral along the Fermi line and $N(p)$ is the density of states. The main temperature dependence in this formula comes from Γ, so in a conventional Fermi liquid in which quantities depend only weakly on position on the Fermi surface, $\rho_c\sim\Gamma(T)$ would have the same temperature dependence as $\rho_{ab}=m\Gamma(T)/(ne^2)$. To explain the observed T dependence of ρ_c/ρ_{ab} one must either assume $Z\to 0$ (i.e., non-Fermi-liquid behavior) or invoke a strong angular dependence in Γ. In this paper we study the consequences of the latter assumption. Because ρ_c is large and has only a weak T dependence in optimally doped materials, over most of the Fermi surface Γ must be large and have only a weak temperature-dependence. Because ρ_{ab} is small and temperature-dependent Γ must be small and temperature dependent in the parts of the Fermi surface where $t_\perp(p)$ vanishes. Therefore we assume that Γ has a large, angular-dependent part that vanishes quadratically for momenta parallel to (π,π) and has negligible frequency and temperature dependence. We parametrize the Fermi surface by a magnitude $p_F(\theta)$ and angle θ, with $\theta=0$ corresponding to $\mathbf{p}$ along the zone diagonal and write

$$\Gamma(\theta,T)=\frac{1}{4}\Gamma_0\sin^2(2\theta)+\frac{1}{\tau_{FL}}. \quad (4)$$

Near the diagonals we approximate

$$\Gamma(\theta,T)=\Gamma_0\theta^2+\frac{1}{\tau_{FL}}. \quad (5)$$

The Fermi-liquid scattering rate, $1/\tau_{FL}$, is taken to be the sum of an impurity part and a T^2 part; explicitly,

$$\frac{1}{\tau_{FL}}=\frac{1}{\tau_{imp}}+\frac{T^2}{T_0}. \quad (6)$$

Here T_0 is an energy scale which in conventional Fermi liquid is of the order of the Fermi temperature but which we take as an adjustable parameter; we will find that it is small, of the order of 10–15 meV for optimally doped YBCO, suggesting that even the diagonal scattering is stronger than in a usual Fermi liquid. Note that τ_{FL} may in principle have ω dependence as well, but we assume that it is weak because fluctuations causing the scattering are slow.

The parameter Γ_0 may be estimated from photoemission experiments. In optimally doped cuprates in the normal state one observes near the $(0,\pi)$ point a very broad weakly dispersing feature with an energy half width >0.1 eV (see e.g., Fig. 6 of Ref. 15), suggesting $\Gamma_0>0.4$ eV. For our numerical estimates we take $\Gamma_0=0.6$ eV because it fits the optical data well.

To study the effects of this unusual scattering on the electrons we write a relaxation-time Boltzmann equation for the quasiparticle distribution function f_p.

$$\left[-i\omega+e\left(\mathbf{E}+\frac{\mathbf{v_p}}{c}\times\mathbf{B}\right)\cdot\partial_{\mathbf{p}}\right]f_p=-\Gamma(f_p-f^0). \quad (7)$$

Because of the strong variation of Γ we shall be mainly interested in states near the zone diagonal. We therefore introduce coordinates along and perpendicular to the diagonal; only the dependence perpendicular is nontrivial. The Boltzmann equation becomes

$$[-i\omega+\Gamma(\theta,T)+\omega_c(\theta)\partial_\theta]\delta f(\theta,\omega)=-e\mathbf{E}\cdot\mathbf{v}(\theta)\frac{\partial f}{\partial \epsilon_p}. \tag{8}$$

This equation may be solved. Because it turns out that the physics is dominated by θ near the zone diagonals we may approximate $\omega_c(\theta)$ and $v(\theta)$ by their values v and ω_c at $\theta=0$. Then in the first quadrant and for **E** parallel to the zone diagonal we have

$$\delta f(\theta,\omega)=-\frac{eEv}{\omega_c}\frac{\partial f_0}{\partial \epsilon_p}\int_{-\infty}^{\theta} d\theta' \exp[-K(\theta,\theta')] \tag{9}$$

with

$$K(\theta,\theta')=\frac{1}{\omega_c}[-i\omega+1/\tau_{FL}](\theta-\theta')+\frac{\Gamma_0}{3\omega_c}(\theta^3-\theta'^3). \tag{10}$$

From the solution one may construct currents in the usual way. We have chosen the electric field to be parallel to the zone diagonal, so we have

$$j_\parallel=\frac{ep_F}{\pi^2}\int d\theta\,\delta f(\theta,\omega), \tag{11}$$

$$j_\perp=\frac{ep_F}{\pi^2}\int d\theta\,\theta\,\delta f(\theta,\omega). \tag{12}$$

At $B=0$, $j_\perp=0$ and $j_\parallel=\sigma_{xx}E$ with

$$\sigma_{xx}=\frac{e^2v_Fp_F}{\pi^2}\int d\theta\frac{1}{-i\omega+1/\tau_{FL}+\Gamma_0\theta^2}$$
$$=\frac{e^2v_Fp_F}{\pi}\sqrt{\frac{\tau_{FL}}{\Gamma_0}}\frac{1}{\sqrt{1-i\omega\tau_{FL}}}. \tag{13}$$

Here p_F is the Fermi wave vector at $\theta=0$. Equation (13) predicts that if $\tau\sim T^{-2}$ then $\sigma\sim 1/T$, as observed in optimally doped cuprates. Qualitatively this behavior occurs because the conductivity is dominated by a small patch (width T) of weakly scattered (lifetime T^{-2}) electrons. Similarly, the weak-field Hall conductivity is obtained by expanding Eq. (8) to order B and is

$$\sigma_{xy}=\frac{\sigma_{xx}}{4}\frac{\omega_c\tau_{FL}}{1-i\omega\tau_{FL}}, \tag{14}$$

while expanding Eq. (8) to OB^2 yields

$$\frac{\delta\rho_{xx}}{\rho}=\frac{5}{32}\omega_c^2\tau_{FL}^3\Gamma_0+\frac{1}{16}(\omega_c\tau_{FL})^2. \tag{15}$$

As we shall see in the next section, all of these formulas agree quantitatively with experiment except for the magnetoresistance. If the leading term in Eq. (15) were absent, it would agree also.

The $\omega=0$ limits of Eqs. (13) and (14) were obtained by Stojkovic and Pines.[12] In a subsequent preprint these authors obtained a magnetoresistance of order $(\omega_c\tau)^2$ (with no Γ_0 term because in their solution of the Boltzmann equation they incorrectly neglected terms arising from derivatives of the scattering rate around the Fermi surface).

These expressions apply in the weak-field limit, in which an electron is scattered many times before the magnetic field bends its orbit appreciably. In the present context ''appreciably'' means that the electron leaves the small patch of size $p$near the diagonal that dominates conduction, in contrast to the usual case in which ''appreciably'' means ''completes of order one cyclotron orbit.'' The criterion for the weak-field limit $\delta\rho/\rho\ll 1$ is

$$\omega_c\ll\omega_c^*=\frac{|1-i\omega\tau_{FL}|^{3/2}}{\tau_{FL}^{3/2}\Gamma_0^{1/2}}. \tag{16}$$

In the high-field, $\omega_c\gg\omega_c^*$ limit, one may calculate by dropping the linear terms in the argument of the exponential. Then calculations give

$$\sigma_{xx}=\frac{2.4e^2v_Fp_F}{\pi^2\sqrt{3}\omega_c}\left(\frac{3\omega_c}{\Gamma_0}\right)^{2/3} \tag{17}$$

and

$$\sigma_{xy}=\frac{e^2v_Fp_F}{\sqrt{3}\pi\Gamma_0}. \tag{18}$$

So $\rho_{xy}\sim\omega_c^{2/3}$. This result may be understood as follows: one expects $\rho_{xy}\sim B/n_{eff}$; here the effective number of carriers is the size of the conducting patch that in the high-field limit may be seen to be of order $\theta\sim B^{-1/3}$ because K [Eq. (10)] is dominated by the second term in which $\theta^3\sim\omega_c$.

III. APPLICATION TO DATA

We now discuss the applicability of our results to data. We begin with clean, optimally doped $YBa_2Cu_3O_7$. In one sample of this material the resistivity at T_c was about 50 $\mu\Omega$ cm;[24] the mean interplane spacing of 5.5 Å implies $\sigma_{xx}=10^{-3}\,\Omega^{-1}$ per plane. Photoemission measurements[14] yield a zone-diagonal velocity $v_F=1.3$ eV Å and $p_F=0.6$ Å^{-1}; use of these values and the observed σ_{xx} per plane implies $\sqrt{\Gamma_0/\tau_{FL}}\approx 60$ meV. Our rough estimate $\Gamma_0\approx 0.6$ eV then implies $\tau_{FL}^{-1}\approx 6$ meV at $T=100$ K and therefore $T_0\approx 12$ meV, although the uncertainties are substantial. The observed linearity of the resistivity then implies $\tau_{FL}^{-1}\approx 24$ meV at $T=200$ K.

As noted above, in our model the observed linear resistivity is not due to the presence in the system of a T-linear scattering rate; rather it comes from the interplay of a T^2 ''nodal'' rate and a strong angular dependence. This may be contrasted to the numerical solution of the Boltzmann equation argued in Sec. V of Ref. 12 to account for the T-linear

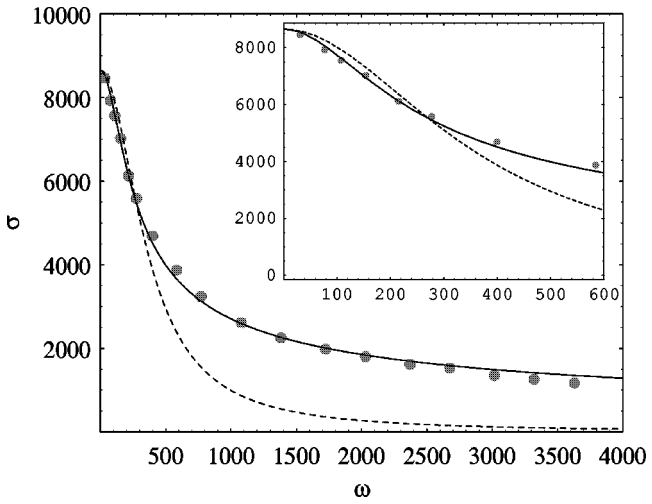

FIG. 1. Optical conductivity $\sigma(\omega)$, at $T=200$ K in the units of Ω^{-1} cm^{-1} plotted vs frequency (cm^{-1}): data from Ref. 17 (shown as dots) and fit to Eq. (13) with $\tau_{FL}^{-1}=21$ meV shown as solid line. Drude fit with $1/\tau=2T$ shown as dashed line.

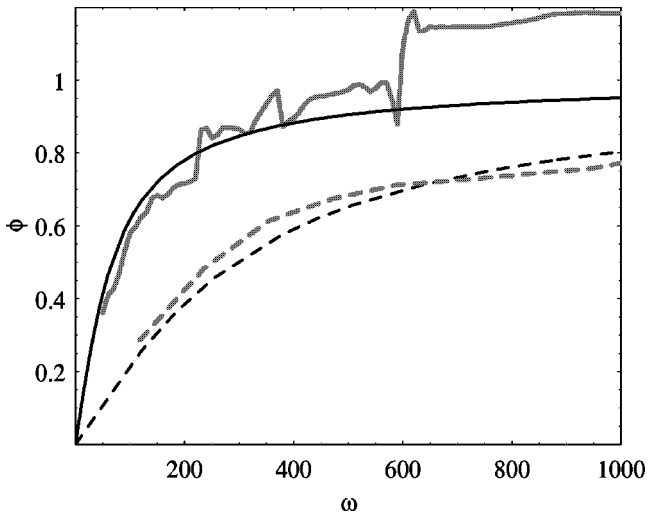

FIG. 2. Optical conductivity phase angle $\phi=\sigma_2/\sigma_1$ plotted vs frequency (cm^{-1}): data at 95 K from Ref. 29 and at 250 K from Ref. [19] (thick dashed curve) and fit to Eq (13) with $\tau_{FL}^{-1}=6$ meV, (thin curve) and $\tau_{FL}^{-1}=28$ meV (thin dashed curve).

resistivity observed in $YBa_2Cu_3O_{7-\delta}$. In that calculation parameters were such that the basic scattering rate was linear in T over all of the Fermi surface.

We now consider the optical conductivity, which provides significant evidence against a T-linear scattering rate. We use Eq. (13) to fit the observed frequency-dependent conductivity. Calculation and data[24] are shown in Fig. 1 for $T=200$ K. The best fit corresponds to $1/\tau_{FL}=21$ meV, very close to the 24 meV estimated from dc transport. We have also fit 100 and 300 K data; agreement is comparably good and leads to a T^2 dependence of $1/\tau_{FL}$.

The agreement between model and data is very reasonable; we note especially the narrowness of the Drude peak and the fact that upward concavity persists down to a very low frequency. Many authors have argued that the observed $\sigma(\omega)$ should be interpreted in terms of a frequency-dependent scattering rate τ_*^{-1} defined as

$$\sigma(\omega)=\frac{ne^2}{m^*(\omega)}\frac{1}{\tau_*^{-1}(\omega)-i\omega}. \tag{19}$$

In the dc limit $\tau_*^{-1}(\omega)$ is supposed to be proportional to T, with a constant of proportionality determined from the requirement that ne^2/m^* reproduce the London penetration depth. In the $YBa_2Cu_3O_7$ samples of Ref. 24 this implies $\tau_*^{-1}\approx 2T$. In the inset to Fig. 1 we compare the data to the naive ansatz of a frequency-independent scattering rate $\hbar/\tau=2T$. As can be seen in the inset, this leads to a conductivity that has too little upward concavity at low frequencies, in contrast to Eq. (13) that fits well over the entire frequency range. The observed upward concavity implies that the intrinsic low-frequency scattering rate is much smaller at $T=200$ K than $\tau_*^{-1}=2T$ obtained from the naive analysis. The discussion of the ratio of imaginary part (σ_2) to real part (σ_1) of the conductivity to be presented below will show that the intrinsic rate varies as T^2, not as T. Both of these facts follow directly from our model but seem otherwise difficult to understand.

We now turn to the high-frequency behavior. The deviation between data and the naive Drude model visible in Fig. 1 has been previously attributed to an anomalous frequency-dependent scattering although a generally accepted microscopic derivation has not been found. One difficulty is that many mechanisms for producing a T-linear resistivity involve scattering off quasistatic fluctuations; such scattering leads to a too weak frequency dependence of the scattering rate and therefore to a too rapid decrease of $\sigma_1(\omega)$. For example, in the gauge-theory approach one finds $\sigma_1(\omega)\sim 1/(T+\omega^\alpha)$ with $\alpha=4/3$ or $3/2$ depending on whether spinons or holons dominate the transport.[25] As another example, the calculations of Stojkovic and Pines that produced a linear resistivity arising from an approximately T-linear scattering rate also produce[26] a $\sigma(\omega)$ that falls too rapidly as frequency is increased, especially at low T. Another difficulty is that in most models the ω and T dependences add, leading to a T dependence of the high frequency σ that is inconsistent with the data. By contrast, in the present approach the frequency dependence is essentially a phase-space effect and there is therefore only little T dependence at large ω. An alternative model that gives the correct high ω behavior is the Luttinger model of Anderson.[27]

To further characterize the conductivity we consider the conductivity phase angle $\sigma_2(\omega)/\sigma_1(\omega)$. In Fig. 2 we show data for this quantity for $YBa_2Cu_3O_7$ at $T=95$ K that have been provided to us by Basov *et al.*[29] and data at $T=250$ K that we extracted from Ref. 28.

Before proceeding with a more detailed analysis it should be noted that the Basov *et al.*[29] conductivity data were obtained on untwinned crystals with the light polarized in the direction perpendicular to the chains and so this conductivity does not include contributions from the chains in contrast to the data of Refs. 24 and 28 obtained on twinned samples. This is why the absolute magnitude of σ_1 at $\omega\lesssim 500$ cm^{-1} obtained by Basov *et al.* are smaller in magnitude than those reported by Orenstein *et al.* by a factor of 2 at $T\approx 100$ K. This makes a detailed comparison difficult. Qualitatively, however, we see that the phase angle tends to a value of the

order of unity at high frequencies, and that the crossover from the high-frequency limit occurs at a relatively low frequency. The fact that the phase angle tends to a constant implies that the real and imaginary parts of the conductivity vary as $\omega^{-\alpha}$; that the constant is near unity imples that $\alpha \approx 1/2$ [in general if $\sigma \propto \omega^{-\alpha}$, $\sigma_2/\sigma_1 \rightarrow \tan(\alpha\pi/2)$] consistent with our form Eq. (13). We also note that high-frequency limit $\sigma \propto 1/\sqrt{\omega}$ is in a reasonable agreement with the empirical observation[30] that for many high-T_c materials the high frequency conductivity obeys a scaling $\sigma_1(\omega) \sim \omega^{-\alpha}$ with $\alpha \approx 0.6$. We further note that the widespread practice[30] of plotting $1/\tau^* \equiv \omega\sigma_1(\omega)/\sigma_2(\omega)$ is not very informative in such cases; the constant high-frequency limit of the ratio guarantees an apparently linear ''scattering rate.'' Despite the uncertainties we have fit the observed frequency dependence of the phase angle to our theoretically expected form $\tan[\frac{1}{2}\tan^{-1}(\omega\tau)]$; the results are shown in Fig. 2 along with the data. Surprisingly, $1/\tau_{FL}$ used in these fits agrees well in magnitude with that determined from the dc conductivity or the data of Ref. 24. We emphasize that one sees directly from the data in Fig. 2 that there is an intrinsic frequency scale in the problem that varies more nearly as T^2 than as T at low temperatures. This frequency scale occurs naturally in our model.

We now turn to the weak-field Hall conductance. We have

$$\cot(\Theta_H) = \frac{1}{\omega_c \tau_{FL}}[1 - i\omega\tau_{FL}]. \tag{20}$$

Thus the model yields the $\cot(\Theta_H) \sim T^2$ law found experimentally. More interestingly, it implies that over a wide frequency range Re $\cot(\Theta_H) \sim$const and Im $\cot(\Theta_H) \sim -\omega/\omega_c$. Precisely this behavior has been observed experimentally (see Fig. 4 of Ref. 31). From these data we may infer that at $T = 100$ K, $B = 8$ T, $\omega_c \approx 1.25$ cm^{-1}, and $\tau_{FL}^{-1} \approx 6$ meV, consistent with τ_{FL}^{-1} inferred from the σ_{xx} conductivity. Thus, at $T = 100$ K the scattering rate determined from the ac Hall effect is numerically the same as that determined from the zero-field ac conductivity. ac Hall data at other temperatures are not available but the observed T^2 dependence of the dc Hall angle suggests to us that at all temperatures the ''Hall scattering rate'' is identical to our τ_{FL}^{-1}, and is not an independent quantity. Extension of the optical Hall angle data to higher temperatures and frequencies would be of great interest: the present model predicts the frequency dependence should persist to high frequencies until the ω^2 term in τ_{FL}^{-1} becomes important; a hint of this behavior may be discerned in the data of Ref. 31 although the measured $\tau^{-1}(\omega)$ is constant within error bars. Further, the only temperature dependence should be a T^2 variation of Re $\cot(\Theta_H)$.

An interesting question concerns the thermopower, which is also anomalous in high-T_c materials. The anomalies in the thermopower have been argued to scale in the same way as the cotangent of the Hall angle.[32] However, we do not have a reliable calculation of thermopower in our model.

We turn now to the magnetoresistance, where a troubling discrepancy exists between model and data. Assuming the magnetic field does not affect the scattering mechanism we find

$$\frac{\delta\rho_{xx}}{\rho_{xx}} = (2.5\Gamma_0\tau_{FL} + 1)\tan^2(\Theta_H). \tag{21}$$

Thus in this model the weak-field magnetoresistance is very large compared to the square of the tangent of the Hall angle and has an additional temperature dependence. These results strongly contradict available data[33] that suggest that $\delta\rho/\rho \tan^2(\theta_H)$ is about 1 at all temperatures in optimally doped and underdoped $YBa_2Cu_3O_{7-\delta}$ and optimally doped and underdoped $Tl_2Ba_2CuO_{6+\delta}$. The observed ratio $\delta\rho/\rho \tan^2(\theta_H)$ is much larger in $LaSrCuO_4$ but because this is a very resistive material and we do not know how to interpret these data within our model. We regard the $YBa_2Cu_3O_{7-\delta}$ and $Tl_2Ba_2CuO_{6+\delta}$ as more representative of the intrinsic behavior of high-T_c materials.

There are two possible resolutions of this discrepancy. Either the model is simply inapplicable, which seems unlikely in the view of the photoemission results and the successes of the optics, or the anomalous scattering rate is itself field dependent in such a way as to cancel the $\Gamma_0\tau$ term in Eq. (15). Because it is likely that the anomalous scattering rate is related to pairing fluctuations a strong B dependence of the scattering rate is possible.

We briefly discuss impurity dependence. We expect $\tau_{FL}^{-1} = \tau_{imp}^{-1} + T^2/T_0$ with the impurity scattering rate τ_{imp}^{-1} linearly dependent on the defect density n_d. Equation (13) for σ_{xx} then predicts strong violations of Matthiesson's rule: the $T = 0$ residual resistivity is predicted to go as the square root of the defect density and the difference $\rho(T,n_{d1}) - \rho(T,n_{d2})$ in resistivity between two otherwise identical samples with different defect density should be temperature dependent, decreasing as T is increased. Neither of these predictions is consistent with available data.[34] We note, however, that the data mostly involve doping on the Cu site. Such doping is known to drastically change the local environment (e.g., by inducing magnetic moments on nearby Cu sites[35]). Further, we note that in all available data, doping increases the slope $d\rho/dT$, presumably because the doping changes an effective carrier number. These effects cannot be modeled by simply adding a $1/\tau_{imp}$ to a theory; therefore we do not regard the inconsistency between the predicted and observed doping dependence of ρ as a definitive disproof of the model we have proposed.

IV. MICROSCOPIC ORIGIN

In this section we present a qualitative discussion of a possible origin for the unusual lifetime we proposed. We have already noted that interaction of electrons with antiferromagnetic fluctuations cannot produce it, and this may be seen explicitly from the results presented in Ref. 12. Because the angular dependence is reminiscent of that of the $d_{x^2-y^2}$ superconducting gap we propose that the lifetime is caused by interaction of electrons with nearly singular $d_{x^2-y^2}$ pairing fluctuations. Similar ideas have been proposed by Chubukov.[36] As an example of an interaction that yields the desired behavior we consider

$$\Gamma(p,p',q) = \Gamma \sin 2\theta_p \sin 2\theta_{p'} D(\omega,q) \tag{22}$$

with the pair-fluctuation propagator $D(\omega,q)$ given by

$$D(\omega,q)=\frac{\phi[\omega^2/u^2(q^2+\xi^{-2})]}{q^2+\xi^{-2}}. \tag{23}$$

Here Γ is an energy scale, $\sin(2\theta_p)$ is the usual d-wave form factor, and ϕ is a scaling function. The form of ϕ is not important; it is however necessary to assume $\int dx\,\phi''(x)$ is convergent and indeed of order unity. Note that we have assumed $\omega\sim uq$ scaling. This is a nontrivial (but widely made) assumption for which we do not have a theoretical justification.

We may construct a one-loop self-energy from the interaction Γ in the usual way. We have (here p and q represent both momentum and frequency)

$$\Sigma(p)=\int (dq)\Gamma(p-q/2,p-q/2,q)G(p+q). \tag{24}$$

We are interested in momenta near the zone diagonal where the self-energy is small. We assume $p_F\xi\gg 1$. We may therefore approximate $G(p+q)$ by its weak-coupling form $G(p)=[\omega-v_F(|p|-p_F)]^{-1}$. After integration over the magnitude of $|p+q|$ we obtain for the imaginary part Σ'' as a function of position θ on the Fermi line

$$\Sigma''(\theta)=\frac{p_F\Gamma}{v_F}\int\frac{d\theta'}{2\pi}\frac{d\omega}{2\pi}\times\sin^2[2(\theta-\theta')]\frac{\phi''\left(\dfrac{\omega^2}{p_F^2\theta'^2+\xi^{-2}}\right)}{p_F^2\theta'^2+\xi^{-2}}. \tag{25}$$

Now ϕ'' is peaked at $\omega\sim u\sqrt{(p_F\theta)^2+\xi^{-2}}$ and vanishes as $\omega\to 0$; thus the integral is dominated by $\theta'\sim(p_F\xi)^{-1}$. For $\theta\gg(p_F\xi)^{-1}$ we find $\Sigma''(\theta)\sim(u\Gamma/v_F)\sin^2(2\theta)$ while for $\theta\ll(p_F\xi)^{-1}$ we get $\Sigma''(\theta)\sim(u\Gamma/v_F)(p_F\xi)^{-2}$. Further, the $\omega\sim uq$ scaling implies $\xi\sim u/T$. Thus the interaction leads to a scattering rate which is roughly of the form $\max(\Gamma_0\theta^2,T^2/T_0)$ with $\Gamma_0\sim(u/v_F)\Gamma$ and $T_0\sim(up_F)^2/\Gamma_0$ as required for the previous analysis. The empirically determined $T_0\approx 12$ meV and $\Gamma_0\approx 0.6$ eV implies $u\sim 0.15$ eV Å $\sim 0.1v_F$. Therefore the one-loop approximation may be justified by the usual Migdal arguments. Within the one-loop approximation the frequency dependence of Σ may be determined; it is very weak (logarithmic), justifying its neglect in the phenomenological analysis.

The crucial features of the interaction are (i) it must be rather sharply peaked about $q=0$ so as not to mix different parts of the Fermi surface too strongly, (ii) it must be a singular function of ω, so as to produce a large, essentially T-independent scattering rate for θ away from the zone diagonal, and (iii) the basic scale (set by ξ^{-1}) is T. We expect these assumptions to apply for optimally doped materials. For underdoped materials we conjecture that the system is closer to an instability, i.e., $\xi^{-1}\ll T$ implying a more singular interaction that leads among other things to a pseudogap in the density of states (see Ref. 37 for a similar calculation). For overdoped materials we expect at low T that $\xi^{-1}\gg T$; in this case the analysis leads to a scattering rate proportional to $T^2\xi^2(\theta^2+\xi^{-2})$, i.e., one which is T^2 everywhere on the Fermi surface but is very anisotropic.

Here we have also made the additional assumption that this interaction exists in the pairing channel, i.e., it is due to multiple scattering with virtual Cooper pairs. These virtual Cooper pairs will lead to a paraconductivity; we now show that this paraconductivity is negligible in comparison to the normal carrier conductivity found in the previous section. The paraconductivity σ_{para} is

$$\sigma_{para}=\int\frac{q^2d^2qd\omega}{(2\pi)^3}\frac{[\mathrm{Im}\,D(\omega,q)]^2}{2T\sinh^2(\omega/2T)}. \tag{26}$$

Evaluation and restoration of dimensional factors leads to $\sigma_{para}\approx\sigma_0(T\xi)^a$ with $\sigma_0=e^2/h$ and $a=1$ if $(T\xi)\gg 1$ and $a=2$ if $(T\xi)\ll 1$; because we expect $(T\xi)\sim 1$ this paraconductivity is small compared to the quasiparticle conductivity calculated previously, $\sigma\sim e^2/h\,v_Fp_F\sqrt{\tau/\Gamma_0}\gg e^2/h$.

V. CONCLUSION

We have considered a model based on ''cold spots''; i.e., the idea that the quasiparticle lifetime τ is unusually short everywhere on the Fermi line except near the zone diagonals. Direct evidence for this behavior is found in photoemission experiments. Using a Boltzmann equation analysis we have shown that the ansatz $\tau(\theta,T)=1/(\Gamma_0\theta^2+T^2/T_0)$ with $\Gamma\approx 0.5$ eV and $T_0\approx 12$ meV reproduces quantitatively the observed dc and ac, longitudinal, and Hall conductivities of optimally doped materials. We regard it as particularly significant that the model reproduces the non-Drude form of the observed optical conductivity (both the upward curvature at low frequencies and the roughly $1/\sqrt{\omega}$ behavior of $\mathrm{Re}\,\sigma$ and $\mathrm{Im}\,\sigma$ at $\omega>400$ cm^{-1}) and the difference between longitudinal and Hall scattering rates.

There are two differences between model predictions and the data. The effects of impurity scattering are different than predicted; however, a comparison is difficult because published doping studies have involved substitution on Cu site which produces many confusing changes in the material. Electron damage or light doping on sites away from the CuO_2 planes in *very clean* samples would provide a more definite test. A more troubling discrepancy is the magnetoresistance that is predicted to have a much larger magnitude and stronger temperature dependence than is observed. We do not at present have a resolution, but the other successes of the phenomenology lead us to believe one may be found. We also showed that the anomalous lifetime could arise from the exchange of virtual Cooper pair fluctuations. If this is the case, then the anomalous scattering would get weaker in an applied magnetic field, perhaps reducing magnetoresistance.

Finally, we comment on the energy scale T_0, characterizing the ''Fermi-liquid'' scattering rate $1/\tau_{FL}=T^2/T_0$. The data imply $T_0\approx 12$ meV, a surprisingly small value corresponding to strong scattering even along the diagonals. (Note that the ac Hall measurements of Ref. 31 combined with the observed T^2 Hall angle dependence directly implies the existence of such a small energy scale in the material.) Thus for T of order room temperature T^2/T_0 is of the order of the smallest Fermion Matsubara frequency πT, suggesting that

even along the diagonals Fermi-liquid behavior breaks down for $T>300$ K. The small value of T_0 implies that even the ''Fermi-liquid'' T^2 scattering must be due to some anomalous singular scattering mechanism, for example, the virtual Cooper pair fluctuations considered in Sec. IV.

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